3.5

Analysis and Conclusions

Link to Prerequisite Skills
Students should complete all the Prerequisite Skills questions before
proceeding with this section.

Warm-Up
1. For each independent variable, identify a dependant variable that
   might form a cause and effect relationship.
   a) The amount of time students study for an exam.
   b) The cost of gasoline.
   c) The amount of space used to display a product in a store.
   d) The amount of time a person exercises per week.
   e) The average number of cars driven in a city per day.

Warm-Up Answers
1. Answers may vary. For example:
   a) The marks students receive on the exam.
   b) The volume of gasoline sold.
   c) The amount of the product sold.
   d) The person's fitness level.
   e) The average daily smog levels.

Teaching Suggestions
• In this section, students develop the skills to critically analyse
  statistical studies and statements. They also learn about cause and effect
  relationships and the process of making appropriate conclusions.
• There is no Investigate for this section.

Warm-Up
• Display the Warm-Up question. Have students complete the question
  independently. Then, discuss the solutions as a class.

Section Opener
• Introduce critical analysis as the process of asking the right questions to
  ensure that statistical analysis and conclusions are accurate and fair.

Examples
• Example 1 illustrates a cause and effect relationship. Point out to students
  that there are other factors that might affect a relationship, as shown
  in part c). Remind students that not all relationships can be accurately
  extrapolated, as explained in part d).
• Example 2 discusses the effect of an influential point. Students should
  see that it is important to analyse the data with and without the
  influential point to understand how they change the trend.
• Example 3 illustrates the importance of performing regression analysis
  appropriately. Students often make the mistake of assuming a value of r
  that is close to $+1$ or -1 proves a cause and effect relationship. Point out
  to students that exponential or quadratic data will often have a strong
  linear correlation but no linear cause and effect relationship will exist.

Key Concepts
- As a class, review the Key Concepts.
- Stress that a high correlation does not mean a cause and effect relationship exists. Give students the example of average daily beach attendance compared to average daily energy consumption. Explain that a scatter plot of the data might show a strong correlation between the two variables but that there is no real cause and effect relationship. Students should see that the real independent variable affecting both variables is the temperature. Remind students to watch for factors that could influence one or both of the variables they are examining.

Discuss the Concepts
- Discuss the questions as a class.
- **Question D2** makes an important point. Students should understand that to confirm a cause and effect relationship, they must look at how a *change* in the independent variable affects the dependant variable.

Discuss the Concepts Suggested Answers (page 186)

D1. The relationship is still linear because as the fathers' heights increase, the heights of the sons increase by approximately the same amount. There are not many points on the graph. If there were more data, the correlation coefficient would increase.

D2. How an increase in study time affects marks is an indicator of a cause and effect relationship, because the effect on the dependent variable (marks) is being assessed based on a change in the independent variable (study time).

Practise (A)
- You may wish to have students work in pairs or small groups to complete the Practise questions.
- Encourage students to refer to the Examples before asking for assistance.
- Consider using a think-pair-share activity for **questions 1 to 4**.
- These questions contain many interesting contextual situations. Select questions that interest your students.
- Each part of each question focuses on a component of critical analysis. Ask students to provide detailed answers to each question. The depth of their answers is also important.

Apply (B)
- **Question 7** is a **Literacy Connect** question. You may wish to assign this question as a journal entry.
- **Question 8, part d)**, asks students to consider other models for the data. They could use a graphing calculator, Fathom™ or *Excel®*, to perform quadratic and exponential regression. Note: with Fathom™, students will see the equation and r-value but the curve of best fit will not appear on the graph. Students will learn more about different regression models in Chapter 5.
- **Question 9** is an Achievement Check question. You may wish to use **BLM 3-10 Section 3.5 Achievement Check Rubric** to assist you in assessing your students' responses.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the Apply questions.
- **Question 11** discusses the double-blind study. This could make a good research project for your students.

Achievement Check Answers (page 189)

  9. a) It is easier to see if a cause and effect relationship exists by examining changes in the variables.

      b) $r = 0.90$. There is a strong linear relationship.

      c) Answers may vary. For example: Yes. An outlier is (21.3, 42.5). This data point should be checked to see if it is accurate. An influential point is $(-10.6, -8.8)$. This data point should be removed from the data set.

      d) Without removing outliers and influential points: $y = 1.38x - 0.1$, where x is the percent change in advertising budget, and y is the percent change in sales.

      e) **i)** 13.7%
        ii) 27.5%
       iii) 137.9%

Literacy Connect

- Have one or two students read the section opener out loud.
- You may wish to demonstrate the steps required to determine the line of best fit using Fathom™ in **Example 2**.
- Discuss the **Literacy Connect** as related to **question 8** on page 188.
- Discuss the meaning of new terms in this section: *cause and effect relationship* and *influential point*. Encourage students to add these definitions to their personal math dictionaries.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	9–11
Reasoning and Proving	2–10
Reflecting	7–9
Selecting Tools and Computational Strategies	5, 6, 8, 9
Connecting	2–11
Representing	5, 6, 8, 9
Communicating	1, 2, 4–11

Ongoing Assessment

- You can use the Achievement Check question and its rubric as formative assessment tools. Most of the Discuss the Concepts questions can be used as short quizzes to determine if students have understood the Examples.

Extra Practice

- Use **BLM 3-9 Section 3.5 Analysis and Conclusions** for extra practice or remediation.