1.4 Optimize Perimeter and Area

Student Text Pages
36–45

Suggested Timing
80–160 min

Tools
- toothpicks, geoboards, colour tiles, or linking cubes
- graphing calculators
- grid paper

Optional
- computers with *The Geometer's Sketchpad*® and spreadsheet software
- TI-Nspire™ CAS graphing calculators
- string or yarn with markings

Related Resources
BLM 1-13 Section 1.4 Practise Questions Rectangles
BLM 1-14 Section 1.4 Optimize Perimeter and Area

Link to Prerequisite Skills
Students should complete all the Prerequisite Skills questions before proceeding with this section.

Warm-Up
Is it possible for two rectangles to have the same perimeter but different areas? Use an example to explain.

Warm-Up Answers

Yes. Examples may vary. For example: Suppose rectangle A is 3 cm by 4 cm and rectangle B is 2 cm by 5 cm. Both rectangles have a perimeter of 14 cm, but rectangle A has an area of 12 cm^2 while rectangle B has an area of 10 cm^2.

Teaching Suggestions
- You may wish to conduct this lesson over two periods. Monitor students' progress as they work through the Investigate, Discuss the Concepts questions, and Practise questions. If necessary, work through the Example and the Apply and Extend questions the next day.

Warm-Up
- Display the Warm-Up question. Have students complete the question independently. Then, discuss the solution as a class.

Investigate
- The key idea is that rectangles of the same perimeter can have different areas, and vice versa. Students should recognize that there are instances where optimizing area or perimeter is important. The Investigate involves maximizing the area of a rectangle with a given perimeter.
- Have students work in small groups. Part A revisits concepts that students encountered in previous grades. Contextual elements that complicate the problem are introduced as the activity progresses.
- Provide students with a variety of tools, such as toothpicks, colour tiles, grid paper, string, and computers with *The Geometer's Sketchpad*®. Encourage students to select their own tools and strategies for both parts of the Investigate. Provide opportunities for groups to share how they solved the problems and to compare results.

Investigate Answers (pages 36–38)

Part A

1. Answers and sketches may vary. For example:

Length (m)	Width (m)	Perimeter (m)	Area (m^2)
10	10	40	100
15	5	40	75
12.5	7.5	40	93.75

2. a) length and width: 10 m

b) 100 m^2

c) It is a square; the length and width are equal.

3. a) The perimeter will increase, so adding stairs will increase the maximum possible area of the patio.

b) Answers may vary. For example: The maximum area of the patio can be obtained by placing the staircases on the same side of the patio, because the dimension of that side can be extended by the greatest amount and the area is increased the most.

4. a) 121 m^2. The maximum area is obtained regardless of the locations of the staircases.

b) Sketches may vary. 121 m^2; length and width: 11 m

5. A square provides the maximum area for a given perimeter. This is true unless there is some constraint on the dimensions of the rectangle.

Part B

1. a) Answers and sketches may vary. For example:

Length (ft)	Width (ft)	Perimeter (ft)	Area (ft^2)
100	103	406	10 300
125	90.5	431	11 312.5
153	76.5	459	11 704.5

b) 11 704.5 ft^2

2. A rectangle with two 76.5 ft sides and one 153 ft side opposite the lake.

3. The swimming area is a rectangle with a length double the width.

4. No. When enclosing three sides, the rectangle is twice as long as it is wide. When enclosing four sides, the rectangle is a square.

Example

- The Example is the reverse of the Investigate: minimizing the perimeter for a rectangle with given area. Method 1 appeals to intuition and experimentation. The systematic trial approach shown can be effective when the numbers are simple. A more rigorous approach is illustrated in Method 2, which requires some sophisticated algebraic reasoning and use of the TI-84 Plus or the TI-Nspire™ CAS graphing calculator. Students bound for college programs requiring some mathematics beyond the high school level will benefit from understanding Method 2.

Key Concepts

- Review the Key Concepts as a class. Ask students to record the concepts in their notebooks and provide sketches to illustrate each point.

Discuss the Concepts

- These questions provide opportunities to assess students' abilities to apply reasoning, proving, and communication skills. Have students discuss these questions in pairs and then write individual answers in their notebooks.

Discuss the Concepts Suggested Answers (page 42)

D1. a) No. Explanations may vary. For example: A 1 m by 4 m rectangle has a perimeter of 10 m and an area of 4 m^2. A 3 m by 2 m rectangle also has a perimeter of 10 m, but its area is 6 m^2.

b) Rectangles with different perimeters can have different areas. Examples may vary. For example: A 2 cm by 5 cm rectangle has a perimeter of 14 cm and an area of 10 cm^2. A 5 cm by 4 cm rectangle has a perimeter of 18 cm and an area of 20 cm^2.

D2. Answers may vary. For example; Back the garden against the garage, since you want the maximum area for a given amount of fencing. You should make the garden in the shape of a rectangle with length 5 m, opposite the garage, and width 2.5 m.

Practise (A)

- You may wish to have students work in pairs or small groups to complete the Practise questions.
- Encourage students to refer to the Examples before asking for assistance.
- Some students may benefit from using **BLM 1-13 Section 1.4 Practise Questions Rectangles** to answer **questions 1 and 3**.
- Students may benefit from using grid paper to draw their nets for **questions 2** and **4**.

Apply (B)

- Provide students with a variety of tools to work with, such as toothpicks, colour tiles, grid paper, string, and computers with *The Geometer's Sketchpad*®.
- For **question 5**, students need to remember that each segment of fencing represents 2 m when expressing their final answers.
- **Questions 7 to 10** represent a natural progression from previous questions posed on optimizing area for a fixed perimeter of a rectangle. Students should recognize that fewer fenced sides allows the material to go further, which will result in a greater enclosed area.
- Although the term *domain* is beyond the scope of this course, **question 11** illustrates a scenario in which the practical domain of a mathematical model must be restricted. You may wish to engage your students in a general discussion of this concept without using the term.
- **Question 13** links to the Chapter Problem. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up. When it gets busy at ski hills, lift lines are often expanded laterally to the direction of flow to accommodate large groups waiting at the bottom of the hill. It is perhaps a psychological phenomenon that a wide, converging line appears less objectionable than a single, long line, even if the wait times are the same. These are some points that could arise from discussion of this problem.
- Students may find that manipulatives and/or technology are helpful for solving **question 14**.

Extend (C)

- For **questions 15** and **16**, some students may discover that the maximum area increases with the number of sides of the polygon.

Common Errors

- Some students may struggle with the algebraic manipulations required to solve optimization problems.

R_x For some students, such a skill will not be important in their post-secondary studies, depending on their plans. Allowing them to use a CAS graphing calculator to perform algebraic calculations might be helpful. Students proceeding to programs such as engineering technology should strengthen their skills in algebraic manipulation and perhaps use a CAS graphing calculator to verify their work.

Accommodations

Visual—copy the table from the Investigate onto an overhead. Complete one row of the table with the class and show a sample sketch, including dimensions.

Perceptual—in Investigate, Part A, use diagrams or models to represent the placement of the staircases in step 3, part b)

Spatial—in Investigate, Part B, use string or geoboards to represent the rectangles

ESL—allow students to work in pairs when using the graphing calculator in the Example

Motor—provide a partner to assist with manipulatives or technology

Language—provide diagrams and a partner to clarify word problems

Literacy Connect

- Have one or two students read the section opener and discuss the meaning of *constraints* and *optimize*.
- Ask individual students to read the steps of the Investigate and clarify the instructions if necessary.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	13–16
Reasoning and Proving	1–16
Reflecting	5, 10, 11, 14–16
Selecting Tools and Computational Strategies	11, 12, 14–16
Connecting	10, 13–16
Representing	2, 4, 6, 11–16
Communicating	2, 4, 6, 10, 11, 13–16

Ongoing Assessment

- You may wish to collect students' responses to the Discuss the Concepts questions to use as a formative assessment tool.

Extra Practice

- Use **BLM 1-14 Section 1.4 Optimize Perimeter and Area** for remediation or extra practice.