1.6 Analyse Optimum Volume and Surface Area

Student Text Pages
54–63

Suggested Timing
80 min

Tools
- graphing calculators
- computers with *The Geometer's Sketchpad®*

Related Resources
BLM 1-15 Section 1.6 Analyse Optimum Volume and Surface Area
BLM 1-16 Section 1.6 Achievement Check Rubric
BLM A-17 Learning Skills Checklist

Link to Prerequisite Skills

Students should complete all the Prerequisite Skills questions before proceeding with this section.

Warm-Up

1. Under what conditions will a square-based prism with a given volume have a minimum surface area?
2. Under what conditions will a square-based prism with a given surface area have a maximum volume?
3. Do you think there is a way to optimize measurements for other three-dimensional objects?

Warm-Up Answers

1. When the square-based prism is in the shape of a cube.
2. When the square-based prism is in the shape of a cube.
3. Answers may vary.

Teaching Suggestions

- In section 1.5, students discovered the optimal conditions related to volume and surface area of a square-based prism, using a variety of tools and strategies. In this section, they confirm these results analytically using algebraic and graphical reasoning. They also extend this process to optimizing these measures for other types of prisms.
- The level of algebraic reasoning required in this section is considerable. Some students may benefit from using a CAS graphing calculator to assist with some of the steps.

Warm-Up

- Pose these questions orally, one at time. Invite student responses using a think-pair-share technique.

Examples

- For Example 1, students develop a mathematical argument to confirm that the optimum volume for a given surface area for a square-based prism occurs when the prism is a cube. Students use known relationships between surface area and volume to eliminate one variable, in this case height, to develop a function relating volume to the single remaining variable, in this case the side length of the square base.
- Students planning to study a math-rich program at college may encounter an algebraic method to optimize such functions.
- For Example 2, students apply the concept of optimizing surface area for a given volume for a right isosceles triangular prism. Ensure that students understand why the base is double the height in this situation. The properties of isosceles triangles mean that the two sides of the same length are opposite the 45° angles. This relationship means one variable, base length, can be eliminated in the area calculations that follow.
- For the areas of the triangular faces, students use the Pythagorean theorem to eliminate the variable x so the surface area can be expressed in terms of h and l. The known volume of the tent is substituted so l can

be expressed in terms of h, which allows surface area to be expressed in terms of the height of the tent only. This function is then optimized using graphing technology.

- After working through Example 2, it might be beneficial to have students summarize the steps of the solution and explain why each step was necessary.

Key Concepts

- Ensure students understand the Key Concepts. You might wish to have students provide an example of each manipulative, technological, and algebraic tool and strategy that can be used to optimize surface area and volume.

Discuss the Concepts

- Have students work in pairs to answer these questions. Alternatively, discuss the answers as a class.

Discuss the Concepts Suggested Answers (page 60)

D1. No. Examples may vary. For example: A square-based prism with volume 48 cm^3 could have dimensions $w = 4$ cm, $l = 4$ cm, and $h = 3$ cm, and surface area 80 cm^2. Another square-based prism with volume 48 cm^3 could have dimensions $w = 2$ cm, $l = 2$ cm, and $h = 12$ cm, and surface area 104 cm^2.

D2. a) Answers may vary. For example: I think the container made by rolling the paper lengthwise will hold more popcorn. The maximum volume of a cylinder occurs when the diameter is equal to the height, and rolling the paper lengthwise is closer to this relationship between diameter and height than rolling the paper widthwise.

b) Maximum volume occurs when paper is rolled lengthwise.

D3. No. The containers have different surface areas because the bottoms have different areas. Only the surface areas of the lateral faces are the same.

Practise (A)

- You may wish to have students work in pairs or small groups to complete the Practise questions.
- Encourage students to refer to the Examples before asking for assistance.

Apply (B)

- For **questions 3** and **4**, encourage students to sketch a diagram.
- For **question 6**, **part c)**, some students may need to review how to calculate percent difference.
- For **questions 7** and **8**, remind students to convert millilitres to cubic centimetres.
- Some students may wish to use a computer with *The Geometer's Sketchpad*® or TABS+ to explore **question 8**.
- **Question 9** is an Achievement Check question. It can be used for diagnostic or formative assessment, or assigned as a small summative assessment piece. You may wish to use **BLM 1-16 Section 1.6 Achievement Check Rubric** to assist you in assessing your students.
- **Question 10** is similar in concept to the problems in section 1.4 involving fencing two or three sides of an area when optimizing area for a given perimeter.
- For **question 11**, you may wish to supply students with a variety of packages to examine.
- For **question 12**, have photographs of buildings in large urban centres to illustrate this concept. This question provides an opportunity to make connections to urban planning in social sciences.

Common Errors

- Some students might have difficulty with the level of algebraic reasoning required.

R_x Consider having students use a CAS graphing calculator to perform some of the steps or to check their work.

Accommodations

Visual—construct models of the shapes in Examples 1 and 2**.** Label the sides with the variables used in the Examples.

Perceptual—provide additional scaffolding for the Examples. Complete each step on a separate sheet of poster paper. Use a different colour marker to explain the reasoning behind each step to help students follow the process.

Language—have students work in groups and follow the steps from the Examples to complete each question. Assign each student to a specific focus (diagrams, formulas, algebra, technology) to help students interpret and solve the problems.

Motor—provide construction paper to build the cylinders in Discuss the Concepts **question D2**. Have students fill the containers with small objects such as dried beans or rice to test their conjecture.

ESL—encourage students to draw diagrams when solving the Practise questions.

Spatial—provide models for **questions 1 and 2** and allow students to measure to check their answers. Have students construct their own models for **question 15** and use small objects such as dried beans or rice to check their answers.

Gifted and Enrichment—have students create a presentation for the class based on their solutions and insights from **questions 11** and **13**

Memory—review simplifying and solving algebraic expressions, solving by substitution, and finding maximum or minimum values of quadratic functions

Extend (C)

- Some students may wish to use a computer with *The Geometer's Sketchpad*® or TABS+ to explore **question 15**.
- **Question 17** provides an opportunity to assess students' reasoning and communication skills.

Achievement Check Answers (page 62)

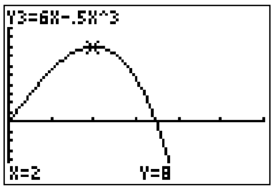
9. Let the dimensions for the base of the prism be x m by x m and the height be h m. The prism has six faces. Calculate the area of each face and substitute 24 m^2 for the known surface area.

$$2x^2 + 4xh = 24$$
$$4xh = 24 - 2x^2$$
$$h = \frac{24 - 2x^2}{4x}$$

The volume of this prism is:

$$V = x^2h$$
$$= x^2\left(\frac{24 - 2x^2}{4x}\right)$$
$$= 6x - 0.5x^3$$

Graph the volume equation to determine the base and height that maximizes the volume of the prism.



The maximum point on this graph is (2, 8). This means the maximum volume Jessie can enclose is 8 m^3, when the base of the tree house is 2 m square. The height of the tree house is 2 m, which can be found by substituting $x = 2$ in the equation for h. The tree house is in the shape of a cube.

Literacy Connect

- Have one or two students read the section opener and ensure that they understand the meaning of, and difference between, an algebraic model and a graphical model.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	4, 6, 8, 10, 11, 16
Reasoning and Proving	1, 2, 4, 6, 8, 10–17
Reflecting	8, 10, 16
Selecting Tools and Computational Strategies	4, 5, 7, 8, 10, 16
Connecting	5–8, 10–17
Representing	4, 5, 8, 10, 14, 16
Communicating	1, 2, 4, 6, 8, 10–13, 15–17

Ongoing Assessment

- While students are working, circulate and see how well each student works. This may be an opportunity to observe and record individual students' learning skills. Use **BLM A-17 Learning Skills Checklist**.

Extra Practice

- Use **BLM 1-15 Section 1.6 Analyse Optimum Volume and Surface Area** for extra practice or remediation.