5.2

Quadratic Models

Student Text Pages
282–293

Suggested Timing
80 min

Tools
- metre sticks
- marbles or ball bearings
- stopwatches
- graphing calculators

Optional
- computers with motion sensors
- large balls, such as basketballs

Related Resources
BLM 5-7 Section 5.2 Quadratic Models
BLM 5-8 Section 5.2 Investigate Table
BLM 5-9 Section 5.2 Example 3 Use Technology
BLM 5-10 Section 5.2 Question 10
BLM 5-11 Section 5.2 Achievement Check Rubric
BLM A-10 Observation General Scoring Rubric

Link to Prerequisite Skills

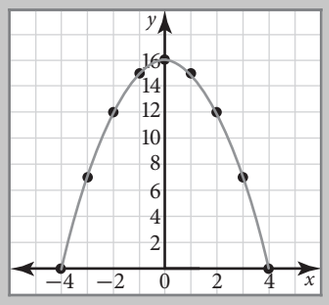
Students should complete all sections in the Prerequisite Skills except for Simple and Compound Interest before proceeding with this section.

Warm-Up

1. Consider the relation $y = 16 - x^2$.
- **a)** Make a table of values for x-values from -4 to 4.
- **b)** Construct a scatter plot of the data.
- **c)** Draw a smooth line or curve through the points on the scatter plot.
- **d)** Does the relation appear to be linear or non-linear? Explain your reasoning

Warm-Up Answers

1. a) to c)



d) The relation appears to be non-linear. The points do not lie in a straight line.

Teaching Suggestions

Warm-Up

- Display the Warm-Up question. Have students complete the question independently. Then, discuss the solutions as a class.

Section Opener

- Video clips of a free-fall theme park ride can be found on video clip sharing Web sites. Alternatively, you can prepare your own video clip of a free-falling object using almost any digital camera. Show the clip frame by frame to demonstrate the increasing distances.

Investigate

- If available, you may wish to use a Calculator Based Ranger (CBR) or similar technology for the Investigate.
- Supply students with **BLM 5-8 Section 5.2 Investigate Table**.

Investigate Answers (pages 282–283)

2. to 5. Answers may vary. For example:

Time (s)	Distance (m)	First Differences
0	0.00	
1	0.05	0.05
2	0.15	0.10
3	0.30	0.15
4	0.50	0.20
5	0.75	0.25
6	1.05	0.30

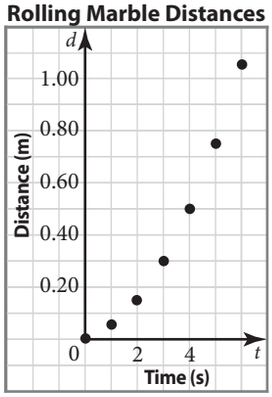
6. Answers may vary. For example:

a) The distance travelled during the second second is twice the distance travelled during the first second. The distance travelled during any given second is an additional 0.05 m more than the previous second.

b) I predict the graph will be non-linear, because the first differences are not equal. The graph will still be increasing though, because the distance travelled increases with increasing time.

7. Answers may vary. For example:

a)



b) Yes, I was correct. The graph is non-linear, but still increasing.

8. a) The distances travelled for equal time intervals are not equal.

b) The units are metres per second.

c) Increasing. First differences are increasing.

9., 10. The second differences are all 0.05.

Examples

- Consider having students use a think-pair-share strategy to work through Example 1.
- The stopping distance for a car is a measure of how much distance it takes to bring the car, moving at a given speed, to a complete stop. Tables used by traffic control agencies such as the police include driver reaction time, as well as reaction time when impaired by alcohol or drugs. Tables may also be constructed for varying road conditions, such as rain, snow, or ice. The data for Example 1 does not include driver reaction time, and assumes maximum braking on a dry road surface.
- The free-fall ride in Example 2 may have different names in different amusement parks.
- In Example 3, the revenue function appears to be quadratic whenever the demand function is linear. Explain that this is not always the case. Caution students that they must ensure that the data really is quadratic by inspecting the coefficient of determination.
- Note: To see the value of r on a graphing calculator, go to **Catalog** and select **Diagnostic: On**.

Technology

- If you are using the TI-Nspire™ CAS graphing calculator, you may wish to use **BLM 5-9 Section 5.2 Example 3 Use Technology** for **Example 3**.

Key Concepts

- When working with real data, the second differences may show some variation. If the variation is not great, the relation can still be modelled using a quadratic function. Advise students to use the coefficient of determination to evaluate how well the quadratic function fits the data.
- Caution students that interpolating and extrapolating data can sometimes lead to error, and discuss why extrapolation is the less reliable of the two.

Discuss the Concepts

- It may help students to see a graph showing a positive rate of change for **question D2** and a negative rate of change for **question D3**.
- Consider asking students to draw a graph for **question D4**, Discuss how the equation would change if the data had a maximum.

Discuss the Concepts Suggested Answers (page 289)

D1. The second differences are all zero.

D2. The rate of change of the data is increasing.

D3. The rate of change of the data is decreasing.

D4. Minimum. Whether n is positive or negative, n^2 is always positive. Therefore, C is always positive and will have a minimum value.

Practise (A)

- You may wish to have students work in pairs or small groups to complete the Practise questions.
- Encourage students to refer to the Examples before asking for assistance.

Apply (B)

- For **questions 4 and 5**, ask students why pilots need to know how to calculate the maximum range from the fuel remaining in the tanks. (So they can safely deal with landing delays due to bad weather or being diverted to a different airport.)
- For **question 6**, review the formula for the volume of a cylinder and discuss how this formula relates to the problem. Ask why a company might manufacture a line of containers with a fixed height.
- **Question 7** is an Achievement Check question. You may wish to use **BLM 5-11 Section 5.2 Achievement Check Rubric** to assist you in assessing your students.
- **Question 8** links to the Chapter Problem. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up.

Extend (C)

- For **Question 9**, some students may benefit from using concrete materials such as linking cubes or dice to construct a model of a typical Roman villa.
- You can find video clips of jumps such as the one described in **question 10** on video sharing Web sites. An interesting example is the car jump, which includes a rotation, from the movie *The Man With the Golden Gun*. Supply students with **BLM 5-10 Section 5.2 Question 10** if they need an enlargement of the photograph.

Common Errors

- Some students may find it confusing when the function is increasing while the rate of change is decreasing, as in **question 3**. As a result, they may determine the rate of change incorrectly.

R_x Try a visual argument using the slope of the tangent to the curve. Show that the slope is highly positive at first, but decreases towards zero at the maximum and then becomes negative. Then show an example of a function that is decreasing while the rate of change is increasing.

Accommodations

Visual—once the **Investigate** data is collected, use the overhead to show students how to set up their data table and calculate the first and second differences

Perceptual—use one group to model the **Investigate** for the class before the groups spread out to their assigned spaces

Spatial—assign groups to specific locations, including the hallway, to ensure groups have sufficient space to collect their data for the **Investigate**

Motor—have students work in pairs or groups for the **Investigate** to assist with set up and measuring

Language—allow a reading partner to assist with interpreting and completing the questions

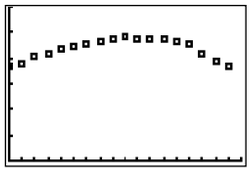
Memory—review the key characteristics of a quadratic function

ESL—have students work with a partner to help read and understand the Investigate, Examples, and Practise questions

Gifted and Enrichment—have the students search the Internet for an example of a quadratic model not described in this section to share with the class

Achievement Check Answers (page 292)

7. a)



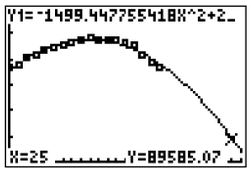
Xmin = 0, Xmax = 18, Xscl = 1, Ymin = 0, Ymax = 600 000, Yscl = 100 000

b) The birth rate increases from 1950 to 1959 then it decreases until 1967.

c) A quadratic model. The data does not seem to be linear but appears to be parabolic.

d) The equation of the quadratic model is: $y = -500x^2 + 26\,800x + 357\,000$.

e) Using the graph with the quadratic regression equation included, the predicted number of births in 1975 ($x = 25$) is 89 585 births. (The simplified function gives 89 500 births.)



f) The model did not predict the birth rate for 1975 very well. Models are generally better for predicting interpolated values than extrapolated values. The year 1975 is outside the domain of the given data. Although the birth rate increased during the Baby Boom years, it returned to its long-term average rate, which was approximately 375 000 births per year.

Literacy Connect

- Have one or two students read the section opener and the Investigate out loud. Allow them to ask questions to clarify the instructions.
- Encourage students to add new terms to their personal math dictionaries.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	7–10
Reasoning and Proving	2–4, 6–9
Reflecting	4, 6, 7
Selecting Tools and Computational Strategies	5–8
Connecting	3–10
Representing	4, 6–10
Communicating	2–9

Ongoing Assessment

- While students are working, circulate and see how well each person works. This may be an opportunity to observe and record individual students' learning skills. Use **BLM A-10 Observation General Scoring Rubric**.

Extra Practice

- Use **BLM 5-7 Section 5.2 Quadratic Models** for extra practice or remediation.