2.4 Cosine Law

Student Text Pages
104–119

Suggested Timing
80 min

Tools
- scientific calculators
- computers with *The Geometer's Sketchpad®*

Optional
- TI-Nspire™ CAS graphing calculators
- TI-84 Plus graphing calculators

Related Resources
BLM 2-8 Section 2.4 Cosine Law
BLM 2-9 Section 2.4 Achievement Check Rubric
BLM A-9 Communication General Scoring Rubric

Link to Prerequisite Skills

Students should complete all the Prerequisite Skills questions before proceeding with this section.

Warm-Up

1. Evaluate to two decimal places.
a) $4 \cos 35°$ **b)** $2.6 \cos 18°$
2. Evaluate. Round your answers to two decimal places.
a) $8.1^2 + 4.7^2 - 16.2 \cos 21°$
b) $1.9^2 + 6.3^2 - 5.4 \cos 76°$

Warm-Up Answers

1. a) 3.28 **b)** 2.47
2. a) 72.58 **b)** 41.99

Teaching Suggestions

Section Opener

- Discuss the scenario in the section opener. Review the cosine law and ensure students understand the definition of a contained angle.

Warm-Up

- Display the Warm-Up questions. Have students complete the Warm-Up questions independently. Then, discuss the solutions as a class.

Investigate

- In the Investigate, students should discover that the cosine law holds true for obtuse triangles, as well as acute triangles.
- Students will be familiar with using the sine law and the cosine law for acute triangles from grade 11. You may wish to have students carry out the Investigates for sections 2.3 and 2.4 on the same day, then work through the remaining parts of each section over one or two days.
- This Investigate could be completed with paper and pencil. However, the results may not be as compelling due to measurement error, and the Investigate would be more time consuming.

Investigate Answers (page 105)

2. Answers may vary. For example:

b) AB = 5.3 cm, BC = 3.2 cm, AC = 4.6 cm, ∠A = 37°, ∠B = 60°, ∠C = 83°

3. The measures will be equal. Explanations may vary.

4. Answers may vary. For example:

a) 10.24

b) 10.31

c) Yes. The cosine value for an obtuse angle is negative so the right side equation is greater than $b^2 + c^2$. This means that side a is the longest, which makes sense since it is opposite the obtuse angle.

5. a) Answers may vary. For example: The calculated measures from step 4 will change but will remain equal to one another.

b) The measures change but remain equal to one another.

6. Yes. The values in the equation are still equal for an obtuse angle, so the cosine law holds for obtuse triangles.

Technology

- The Use Technology sections on pages 112 to 115 and pages 116 to 119 show how to use the TI-Nspire™ CAS graphing calculator and the TI-84 Plus graphing calculator to apply the sine and cosine laws. These sections can be used at any time in this lesson. Have students refer to the Technology Appendix in their textbooks if they need assistance using the calculators.
- Using an interactive whiteboard, such as a Smartboard®, or projector would be an efficient way to demonstrate the CAS features as part of the lesson.

Use Technology Answers (pages 116–119)

3. b) 50°

4. a) 44

5. b) It uses the cosine law to solve for a side length.

c) 18

6. b) 56°

7. It saves time and reduces the chance of errors.

Examples

- In Example 1, the cosine law is used to determine the length of an unknown side. Remind students to follow the correct order of operations when using a scientific or graphing calculator.
- In Example 2, the cosine law is used to determine the measure of an unknown angle. Methods 1 and 2 show two ways to rearrange the cosine law in terms of the unknown angle. Have students who struggle with algebraic manipulation use a CAS graphing calculator to assist with the steps in the equations.
- Note that the curriculum expectations of this course only deal with unambiguous cases, so the triangle in Example 2 has only one solution. It is possible for some oblique triangles to have two solutions depending on the order in which the angles are determined.
- Example 3 demonstrates the use of the cosine law, the sine law, and geometric reasoning to solve a triangle. Point out to students that after the first calculation, there will always be enough information to use the sine law and/or the sum of the angles in a triangle relationship.

Key Concepts

- Have students make memory aid diagrams associating the type of triangle with the tool or formula that solves it. Ensure they include the necessary information. You may wish to have students copy this chart or make their own version.

Given Information	Tool	Solve for
side-angle-side	cosine law	side
angle-side-angle	sine law	side
angle-angle-side	sine law	side
side-side-angle	sine law	angle
side-side-side	cosine law	angle
angle-angle-angle	Not enough information given. Cannot solve.	

Discuss the Concepts

- These questions provide a good opportunity to discuss the need for both the sine law and the cosine law. The sine law involves simpler calculations, so most students will choose the sine law in situations where either could be used.

Discuss the Concepts Suggested Answers (page 109)

D1. No. There is no known side-angle pair.

D2. a) No. No side lengths are known.

b) No. Only one side length is known.

D3. a) Yes. All three side lengths are known.

b) No. Only two side lengths are known. However, the sine law could be used to determine the length of the third side, and then the cosine law could be used to determine the measure of the unknown angle.

Practise (A)

- You may wish to have students work in pairs or small groups to complete the Practise questions.
- Encourage students to refer to the Examples before asking for assistance.
- Remind students to use the correct units in their answers.

Apply (B)

- **Questions 7 to 9** provide good opportunities for students to practise applying the cosine law in conjunction with other mathematical skills, and to select tools and strategies and apply reasoning.
- **Question 10** provides an opportunity to assess students' reasoning and communication skills.
- For **question 11**, you may need to remind some students to add the distances together to determine their final answer.
- **Question 12** is an Achievement Check question. It can be used for diagnostic or formative assessment, or assigned as a small summative assessment piece. You may wish to use **BLM 2-9 Section 2.4 Achievement Check Rubric** to assist you in assessing your students.

Common Errors

- Some students get answers that do not make sense when evaluating an expression involving the cosine law.

R_x Have students carefully check their calculator keystrokes if they are entering the entire calculation in a single step. They might be missing brackets or operation signs. Some students may want to perform the calculations in multiple steps. Students should also check that the calculator is set to degree mode.

Accommodations

Perceptual—complete the **Discuss the Concepts** questions as a class to practise recognizing when to use the sine or cosine law

Gifted and Enrichment—have students explore the school for a length that can be calculated using the sine law, the cosine law, or both. Students can develop their problem for the class to use in the next section. Have students research the formal derivation of the cosine law.

Visual—provide a handout with a worked example of solving an obtuse triangle. Include a written explanation beside each step in the solution. Create a similar handout with parts of the solution missing. Have students fill in the blanks according to the written explanations provided.

Spatial—have students check their diagrams with a partner before completing each question

Language—add a diagram and a definition for *contained angle* to the Word Wall

Memory—post the variations of the cosine law around the classroom

ESL—ensure students understand when it is best to use the cosine law and when it is best to use the sine law. Remind them that both might be needed to solve a problem.

Extend (C)

- Assign the Extend question to students who are not being challenged by the Apply questions.
- For **question 13**, encourage students to draw and label a diagram. This question has connections to vectors, which some students may encounter if they study math-rich programs, such as engineering technology, at the college level.

Achievement Check Answers (page 111)

12. a) 350 m

b) Monday: Cottage → Big Duck Island → Froggy Island → Cottage
Or Cottage → Froggy Island → Big Duck Island → Cottage
Wednesday: Cottage → Froggy Island → Big Duck Island → Froggy Island → Cottage
Or Cottage → Big Duck Island → Cottage → Big Duck Island → Cottage
Friday: Cottage → Big Duck Island → Froggy Island → Big Duck Island → Cottage

c) Yes. Alternate routes are found by trial and error. There does not appear to be a second possible route for Friday.

Literacy Connect

- Have one or two students read the section opener out loud. Discuss the need for the cosine law and when it should be used.
- Allow students to work with a partner when solving problems. Remind them that both the cosine law and sine law may be required to solve some of the problems.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	10, 12, 13
Reasoning and Proving	10, 12
Reflecting	10, 11
Selecting Tools and Computational Strategies	5–10, 13
Connecting	11–13
Representing	2, 3, 5, 6, 8, 9, 13
Communicating	10, 12

Ongoing Assessment

- Assess students' ability to communicate mathematically and to justify their thinking. You may wish to use **BLM A-9 Communication General Scoring Rubric** to assist you in assessing your students.

Extra Practice

- Use **BLM 2-8 Section 2.4 Cosine Law** for extra practice or remediation.