2.3

Sine Law

Student Text Pages
96–103

Suggested Timing
80 min

Tools
- scientific calculators
- computers with *The Geometer's Sketchpad®*

Related Resources
BLM 2-7 Section 2.3 Sine Law
BLM A-8 Application General Scoring Rubric

Link to Prerequisite Skills

Students should complete all the Prerequisite Skills questions before proceeding with this section.

Warm-Up

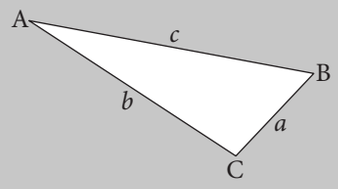
1. Draw an acute triangle, ABC. Label the sides and vertices.

2. Solve for x. Round each answer to one decimal place.

a) $\frac{12.8}{21.2} = \frac{x}{18.7}$ **b)** $\frac{3.6}{8.1} = \frac{5.4}{x}$

Warm-Up Answers

1. Answers may vary. For example:



2. a) 11.3 **b)** 12.2

Teaching Suggestions

Warm-Up

- Display the Warm-Up questions. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Review the sine law and ensure students understand the definitions of oblique, acute, and obtuse triangles.

Investigate

- In the Investigate, students should discover that the sine law holds true for obtuse triangles, as well as acute triangles. You may wish to review *The Geometer's Sketchpad®* notation shown in the **Literacy Connect.**
- Students will be familiar with using the sine law and the cosine law for acute triangles from grade 11. You may wish to have students carry out the Investigates for sections 2.3 and 2.4 on the same day, then work through the remaining parts of each section over one or two days.
- This Investigate could be completed with paper and pencil. However, the results may not be as compelling due to measurement error, and the Investigate would be more time consuming.

Investigate Answers (page 97)

2. Answers may vary. For example:
b) AB = 4.0 cm, BC = 5.4 cm, AC = 5.0 cm, ∠A = 72°, ∠B = 63°, ∠C = 45°

3. Answers may vary. For example:
b) 5.68
c) 5.68, 5.68; actual values: 5.61, 5.66
d) cosine ratio: 17.47; tangent ratio: 1.75

4. a) If the shape of the triangle changes, the ratios will change but will still all be equal.
b) The sine ratio is constant for each triangle.

5. Yes. The sine value of an obtuse angle is still positive, so the sine law holds for obtuse triangles.

Examples

- In Example 1, the sine law is used to solve for an unknown angle in an obtuse triangle. Some students may need to review the algebraic steps and the calculator keystrokes. You may wish to discuss with the class the alternate form of the sine law that could have been used and the extra algebra involved.
- In Example 2, the sine law is used to solve for an unknown side in an obtuse triangle. Point out to students that the geometric relationship of interior angles of a triangle is used before the sine law can be applied.
- Example 3 illustrates the process of solving a triangle, which is to determine all the side lengths and angle measures. A point of discussion could focus on the option of using the sine law three times, instead of the middle step involving geometric reasoning, and the extra steps that would be required.

Key Concepts

- Read through the Key Concepts. If needed, have students summarize these points in their own words and write these summaries in their notebooks. You could also have students brainstorm as a class and then work in pairs to summarize the material.

Discuss the Concepts

- Have students work through these questions using a think-pair-share format, and then discuss the answers as a class. Discuss the triangle properties (ASA, AAS, SSA) as they are an effective tool in remembering when to use the sine law or the cosine law, which appears in the next section.

Discuss the Concepts Suggested Answers (page 100)

D1. a) Yes. The corresponding opposite angle and one other side-angle pair is known.
b) No. All three angle measures are known but no side lengths are known.

D2. a) No. No side-angle pair is known.
b) Yes. The corresponding opposite side length and one other side-angle pair is known.

D3. Yes. Use the angle sum of a triangle property to solve for ∠B and then use the sine law with the side length c and ∠C to solve for the side length b.

Common Errors

- Some students get answers that do not make sense when evaluating an expression involving the sine law.

R_x Have students carefully check their calculator keystrokes if they are entering the entire calculation in a single step. They might be missing brackets or operation signs. Some students may want to perform the calculations in multiple steps. Students should also check that the calculator is set to degree mode.

Accommodations

Motor—provide a partner to read instructions and/or manipulate the computer for the **Investigate**

Perceptual—answer the **Discuss the Concepts** questions as a class to practise matching sides to their angles and writing the ratios for the sine law

ESL—allow students to work in pairs to interpret and solve the word problems. Ensure students draw a diagram to help them solve the problem.

Gifted and Enrichment—have students research the formal derivation of the sine law

Visual—post a worked example on chart paper to summarize the steps for solving ratios when using the sine law

Memory—add definitions for the sine law and oblique triangle to the Word Wall

Practise (A)

- You may wish to have students work in pairs or small groups to complete the Practise questions.
- Encourage students to refer to the Examples before asking for assistance.
- Remind students to use the correct units of measure in their final answers.

Apply (B)

- **Questions 7 to 9** are excellent exercises for students to practise applying the sine law in conjunction with other mathematical skills. These questions provide students opportunities to select tools and strategies and apply reasoning.
- For **question 11**, assumptions may include that the ball's path is straight and that there are no players between Hector and the goal.
- **Questions 12 and 13** address, in an indirect way, the concept of the ambiguous case of the sine law, where the given information can lead to two unique triangle solutions. Note that the curriculum expectations for this course only deal with the unambiguous case.
- **Question 14** links to the Chapter Problem. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the Apply questions.
- **Question 15** is a good example of a multi-step problem that requires students to choose tools and strategies. Students might find it helpful to draw a diagram where the two joined triangles are separated.
- For **question 16**, students must apply geometric properties (complementary angles) to determine the measures of two interior angles of the triangle.

Literacy Connect

- Review the different conventions for labelling the sides of a triangle as mentioned in the Literacy Connect on page 97.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	13, 14
Reasoning and Proving	n/a
Reflecting	11
Selecting Tools and Computational Strategies	10, 13, 14
Connecting	10–14
Representing	2, 3, 5, 6, 8, 9
Communicating	n/a

Ongoing Assessment

- You may wish to use **BLM A-8 Application General Scoring Rubric** to assess students' responses to **question 8 or 9**.

Extra Practice

- Use **BLM 2-7 Section 2.3 Sine Law** for extra practice or remediation.