5.1 Linear Models

Student Text Pages
268–281

Suggested Timing
80 min

Tools
- wind-up or battery-powered toy vehicles
- stopwatches
- metre sticks or measuring tapes
- masking tape
- graphing calculators

Related Resources
BBLM 5-3 Section 5.1 Linear Models
BLM 5-4 Section 5.1 Investigate Table
BLM 5-5 Section 5.1 Example 2 Use Technolo
BLM 5-6 Section 5.1 Achievement Check Rubric

Link to Prerequisite Skills

Students should complete all sections in the Prerequisite Skills except for Simple and Compound Interest before proceeding with this section.

Warm-Up

1. The cost, C, of renting an airplane for h hours is shown in the table.

Time (h)	Cost ($)
1	75.00
2	150.00
3	225.00
4	300.00
5	375.00

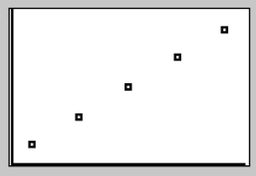
a) Calculate the first differences.
b) Create a scatter plot with cost on the vertical axis and time on the horizontal axis.
c) Use linear regression to determine an equation that relates cost to time.

Warm-Up Answers

1. a)

Time (h)	Cost ($)	First Differences
1	75.00	
2	150.00	**75.00**
3	225.00	**75.00**
4	300.00	**75.00**
5	375.00	**75.00**

b)



c) The equation is $C = 75h$.

Teaching Suggestions

Warm-Up

- Display the Warm-Up question. Have students complete the question independently. Then, discuss the solutions as a class.

Section Opener

- Read the section opener. Ask students to suggest one or two other examples of decision-making that will depend on graphs or tables.

Investigate

- If you have access to a Calculator Based Ranger (CBR) or similar technology, you may wish to use it in the Investigate.
- Supply students with **BLM 5-4 Section 5.1 Investigate Table**.

Investigate Answers (pages 268–269)

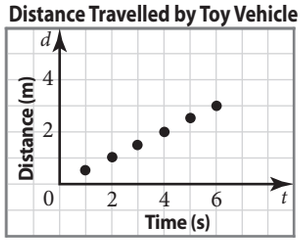
4. to 6. Answers may vary. For example:

Time (s)	Distance (m)	First Differences
0	0	0.5
1	0.5	0.5
2	1	0.5
3	1.5	0.5
4	2	0.5
5	2.5	0.5
6	3	

7. a) Answers may vary. For example: For each additional second travelled, the toy vehicle travelled an additional 0.5 m.

b) The graph will be a straight line that increases as time increases because the distance travelled between each pair of seconds is the same.

8. a)



b) Answers may vary. For example: The graph is a straight line that increases from left to right. This matches my prediction from step 7.

9. a) They are the same.

b) Constant. First differences are the same for each time interval.

Examples

- When working through Example 1, some students may know that the price of electricity is actually a step function; that is, the price per kilowatt-hour changes at certain levels of use. As a result, heavy users pay less per kilowatt-hour overall than light users. If this issue arises, you may wish to draw a graph of this situation for students.
- For Example 2, remind students that 0.25 h is equal to 15 min.
- When working through Example 2, an aviation-savvy student might note that the aircraft gets lighter as it consumes fuel, and thus consumes less fuel per hour the longer it is in flight. Although this is true, the effect is negligible for a light aircraft, since the weight of the fuel is typically less than 10% of the weight of the aircraft. It is not negligible for a large commercial aircraft, where the weight of the fuel is a much more significant fraction of the entire weight of the aircraft.
- The data supplied for Example 3 might appear random at first. Statistical methods reveal an overall trend that might not be obvious from the table. Consider using a think-pair-share activity.

Technology

- If you are using the TI-Nspire™ CAS calculator, you may wish to use **BLM 5-5 Section 5.1 Example 2 Use Technology** for **Example 2**.

Key Concepts

- Ensure that students understand the difference between a graph of constant speed and a constant graph. It may be helpful to refer to the slopes of the graphs.
- Rate of change is a difficult concept for many students. Reinforce the meaning of rate of change, including examples.

Discuss the Concepts

- **Question D1** points out one of the dangers in making predictions using extrapolation: rates of change are not necessarily constant.
- **Question D3** introduces a non-linear relation.
- When discussing **question D4**, ensure that students understand that a rate of change of zero implies a horizontal line graph.

Discuss the Concepts Suggested Answers (page 275)

D1. No. He has only two years' worth of data. If he wants to be confident in his prediction, he should use more data.

D2. There is a linear relationship between the size of a person's foot and their height. This relationship has a positive rate of change, meaning that the larger a person's foot, the greater their height.

D3. No. Anika changed the speed of her car so the distance travelled will not be equal for each time interval.

D4. Time, in seconds, and the position of a parked car.

Practise (A)

- You may wish to have students work in pairs or small groups to complete the Practise questions.
- Encourage students to refer to the Examples before asking for assistance.
- For **question 3,** discuss the difference between a constant speed and an average speed.

Apply (B)

- For **question 6**, ask students to consider the effect on the share value trend if a competitor introduced an advanced version of a similar product made by Ranjeet's technology company.
- For **question 7**, some students may need to review the formula for the volume of a cylinder.
- For **question 8**, you may wish to explain that *scuba* is an acronym that stands for Self-Contained Underwater Breathing Apparatus. The aqualung was invented in 1943 by the famous underwater explorer and scientist Jacques Cousteau.
- **Question 10** is an Achievement Check question. It can be used for diagnostic or formative assessment, or assigned as a small summative assessment piece. You may wish to use **BLM 5-6 Section 5.1 Achievement Check Rubric** to assist you in assessing your students.
- **Question 11** links to the Chapter Problem. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up.
- For **question 12**, initiate a brief discussion about the barriers to women entering the construction trades that have been removed and what barriers might still be present.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the Apply questions.

Common Errors

- Some students may attempt to determine finite differences for data that does not have equal increases/decreases in the independent variable.

R_x Have students inspect the rate of change of the independent variable to ensure that it is constant before attempting to calculate finite differences.

- Some students might think that a linear model is always valid because they can use a graphing calculator to perform a linear regression on any data, whether it is linear or not.

R_x Have students use the correlation coefficient to assess the fit of the regression equation. Point out that unless the correlation coefficient is reasonably close to 1, the regression equation is likely not valid for the data.

Accommodations

Visual—once the **Investigate** data is collected, use the overhead to show students how to set up their data table and calculate the first differences

Perceptual—use one group to model the **Investigate** for the class before the groups spread out to their assigned spaces

Spatial— assign groups to specific locations, including the hallway, to ensure groups have sufficient space to collect their data for the I**nvestigate**

Motor—have students work in pairs or groups for the **Investigate** to assist with set up and measuring

Language—allow a reading partner to assist with interpreting and completing the questions

Memory— review the key characteristics and steps involved in creating a scatter plot

ESL—have students work with a partner to help read and understand the Investigate, Examples, and Practise questions

Gifted and Enrichment—have students search the Internet for an example of a linear model not described in this section to share with the class

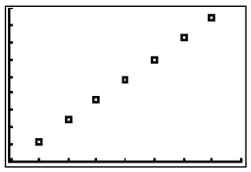
Achievement Check Answers (page 279)

10. a) Use a differences table.

t	s	First Differences	Second Differences
0	0		
		6	
1	6		0
		6	
2	12		0
		6	
3	18		0
		6	
4	24		0
		6	
5	30		0
		6	
6	36		0
		6	
7	42		

First differences are constant, so this is a linear relationship.

b)



Xmin = 0, Xmax = 8, Xscl = 1, Ymin = 0, Ymax = 45, Yscl = 5
The points lie in a straight line, so the relationship is linear.

c) 6 km/h/s

d) 60 km/h

e) The speed after 30 s would be 180 km/h, which is not reasonable.

Literacy Connect

- Have one or two students read the section opener and the Investigate out loud. Allow them to ask questions to clarify the instructions.
- Discuss the **Literacy Connects** on pages 270 and 273.
- Encourage students to add new terms to their personal math dictionaries.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	13–15
Reasoning and Proving	5–14
Reflecting	6, 10
Selecting Tools and Computational Strategies	3, 4, 9, 12, 14, 15
Connecting	3–13
Representing	6–14
Communicating	5–12, 15

Ongoing Assessment

- You may wish to collect students' responses to the Discuss the Concepts questions.

Extra Practice

- Use **BLM 5-3 Section 5.1 Linear Models** for extra practice or remediation