McGraw-Hill Ryerson

Foundations for College Mathematics 12: Teacher's Resource

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Overview of *Foundations for College Mathematics 12*

The McGraw-Hill Ryerson *Foundations for College Mathematics 12* program has six components.

Student Text

The student text introduces topics in real-world contexts. In each numbered section, **Investigate** activities encourage students to develop their own understanding of new concepts. **Examples** present solutions in a clear, step-by-step manner, and then the **Key Concepts** summarize the new principles. **Discuss the Concepts** gives students an opportunity to reflect on the concepts of the numbered section, and helps you assess students' grasp of the new ideas and readiness to proceed with the exercises.

Practise (A) questions are single-step knowledge questions and assist students in building their understanding. **Apply (B)** questions allow students to use what they have learned to solve problems and make connections among concepts. **Extend (C)** questions are more challenging and thought-provoking. Answers to Practise, Apply, and Extend questions are provided at the back of the text. A **Chapter Problem** is introduced in the **Prerequisite Skills** section of each chapter. Students revisit different aspects of the problem in the numbered sections, leading up to the **Chapter Problem Wrap-Up** at the end of the chapter. **Chapter Tasks** are more involved problems that require students to use several concepts from the preceding chapters. Solutions to the Chapter Problem Wrap-Up and Chapter Tasks are provided in this Teacher's Resource.

A **Chapter Review** of skills and concepts is provided at the end of each chapter. Questions are organized by specific numbered sections from the chapter. **Cumulative Reviews** are provided after Chapters 2, 4, 6, and 8 and help prepare students for the Tasks.

A **College Preparation Test** is provided at the end of Chapter 2, 4, 6, and 8. Tests are written in the format often used on college entrance exams. This tool will help students prepare for college entrance.

The **Technology Appendix** provides instructions on the use of *The Geometer's Sketchpad*®, and TI-83/84 Plus and TI-Nspire™ CAS graphing calculators.

The text includes a number of items that can be used as assessment tools:

- **Discuss the Concepts** questions assess student understanding of the concepts
- **Achievement Checks** questions provide opportunities for formative assessment using the four Achievement Chart Categories, Knowledge and Understanding, Thinking, Communication, and Application
- **Practice Tests** contain multiple choice, short response, and extended response questions to help model classroom testing practices
- **Chapter Problem Wrap-Ups** finish each chapter by providing a set of questions that involve all four Achievement Chart Categories
- **Tasks** are presented after each chapter and reinforce concepts from the chapter

Technology is integrated throughout the program and includes the use of scientific calculators, graphing calculators, dynamic geometry programs, statistical software, and the Internet.

Teacher's Resource

This Teacher's Resource provides the following teaching and assessment suggestions:

- **Teaching Suggestions** for all the sections
- **Literacy Link** and Career Profile
- **Practice** and chapter-specific blackline masters
- Answers to the **Investigate** questions
- Responses for the **Discuss the Concepts** questions
- Solutions and rubrics for the **Chapter Problem Wrap-Up** and **Chapter Tasks**
- Students' **Common Errors** and suggested remedies
- Solutions and rubrics for the **Achievement Check** questions
- Suggestions for Ongoing Assessment and Summative Assessment
- **Accommodations** for students with different needs

Practice and Homework Book

The program includes a **Practice and Homework Book** which mirrors the chapters and section organization and sequence of the student text and is cross-referenced to pages in the text. Features include the following:

- Each chapter begins with **Get Set—**a review of Key Skills
- Each section begins with a series of topic-related Warm-Up questions, followed by similarly grouped Practise Questions
- Each chapter concludes with a Chapter Review, including Key Terms
- Practice and Homework Book answers are included on the Teacher's Resource CD ROM

Computerized Assessment Bank CD ROM

The Computerized Assessment Bank CD ROM (CAB) contains questions based on the material presented in the student text, and allows you to create and modify tests. Questions are connected to the chapters in the student text. The question types include: True/False, Multiple Choice, Completion, Matching, Short Answer, and Problem. Each question in the CAB is correlated to the corresponding Achievement Chart Category, specific curriculum expectation, and curriculum strand from the Ontario Mathematics MAP4C Curriculum.

Solutions CD ROM

The Solutions CD ROM provides worked-through solutions for all questions in the numbered sections of the student text, except for Achievement Check questions, which are in this Teacher's Resource. In addition, the Solutions CD ROM provides worked-through solutions for questions in the Review, Practice Test, and Cumulative Review features.

Web site

In addition to our McGraw-Hill Ryerson Web site, teachers can access the password protected site to obtain ready-made files for *The Geometer's Sketchpad®* activities in the text, information about managing TI technology, further support material for differentiated learners, and many other supplemental activities.

To access this site go to:
http://www.mcgrawhill.ca/books/foundations12
username: foundation
password: math12

Structure of the Teacher's Resource

The teaching notes for each chapter have the following structure:

Chapter Opener

The following items are included in the Chapter Opener:

- **Specific Expectations** that apply to the chapter, listed by strand
- **Key Terms** that will be introduced in the chapter, and which are defined in the margin
- **Teaching Suggestions** include notes on the Chapter Opener, and Assessment
- Introduction to a **Chapter Problem** that includes questions designed to help students move toward the **Chapter Problem Wrap-Up** at the end of the chapter

Planning Chart

This table provides an overview of each chapter at a glance, and specifies:

- **Student Text Pages** references and **Suggested Timing** for numbered sections
- Related blackline masters available on the Teacher's Resource CD ROM
- Assessment blackline masters for each section of the chapter
- Special tools and/or technology tools that may be needed

Blackline Masters Checklist

- A useful organizer, by Chapter and Section which lists relevant BLMs and their purpose

Prerequisite Skills

The following items are included in the margin:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) for extra practice or remediation, assessment, or enhancement
- **Common Errors** and remedies to help you anticipate and deal with common errors that may occur
- **Accommodations** for students having difficulties or needing enrichment

The key items in this section include:

- **Teaching Suggestions** for how to use the **Prerequisite Skills** section
- **Assessment** ideas on how to ascertain that students are ready for this chapter

Numbered Sections

The following items are listed in the margin:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) for extra practice or remediation, assessment, or enhancement
- **Common Errors** and remedies give you ideas on how to help students who make typical mistakes
- **Accommodations** provide ideas for how to provide assistance to students having difficulties or needing enrichment

The notes in each section include the following key elements:

- **Link to Prerequisite Skills** refers back to the relevant part of the Prerequisite Skills section (included in some numbered sections)
- **Warm-Up** and **Warm-Up Answers** provide a short check of the prerequisite skills needed for the section and often include a few Mental Math questions

- **Teaching Suggestions** give insights or point out connections on how to present the material from the text
- **Investigate Answers** let you know the expected outcomes of these activities
- **Discuss the Concepts** answers help consolidate students' understanding of the **Key Concepts** that are presented in the student text
- Notes for the **Practise**, **Apply**, and **Extend** questions in the text provide: comments on specific questions to anticipate any difficulties; ways to deal with students' questions; and hints on how to help students answer the questions
- **Achievement Check Answers** are included as are Achievement Check rubrics (as Blackline masters)
- **Ongoing Assessment** suggestions give a variety of strategies that can be used to assess the students' learning

End of Chapter Items

The **Chapter Reviews** in this Teacher's Resource include the following items:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) for extra practice or remediation, assessment, or enhancement
- Using the **Student Book Review** and **Teacher's Resource BLM Review** gives insights on how to present the information in the **Chapter Reviews**
- **Ongoing Assessment** suggestions give a variety of strategies you can use to assess the students' learning

The **Practice Tests** in this Teacher's Resource have the following key features:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) for extra practice or remediation, assessment, or enhancement
- **Study Guide** directs students who have difficulty with specific questions to appropriate examples to review
- **Summative Assessment** refers you to the **Chapter Test** to assess student performance
- **Accommodations** provide ideas for how to provide assistance to students having difficulties or needing enrichment
- **Using the Practice Tests** gives you insights on how to present the information in the Practice Tests

The **Chapter Problem Wrap-Up** includes the following elements:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) for extra practice and remediation, assessment, or enhancement
- **Using the Chapter Problem Wrap-Up** includes teaching suggestions specific to the problem
- **Summative Assessment** refers you to the **Chapter Problem Rubric** to assess student achievement
- **Sample Response** provides a typical level 3 answer and distinguishes it from a level 2 and level 4 response

A **Task** occurs at the end of each chapter and includes:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed for the section
- **Related Resources** (Blackline masters) useful for extra practice or remediation, assessment, or enhancement
- **Specific Expectations** covered in the Chapter Tasks

- **Teaching Suggestions** with steps for you to follow
- **Prompts for Getting Started** provides a list of questions you can use to help students begin the Task
- **Hints for Evaluating a Response** provides a list of questions you should consider when assessing students' responses
- **Accommodations** provide ideas for how to provide assistance to students having difficulties or needing enrichment
- **Ongoing Assessment** refers you to the **Chapter Task Rubric** to assess student achievement
- **Level 3 Sample Response** provides a typical level 3 answer and distinguishes it from a level 2 and level 4 answer

Cumulative Reviews are included at the end of Chapters 2, 4, 6, and 8. The following information is provided:

- **Student Text Pages** references and **Suggested Timing**
- **Tools** and **Technology Tools** needed
- **Related Resources** (Blackline masters) useful for extra practice or remediation, assessment or enhancement
- **Using the Cumulative Chapter Reviews** includes specific teaching suggestions
- **Ongoing Assessment** suggestions give a variety of strategies you can use to assess student's learning

The **Teacher's Resource CD ROM** provides various blackline masters in PDF and Word format, including:

- Generic Masters
- Technology Masters
- Practice Masters
- Assessment Masters
- Chapter-specific Masters
- This TR CD also contains all **Student Text** answers that were not included in the text itself, all Practice and Homework Book answers, and the entire TR in PDF format.

Program Philosophy

The *Foundations for College Mathematics 12* is an exciting new resource for intermediate learners.

The *Foundations for College Mathematics 12* program is designed to:

- provide full support in teaching the Ontario MAP4C mathematics curriculum
- enable and guide students' progress from concrete to representational and then to abstract thinking
- offer a diversity of options that collectively deliver student and teacher success

> *Given the changes occurring during adolescence, school administrators and teachers need to consider how best to match instruction to … the developing capabilities and varied needs of students…*
>
> *The (Foundations for College Mathematics 12) program is based on a view that all students can be successful in mathematics… [It] reflects principles of effective practice and research on how adolescents learn, prerequisites for achieving a balanced approach to mathematics.*
>
> *Creating Pathways: Mathematical Success for Intermediate Learners*, Folk, McGraw-Hill Ryerson, 2004

During grades 7 to 10, most students progress from solely concrete thinking toward more sophisticated forms of cognition, as shown in the diagram:

Concrete Thinking

- typically work with physical objects
- focus of thinking is specific
- little or no reflection on thought processes
- able to solve very simple problems

▶

Representative Thinking

- sometimes called "semi-concrete"
- typically work with diagrams
- thinking focus becoming more general and systematic
- meta-cognitive thinking about thought processes begins to develop
- explore hypothetical or "what-if" thinking, with support
- able to solve moderately challenging problems
- use problem-solving strategies effectively, with some guidance

▶

Abstract Thinking

- able to work with or without materials or diagrams
- thinking focus instinctively general and systematic
- meta-cognitive thinking is well developed
- naturally explore hypothetical or "what-if" thinking
- able to solve problems that extend or deepen thinking
- confidently select and adapt problem-solving strategies

In *Foundations for College Mathematics 12*, students most often start with representational thinking. Concrete models are used in some sections, particularly geometry. They may be helpful to some students for other sections, for example algebra tiles can help some students understand the process of factoring quadratic expressions. Only when students are comfortable with the concrete and representative do they begin to move toward the abstract. Suggestions for alternative ways to approach some key topics provide students with the opportunity to learn in a manner that may engage them and increase their chances of success.

Approaches to Teaching Mathematics

The following assumptions and beliefs form the foundation of the *Foundations for College Mathematics 12* program:

1. Students demonstrate a wide range of prior knowledge and experiences, and learn via various styles and different rates.
2. Learning is most effective when students are given opportunities to investigate concepts before being introduced to the abstract mathematics involved.
3. Learning is most likely when familiar, meaningful contexts are used to illustrate ideas and applications of concepts.
4. Students benefit when different learning approaches are used–independent, cooperative, hands-on and teacher guided.

> *Learning is enhanced when students experience a variety of instructional approaches, ranging from direct instruction to inquiry-based learning.*
>
> *Ontario Ministry of Education and Training, 2004*

The concrete and abstract progression is exemplified in the following styles of mathematics teaching.

Most applied students learn best by using a concrete, discovery-oriented approach to develop concepts. Once these concepts have been developed, a connectionist approach helps students consolidate their learning.

Transmission-Oriented

- teaching involves "delivering" the curriculum
- focuses on procedures and routines
- emphasizes clear explanations and practice
- "chalk-and-talk"

▶

Connectionist-Oriented

- teaching involves helping students develop and apply their own conceptual understandings
- focuses on different models and methods and the connections among them
- emphasizes "problematic" challenges and teacher-student dialogue

▶

Discovery-Oriented

- teaching involves helping students learn by "doing"
- focuses on applying strategies to practical problems and using concrete materials
- emphasizes student-determined pacing
- "hands-on"

At this level, some transmission-oriented learning is also useful. This variety of approaches can be seen in the *Foundations for College Mathematics 12* program design.

Feature	Teaching Style(s) Supported
Chapter Problem	connectionist
Investigate	discovery, connectionist
Examples	transmission, connectionist
Key Concepts	transmission
Discuss the Concepts	connectionist
Practise the Concepts	connectionist, transmission
Apply the Concepts	connectionist, transmission
Extend the Concepts	connectionist, transmission
Review	transmission, connectionist
Task	discovery, connectionist

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Instructional Practice

The resources available in today's classroom offer opportunities and challenges. Indeed, the principal challenge—one that many teachers of mathematics are reluctant to confront—is to teach successfully to the opportunities available.

Grouping

> *Instructional practice that incorporates a variety of grouping approaches enhances the richness of learning for students.*
>
> *Creating Pathways: Mathematical Success for Intermediate Learners, Folk, McGraw-Hill Ryerson, 2004*

At one end of the scale, individual work provides an opportunity for students to work on their own, at their own pace. At the other extreme, class discussion of problems and ideas creates a synergistic learning environment. In between, carefully selected groups bring cooperative learning into play.

Manipulatives and Materials

> *Effective use of manipulatives helps students move from concrete and visual representations to more abstract cognitive levels.*
>
> *Ontario Ministry of Education and Training, 2003*

Although many teachers feel unsure about teaching with manipulatives and other concrete materials, many students find them a powerful way to learn. The *Foundations for College Mathematics 12* program supports the use of manipulatives, helps teachers adapt to this kind of teaching. The Teaching Suggestions sections in the Teacher's Resource provide suggestions for developing student understanding using semi-concrete materials, such as diagrams and charts.

Technology

Special computer software designed for the classroom and licensed by the Ministry of Education for use in Ontario classrooms, such as *The Geometer's Sketchpad*®, provides a powerful tool for teaching and learning. The *Foundations for College Mathematics 12* program supports the use of such software as an enhancement to the classroom experience. In addition, support for Computer Algebra Systems is included. Graphing calculator instructions are provided in the Investigate activities and Technology Appendix. Instructions for the TI-Nspire™ CAS graphing calculator are for OS 1.6. Be sure to download the latest operating system from the Texas Instruments downloads site. Allow students to familiarize themselves with the calculator if it is new to them. The instructions provided in the student textbook assume that students know how to turn on the calculator and how to open an application page. Multiple solutions for worked-through examples in the text allow teachers to enjoy wide flexibility in lesson planning. As a result, you can plan activities using manipulatives, pencil and paper, graphing calculators, software, or any combination of these.

The Internet provides great opportunities for enhancing learning. As with many other sources of information, students must be protected from inappropriate content. The McGraw-Hill Web site at *http://www.mcgrawhill.ca/links/foundations12* (for students) has been designed to offer only safe and reliable Web site links for students to explore as an integrated part of the *Foundations for College Mathematics 12* program. The companion Web site for teachers is *http://www.mcgrawhill.ca/books/foundations12*.

Literacy

Effective mathematics classrooms show students that math is everywhere in their world. For example, students should see that knowledge of probability is useful when learning about the electoral process in Social Studies class. Their work in graphing can be used in Science class. Their written explanations are also a language arts product. When connections such as these are made, students begin to see that math is not an isolated subject, but rather a vital part of everyday life. Contextual examples and problems can be linked to students' everyday experiences outside the classroom, as well.

Writing and Mathematics

Being able to communicate ideas clearly is an important part of the *Foundations for College Mathematics 12* program. Students are asked to write about the mathematics they are learning, and communicate their understanding about what they are learning.

Take time to discuss the importance of being able to communicate understanding. The students' responses are meant to communicate with the teacher and are assessed as part of the mathematics work.

Literacy Connections

There are Literacy Connection suggestions in several chapters of this Teacher's Resource. These provide ideas as to how you might assist students to improve their mathematical literacy by using and extending the Literacy Connect questions that are in most numbered sections of the student text. BLMs are provided for additional student support.

Cooperative Learning

Students learn effectively when they are actively engaged in the process of learning. Many of the sections in *Foundations for College Mathematics 12* include Investigate activities that foster this approach. These activities are best done through cooperative learning during which students work together—either with a partner or in a small group of three or four—to complete the activity and develop generalizations about the topic or process.

Group learning such as this is an important aspect of a constructivist educational approach. It encourages interactions and increases chances for students to communicate and learn from each other (Sternberg & Williams, 2002).

Teacher's Role

In classrooms where students are adept at cooperative learning, the teacher becomes the facilitator, guide, and progress monitor. Until students have reached that level of group cooperation, however, you will need to coach them in how to learn cooperatively. This may include:

- Making sure that the materials are at hand and directions are perfectly clear so that students know what they are doing before starting group work
- Carefully structuring activities so that students can work together
- Providing coaching in how to provide peer feedback in a way that allows the listener to hear and attend
- Constantly monitoring student progress and providing assistance to groups having problems either with group cooperation or the math at hand

Types of Groups

The size of group you choose to use may vary from activity to activity. Small-group settings allow students to take risks that they might not take in a whole class setting (Van de Walle, 2000). Research suggests that small groups are fertile environments for developing mathematical reasoning (Artz & Yaloz-Femia, 1999).

Results of international studies suggest that groups of mixed ability work well in mathematics classrooms (Kilpatrick, Swafford, & Findell, 2001). If the class is new to cooperative learning, you may wish to assign students to groups according to the specific skills of each individual. For example, you might pair a student who is talkative but weak in number sense and numeration with a quiet student who is strong in those areas. You might pair a student who is weak in many parts of mathematics but has excellent spatial sense with a stronger mathematics student who has poor spatial sense. In this way, student strengths and weaknesses complement each other and peers have a better chance of recognizing the value of working together.

Cooperative Learning Skills

When coaching students about cooperative learning, you may want to consider task skills and working relationship skills, as indicated in the table below.

Task Skills	Working Relationship Skills
• Following directions • Communicating information and ideas • Seeking clarification • Ensuring that others understand • Actively listening to others • Staying on task	• Encouraging others to contribute • Acknowledging and responding to the contributions of others • Checking for agreement • Disagreeing in an agreeable way • Mediating disagreements within the group • Sharing • Showing appreciation for the efforts of others

Class discussions, modelling, peer coaching, role-playing, and drama can be used to provide positive task skills. For example, you might role-play different ways to provide feedback and have a class discussion on which ones students like and why. You might discuss common group roles and how group members can use them. Students also need to understand that the same person can play more than one role.

Role	Math Connection	Sample Comment
Leader	• Makes sure the group is on task and everyone is participating • Pushes group to come to a decision	Let's do this. Can we decide…? This is what I think we should do…
Recorder	• Manages materials • Writes down data collected or measurements made	This is what I wrote down. Is that what you mean?
Presenter	• Presents the group's results and conclusions	We feel that… These are our conclusions… Our group found…
Organizer	• Watches time • Keeps on topic • Encourages getting the job done	Let's get started. Where should we start? So far we've done the following… Are we on topic? What else do we need to do?
Clarifier	• Checks that members understand and agree	Does everyone understand? So, what I hear you saying is… Do you mean that…?

Types of Strategies

A number of different types of cooperative learning strategies can be used in the mathematics classroom, and many are suggested in this Teacher's Resource. The *Foundations for College Mathematics 12* program includes selected blackline masters (BLMs) to use with some but not all of these strategies.

Think-Pair-Share

Students individually think about a concept, and then pick a partner to share their ideas. For example, students might work on the Discuss the Concepts questions, and then choose a partner to discuss the concepts with. Working together, the students could expand on what they understood individually. In this way, they learn from each other, learn to respect each other's ideas, and learn to listen.

Cooperative Task Group

Task groups of two to four students work on activities in the Investigate section. As a group, students share their understanding of what is happening during the activity and how that relates to the mathematics topic, at the same time as they develop group cooperation skills.

Jigsaw

Individual group members are responsible for researching and understanding a specific part of the information for a project. Individual students then share what they have learned so that the entire group gets information about all areas being studied. For example, during data management, this type of group might have "experts" in making various types of graphs using technology. Group members could then coach each other in making each kind of graph.

Another way of using the Jigsaw method is to assign "home" and "expert" groups during a large project. For example, students researching the shapes of various sports' surfaces might have a home group of four in which each member is responsible for researching one of soccer, baseball, hockey, or basketball. Individual members then move to expert groups. Expert groups include all of the students responsible for researching one of the sports. Each of the expert groups researches their particular sport. Once the information has been gathered and prepared for presentation, individual members of the expert group return to their home group and teach other members about their sport.

Placemat

In groups of four, students individually complete their section of a placemat. The group then pools their responses and completes the centre portion of the placemat with group responses. This method can be used for pre-assessment (diagnostic), review, or to summarize a topic.

Concept Attainment

Based on a list of examples and non-examples of a concept, students identify and define the concept. Then, they determine the critical attributes of the concepts and apply their defined concept to generate their own examples and non-examples.

Think Aloud

Work through a problem in front of the class, verbalizing your thinking throughout. This method can help develop process thinking in students.

Decision Tree

Students use a graphic organizer flow chart to identify key decisions and consequences.

Carousel
Students at different stations display and explain topics or concepts to other classmates who rotate through the stations, usually in order.

Timed Retell
Students sit in pairs facing each other. After some preparation time, Student A has 30 s to tell what she or he knows about the topic to Student B. Student B then retells the talk for about 30 s and adds additional information. Both students then write a summary of the talk.

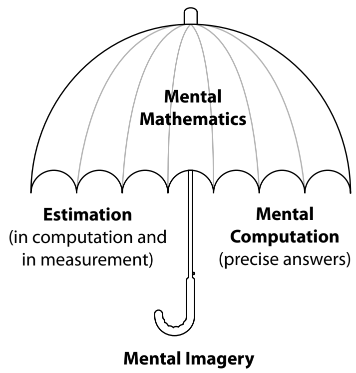
Frayer Model
Students complete four quadrants for a specified topic: definition, facts/characteristics, examples, and non-examples. Variation: Give students a completed model and ask them to identify the topic/concept.

Word Wall
Individually or in groups, students complete cards for words or symbols, and then post the cards to use during future studies. One side of the card has the word or symbol, while the other side has four quadrants: the word, definition, picture or diagram, and an example or application.

Three-Step Interview
In triads, label students A, B, and C. Have students individually compose interview questions. Assign roles to the three groups: A = Interviewer; B = Interviewee; C = Recorder. Student A interviews Student B, while Student C records the information. Then the students rotate roles. After all the interviews are complete, students share the recorded information in a Round Robin format.

Mental Mathematics

A major goal of mathematics instruction for the twenty-first century is for students to make sense of the mathematics in their lives. The development of all areas of mental mathematics is a major contributor to this comfort and understanding.



The diagram above shows the various components under the umbrella of Mental Mathematics. All three are considered mental activities and interact with each other to make the connections required for mathematics understanding.

Computational Estimation

Computational estimation refers to the approximate answers for calculations, a very practical skill in today's world. The development of estimation skills helps refine mental computation skills, enhances number sense, and fosters confidence in math abilities, all key in problem solving. Over 80% of out-of-school problem-solving situations involve mental computation and estimation (Reys & Reys, 1986).

Computational estimation does not mean guessing at answers. Rather, it involves a host of computational strategies that are selected to suit the numbers involved. The goal is to refine these strategies over time with regular practice, so that estimates become more precise. The ultimate goal is for students to estimate automatically and quickly when faced with a calculation. These estimations are a check for reasonableness and provide learners with a strategy for checking their actual calculations.

Measurement Estimation

This skill relies on awareness of the measurement attributes (e.g., metre, kilometre, litre, kilogram, hour). Just as computational estimation enhances number sense, practice in measurement estimation enhances measurement sense.

A *referent* is a personal mental tool that students can develop for use in thinking about measurement situations. Tools could include the distance from home to school, a 100-km trip, the capacity of a can of juice, the duration of 30 min, and the area of the math textbook cover. These referents develop with measurement practice, and specifically with practice that encourages students to form these frames of reference. Students can compare other measurements to these referents. By doing so, they can gain a better understanding of what may be happening in a problem-solving situation.

You can help students develop referents by doing activities such as asking students to use their fingers or hands to show such measurements as: 6 cm, 260 mm, 0.4 m, a 60° angle, or 2000 cm^3.

Mental Imagery

Mental imagery in mathematics refers to the images in the mind when one is doing mathematics. It is these mental representations, or conceptual knowledge, that need to be developed in all areas of mathematics. Capable math students "see" the math and are able to perform mental manoeuvres in order to make connections and solve problems. These images are formed when students manipulate objects, explore numbers and their meanings, and talk about their learning. Students must be encouraged to look into their mind's eye and "think about their thinking."

Asking, *What do you see in your mind's eye* when asked to visualize, encourages students to think about the images they are using to help them solve problems. Students are often surprised when fellow students share their personal images; the discussion generated is very worthwhile.

Try these Mental Imaging Activities with your students.

Example 1:
Draw the mental image you have for each of the following:
- $\frac{5}{8}$
- 345 500 in relation to a million
- a 280° angle
- 0.67 m
- 42 cm
- a 7.2-kg fish
- a 7-g fish

Example 2:
Use mental imagery to answer the following:
1. How many edges does a triangular prism have?
2. If I am facing west, what direction is to my left?
3. How many sides does a pentagonal pyramid have?
4. Imagine a 6-cm cube. What is its volume?
5. You cut off the top of a right cone. What shape is exposed?

Mental Computation

Mental computation refers to an operation used to obtain the precise answer for a calculation. Unlike traditional algorithms, which involve one method of calculation for each operation, mental computations include a number of strategies—often in combination with others—for finding the exact answer.

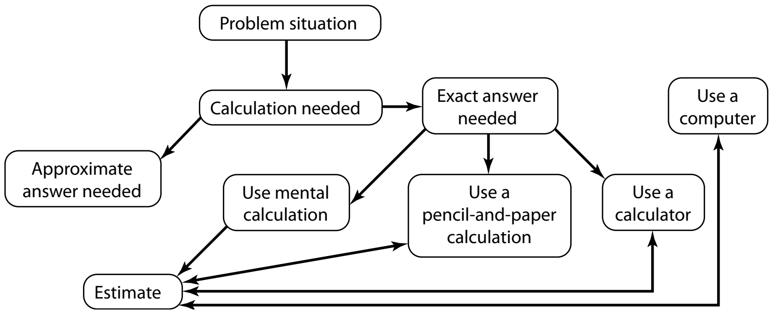
Some Points Regarding Mental Mathematics

- Students must have knowledge of the basic facts (addition and multiplication) in order to estimate and calculate mentally. Without knowing the basic facts, it is unlikely that students will ever attempt to employ any estimation or mental math strategies, as these will be too tedious.
- Mental math strategies are flexible; you need to select one that is appropriate for the numbers in the computation. Students should select appropriate strategies for a variety of computation examples, and use the strategies in problem-solving situations.
- Sometimes mental math strategies are used in conjunction with pencil and paper tasks. The questions are rewritten to make the calculation easier.
- The ultimate goal of mental mathematics is for students to estimate for reasonableness, and to look for opportunities to calculate mentally.

Keep in Mind

Capable students of mathematics are comfortable with numbers. This comfort means that the students see patterns in numbers and intuitively know how they relate to each other and how they will behave in computational situations. Due to their comfort with numbers, these students have developed strong skills in estimation and mental math. Because of this, their understanding of numbers is further strengthened. We say they have "number sense." This sense of numbers develops gradually and varies as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms.

The position of the National Council of Teachers of Mathematics (NCTM) on how to proceed when faced with a problem that requires a calculation is best explained with this chart.



The chart tells us that, given a problem requiring calculation, students should ask themselves the following questions:

- Is an approximate answer adequate or do I need the precise answer?
- If an estimate is sufficient, what estimation strategy best suits the numbers provided?
- If an exact answer is needed, can I use a mental strategy to solve it?
- If the numbers don't lend themselves to a mental strategy, can I do the calculation using a paper-and-pencil method?
- If the calculation is too complex, I will use a calculator. What is a good estimate for the answer?

NCTM's Number and Operations Standard states that, "Instructional programs from kindergarten through grade 12 should enable all students to compute fluently and make reasonable estimates" (Principles and Standards for School Mathematics, 2000). Whether the students select an estimation strategy, a mental strategy, a paper-and-pencil method, or use the calculator, they must use their estimation skills to judge the reasonableness of any answer.

Mental Math Strategies

In *Foundations for College Mathematics 12*, mental math strategies are explicitly practised in some of the Warm-Up questions that are presented in this Teacher's Resource for each section within individual chapters. In addition, even though not always explicitly mentioned, students use mental math strategies throughout many parts of the text.

Problem Solving

> *Solving problems is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking.*
>
> *National Council of Teachers of Mathematics, 2000*

Problem solving is an integral part of mathematics learning. The National Council of Teachers of Mathematics recommends that problem solving be the focus of all aspects of mathematics teaching because it encompasses skills and functions, which are an important part of everyday life.

> *NCTM Problem-Solving Standard*
> Instructional programs should enable all students to—
> • Build new mathematical knowledge through problem solving
> • Solve problems that arise in mathematics and in other contexts
> • Apply and adapt a variety of appropriate strategies to solve problems
> • Monitor and reflect on the process of mathematical problem solving

Problem solving is, however, more than a vehicle for teaching and reinforcing mathematical knowledge and helping to meet everyday challenges. It is also a skill that can enhance logical reasoning. It requires students to make logical deductions, connections, and to apply their mathematical understanding to situations outside the classroom. For these reasons problem solving can be developed as a valuable skill in itself, a way of thinking, rather than just the means to an end of finding the correct answer.

In *Foundations for College Mathematics 12*, a variety of problem-solving opportunities are provided for students.

The problem-solving model involves four steps:

1. understand–indentify what the problem is asking
2. plan–choose which strategy or combination of strategies to use
3. solve–carry out the plan
4. look back–determine if the answer is reasonable

The **Examples** in the student text often provide **Solutions** using different methods. Students are encouraged to try different methods to solve problems. Common problem-solving strategies include the following: draw a diagram; make an organized list; look for a pattern; make a model; work backward; make a table or chart; act it out; use systematic trial; make an assumption; find needed information; choose a formula; solve a simpler problem.

- Each chapter includes the investigation of a specific real-life problem. The **Chapter Problem** is then revisited throughout the chapter through **Chapter Problem** questions, and ends with the **Chapter Problem Wrap-Up**.
- Questions that involve the **Mathematical Process Expectations** are embedded throughout the chapters.
- At the end of each chapter, students are presented with a **Task** where the solution path is not readily apparent and where solving the problem requires more than just applying a familiar procedure. These cross-curricular tasks require students to apply what they have learned in the current chapter and the previous chapters to solve real-life, broad-based problems.

Mathematical Processes

The seven expectations presented at the start of the mathematics curriculum in Ontario describe the mathematical processes that students need to learn and apply as they investigate mathematical concepts, solve problems, and communicate their understanding. Although the seven processes are categorized, they are interconnected and are integrated into student learning in all areas of the *Foundations for College Mathematics 12* program.

Problem Solving

Problem solving is the basis of the *Foundations for College Mathematics 12* program. Students can achieve the expectations by using this essential process, and it is an integral part of the mathematics curriculum in Ontario. Useful problem-solving strategies include: making a model, picture, or diagram; looking for a pattern; guessing and checking; making assumptions; making an organized list; making a table or chart; making a simpler problem; working backwards; using logical reasoning.

Reasoning and Proving

Critical thinking is an essential part of mathematics. As the students investigate mathematical concepts in *Foundations for College Mathematics 12*, they learn to: employ inductive reasoning; make generalizations based on specific findings; use counter-examples to disprove conjectures; use deductive reasoning.

Reflecting

Students are given opportunities to regularly and consciously reflect on their thought processes as they work through the **Investigates** and exercises in *Foundations for College Mathematics 12*. As they reflect, they learn to: recognize when the technique they are using is not helpful; make a conscious decision to switch to a different strategy; rethink the problem; search for related knowledge; determine the reasonableness of an answer.

Selecting Tools and Computational Strategies

Students are given many opportunities to use a variety of manipulatives, electronic tools, and computational strategies in the *Foundations for College Mathematics 12* program. The student text provides examples of and ways to use various types of technology, such as calculators, computers, and communications technology, to perform particular mathematical tasks, investigate mathematical ideas, and solve problems. These important problem-solving tools can be used to: investigate number and graphing patterns, geometric relationships, and different representations; simulate situations; collect, organize, and sort data; extend problem solving.

Connecting

Foundations for College Mathematics 12 is designed to give students many opportunities to make connections between concepts, skills, mathematical strands, and subject areas. These connections help them see that mathematics is much more than a series of isolated skills and concepts. Connecting mathematics to their everyday lives also helps students see that mathematics is useful and relevant outside the classroom.

Representing

Throughout the *Foundations for College Mathematics 12* program, students represent mathematical ideas in various forms: numeric, geometric, graphical, algebraic, pictorial, and concrete representations, as well as representation using dynamic software. Students are encouraged to use more than one representation for a single problem, seeing the connections between them.

Communicating

Students use many different ways of communicating mathematical ideas in the *Foundations for College Mathematics 12* program, including: oral, visual, writing, numbers, symbols, pictures, graphs, diagrams, and words. The process of communication helps students reflect on and clarify ideas, relationships, and mathematical arguments.

Using Mathematical Processes

You can encourage students to use the mathematical processes in their work by prompting them with questions such as the following:

- *How can you tell whether your answer is correct/reasonable?* This promotes reasoning and reflection.
- *Why did you choose this method?* This promotes reflection, reasoning, selecting tools and computational strategies, and communication.
- *Could you have solved the problem another way?* This promotes reasoning, reflection, selecting tools and computational strategies, representing, and communication.
- *In what context have you solved a problem like this before?* This promotes connecting.

You can also encourage students to use a Think-Pair-Share approach to problem solving (see the **Cooperative Learning** section in this Program Overview). They will benefit greatly from brainstorming ideas and comparing methods of approach. A useful life skill is willingness to try different methods of solving a problem, learning from methods that perhaps do not reach the final goal, and being able to change their approach to reach the solution.

Technology

> *The use of technology in instruction should further alter both the teaching and the learning of mathematics. Computer software can be used effectively for class demonstrations and independently by students to explore additional examples, perform independent investigations, generate and summarize data as part of a project, or complete assignments. Calculators and computers with appropriate software transform the mathematics classroom into a laboratory much like the environment in many science classes, where students use technology to investigate, conjecture, and verify their findings.*
>
> *In this setting, the teacher encourages experimentation and provides opportunities for students to summarize ideas and establish connections with previously studied topics.*
>
> *Curriculum and Evaluation Standards for School Mathematics, NCTM, 1989*

Foundations for College Mathematics 12 taps the full power of today's interactive technologies to engage students in math inquiry, research, and problem solving. Technology is a major focus in several of the chapters, providing students with hands-on experience in creating graphs, and constructing and manipulating geometric figures. If at all possible, a classroom environment should be in place in which students are encouraged to reach for and apply technology whenever they feel the situation calls for it. In such an environment, the ongoing use of technology becomes another tool in the student's problem-solving tool kit, rather than a discrete event.

The *Foundations for College Mathematics 12* program includes opportunities for students to do research in the library or on the Internet. Consider having a class discussion on Internet Web sites and appropriate sources. Remind students that anyone can create a Web site on any topic on the Internet. Ask students to raise their hands if they have a personal web site or keep an Internet journal (a *blog*). Explain that Web sites like these contain personal opinions and information contained on them should be looked at critically. This also may provide an opportunity to remind students that personal information should never be revealed over e-mail, in an on-line journal, or a chat-room, and that anything that makes them uncomfortable should be reported immediately to their parent or guardian.

Types of Programs

Several types of software programs are used in *Foundations for College Mathematics 12*.

Technology BLMs are also available, providing students with step-by-step directions on how to use technology, such as software and Computer Algebra System calculators, to explore the mathematical concepts of the lesson. These BLMs include:

- BLM T-1 Microsoft® Excel
- BLM T-2 *The Geometer's Sketchpad®* 3
- BLM T-3 *The Geometer's Sketchpad®* 4
- BLM T-4 *Fathom*™
- BLM T-5 *The Computer Algebra System®*
- BLM T-6 Using the CBR™

The **Technology Appendix**, on pages 498–536, of the student text provides clear step-by-step instruction in the basic functions of the TI-83 Plus, TI-84 Plus and TI-Nspire™ CAS graphing calculators and the basic features of *The Geometer's Sketchpad®*.

Assessment

The main purpose of assessment is to improve student learning. Assessment data helps you determine the instructional needs of your students during the learning process. Some assessment data is used to evaluate students for the purpose of reporting.

Assessment must be purposeful and inclusive for all students. It should be varied to reflect learning styles of students and be clearly communicated with students and parents. Assessment can be used diagnostically to determine prior knowledge, formatively to inform instructional planning, and in a summative manner to determine how well the students have achieved the expectations at the end of a learning cycle.

Diagnostic Assessment

Assessment for diagnostic purposes can determine where individual students will need support and will help to determine how the classroom time needs to be spent. *Foundations for College Mathematics 12* provides you with diagnostic support at the start of the text and the beginning of every chapter.

- The **Prerequisite Skills** section at the beginning of each chapter provides coaching on essential concepts and skills needed for the upcoming chapter. **Prerequisite Skills Self-Assessment** blackline masters are also provided for each chapter.
- For students needing support beyond the Prerequisite Skills, the **Practice Masters** provided in this Teacher's Resource help to develop conceptual understanding and improve procedural efficiency.

Diagnostic support is also provided at the start of every section.

- Each section begins with an introduction to facilitate open discussion in the classroom.
- Each activity starts with a question that stimulates prior knowledge and allows you to monitor students' readiness.

Formative Assessment

Formative assessment tools are provided throughout the text and Teacher's Resource. Formative assessment allows you to determine students' strengths and weaknesses and guide your class towards improvement. *Foundations for College Mathematics 12* provides blackline masters for student use that complement the text in areas where formative assessment indicates that students need support.

The **Chapter Opener**, visual, and the introduction to the **Chapter Problem** at the beginning of each chapter in the student book provide opportunities for you to do a rough formative assessment of student awareness of the chapter content.

Within each lesson:

- **Key Concepts** can be used as a focus for classroom discussion to determine the students' readiness to continue.
- **Discuss the Concepts** questions allow you to determine if the student has developed the conceptual understanding and/or skills that were the goal of the section.
- **Practise (A)** questions allow you to determine whether students have basic knowledge skills related to the expectation(s) of the section.
- **Apply (B)** questions offers you an opportunity to determine students' understanding of concepts through conversations and written work. It also allows you to monitor students' procedural skills, their application of procedures, their ability to communicate their understanding of concepts, and their ability to solve problems related to the section's Key Concepts.

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- **Achievement Check** questions allow students to demonstrate their knowledge and understanding and their ability to apply, think of, and communicate what they have learned.
- **Chapter Problem** questions provide opportunities to verify that students are developing the skills and understanding they need to complete the **Chapter Problem Wrap-Up** questions.
- **Extend the Concepts** questions are more challenging and thought-provoking, and are aimed at Level 3 and 4 performance.
- **Chapter Reviews** and **Cumulative Reviews** provide an opportunity to assess Knowledge/Understanding, Thinking, Communication, and Application.

Summative Assessment

Summative data is used for both planning and evaluation.

- A **Practice Test** (Text and BLM) and a **Chapter Test** (BLM only) in each chapter assess students' achievement of the expectations in the areas of Knowledge/Understanding, Thinking, Communication, and Application.
- The **Chapter Problem** provides a problem-solving opportunity using an open-ended question format that is revisited in the **Chapter Problem Wrap-Up** questions. The **Chapter Problem** can be used to evaluate students' understanding of the expectations under the categories of Knowledge and Understanding, Thinking, Communication, and Application.
- **Tasks** are open-ended investigations with rubrics provided. They are presented at the end of each chapter. The Tasks require students to use and make connections among several concepts from the chapter.
- BLMs of rubrics for Chapter Problems and Tasks are provided in the Teacher's Resource CD ROM

Portfolio Assessment

Student-selected portfolios provide a powerful platform for assessing students' mathematical thinking. Portfolios:

- Help teachers assess students' growth and mathematical understanding
- Provide insight into students' self-awareness about their own progress
- Help parents understand their child's growth

Foundations for College Mathematics 12 has many components that provide ideal portfolio items. Inclusion of all or any of these chapter items provides insight into students' progress in a non-threatening, formative manner. These items include:

- Students' responses to the **Chapter Opener**
- Students' responses to the **Chapter Problem Wrap-Up** assignments
- Responses to **Discuss the Concepts** questions, which allow students to explore their initial understanding of concepts
- Answers to **Achievement Check** questions, which are designed to show students' mastery of specific expectations
- **Task** assignments, which show students' understanding across several chapters

Assessment Masters

Foundations for College Mathematics 12 provides a variety of assessment tools with the chapter-specific blackline masters, such as Chapter Tests, Chapter Problem Wrap-Up rubrics, and Task rubrics. In addition, the program offers a wide variety of generic assessment blackline masters. These BLMs will help you to effectively monitor student progress and evaluate instructional needs.

Generic Assessment BLM	Type	Purpose
BLM A-1 Assessment Recording Sheet	Chart	Organize comments for assessment of students observations, portfolios, and presentations
BLM A-2 Attitudes Assessment Checklist	Checklist	Assess students' attitude as they work on a task
BLM A-3 Portfolio Checklist	Checklist	Assess students' portfolios
BLM A-4 Presentation Checklist	Checklist	Assess students' oral and written presentations
BLM A-5 Problem Solving Checklist	Checklist	Assess students' problem solving skills
BLM A-6 Knowledge and Understanding General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Knowledge and Understanding category
BLM A-7 Thinking General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Thinking category
BLM A-8 Application General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Application category
BLM A-9 Communication General Scoring Rubric	Rubric	Evaluate students' understanding of expectations under the Communication category
BLM A-10 Observation General Scoring Rubric	Rubric	Assess students' understanding of the expectations under all four categories
BLM A-11 Group Work Assessment Recording Sheet	Worksheet	Record comments as students work on group tasks
BLM A-12 Group Work Assessment General Scoring Rubric	Rubric	Assess students' group-related work
BLM A-13 Self-Assessment Recording Sheet	Worksheet	Students self-assess their understanding of chapter material
BLM A-14 Self-Assessment Checklist	Checklist	Students self-assess their understanding of chapter material
BLM A-15 Teamwork Self Assessment	Worksheet	Students evaluate their work as part of a team
BLM A-16 Assessing Work in Progress	Worksheet	Student groups assess their progress as they work to complete a task
BLM A-17 Learning Skills Checklist	Checklist	Assess students' work habits and learning skills
BLM A-18 Opinion Piece Checklist	Checklist	Assess students' work on an opinion piece
BLM A-19 Report Checklist	Checklist	Assess students' work on a report

Intervention

Foundations for College Mathematics 12 accommodates a broad range of needs and learning styles, including those students requiring accommodations, and students with limited proficiency in English. This Teacher's Resource provides support in addressing multiple intelligences and learning styles through a variety of strategies.

- Excellent visuals and multiple representations of concepts and instructions support visual learners, ESL students, and struggling readers
- Relevant contexts, including multicultural examples, engage students and provide a purpose for the mathematics being learned
- **Extend** questions in the student text provide additional challenge for those students who can complete the Practise and Apply questions with no difficulties.
- **Accommodations** in the margin provide suggestions for students having difficulties or needing enrichment

Reaching all Students

Students may experience difficulty meeting provincial standards for a variety of reasons. General cognitive delays, social-emotional issues, behavioural difficulties, health-related factors, and extended or sporadic absences from instruction underlie the math difficulties experienced by some students. These factors do not explain the challenges other students encounter, however. For these students, math difficulties are usually related to three key areas: language, visual/perceptual/spatial/motor, or memory.

Language

Students with language learning difficulties demonstrate difficulty reading and understanding math vocabulary and math story problems, and determining saliency (e.g., picking out the most important details from irrelevant information). Processing information that is presented using oral or written language is often difficult for these students, who may be more efficient learners when information is presented in a non-verbal, visual format. Diagrams and pictorial representations of math concepts are usually more meaningful to these students than lengthy verbal or written descriptions.

Visual/Perceptual/Spatial/Motor

Some students demonstrate difficulties understanding and processing information that is presented visually and in a non-verbal format. Language support to supplement and make sense of visually presented information is often beneficial (e.g., verbal explanation of a visual chart). Visual, perceptual, spatial, and motor difficulties may be evident in students' written output, as well as in their ability to process visually inputted information. Difficulties with near and far point copying, accurately aligning numbers in columns, properly sequencing numbers, and illegible handwriting are examples of output difficulties in this area.

Memory (Short-Term, Working, and Long-Term Memory)

Students with short-term memory difficulties find it hard to remember what they have just heard or seen (e.g., auditory short-term memory, visual short-term memory). A weak working or active memory makes it difficult for students to hold information in their short-term memory and manipulate it (e.g., hold what they have just heard and then perform a mathematical operation with that information). For others, the retrieval of information from long-term memory (e.g., remembering number facts and previously taught formulae) is difficult. Students with long-term memory difficulties may also have difficulty storing information in their long-term memory, as well as retrieving it.

Modifications, Individual Education Plans (IEP), and Accommodations

A modification changes what is being taught by reaching well below or well above grade level, or by reducing the number of curriculum expectations. Students with a modified math program have an Individual Education Plan (IEP) describing how their program differs from classmates in their grade. An IEP also describes strategies, resources, and how the student will be evaluated. Modifying a student's program is a well-defined process involving the principal, teachers, parents, and student. Addressing a student's need for program modification falls outside the scope of this Teacher's Resource.

Accommodations

Accommodations do not change what is being taught. Rather, an accommodation to a student's program alters the "how," "when," or "where" the student is taught or assessed without changing curriculum expectations. This Teacher's Resource provides suggested accommodations based on the student's identified area of difficulty. Three types of accommodations are provided.

- Instructional accommodations refer to changes in teaching strategies that allow the student to access the curriculum.
- Environmental accommodations refer to changes that are required to the classroom and/or school environment.
- Assessment accommodations refer to changes that are required in order for the student to demonstrate learning.

The following three charts provide accommodations for the three key areas underlying math difficulties. Accommodations have been grouped under the headings of instructional, environmental, and assessment.

Chart I: Accommodations for Students with Language Difficulties

Instructional	Environmental	Assessment
• Pre-teach vocabulary • Give concise, step-by-step directions • Teach students to look for cue words, highlight these words • Use visual models • Use visual representations to accompany word problems • Encourage students to look for common patterns in word problems	• Provide reference charts with operations and formulae stated simply • Post reference charts with math vocabulary • Reinforce learning with visual aids and manipulatives • Using a visual format, post strategies for problem solving • Use a peer tutor or buddy system	• Read instructions/word problems on tests to students • Extend time lines

Chart II: Accommodations for Students with Visual/Perceptual/Spatial/Motor Difficulties

Instructional	Environmental	Assessment
• Reduce copying • Provide worksheets • Provide grid paper • Provide concrete examples • Allow use of a number line • Provide a math journal • Encourage and teach self-talk strategies • Chunk learning and tasks	• visual bombardment • a work carrel or work area that is not visually distracting • rest periods and breaks	• Provide graph paper for tests • Extend time lines • Provide consumable tests • Reduce the number of questions required to indicate competency • Provide a scribe when lengthy written answers are required

Chart III: Accommodations for Students with Memory Difficulties

Instructional	Environmental	Assessment
• Regularly review concepts • Activate prior knowledge • Teach mnemonic strategies (e.g., SOHCAHTOA) • Teach visualization strategies • Allow use of multiplication tables • Colour-code steps in sequence • Teach functional math concepts related to daily living	• Provide reference charts with commonly used facts, formulae, and steps for problem-solving • Allow use of a calculator • Use games and computer programs for practice of knowledge-based skills	• Allow use of formula lists • Allow use of other reference charts as appropriate • Allow use of calculators • Extend time lines • Present one concept-type of question at a time

Accommodations for ESL Students

For ESL students, language issues are pervasive throughout all subject areas, including math. Non-math words are often more problematic for ESL students because understanding the meaning of these words is often taken for granted. Everyday language is laden with vocabulary, comparative forms, figurative speech, and complex language structures that are not explained. By contrast, key words in math are usually highlighted in the text and carefully explained by the teacher. Accommodations to the programs of ESL students do not change the curriculum expectations.

Accommodations for ESL Students

Instructional	Environmental	Assessment
• Pre-teach vocabulary • Explain colloquial expressions and figurative speech • Review comparative forms of adjectives	• Display reference charts with mathematical terms and language • Encourage personal math dictionaries with math terms and formulae	• Allow access to personal math dictionaries • Read instructions to students and clarify terms • Allow additional time

Accommodations for Learning-Disabled Students

A student with a learning disability usually suffers from an inability to think, listen, speak, write, spell, or calculate that is not obviously caused by any mental or physical disability. There seems to be a lag in the developmental process and/or a delay in the maturation of the central nervous system. Providing simplified presentations, repetitions, more specific examples, or breaking content blocks into simpler sections may help in minor cases of learning disability.

Accommodations for At-Risk Students

Students learn in different ways. For all students to have the opportunity to succeed, we need to have alternative ways of delivering program. For example, a student whose dominant learning modality is kinesthetic/tactile needs active, hands-on investigations. A student with strong social/emotional intelligence benefits more from interpersonal interactions and needs instructional strategies like Jigsaw or Think-Pair-Share to optimize their chances of acquiring the skills and knowledge in the curriculum (see the **Cooperative Learning** section in this Teacher's Resource). These students underachieve and become at-risk not because they have acquired concepts imperfectly (and need remediation), but because they have not become engaged in their own learning, and often have failed to acquire concepts at all. At-risk students are in danger of completing their schooling without adequate skills development to function effectively in society. Risk factors include low achievement and retention, behaviour problems, poor attendance, and low socio-economic status.

By addressing topics in a new or different way, teachers can provide at-risk students with the opportunity to learn in a manner that may engage them and increase their chances of success.

Neither failing such students nor putting them in pullout programs has produced much gain in achievement, but there are certain approaches that do help.

- Allow students to proceed at their own pace through a well-defined series of instructional objectives.
- Place students in small, mixed-ability learning groups to master the material first presented by the teacher. Reward teams based on the individual learning of all team members.
- Have students serve as peer tutors, as well as being tutored. This helps raise their self-esteem and makes them feel they have something to contribute.
- Involve students in learning about something that is relevant to them, such as money management or wise shopping.
- Get parents involved in their child's learning as much as possible.

Curriculum Correlation between McGraw-Hill Ryerson *Foundations for College Mathematics 12* and The Ontario Curriculum Foundations for College Mathematics, Grade 12, College Preparation (MAP4C)

This course enables students to broaden their understanding of mathematics as a problem-solving tool in the real world. Students will extend their understanding of quadratic relations; investigate situations involving exponential growth; solve problems involving compound interest; solve financial problems connected with vehicle ownership; develop their ability to reason by collecting, anlysing, and evaluating data involving one variable; connect probability and statistics; and solve problems in geometry and trigonometry. Students consolidate their mathematical skills as they solve problems and communicate their thinking

Mathematical Process Expectations

The mathematical processes are to be integrated into student learning in all areas of this course.

	Throughout this course, students will:
Problem Solving	• develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
Reasoning and Proving	• develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
Reflecting	• demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
Selecting Tools and Computational Strategies	• select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical idea and to solve problems;
Connecting	• make connections among mathematical concepts and procedures, and realter mathematical ideas or situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
Representing	• create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; on screen dynamic representations), connect and compare them, and select and apply the appropriate representation to solve problems;
Communicating	• communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

The mathematical process expectations are integrated throughout *Foundations for College Mathematics 12*.

The codes for the curriculum expectations used here are consistent with the codes used in the PDF document for Foundations for College Mathematics Expecatations (MAP4C) that is available on-line from The Ontario Curriculum Unit Planner (OCUP), in the section Grade by Grade PDFs of Ontario Curriculum Expectations. Go to http://elearningontario.ca/ and search using the key work "ocup."

Mathematical Models

Overall Expectations

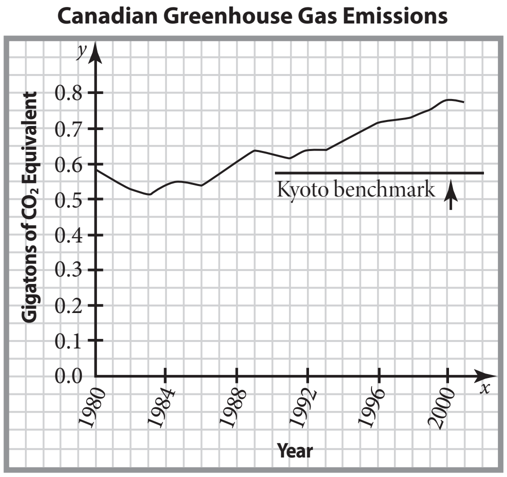
By the end of this course, students will:

1. evaluate powers with rational exponents, simplify algebraic expressions involving exponents, and solve problems involving exponential equations graphically and using common bases;
2. describe trends based on the interpretation of graphs, compare graphs using initial conditions and rates of change, and solve problems by modelling relationships graphically and algebraically;
3. make connections between formulas and linear, quadratic, and exponential relations, solve problems using formulas arising from real-world applications, and describe applications of mathematical modelling in various occupations.

Specific Expectations

	Chapter/Section	Pages
Solving Exponential Equations		
By the end of this course, students will:		
MM1.01 – determine, through investigation (e.g., by expanding terms and patterning), the exponent laws for multiplying and dividing algebraic expressions involving exponents [e.g., $(x^3)(x^2)$, $x^3 \div x^5$] and the exponent law for simplifying algebraic expressions involving a power of a power [e.g. $(x^6y^3)^2$]	**6.1**	**342–351**
MM1.02 – simplify algebraic expressions containing integer exponents using the laws of exponents ***Sample problem:*** Simplify $\frac{a^2b^5c^5}{ab^{-3}c^4}$ and evaluate for $a = 8$, $b = 2$, and $c = -30$.	**6.1**	**342–351**
MM1.03 – determine, through investigation using a variety of tools (e.g., calculator, paper and pencil, graphing technology) and strategies (e.g., patterning; finding values from a graph; interpreting the exponent laws), the value of a power with a rational exponent (i.e., $x^{\frac{m}{n}}$, where $x > 0$ and m and n are integers) ***Sample problem:*** The exponent laws suggest that $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$. What value would you to $4^{\frac{1}{2}}$? What value would you assign to $27^{\frac{1}{3}}$? Explain your reasoning. Extend your reasoning to make a generalization about the meaning of $x^{\frac{1}{n}}$, where $x > 0$ and n is a natural number.	**6.1**	**342–351**
MM1.04 – evaluate, with or without technology, numerical expressions involving rational exponents and rational bases [e.g., 2^{-3}, $(-6)^3$, $4^{\frac{1}{2}}$, 1.01^{120}]*	**6.1**	**342–351**
MM1.05 – solve simple exponential equations numerically and graphically, with technology (e.g., use systematic trial with a scientific calculator to determine the solution to the equation $1.05^x = 1.276$), and recognize that the solutions may not be exact ***Sample problem:*** Use the graph of $y = 3$ to solve the equation $3^x = 5$.	**6.4**	**368–375**

MM1.06 – solve problems involving exponential equations arising from real-world applications by using a graph or table of values generated with technology from a given equation [e.g., $h = 2(0.6)^n$, where h represents the height of a bouncing ball and n represents the number of bounces] ***Sample problem:*** Dye is injected to test pancreas function. The mass, R grams, of dye remaining in a healthy pancreas after t minutes is given by the equation $R = I(0.96)^t$, where I grams is the mass of dye initially injected. If 0.50 g of dye is initially injected into a healthy pancreas, determine how much time elapses until 0.35 g remains by using a graph and/or table of values generated with technology.	**6.4, 6.5**	**368–389**
MM1.07 – solve exponential equations in one variable by determining a common base (e.g., $2^x = 32$, $4^{5x-1} = 2^{2(x+11)}$, $3^{5x+8} = 27^x$) ***Sample problem:*** Solve $3^{5x+8} = 27^x$ by determining a common base, verify by substitution, and make connections to the intersection of $y = 3^{5x+8}$ and $y = 27^x$ using graphing technology.	**6.3**	**362–367**
* The knowledge and skills described in this expectation are to be introduced as needed, and applied and consolidated, where appropriate, throughout the course.		
Modelling Graphically		
By the end of this course, students will:		
MM2.01 – interpret graphs to describe a relationship (e.g., distance travelled depends on driving time, pollution increases with traffic volume, maximum profit occurs at a certain sales volume), using language and units appropriate to the context	**5.1, 5.2, 5.3**	**268–305**

MM2.02 – describe trends based on given graphs, and use the trends to make predictions or justify decisions (e.g., given a graph of the men's 100-m world record versus the year, predict the world record in the year 2050 and state your assumptions; given a graph showing the rising trend in graduation rates among Aboriginal youth, make predictions about future rates) ***Sample problem:*** Given the following graph, describe the trend in Canadian greenhouse gas emissions over the time period shown. Describe some factors that may have influenced these emissions over time. Predict the emissions today, explain your prediction using the graph and possible factors, and verify using current data.  Source: Environment Canada, Greenhouse Gas Inventory 1990–2001, 2003	**5.1, 5.2, 5.3**	**268–305**
MM2.03 – recognize that graphs and tables of values communicate information about rate of change, and use a given graph or table of values for a relation to identify the units used to measure rate of change (e.g., for a distance–time graph, the units of rate of change are kilometres per hour; for a table showing earnings over time, the units of rate of change are dollars per hour)	**5.1, 5.2, 5.3**	**268–305**
MM2.04 – identify when the rate of change is zero, constant, or changing, given a table of values or a graph of a relation, and compare two graphs by describing rate of change (e.g., compare distance–time graphs for a car that is moving at constant speed and a car that is accelerating)	**5.1, 5.2, 5.3**	**268–305**
MM2.05 – compare, through investigation with technology, the graphs of pairs of relations (i.e., linear, quadratic, exponential) by describing the initial conditions and the behaviour of the rates of change (e.g., compare the graphs of amount versus time for equal initial deposits in simple interest and compound interest accounts) ***Sample problem:*** In two colonies of bacteria, the population doubles every hour. The initial population of one colony is twice the initial population of the other. How do the graphs of population versus time compare for the two colonies? How would the graphs change if the population tripled every hour, instead of doubling?	**5.4**	**310–319**

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<table>
<tr><td>MM2.06 –recognize that a linear model corresponds to a constant increase or decrease over equal intervals and that an exponential model corresponds to a constant percentage increase or decrease over equal intervals, select a model (i.e., linear, quadratic, exponential) to represent the relationship between numerical data graphically and algebraically, using a variety of tools (e.g., graphing technology) and strategies (e.g., finite differences, regression), and solve related problems
Sample problem: Given the data table at the top of page 139, determine an algebraic model to represent the relationship between population and time, using technology. Use the algebraic model to predict the population in 2015, and describe any assumptions made.
<table>
<tr><th>Years after 1955</th><th>Population of Geese</th></tr>
<tr><td>0</td><td>5 000</td></tr>
<tr><td>10</td><td>12 000</td></tr>
<tr><td>20</td><td>26 000</td></tr>
<tr><td>30</td><td>62 000</td></tr>
<tr><td>40</td><td>142 000</td></tr>
<tr><td>50</td><td>260 000</td></tr>
</table></td><td>5.5,
6.5</td><td>320–331,
378–389</td></tr>
<tr><td colspan="3">Modelling Algebraically</td></tr>
<tr><td colspan="3">By the end of this course, students will:</td></tr>
<tr><td>MM3.01 – solve equations of the form $x^n = a$ using rational exponents (e.g., solve $x^3 = 7$ by raising both sides to the exponent $\frac{1}{3}$)</td><td>6.4</td><td>362–375</td></tr>
<tr><td>MM3.02 – determine the value of a variable of degree no higher than three, using a formula drawn from an application, by first substituting known values and then solving for the variable, and by first isolating the variable and then substituting known values
Sample problem: Use the formula $V = \pi r^3$ to determine the radius of a sphere with a volume of 1000 cm^3.</td><td>7.4, 7.5, 7.6</td><td>382–413</td></tr>
<tr><td>MM3.03 – make connections between formulas and linear, quadratic, and exponential functions [e.g., recognize that the compound interest formula, $A = P(1 + i)^n$, is an example of an exponential function $A(n)$ when P and i are constant, and of a linear function $A(P)$ when i and n are constant], using a variety of tools and strategies (e.g., comparing the graphs generated with technology when different variables in a formula are set as constants)
Sample problem: Which variable(s) in the formula $V = \pi r^2 h$ would you need to set as a constant to generate a linear equation? A quadratic equation? Explain why you can expect the relationship between the volume and the height to be linear when the radius is constant.</td><td>Throughout Chapter 5,
6.2</td><td>268–331,
352–361</td></tr>
<tr><td>MM3.04 – solve multi-step problems requiring formulas arising from real-world applications (e.g., determining the cost of two coats of paint for a large cylindrical tank)</td><td>6.5</td><td>378–389</td></tr>
<tr><td>MM3.05 – gather, interpret, and describe information about applications of mathematical modelling in occupations, and about college programs that explore these applications</td><td>6.2,
6.4, 6.5,
Chapter 6 Task</td><td>342–361,
368–389,
396</td></tr>
</table>

Personal Finances

Overall Expectations

By the end of this course, students will:

1. demonstrate an understanding of annuities, including mortgages, and solve related problems using technology;
2. gather, interpret, and compare information about owning or renting accommodation, and solve problems involving the associated costs;
3. design, justify, and adjust budgets for individuals and families described in case studies, and describe applications of the mathematics of personal finance.

Specific Expectations

	Chapter/Section	Pages
Understanding Annuities		
By the end of this course, students will:		
PF1.01 – gather and interpret information about annuities, describe the key features of an annuity, and identify real-world applications (e.g., RRSP, mortgage, RRIF, RESP)	**7.1**	**402–411**
PF1.02 – determine, through investigation using technology (e.g., the TVM Solver on a graphing calculator; online tools), the effects of changing the conditions (i.e., the payments, the frequency of the payments, the interest rate, the compounding period) of an ordinary simple annuity (i.e., an annuity in which payments are made at the *end* of each period, and compounding and payment periods are the same) (e.g., long-term savings plans, loans) ***Sample problem:*** Given an ordinary simple annuity with semi-annual deposits of $1000, earning 6% interest per year compounded semi-annually, over a 20-year term, which of the following results in the greatest return: doubling the payments, doubling the interest rate, doubling the frequency of the payments and the compounding, or doubling the payment and compounding period?	**7.2**	**414–419**
PF1.03 – solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary simple annuity ***Sample problem:*** Using a spreadsheet, calculate the total interest paid over the life of a $10 000 loan with monthly repayments over 2 years at 8% per year compounded monthly, and compare the total interest with the original principal of the loan.	**7.1**	**402–411**
PF1.04 – demonstrate, through investigation using technology (e.g., a TVM Solver), the advantages of starting deposits earlier when investing in annuities used as long-term savings plans ***Sample problem:*** If you want to have a million dollars at age 65, how much would you have to contribute monthly into an investment that pays 7% per annum, compounded monthly, beginning at age 20? At age 35? At age 50?	**7.2**	**414–419**

PF1.05 – gather and interpret information about mortgages, describe features associated with mortgages (e.g., mortgages are annuities for which the present value is the amount borrowed to purchase a home; the interest on a mortgage is compounded semi-annually but often paid monthly), and compare different types of mortgages (e.g., open mortgage, closed mortgage, variable-rate mortgage)	**7.3**	**420–429**
PF1.06 – read and interpret an amortization table for a mortgage ***Sample problem:*** You purchase a $200 000 condominium with a $25 000 down payment, and you mortgage the balance at 6.5% per year compounded semi-annually over 25 years, payable monthly. Use a given amortization table to compare the interest paid in the first year of the mortgage with the interest paid in the 25th year.	**7.3**	**420–429**
PF1.07 – generate an amortization table for a mortgage, using a variety of tools and strategies (e.g., input data into an online mortgage calculator; determine the payments using the TVM Solver on a graphing calculator and generate the amortization table using a spreadsheet), calculate the total interest paid over the life of a mortgage, and compare the total interest with the original principal of the mortgage	**7.3**	**420–429**
PF1.08 – determine, through investigation using technology (e.g., TVM Solver, online tools, financial software), the effects of varying payment periods, regular payments, and interest rates on the length of time needed to pay off a mortgage and on the total interest paid ***Sample problem:*** Calculate the interest saved on a $100 000 mortgage with monthly payments, at 6% per annum compounded semiannually, when it is amortized over 20 years instead of 25 years.	**7.4**	**430–437**
Renting or Owning Accommodations		
By the end of this course, students will:		
PF2.01 – gather and interpret information about the procedures and costs involved in owning and in renting accommodation (e.g., apartment, condominium, townhouse, detached home) in the local community	**8.1, 8.3**	**448–453, 462–467**
PF2.02 – compare renting accommodation with owning accommodation by describing the advantages and disadvantages of each	**8.4**	**468–477**
PF2.03 – solve problems, using technology (e.g., calculator, spreadsheet), that involve the fixed costs (e.g., mortgage, insurance, property tax) and variable costs (e.g., maintenance, utilities) of owning or renting accommodation ***Sample problem:*** Calculate the total of the fixed and variable monthly costs that are associated with owning a detached house but that are usually included in the rent for rental accommodation.	**8.4**	**468–477**

Designing Budgets		
By the end of this course, students will:		
PF3.01 – gather, interpret, and describe information about living costs, and estimate the living costs of different households (e.g., a family of four, including two young children; a single young person; a single parent with one child) in the local community	**8.4**	**468–477**
PF3.02 – design and present a savings plan to facilitate the achievement of a long-term goal (e.g., attending college, purchasing a car, renting or purchasing a house)	**8.1**	**448–453**
PF3.03 – design, explain, and justify a monthly budget suitable for an individual or family described in a given case study that provides the specifics of the situation (e.g., income; personal responsibilities; costs such as utilities, food, rent/mortgage, entertainment, transportation, charitable contributions; long-term savings goals), with technology (e.g., using spreadsheets, budgeting software, online tools) and without technology (e.g., using budget templates)	**8.5**	**478–481**
PF3.04 – identify and describe the factors to be considered in determining the affordability of accommodation in the local community (e.g., income, long-term savings, number of dependants, non-discretionary expenses), and consider the affordability of accommodation under given circumstances ***Sample problem:*** Determine, through investigation, if it is possible to change from renting to owning accommodation in your community in five years if you currently earn \$30 000 per year, pay \$900 per month in rent, and have savings of \$20 000.	**8.5**	478–481
PF3.05 – make adjustments to a budget to accommodate changes in circumstances (e.g., loss of hours at work, change of job, change in personal responsibilities, move to new accommodation, achievement of a long-term goal, major purchase), with technology (e.g., spreadsheet template, budgeting software)	**8.5**	478–481
PF3.06 – gather, interpret, and describe information about applications of the mathematics of personal finance in occupations (e.g., selling real estate, bookkeeping, managing a restaurant, financial planning, mortgage brokering), and about college programs that explore these applications	**Chapter 8 Task**	486

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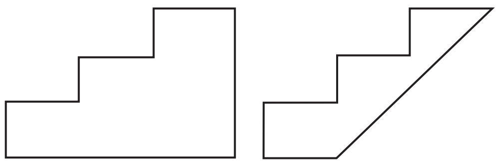
Geometry and Trigonometry

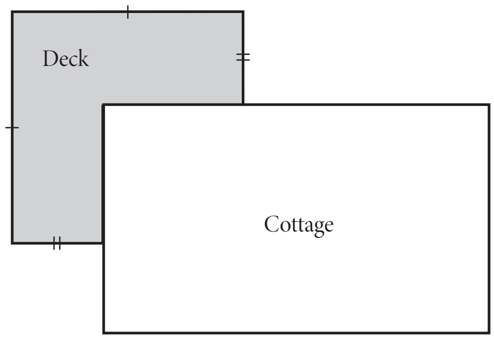
Overall Expectations

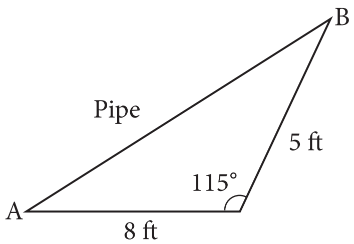
By the end of this course, students will:

1. solve problems involving measurement and geometry and arising from real-world applications;
2. explain the significance of optimal dimensions in real-world applications, and determine optimal dimensions of two-dimensional shapes and three-dimensional figures;
3. solve problems using primary trigonometric ratios of acute and obtuse angles, the sine law, and the cosine law, including problems arising from real-world applications, and describe applications of trigonometry in various occupations.

Specific Expectations

	Chapter/Section	Pages
Solving Problems Involving Measurement and Geometry		
By the end of this course, students will:		
GT1.01 – perform required conversions between the imperial system and the metric system using a variety of tools (e.g., tables, calculators, online conversion tools), as necessary within applications	**1.1, 1.2, 1.3**	**6–35**
GT1.02 – solve problems involving the areas of rectangles, triangles, and circles, and of related composite shapes, in situations arising from real-world applications ***Sample problem:*** A car manufacturer wants to display three of its compact models in a triangular arrangement on a rotating circular platform. Calculate a reasonable area for this platform, and explain your assumptions and reasoning.	**1.1**	**6–15**
GT1.03 – solve problems involving the volumes and surface areas of rectangular prisms, triangular prisms, and cylinders, and of related composite figures, in situations arising from real-world applications ***Sample problem:*** Compare the volumes of concrete needed to build three steps that are 4 ft wide and that have the cross-sections shown below. Explain your assumptions and reasoning. 	**1.2, 1.3**	**18–35**

Investigating Optimal Dimensions		
By the end of this course, students will:		
GT2.01 – recognize, through investigation using a variety of tools (e.g., calculators; dynamic geometry software; manipulatives such as tiles, geoboards, toothpicks) and strategies (e.g., modelling; making a table of values; graphing), and explain the significance of optimal perimeter, area, surface area, and volume in various applications (e.g., the minimum amount of packaging material, the relationship between surface area and heat loss) ***Sample problem:*** You are building a deck attached to the second floor of a cottage, as shown below. Investigate how perimeter varies with different dimensions if you build the deck using exactly 48 1-m x 1-m decking sections, and how area varies if you use exactly 30 m of deck railing. *Note:* the entire outside edge of the deck will be railed. 	**1.4, 1.5, 1.6**	**36–63**
GT2.02 – determine, through investigation using a variety of tools (e.g., calculators, dynamic geometry software, manipulatives) and strategies (e.g., modelling; making a table of values; graphing), the optimal dimensions of a two-dimensional shape in metric or imperial units for a given constraint (e.g., the dimensions that give the minimum perimeter for a given area) ***Sample problem:*** You are constructing a rectangular deck against your house. You will use 32 ft of railing and will leave a 4-ft gap in the railing for access to stairs. Determine the dimensions that will maximize the area of the deck.	**1.4**	**36–45**
GT2.03 – determine, through investigation using a variety of tools and strategies (e.g., modelling with manipulatives; making a table of values; graphing), the optimal dimensions of a right rectangular prism, a right triangular prism, and a right cylinder in metric or imperial units for a given constraint (e.g., the dimensions that give the maximum volume for a given surface area) ***Sample problem:*** Use a table of values and a graph to investigate the dimensions of a rectangular prism, a triangular prism, and a cylinder that each have a volume of 64 cm^3 and the minimum surface area.	**1.5, 1.6**	**46–63**

Solving Problems Involving Trigonometry		
By the end of this course, students will:		
GT3.01 – solve problems in two dimensions using metric or imperial measurements, including problems that arise from real-world applications (e.g., surveying, navigation, building construction), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios, and of acute triangles using the sine law and the cosine law	**2.1, 2.3, 2.4, 2.5**	**74–83, 96–129**
GT3.02 – make connections between primary trigonometric ratios (i.e., sine, cosine, tangent) of obtuse angles and of acute angles, through investigation using a variety of tools and strategies (e.g., using dynamic geometry software to identify an obtuse angle with the same sine as a given acute angle; using a circular geoboard to compare congruent triangles; using a scientific calculator to compare trigonometric ratios for supplementary angles)	**2.2, 2.5**	**84–95, 120–129**
GT3.03 – determine the values of the sine, cosine, and tangent of obtuse angles	**2.2, 2.5**	**84–95, 120–129**
GT3.04 – solve problems involving oblique triangles, including those that arise from real-world applications, using the sine law (in nonambiguous cases only) and the cosine law, and using metric or imperial units ***Sample problem:*** A plumber must cut a piece of pipe to fit from *A* to *B*. Determine the length of the pipe.  	**2.3, 2.4, 2.5**	**96–129**
3.5 – gather, interpret, and describe information about applications of trigonometry in occupations, and about college programs that explore these applications ***Sample problem:*** Prepare a presentation to showcase an occupation that makes use of trigonometry, to describe the education and training needed for the occupation, and to highlight a particular use of trigonometry in the occupation.	**Chapter 2 Task**	**136**

Data Management

Overall Expectations

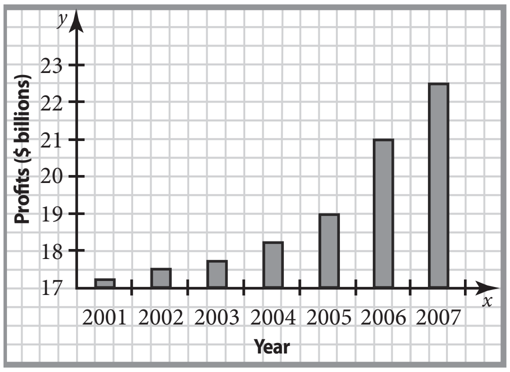
By the end of this course, students will:

1. collect, analyse, and summarize two-variable data using a variety of tools and strategies, and interpret and draw conclusions from the data;
2. demonstrate an understanding of the applications of data management used by the media and the advertising industry and in various occupations.

Specific Expectations

	Chapter/Section	Pages
Working With Two-Variable Data		
By the end of this course, students will:		
DM1.01 – distinguish situations requiring one-variable and two-variable data analysis, describe the associated numerical summaries (e.g., tally charts, summary tables) and graphical summaries (e.g., bar graphs, scatter plots), and recognize questions that each type of analysis addresses (e.g., What is the frequency of a particular trait in a population? What is the mathematical relationship between two variables?) ***Sample problem:*** Given a table showing shoe size and height for several people, pose a question that would require one-variable analysis and a question that would require two-variable analysis of the data.	**3.1**	**142–151**
DM1.02 – describe characteristics of an effective survey (e.g., by giving consideration to ethics, privacy, the need for honest responses, and possible sources of bias, including cultural bias), and design questionnaires (e.g., for determining if there is a relationship between age and hours per week of Internet use, between marks and hours of study, or between income and years of education) or experiments (e.g., growth of plants under different conditions) for gathering two-variable data	**3.2**	**152–159**
DM1.03 – collect two-variable data from primary sources, through experimentation involving observation or measurement, or from secondary sources (e.g., Internet databases, newspapers, magazines), and organize and store the data using a variety of tools (e.g., spreadsheets, dynamic statistical software) ***Sample problem:*** Download census data from Statistics Canada on age and average income, store the data using dynamic statistics software, and organize the data in a summary table.	**3.3**	**160–167**
DM1.04 – create a graphical summary of two-variable data using a scatter plot (e.g., by identifying and justifying the dependent and independent variables; by drawing the line of best fit, when appropriate), with and without technology	**3.1, 3.3**	**142–151, 160–167**

DM1.05 – determine an algebraic summary of the relationship between two variables that appear to be linearly related (i.e., the equation of the line of best fit of the scatter plot), using a variety of tools (e.g., graphing calculators, graphing software) and strategies (e.g., using systematic trials to determine the slope and y-intercept of the line of best fit; using the regression capabilities of a graphing calculator), and solve related problems (e.g., use the equation of the line of best fit to interpolate or extrapolate from the given data set)	**3.4**	**168–179**
DM1.06 – describe possible interpretations of the line of best fit of a scatter plot (e.g., the variables are linearly related) and reasons for misinterpretations (e.g., using too small a sample; failing to consider the effect of outliers; interpolating from a weak correlation; extrapolating nonlinearly related data)	**3.4, 3.5**	**168–189**
DM1.07 – determine whether a linear model (i.e., a line of best fit) is appropriate given a set of two-variable data, by assessing the correlation between the two variables (i.e., by describing the type of correlation as positive, negative, or none; by describing the strength as strong or weak; by examining the context to determine whether a linear relationship is reasonable)	**3.4**	**168–179**
DM1.08 – make conclusions from the analysis of two-variable data (e.g., by using a correlation to suggest a possible cause-and-effect relationship), and judge the reasonableness of the conclusions (e.g., by assessing the strength of the correlation; by considering if there are enough data)	**3.5**	**182–189**
Applying Data Management		
By the end of this course, students will:		
DM2.01 – recognize and interpret common statistical terms (e.g., percentile, quartile) and expressions (e.g., accurate 19 times out of 20) used in the media (e.g., television, Internet, radio, newspapers)	**4.1**	**200–211**
DM2.02 – describe examples of indices used by the media (e.g., consumer price index, S&P/TSX composite index, new housing price index) and solve problems by interpreting and using indices (e.g., by using the consumer price index to calculate the annual inflation rate) ***Sample problem:*** Use the new housing price index on E-STAT to track the cost of purchasing a new home over the past 10 years in the Toronto area, and compare with the cost in Calgary, Charlottetown, and Vancouver over the same period. Predict how much a new home that today costs $200 000 in each of these cities will cost in 5 years.	**4.2**	**214–225**

DM2.03 – interpret statistics presented in the media (e.g., the UN's finding that 2% of the world's population has more than half the world's wealth, whereas half the world's population has only 1% of the world's wealth), and explain how the media, the advertising industry, and others (e.g., marketers, pollsters) use and misuse statistics (e.g., as represented in graphs) to promote a certain point of view (e.g., by making a general statement based on a weak correlation or an assumed cause-and-effect relationship; by starting the vertical scale on a graph at a value other than zero; by making statements using general population statistics without reference to data specific to minority groups)	**4.3**	**226–235**
DM2.04 – assess the validity of conclusions presented in the media by examining sources of data, including Internet sources (i.e., to determine whether they are authoritative, reliable, unbiased, and current), methods of data collection, and possible sources of bias (e.g., sampling bias, non-response bias, a bias in a survey question), and by questioning the analysis of the data (e.g., whether there is any indication of the sample size in the analysis) and conclusions drawn from the data (e.g., whether any assumptions are made about cause and effect) ***Sample problem:*** The headline that accompanies the following graph says "Big Increase in Profits". Suggest reasons why this headline may or may not be true. **Big Increase in Profits** 	**4.4, 4.5**	**244–255**
DM2.05 – gather, interpret, and describe information about applications of data management in occupations, and about college programs that explore these applications	**Chapter 4 Task**	**262**