# 5

## **Strand** Mathematical Models

Student Text Pages 264–337

Suggested Timing 10–15 min

**Related Resources** BLM A-4 Presentation Checklist

## **Key Terms**

coefficient of determination exponential model linear model quadratic model rate of change

## **Graphical Models**

## Chapter Curriculum Specific Expectations Modelling Graphically

In this chapter, students will

**MM2.01** interpret graphs to describe a relationship (e.g., distance travelled depends on driving time, pollution increases with traffic volume, maximum profit occurs at a certain sales volume), using language and units appropriate to the context

**MM2.02** describe trends based on given graphs, and use the trends to make predictions or justify decisions (e.g., given a graph of the men's 100-m world record versus the year, predict the world record in the year 2050 and state your assumptions; given a graph showing the rising trend in graduation rates among Aboriginal youth, make predictions about future rates) **MM2.03** recognize that graphs and tables of values communicate information about rate of change, and use a given graph or table of values for a relation to identify the units used to measure rate of change (e.g., for a distance–time graph, the units of rate of change are kilometres per hour; for a table showing earnings over time, the units of rate of change are dollars per hour)

**MM2.04** identify when the rate of change is zero, constant, or changing, given a table of values or a graph of a relation, and compare two graphs by describing rate of change (e.g., compare distance–time graphs for a car that is moving at constant speed and a car that is accelerating)

**MM2.05** compare, through investigation with technology, the graphs of pairs of relations (i.e., linear, quadratic, exponential) by describing the initial conditions and the behaviour of the rates of change (e.g., compare the graphs of amount versus time for equal initial deposits in simple interest and compound interest accounts)

**MM2.06** recognize that a linear model corresponds to a constant increase or decrease over equal intervals and that an exponential model corresponds to a constant percentage increase or decrease over equal intervals, select a model (i.e., linear, quadratic, exponential) to represent the relationship between numerical data graphically and algebraically, using a variety of tools (e.g., graphing technology) and strategies (e.g., finite differences, regression), and solve related problems

## **Modelling Algebraically**

In this chapter, students will

**MM3.03** make connections between formulas and linear, quadratic, and exponential functions [e.g., recognize that the compound interest formula,  $A = P(1 + i)^n$ , is an example of an exponential function A(n) when P and i are constant, and of a linear function A(P) when i and n are constant], using a variety of tools and strategies (e.g., comparing the graphs generated with technology when different variables in a formula are set as constants)

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## **Teaching Suggestions** Chapter Opener

## • Have students read and discuss the chapter opener. Write a summary of their comments on the board. Some prompts could be:

- What kinds of jobs involve investing money, planning flights, diagnosing leaks, or monitoring the spread of infections?
- Suggest a specific problem that might be faced in each of these jobs.
- What kinds of predictions would be useful for each of these problems?
- Ask students if they recognize any of the key terms.

## **Career Profile**

Have students discuss what they know about a career as an environmental technician. As an extension to the discussion, have students research this career and other careers that are related to graphical modelling, and present their findings to the class. You may wish to use **BLM A-4 Presentation Checklist** to assess students' presentations.

Using their research, have students discuss:

- The tasks of an environmental technician.
- The type of education and training needed for this career.
- Other careers that use graphical modelling.
- The differences in the training and education required for a similar career.

You may wish to have students include their research in their Portfolios.

## **Chapter 5 Planning Chart**

Section Suggested Timing	Student Text Page(s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 5 Opener • 10–15 min	264–265		• BLM A-4 Presentation Checklist	
Prerequisite Skills • 40–80 min	266–267	• BLM 5-1 Prerequisite Skills	• BLM 5-2 Prerequisite Skills Self Assessment Checklist	• graphing calculators
5.1 Linear Models • 80 min	268–281	<ul> <li>BLM 5-3 Section 5.1 Linear Models</li> <li>BLM 5-4 Section 5.1 Investigate Table</li> <li>BLM 5-5 Section 5.1 Example 2 Use Technology</li> </ul>	• BLM 5-6 Section 5.1 Achievement Check Rubric	<ul> <li>wind-up or battery- powered toy vehicles</li> <li>stopwatches</li> <li>metre sticks or measuring tapes</li> <li>masking tape</li> <li>graphing calculators</li> </ul>
5.2 Quadratic Models • 80 min	282–293	<ul> <li>BLM 5-7 Section 5.2 Quadratic Models</li> <li>BLM 5-8 Section 5.2 Investigate Table</li> <li>BLM 5-9 Section 5.2 Example 3 Use Technology</li> <li>BLM 5-10 Section 5.2 Question 10</li> </ul>	<ul> <li>BLM 5-11 Section 5.2 Achievement Check Rubric</li> <li>BLM A-10 Observation General Scoring Rubric</li> </ul>	<ul> <li>metre sticks</li> <li>marbles or ball bearings</li> <li>stopwatches</li> <li>graphing calculators</li> <li>Optional</li> <li>computers with motion sensors</li> <li>large balls, such as basketballs</li> </ul>
5.3 Exponential Models • 80 min	294–309	<ul> <li>BLM 5-12 Section 5.3 Exponential Models</li> <li>BLM 5-13 Section 5.3 Investigate Table</li> <li>BLM T-6 Using the CBR<sup>TM</sup></li> </ul>	<ul> <li>BLM 5-14 Section 5.3 Achievement Check Rubric</li> <li>BLM A-9 Communication General Scoring Rubric</li> </ul>	<ul> <li>foam or cardboard cups of hot water</li> <li>digital thermometers</li> <li>stopwatches</li> <li>graphing calculators</li> <li>Optional</li> <li>computer or calculator with temperature probe</li> <li>TI-Nspire<sup>TM</sup> CAS graphing calculators</li> <li>computers with spreadsheet software</li> </ul>
5.4 Analyse Graphical Models • 80 min	310–319	<ul> <li>BLM 5-15 Section 5.4 Analyse Graphical Models</li> <li>BLM 5-16 Section 5.4 Example 2 Use Technology</li> </ul>	<ul> <li>BLM 5-17 Section 5.4 Achievement Check Rubric</li> <li>BLM A-8 Application General Scoring Rubric</li> </ul>	• graphing calculators
5.5 Select a Mathematical Model • 80 min	320–331	<ul> <li>BLM 5-18 Section 5.5 Select a Mathematical Model</li> <li>BLM 5-19 Section 5.5 Example 1 Use Technology</li> <li>BLM 5-20 Section 5.5 Example 2 Use Technology</li> </ul>	• BLM 5-21 Section 5.5 Achievement Check Rubric	<ul> <li>masses, string, and supports for pendulums</li> <li>stopwatches</li> <li>graphing calculators</li> </ul>
Chapter 5 Review • 40–80 min	332–333	<ul><li>BLM 5-22 Chapter 5 Literacy</li><li>BLM 5-23 Chapter 5 Review</li></ul>		• graphing calculators
Chapter 5 Practice Test • 40–80 min	334–335	• BLM 5-24 Environmental Investigation	<ul> <li>BLM 5-25 Chapter 5 Practice Test</li> <li>BLM 5-26 Chapter 5 Test</li> <li>BLM A-4 Presentation Checklist</li> <li>BLM A-5 Problem Solving Checklist</li> </ul>	• graphing calculators
Chapter 5 Problem Wrap-Up • 20–40 min	335		• BLM 5-27 Chapter 5 Problem Wrap-Up Rubric	• graphing calculators
Chapter 5 Task • 80 min	336–337		• BLM 5-28 Chapter 5 Task Rubric	<ul> <li>graphing calculators</li> <li>computers with Internet access and spreadsheet software</li> </ul>

## **Chapter 5 Blackline Masters Checklist**

	Title		Purpose		
	BLM A-4	Presentation Checklist	Assessment		
Prerequisite Skills					
	BLM 5-1	Prerequisite Skills	Practice		
	BLM 5-2	Prerequisite Skills Self-Assessment Checklist	Self-Assessment		
5.1 Linear Models					
	BLM 5-3	Section 5.1 Linear Models	Practice		
	BLM 5-4	Section 5.1 Investigate Table	Student Support		
	BLM 5-5	Section 5.1 Example 2 Use Technology	Technology		
	BLM 5-6	Section 5.1 Achievement Check Rubric	Assessment		
5.2 Quadratic Models					
	BLM 5-7	Section 5.2 Quadratic Models	Practice		
	BLM 5-8	Section 5.2 Investigate Table	Student Support		
	BLM 5-9	Section 5.2 Example 3 Use Technology	Technology		
	BLM 5-10	Section 5.2 Question 10	Student Support		
	BLM 5-11	Section 5.2 Achievement Check Rubric	Assessment		
	BLM A-10	Observation General Scoring Rubric	Assessment		
5.3 Exponential Models					
	BLM 5-12	Section 5.3 Exponential Models	Practice		
	BLM 5-13	Section 5.3 Investigate Table	Student Support		
	BLM 5-14	Section 5.3 Achievement Check Rubric	Assessment		
	BLM T-6	Using the CBR™	Technology		
	BLM A-9	Communication General Scoring Rubric	Assessment		
5.4 Analyse Graphical Models					
	BLM 5-15	Section 5.4 Analyse Graphical Models	Practice		
	BLM 5-16	Section 5.4 Example 2 Use Technology	Technology		
	BLM 5-17	Section 5.4 Achievement Check Rubric	Assessment		
	BLM A-8	Application General Scoring Rubric	Assessment		
5.5 Select a Mathematical Model					
	BLM 5-18	Section 5.5 Select a Mathematical Model	Practice		
	BLM 5-19	Section 5.5 Example 1 Use Technology	Technology		
	BLM 5-20	Section 5.5 Example 2 Use Technology	Technology		
	BLM 5-21	Section 5.5 Achievement Check Rubric	Assessment		
Chapter 5 Review					
	BLM 5-22	Chapter 5 Literacy	Literacy		
	BLM 5-23	Chapter 5 Review	Review		
Chapter 5 Practice Test					
	BLM 5-24	Environmental Investigation	Assessment		
	BLM A-4	Presentation Checklist	Assessment		
	BLM A-5	Problem Solving Checklist	Assessment		
	BLM 5-25	Chapter 5 Practice Test	Diagnostic Assessment		
	BLM 5-26	Chapter 5 Test	Summative Assessment		
Chapter 5 Problem Wrap-Up					
	BLM 5-27	Chapter 5 Problem Wrap-Up Rubric	Summative Assessment		
Chapter 5 Task					
	BLM 5-28	Chapter 5 Task Rubric	Summative Assessment		
	BLM 5-29	BLM Answers			

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## Student Text Pages 266–267

Suggested Timing 40–80 min

#### Tools

• graphing calculators

#### Related Resources

BLM 5-1 Prerequisite Skills BLM 5-2 Prerequisite Skills Self-Assessment Checklist

#### **Common Errors**

- Some students may enter calculations involving exponents into their calculators incorrectly, especially when a constant is multiplied by a power, as in question 7, part f), or in problems involving compound interest.
- R<sub>x</sub> Have students work in small groups, and compare how these expressions are entered. Scientific calculators often vary in the keystrokes required. Ensure that students are familiar with their calculators.
- Some students have difficulty determining which quantity belongs on which axis when making a scatter plot.
- **R**<sub>x</sub> Review the concepts of dependent and independent variables.

#### Accommodations

**Visual**—have students work in groups to create and present posters on the topics in the Prerequisite Skills to review key concepts

**Motor**—complete **question 6** as a class using an LCD projector to model and review graphing calculator keystrokes for regression

**Memory**— post the formulas for simple and compound interest

## **Teaching Suggestions**

- Consider having students work in pairs or small groups. Each group can keep a record of concepts that caused particular problems, and report a summary to the class when they are finished. This is useful information for planning strategic reinforcement of prerequisite skills as you work through the chapter.
- The graph in **question 2** implies that the price of the motorcycle decreases continuously. You can ask students to explain why this might not be practical. Have them suggest a better strategy for the owner. (Decreasing the price at the beginning of each week.) Point out that a model is really just a model, and needs to be used carefully when applying its predictions to the real world.
- For **questions 3 and 4**, review how to use graphing calculators to calculate finite differences.
- For **question 5**, review how to use a graphing calculator to create a scatter plot. Refer to the Technology Appendix in the student textbook.
- Before assigning **questions 9 and 10**, students may need a brief review of the formulas for simple and compound interest, and the meaning of the variables.

## Assessment

- Assess student readiness to proceed by informal observation as students are working on the questions. A formal test is inappropriate since this material is not part of the grade 12 curriculum for this chapter.
- Student self-assessment is also an effective technique; students can place a checkmark beside topics in the Prerequisite Skills in which they feel confident with the necessary skills. Use **BLM 5-2 Prerequisite Skills Self-Assessment Checklist** as a self-assessment for students.

## **Extra Practice**

• Use BLM 5-1 Prerequisite Skills for extra practice or remediation.

## **Chapter Problem**

- The Chapter Problem is introduced on page 267. The managementoriented theme of the Chapter Problem is well suited to this chapter, providing students with a variety of contextual opportunities to apply and model with linear, quadratic, or exponential functions. You may wish to have students complete the Chapter Problem revisits that occur throughout the chapter. These questions are designed to help students move toward the chapter Problem Wrap-Up on page 335.
- Alternatively, you may wish to assign the Chapter Problem questions and Chapter Problem Wrap-Up when students have completed the chapter, as part of a summative assessment.