Date: _____

CHAPTER 6

Algebraic Models

Get Set

Answer these questions to check your understanding of the Prerequisite Skills concepts on pages 340–341 of the *Foundations for College Mathematics 12* textbook.

Powers and Square Roots

1. For each power:
- identify the base
- identify the exponent
- evaluate the power

Hint: Express the right side as a power with the same base as the expression on the left side. Then solve for x.

a) 2^3 **b)** $(-3)^2$ **c)** -3^2 **d)** 3^{-2}

2. Determine the value of x.

a) $3^x = 27$ **b)** $7^x = 49$

c) $10^x = 10\,000$ **d)** $5^x = 3125$

3. Evaluate each root.

a) $\sqrt{36}$ **b)** $\sqrt{81}$

c) $\sqrt{121}$ **d)** $\sqrt{169}$

4. Use a calculator to evaluate each expression to one decimal place.

a) $\sqrt{30}$ **b)** $\sqrt{60.9}$

c) $\sqrt{81.4}$ **d)** $\sqrt{125.9}$

Exponent Laws

5. Write each expression as a single power, then evaluate.

a) $5^2 \times 5^3$ **b)** $(-2)^5 \times (-2)^3$

c) $7^5 \div 7^3$ **d)** $(9^3)^2$

Hint:
$b^m \times b^n = b^{m+n}$
$b^m \div b^n = b^{m-n}$
$(b^m)^n = b^{mn}$

6. Simplify.

a) $(x^2)(x^2)(x)$ **b)** $a^2x^3(4ax^2)$

c) $\dfrac{1}{(2a^4)^3}$ **d)** $\dfrac{c^3b^4}{cb^2}$

Date: _____

Zero and Negative Exponents

7. Evaluate. Hint: $b^0 = 1$.

a) 5^0 **b)** -3^0

c) $(-6)^0$ **d)** 0^0

8. Write each expression with a positive exponent, then evaluate. Hint: $b^{-m} = \frac{1}{b^m}$.

a) 5^{-1} **b)** 3^{-2}

c) 6^{-3} **d)** $\left(\frac{1}{3}\right)^{-2}$

Algebraic and Graphical Models

9. **a)** Copy and complete the table of values for the relation $y = 3x^2$.

x	y	First Differences	Second Differences	Ratio of Successive y-Values
0				
1				
2				
3				
4				
5				

b) Is the relation linear, quadratic, exponential, or none of these? Explain how you know.

c) Graph the relation. Does the graph confirm your answer to part b)? Explain.

10. The table shows the relationship between the width of a regulation Canadian flag and its area.

Width (cm)	Area (cm^2)
0	0
10	200
20	800
30	1800
40	3200

Is the relation linear, quadratic, exponential, or none of these? Explain.

Date: _____

6.1 Exponent Laws

Textbook pp. 342–351

Warm-Up

1. Number Skills

Find the GCF and LCM of 56 and 84 by expressing each number as a product of primes.

2. Algebra

Expand and collect like terms.

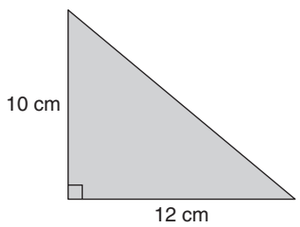
$5(x^2 - 3y + 2) - (x - 1)(x + 3) - 7y$

3. Relations

Rearrange the equation $y = \frac{5x + 3}{a^2}$ to express x as a function of y.

4. Geometry/Measurement

What is the surface area of a triangular prism that is 10 cm wide and has the following base? Round your answer to the nearest square centimetre.



5. Data/Probability

If you toss a fair coin twice, will you always get one head and one tail? Explain.

6. Modelling

Use finite differences to determine whether the data in this table is best modelled by a linear, quadratic, or exponential function.

x	*y*
2	12
3	27
4	48
5	75
6	108

7. Math Literacy

What is the base of the exponent in the expression -7.2^3?

A 7

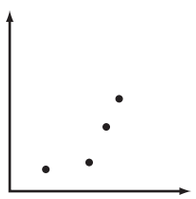
B 7.2

C −7.2

D 2

8. Previous Section

Which model—linear, quadratic, or exponential—seems most appropriate for this scatter plot?



Date: _____

Practise

1. Use the product rule to simplify.

a) $12^2 \times 12^{-2}$ **b)** $4^{-2} \times 4^{-3}$

c) $10^4 \times 10^0 \times 10^{-2}$ **d)** $5^{-1} \times 5^{-3} \times 5^{-2}$

2. Use the product rule to simplify, then evaluate each expression for the given values. Give your answer as an integer or a fraction.

a) $(y^{-2})(y^4)(y^{-3})$, for $y = 13$

b) $(z^{-6})(z^4)(z^{-2})$, for $z = 2$

c) $(s^5r^2)(s^{-3}r^{-4})$, for $s = 5$ and $r = 6$

d) $(h^{-7}k^2)(h^5k^{-5})$, for $h = 7$ and $k = 3$

3. Use the quotient rule to simplify, then evaluate.
Give your answer as an integer or a fraction.

a) $2^2 \div 2^4$ **b)** $5^{-2} \div 5^{-1}$ **c)** $\dfrac{4^{-1}}{4^3}$ **d)** $\dfrac{(-3)^{-1}}{(-3)^{-2}}$

4. Use the quotient rule to simplify, then evaluate each expression for the given values. Give your answer as an integer or a fraction.

a) $(w^{-2} \div w^{-3})$, for $w = 5$

b) $(p^{-1} \div p^4)$, for $p = 2$

c) $\dfrac{q^2m^8}{q^{-1}m^6}$, for $q = 10$ and $m = 12$

d) $(c^{10}d^{-2} \div c^8d^2)$, for $c = 7$ and $d = 3$

e) $\dfrac{k^5h^2}{k^2}$, for $k = -2$ and $h = -3$

5. Use the power of a power rule to simplify, then evaluate.
Give your answer as an integer or a fraction.

a) $(3^2)^{-3}$ **b)** $(8^{-2})^{-1}$ **c)** $[(-5)^2]^{-2}$ **d)** $[(-2)^{-2}]^{-2}$

6. Use the power of a power rule to simplify, then evaluate each expression for the given values. Give your answer as an integer or a fraction.

a) $(e^4)^{-2}$, for $e = -2$

b) $(4u^3)^{-2}$, for $u = 3$

c) $(xy^4)^{-2}$, for $x = 7$ and $y = -1$

d) $(g^{-3}h^2)^{-2}$, for $g = -2$ and $h = -4$

e) $\left(\dfrac{p^4q^2}{p^2}\right)^{-1}$, for $p = -3$ and $q = 2$

Date: _____

7. Simplify. Write your answer as a power with a positive exponent.

a) $c^4 \times c^5$

b) $u^{-2} \div u^{-7}$

c) $t^2s^{-9} \times t^{-3}s^{-5}$

d) $\dfrac{m^{-1}n^3}{m^{-4}n^{-2}}$

e) $\left(\dfrac{w^{-2}z^{-7}}{w^{-2}z^0}\right)^3$

f) $\left(\dfrac{j^{-1}}{k^3}\right)^{-1}$

8. Simplify, then evaluate to two decimal places, where necessary.

a) $(1.3^4)^{-2}$

b) $\dfrac{7.5^5}{7.5^{-3}}$

c) $(3.34^{-4})^{-3}$

d) $\left(\dfrac{0.2^6}{0.2^{-2}}\right)^{-1}$

e) $\left(\dfrac{3.1}{3.1^{-2}}\right)^{-1}$

f) $(-2.27^2)^{-2}$

9. Consider the expression $(x^3y^{-2})(x^{-3}y^3)$, when $x = 3$ and $y = 2$.
Evaluate the expression using each method.

a) Substitute the values into the expression, then evaluate.

b) Simplify the expression, then substitute the values and evaluate.

10. Consider the expression $\left(\dfrac{d^3m^4}{d^2m^{-1}}\right)^{-2}$, when $d = 6$ and $m = -1$.
Evaluate the expression using each method.

a) Substitute the values into the expression, then evaluate.

b) Simplify the expression, then substitute the values and evaluate.

11. Evaluate $(m^3)^{-2} + (m^{-3})(m^2)$ for $m = 3$.

Date: _____

6.2 Rational Exponents

Textbook pp. 352–361

Warm-Up

1. Number Skills

Calculate. Express your answers in lowest terms.

a) $\frac{3}{15} + \frac{1}{4}$

b) $\frac{1}{3} + \frac{11}{12}$

c) $\frac{6}{5} - \frac{5}{6}$

2. Algebra

Simply. Express your answers as a single fraction.

a) $\frac{r}{s}\left(\frac{s}{q}\right)$

b) $\frac{a}{4}\left(\frac{2}{b}\right)$

c) $\frac{a}{b} \div \frac{cb}{a}$

3. Relations

Graph the functions $y = 2^x$ and $y = x^2$ on the same set of axes. Which function is increasing most rapidly?

4. Geometry/Measurement

Draw a net for a three-dimensional object that has at least one triangular face. What is the name of your object?

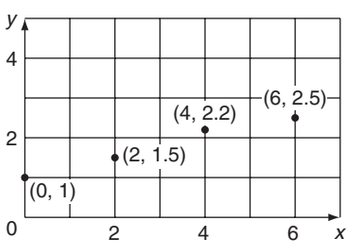
5. Data/Probability

What is the median value of the set of data?

3, 9, 2, 7, 83, 2, 5, 12, 45

6. Modelling

Use linear regression to find a model for the data. Round values to two decimal places.



7. Math Literacy

Does the expression $\left(\frac{1}{2}\right)^3$ involve a rational exponent?

8. Previous Section

Simplify the expression $\left(\frac{u^3v}{u^{-1}}\right)^{-2}$. Write your answer as a power with a positive exponent.

Date: _____

Practise

1. Evaluate.

a) $\sqrt{144}$ **b)** $\sqrt[3]{64}$ **c)** $\sqrt[5]{100\,000}$ **d)** $\sqrt[6]{64}$

2. Evaluate, if possible. If not possible, explain why.
Hint: Even powers of both positive and negative numbers give positive values.

a) $\sqrt{-16}$ **b)** $\sqrt[3]{-343}$

c) $\sqrt[4]{-10\,000}$ **d)** $\sqrt[5]{-1}$

3. Use a calculator to evaluate to two decimal places.

a) $\sqrt[3]{30}$ **b)** $\sqrt[4]{25}$ **c)** $\sqrt[3]{-100}$ **d)** $\sqrt[2]{91}$

4. Write in radical form. Then evaluate.

a) $8^{\frac{1}{3}}$ **b)** $49^{\frac{1}{2}}$ **c)** $1296^{\frac{1}{4}}$ **d)** $3125^{\frac{1}{5}}$

5. Evaluate, if possible. If not possible, explain why.
Hint: It is not possible to evaluate an expression involving a rational exponent, $b^{\frac{1}{n}}$, if the index of the corresponding radical, n, is even and the base, b, is negative.

a) $(-16)^{\frac{1}{2}}$ **b)** $(-4)^{\frac{1}{2}}$

c) $(-512)^{\frac{1}{3}}$ **d)** $(-16)^{\frac{1}{4}}$

6. Evaluate.

a) $8^{\frac{2}{3}}$ **b)** $100\,000^{\frac{3}{5}}$ **c)** $6561^{\frac{2}{4}}$ **d)** $4^{\frac{7}{2}}$

7. Evaluate. Round your answer to three decimal places.

a) $14^{\frac{1}{2}}$ **b)** $26.2^{\frac{1}{4}}$ **c)** $17^{\frac{4}{3}}$ **d)** $\sqrt[5]{39}$

e) $81^{\frac{3}{10}}$ **f)** $32^{\frac{2}{3}}$ **g)** $56.7^{\frac{4}{9}}$ **h)** $(-96)^{\frac{2}{7}}$

Date: _____

8. **a)** Predict the shape of the graph of $y = x^{\frac{1}{3}}$. Sketch your predicted graph.

b) Graph the function using graphing technology. How does the graph compare to your prediction?

c) Repeat parts a) and b) for $y = x^{\frac{1}{4}}$.

d) How does the graph of $y = x^{\frac{1}{3}}$ compare to the graph of $y = x^{\frac{1}{4}}$? Explain your observations.

9. The volume, v, in cubic centimetres, of a cube with surface area s, in square centimetres, is given by the equation $v = \left(\frac{s}{6}\right)^{\frac{3}{2}}$.

a) A cube has a surface area of 50 cm^2. What is the volume of the cube, to the nearest tenth of a cubic centimetre?

b) Cube A has surface area 20 cm^2. Cube B has surface area 40 cm^2. Predict whether the volume of cube B will be double the volume of cube A.

c) Use a graphing calculator or graphing software to check your prediction in part b).

10. The formula $i = \left(\frac{A}{P}\right)^{\frac{1}{n}} - 1$ gives the annually compounded interest rate, as a decimal, required to grow an initial investment of P dollars to A dollars after n years.

a) Kara has decided to invest her \$500 tax refund. What rate of interest will Kara need if she wants to grow her investment to \$1000 after 6 years? Give your answer to the nearest tenth of a percent.

b) What rate of interest will Kara need if she wants to grow her investment to \$1000 after 8 years? Give your answer to the nearest tenth of a percent.

11. Simplify $\sqrt{a^8}\ \sqrt[3]{27a^3}\ \sqrt{16a^2b^8}$.

Date: _____

6.3 Represent Exponential Expressions

Textbook pp. 362–367

Warm-Up

1. Number Skills

Order from least to greatest.

$1.04, \frac{101}{100}, 1.4, \frac{104}{102}, \frac{22}{21}$

2. Algebra

Simplify. Express your answers as a single fraction.

a) $\frac{1}{a^3} + \frac{1}{a^2}$

b) $\frac{m^2}{n^3} + \frac{n}{m^2}$

c) $\frac{u^4}{v^2} + \frac{vu}{vu^2}$

3. Relations

Find the point of intersection of the graphs of $y = x^2$ and $y = -(x - 2)^2 + 4$ using

a) a graphical method

b) an algebraic method

4. Geometry/Measurement

a) Write the formula for the volume, v, of a cube in terms of the side length, s.

b) Use the formula from part a) to calculate the volume of a cube with a side length of 5 cm.

c) Predict whether the volume of a cube with a side length of 10 cm is double the volume of a cube with a side length of 5 cm.

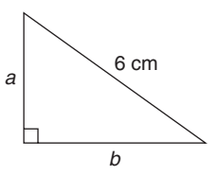
d) Calculate the volume of a cube with a side length of 10 cm to test your prediction.

5. Data/Probability

Anders knows that there is a 50% chance that it will be cloudy tomorrow, and a 30% change of rain. Anders says: "Since there's a 50% chance it will be cloudy, and a 30% chance of rain, there's an 80% chance that it will either be cloudy or rainy tomorrow." Is Anders correct? Explain.

6. Modelling

A right triangle has a hypotenuse that measures 6 cm. Write an equation to express length a in terms of length b.



7. Math Literacy

Write "the fourth root of 15" in the form $\sqrt[n]{b}$.

8. Previous Section

Evaluate.

a) $8^{\frac{2}{3}}$

b) $81^{\frac{3}{4}}$

Date: _____

Practise

1. Write each power as a power with base 5. Verify your answers by evaluating the power.

a) 25^1 **b)** 125^2 **c)** 625^{-1}

2. Write each power as a power with base 6. Verify your answers by evaluating the power.

a) 216^3 **b)** 36^2 **c)** 1296^{-2}

3. Rewrite each power with the indicated base.

a) 8^2 as a power of 2 **b)** 36^{-2} as a power of 6

c) 64^2 as a power of 4 **d)** 64^2 as a power of 2

e) 4^0 as a power of 2 **f)** 4^0 as a power of 3

4. **a)** Use graphing technology to find the point of intersection of the graphs of $y = 3^x$ and $y = 9^{x-1}$.

b) Explain how your answer to part a) gives the solution to the equation $3^x = 9^{x-1}$.

c) Use an algebraic method to solve the equation $3^x = 9^{x-1}$.

5. **a)** Solve $7^{x-12} = 49^{2x}$.

b) Check your solution by substituting into the left and right sides of the equation and evaluating.

6. Solve. Hint: Use graphing technology to check your solution to an exponential equation. Graph each side of the equation and find the point of intersection.

a) $2^{5x-15} = 32^{2x}$

b) $81^{a+13} = 9^a$

c) $216^{y-8} = 6^{4y}$

d) $2^{10(m+1)} = 8^{m-2}$

e) $10^{r-12} = 10\,000^{r+3}$

f) $121^{b-5} = 11^{b+2}$

7. Solve. Check your solutions.

a) $25^{w-2} = 125^{w-7}$ **b)** $216^{3(z+2)} = 36^{z+2}$

Date: _____

8. A scientist is conducting an experiment involving two types of bacteria. The growth of bacteria A can be modelled by the equation $A = 2^{3d}$, where d is the number of days since the start of the experiment, and A is the bacterial population on day d. The growth of bacteria B can be modelled by the equation $B = 4^{d+3}$, where d is the number of days since the start of the experiment, and B is the bacterial population on day d.

a) What is the population of each type of bacteria 4 days after the start of the experiment?

b) The experiment will be completed when the populations of the two types of bacteria are equal. After how many days will the experiment be completed? Use an algebraic method to produce your answer.

c) What is the population of each type of bacteria when the experiment is completed?

d) Use graphing technology to verify your answers to parts b) and c).

e) Examine the graphs of $A = 2^{3d}$ and $B = 4^{d+3}$. Which population is growing the fastest?

9. Consider the equation $36^{2(x+2)} = 216^{x-8}$.
Hint: Some exponential equations can be easily solved, even when one base is not a power of the other base. Look for a number that can express both bases as powers.

a) Solve the equation by expressing each side as a power of 6.

b) Solve the equation using graphing technology.

c) Which method do you prefer? Explain.

10. Consider the equation $2^{3x^2} = 4^{x^2+2}$.

a) Express both sides of the equation as powers of 2.

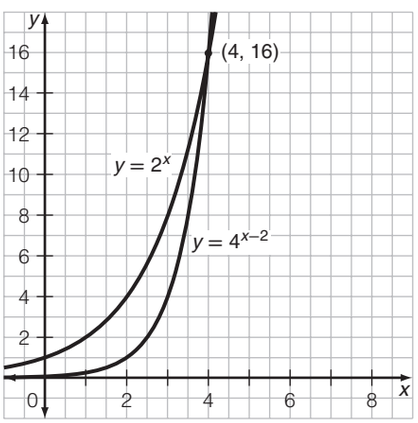
b) Use an algebraic method to find the two solutions for this equation.

c) Predict how the solution(s) would be represented if you graph both sides of the equation on the same set of axes.

d) Test your prediction in part c) by graphing 2^{3x^2} and 4^{x^2+2}.

11. a) This graph gives information about the solution to an exponential equation. Which exponential equation is represented in this graph?

b) Explain how the graph can be used to solve the exponential equation you identified in part a).



Date: _____

6.4 Tools and Strategies to Solve Equations Involving Exponents

Textbook pp. 368–375

Warm-Up

1. Number Skills

Evaluate. Round your answers to the nearest hundredth.

a) 1.3π

b) $4.52 + \frac{1}{3}$

c) $5\pi^2$

2. Algebra

Rearrange each equation to isolate the variable x.

a) $xy = 5x - 12$

b) $z = 5xy + xz^2$

c) $m = 3xa + 4xb^8$

3. Relations

Consider the equation $y = 3(2.3\sqrt{x})$. Use systematic trial to find a value of x that makes the equation true when $y = 100$. Round your answer to the nearest hundredth.

4. Geometry/Measurement

One interior angle of a parallelogram measures 30º. What are the measures of the other three interior angles?

5. Data/Probability

A used-car salesman wants to gather data about customer preferences. The survey he designs asks: "Would you prefer an expensive new car, or a less expensive used car?" Explain why the results of this survey may not be reliable.

6. Modelling

Sasha drops a tennis ball from a height of 10 m and observes the ball until it stops bouncing.

a) Predict what Sasha observes. Sketch a graph of your prediction, showing the height of the ball, h, as a function of the time since the ball is dropped, t. Choose appropriate units for h and t.

b) Explain any assumptions you made when creating your sketch.

7. Math Literacy

Which expression shows a power of 3?

A 10^3

B $3x$

C $3(4^2)$

D $\sqrt[3]{16}$

8. Previous Section

Solve.

a) $2^{4x+4} = 16^{2x}$

b) $3^{3y} = 9^{3y-3}$

c) $10^{z+3} = 100^{z-7}$

Date: _____

Practise

1. Solve. Round your answer to two decimal places.
Hint: Equations with variables raised to an even power often have more than one solution. For example, $x^4 = 16$ has two solutions, 2 and –2.

a) $m^3 = 12$ **b)** $c^3 = -40$

c) $225 = 4r^3$ **d)** $980 = 3h^4$

2. Solve. Round your answer to two decimal places.

a) $900 = \frac{4}{3}{}^{1}r^3$ **b)** $\pi r^2 + 10 = 20$ **c)** $50 = \frac{1}{3}\pi r^3(20)$

3. Estimate the value of x without using a calculator. Provide reasons for your estimates.

a) $2^x = 12$ **b)** $15^k = 100$ **c)** $5^y = 52$ **d)** $2 = 50^b$

4. Use graphing technology to solve each equation in question 3 to two decimal places.

5. Use systematic trial to solve $7 = 2^x$ to one decimal place.

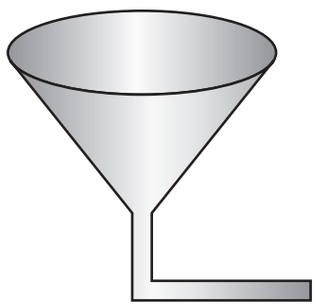
6. Use graphing technology to solve $2^{x-3} = 6^{0.2x}$. Round your answer to one decimal place.

7. Identify the error in the solution for m. Then, provide a correct solution. Check your solution by substituting the value into the original equation.

$$\frac{2m^5}{3} = 400$$
$$2m^5 = 3(400)$$
$$m^5 = \frac{1200}{2}$$
$$m^5 = 400$$
$$\sqrt{m^5} = \sqrt{400}$$
$$m = 20$$

Date: _____

8. A plastics recycling plant has a cone-shaped storage tank used to house chemicals needed for the recycling process. The tank is filled with the chemical and emptied when the recycling process begins. The equation that describes the volume of the tank is $v = \frac{1}{3}\pi r^2 h$, where r is the radius of the circular base of the cone, h is the height of the cone, and v is the volume of the cone.



a) The design of the recycling plant requires a storage tank 10 m tall. Write the equation for the volume of a cone-shaped tank that is 10 m tall.

b) The recycling process requires 300 m^3 of the chemical. Determine the radius of the base of the smallest tank that has the required volume, to the nearest hundredth of a metre.

9. Paolo has $1000 in savings to invest, and he wants $1200 to use to buy a new laptop in a few years. Paolo has two investment options.

a) In option 1, Paolo can earn 4.3% per year, compounded annually. Paolo would invest all of his $1000 savings in this option. The equation that describes Paolo's investment in option 1 is $1200 = 1000(1.043)^n$, where n is the number of years for which the money is invested. Solve the equation for n to determine how long will it take for Paolo's investment to grow to $1200. Round your answer to one decimal place.

b) After some research, Paolo discovered a second investment option. In option 2, Paolo can earn 5.1% interest, compounded annually. Because the interest rate is higher, Paolo thinks he can invest only $900 and still earn $1200 in less time than it takes with option 1. The equation that describes option 2 is $1200 = 900(1.051)^n$, where n is the number of years for which the money earns interest. Solve the equation for n to determine how long will it take for Paolo's investment to grow to $1200 in option 2. Round your answer to one decimal place.

c) Was Paolo's assumption about option 2 correct? Explain.

10. Two insect colonies are growing at an exponential rate. The variable d is the number of days and the variable P is the population of the colony on day d. The growth of colony 1 is modelled by $P = 50(2^{0.5d-5})$ and the growth of colony 2 is modelled by $P = 100(2^{d-10})$.

a) Use graphing technology to determine when the two insect colonies will have the same population.

b) Because the two colonies are competing for resources, the first colony to reach a population of 200 is expected to be the dominant colony. Determine which colony is expected to be dominant.

Date: _____

6.5 Construct and Apply Exponential Models

Textbook pp. 378–389

Warm-Up

1. Number Skills

Evaluate. Round your answers to the nearest hundredth.

a) $1.34(2.8^2)$

b) $7.345 + 1.01(434)$

c) $\left(\frac{1}{3}\right)^2$

2. Algebra

Identify the error in the solution for r. Then provide a correct solution.

$$\frac{3r}{4^2} = 6$$
$$3r = -(4^2)6$$
$$3r = -96$$
$$r = -32$$

3. Relations

Which is true of exponential relations?

A The first differences are constant.

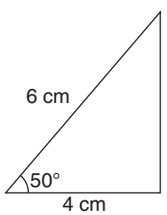
B The second differences are constant.

C The first differences are three times the value of the second differences.

D Neither the first differences nor the second differences are constant.

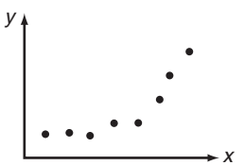
4. Geometry/Measurement

Should the sine law or the cosine law be used to solve this triangle?



5. Data/Probability

Which statement best describes the trend in the data?



A The values are increasing at an increasing rate.

B The values are increasing at a decreasing rate.

C The values show no pattern.

6. Modelling

The height, h, of triangle ABC is equal to the length of its base, b.

a) Write an equation to model the area, A, of triangle ABC.

b) Are there any restrictions on the values of the variables in your equation? Explain.

7. Math Literacy

a) What does "the population growth of a bacterial colony is modelled by an exponential relation" mean?

b) What sort of information might the exponential relation provide about the colony?

8. Previous Section

Solve. Round your answer to two decimal places.

a) $3^x = 12$

b) $7^y = 20$

c) $5 = 3h^2$

Date: _____

Practise

Section 6.5

1. An account pays 4.3% interest, compounded annually. An initial deposit of P dollars is worth A dollars after 7 years have passed. The relation that describes this investment is $A = P(1.043)^7$.

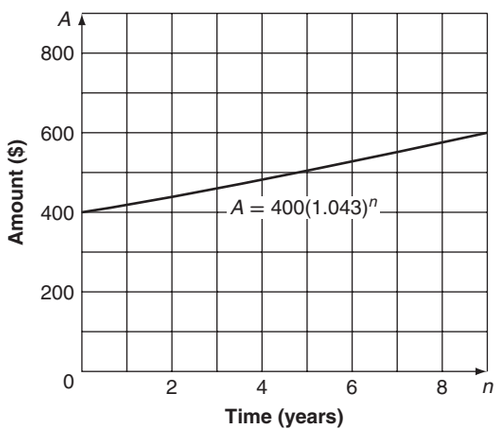
a) Graph the relation.

b) Is the graph linear, quadratic, exponential, or none of these? Explain how you know. Hint: The first differences of a linear function are constant, the second differences of a quadratic function are constant, and the ratio of successive values of an exponential function are constant.

c) What will an initial investment of $400 be worth after 7 years?

d) How much should be invested to earn $600 after 7 years?

2. This graph shows the amount, A, in an account that pays 4.3% interest, compounded annually, when $400 has been left in the account for n years. The relation that describes this investment is $A = 400(1.043)^n$.



a) What does the A-intercept represent?

b) Construct a table of values for the relation $A = 400(1.043)^n$ up to $n = 5$.

c) Calculate the first differences, second differences, and the ratio of successive A-values.

d) Is the relation linear, quadratic, exponential, or none of these? Explain how you know.

e) What is the amount in the account after
i) 5 years? **ii)** 20 years?

f) How long will it take for the amount to triple in value to the nearest tenth of a year?

3. Compare the graphs and equations in questions 1 and 2.

a) What do the relations have in common?

b) How are they different?

Date: _____

4. This table shows the number of cars per 1000 people in one neighbourhood of a large city. Hint: The best model is not always the one that is closest to the given data values. It is also important to consider whether the predictions generated by the model make sense for the situation.

Year	Number of Cars (per 1000 people)
1920	100
1930	120
1940	190
1950	258
1960	289
1970	317
1980	420
1990	500
2000	563

a) Construct a scatter plot of the data. Describe the trend.

b) Use a graphing calculator or other graphing technology to develop an appropriate mathematical model for predicting the number of cars per 1000 people in 2020. Which type of model fits the data best: linear, quadratic, or exponential?

c) Describe any limitations of your model. Are there other factors that might affect the number of cars per 1000 people that are not represented in your model?

5. Stephanie is a professional photographer, and is looking to buy a new camera. Stephanie buys a new camera every 4 years and sells her old camera. She puts the money from the sale towards the purchase of a new camera. Two cameras, the XR and the C3, will fit her needs, and she wants to consider the resale value of each camera.

XR: Depreciates by 15% each year. The value of the camera, V, after n years is given by $V = 2700(0.85)^n$.

C3: Loses \$200 of its value each year. The value of the camera, V, after n years is given by $V = 2600 - 280n$.

a) Graph the relations on the same set of axes. Describe each relation.

b) Which camera has the higher resale value after 4 years?

c) If Stephanie decides to keep the camera for 6 years instead of 4, which camera has the higher resale value? Explain.

Date: _____

6. Vanadium is a very stable but soft metal. Vanadium-50 (V-50) has a half-life of 1.5×10^{17} years. Consider a 1000-mg sample of V-50. After 1.5×10^{17} years, there will be 500 mg of V-50 remaining. After another 1.5×10^{17} years, there will be 250 mg of V-50 remaining.

a) Complete the table of values showing the amount of V-50 remaining after 5 half-life periods have passed, the first and second differences, and the ratios of successive amounts.

Half-Life	Amount Remaining (mg)	First Differences	Second Differences	Ratios
0	1000			
1	500			
2	250			
3				
4				
5				

b) Is the relationship between the number of half-life periods and the amount of V-50 remaining linear, quadratic, or exponential, or none? Explain.

c) Use the table of values to estimate when the amount of V-50 remaining will be 100 mg.

d) Construct a scatter plot of the data. Does the trend confirm your answer to part b)?

e) Determine an equation for the curve of best fit.

f) Determine how long it will take for the initial amount of V-50 to decay to 100 mg.

7. Uranium-239 (U-239) has a half-life of 2 years. Consider a 100-mg sample of U-239. After 2 years, there will be $\frac{1}{2} \times 100 = 50$ mg of U-239 remaining. After another 2 years, there will be 25 mg remaining.

a) Complete a table of values showing the amount of U-239 remaining after 5 half-life periods have passed.

b) Calculate the first differences, second differences, and the ratio of successive values.

c) Is the relationship between the number of half-life periods and the amount of U-239 remaining exponential? Explain.

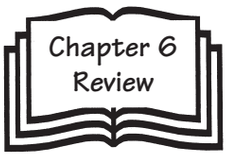
d) Construct a scatter plot of the data. Does the trend confirm your answer?

e) Determine an equation for the curve of best fit.

f) Determine how long it will take for the initial amount of U-239 to decay to 5 mg.

Date: _____

Chapter 6 Review



6.1 Exponent Laws, textbook pages 342–351

1. Simplify, then evaluate. Give your answer as an integer or a fraction.

a) $6^{-2} \div 6^{-5}$ **b)** $7^{-1} \times 7^{5} \times 7^{-8}$ **c)** $\dfrac{10^{8}}{10^{3}}$

d) $(12^{-3})^{2}$ **e)** $\left(\dfrac{6^{4}}{6^{3}}\right)^{-2}$ **f)** $[(-2)^{-4}]^{-1}$

2. Simplify. Write your answer as a power with a positive exponent.

a) $(z^{-2})(z^{-1})(z^{4})$ **b)** $\dfrac{r^{7}}{r^{-2}}$ **c)** $\left(\dfrac{m^{12}}{m^{8}}\right)^{-1}$

d) $(a^{4}b^{2})^{-2}$ **e)** $\left(\dfrac{2s^{4}}{s^{0}}\right)^{-1}$ **f)** $\left(\dfrac{c^{-1}d}{cd^{4}}\right)^{4}$

3. Simplify $(m^{-2}n^{4})^{-1}$, then evaluate for $m = 4$ and $n = -3$. Give your answer as an integer or a fraction.

4. Simplify, then evaluate to three decimal places, where appropriate.

a) $[3^{2}(1.4)]^{-2}$ **b)** $[(7.8)^{-2}]^{-1}$ **c)** $\left(\dfrac{4.5^{-4}}{4.5^{-2}}\right)^{2}$

6.2 Rational Exponents, textbook pages 352–361

5. Evaluate, if possible. If not possible, explain why.

a) $\sqrt{-121}$ **b)** $\sqrt[4]{625}$ **c)** $\sqrt[3]{-64}$

d) $\sqrt[4]{10\,000}$ **e)** $\sqrt[6]{-1\,000\,000}$ **f)** $\sqrt[3]{125}$

6. Write in radical form, then evaluate, if possible. If not possible, explain why.

a) $(-1)^{\frac{1}{4}}$ **b)** $4^{\frac{1}{2}}$ **c)** $(-8)^{\frac{1}{3}}$

d) $32^{\frac{2}{5}}$ **e)** $9^{\frac{5}{2}}$ **f)** $(-16)^{\frac{3}{2}}$

Date: _____

6.3 Represent Exponential Expressions, textbook pages 362–367

7. Write each power as a power with base 5.

a) 25^2 **b)** 125^{-3} **c)** 7^0

8. Solve.

a) $4^{-2} = 2^x$ **b)** $5^x = 25^{-2x-5}$ **c)** $10^{2y} = 1000^{4y-100}$

d) $6^{3s-7} = 36^{s-1}$ **e)** $2^{4(2x-1)} = 32^{2(x-1)}$ **f)** $8^{z-4} = 32^{z+2}$

6.4 Tools and Strategies to Solve Equations Involving Exponents, textbook pages 368–375

9. Leo is studying a species of aquatic insect living in a pond. As the eggs in the pond begin to hatch, the population of that insect, P, is modelled by the equation $P = 100(2^{0.5n})$, where n is number of days after the first eggs begin to hatch.

a) What is the population of insects in the pond

i) after 2 days? **ii)** after 3 days?

b) When will the population reach

i) 500? **ii)** 1000?

10. The population of a newly posted video on a vide-sharing Web site is growing exponentially. The equation that describes the number of times the video has been viewed, V, is given by $V = 2000(3^n)$, where n is the number of days since the video was posted to the site. Later that same day a second video is posted to the site. The popularity of the second video is given by the equation $V = 500(5^n)$, where V is the number of times the video has been viewed, and n is the number of days since the video was posted.

a) Which video will have the most views after 2 days?

b) Which video will be the first to have been viewed 100 000 times?

c) Will the two videos ever have the same number of views? If they will, determine the time at which this will occur, and the number of views at that time.

6.5 Construct and Apply Exponential Models, textbook pages 378–389

11. a) Refer to question 9. Are there any restrictions that might apply to the exponential model of the insect population? Explain.

b) Refer to question 10. Is it likely that the popularity of the videos will continue to increase at an exponential rate? Explain.

12. Ling has $700 to invest, and is considering two investment options.

Option A: A bond that pays 6.2% simple interest each year. The amount, A, after n years is given by the equation $A = 700 + 43.4n$.

Option B: A savings account that pays 5.3% per year, compound annually. The amount, A, after n years is given by the equation $A = 700(1.053)^n$.

a) Graph each relation on the same set of axes. Describe each relation.

b) Compare the options. Which is a better investment if Ling invests her money

i) for 4 years? **ii)** for 10 years?