5.3 Exponential Models

Link to Prerequisite Skills

Students should complete all the Prerequisite Skills questions before proceeding with this section.

Student Text Pages
294–309

Suggested Timing
80 min

Tools
- foam or cardboard cups of hot water
- digital thermometers
- stopwatches
- graphing calculators

Optional
- computer or calculator with temperature probe
- TI-Nspire™ CAS graphing calculators
- computers with spreadsheet software

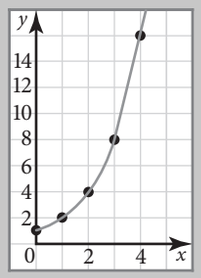
Related Resources
BLM 5-12 Section 5.3 Exponential Models
BLM 5-13 Section 5.3 Investigate Table
BLM 5-14 Section 5.3 Achievement Check Rubric
BLM T-6 Using the CBR™
BLM A-9 Communication General Scoring Rubric

Warm-Up

1. Consider the relation $y = 2^x$.
- **a)** Make a table of values for x-values from -4 to 4.
- **b)** Construct a scatter plot of the data.
- **c)** Draw a smooth line or curve through the points on the scatter plot.
- **d)** Does the relation seem to be increasing, constant, or decreasing? Explain your reasoning.
- **e)** Does the rate of change seem to be increasing, constant, or decreasing? Explain your reasoning.

Warm-Up Answers

1. a) to c)



- **d)** The graph shows an upward trend. The relation is increasing.
- **e)** The slope of the graph is increasing. The rate of change is increasing.

Teaching Suggestions

Warm-Up

- Display the Warm-Up questions. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Have students read the section opener. The three examples cited in the opener at first appear to have nothing in common. Ask students to explain why a quantity like the world's population might grow exponentially.

Investigate

- Waiting for hot water to cool can be a tedious process. If you have access to a Calculator Based Ranger (CBR) or a computer with a temperature probe, you can set up the experiment at the side of the class prior to starting the Investigate. Introduce the experiment, and then let the CBR or computer record the data. The data can be stored and recalled when it is needed. Use **BLM T-6 Using the CBR™** if you need assistance.
- Supply students with **BLM 5-13 Section 5.3 Investigate Table**.

Investigate Answers (page 294–295)

3, 4, 6, 7 Answers may vary. For example:

Time (min)	Temperature (°C)	First Differences	Second Differences	Ratio
0	53.0			
1	51.5	−1.5	0.1	0.97
2	50.1	−1.4	0	0.97
3	48.7	−1.4	0.1	0.97
4	47.4	−1.3	0.1	0.97
5	46.2	−1.2	0	0.97
6	45.0	−1.2	0.1	0.98
7	43.9	−1.1	0	0.97
8	42.8	−1.1	0.1	0.98
9	41.8	−1.0	0	0.98
10	40.8	−1.0	0.1	0.98
11	39.9	−0.9	0	0.98
12	39.0	−0.9	0	0.98
13	38.1	−0.9	0	0.98
14	37.3	−0.8	0.1	0.98
15	36.6	−0.7	0.1	0.98
16	35.8	−0.8	−0.1	0.98
17	35.1	−0.7	0.1	0.98
18	34.5	−0.6	0.1	0.98
19	33.9	−0.6	0	0.98
20	33.3	−0.6		0.98

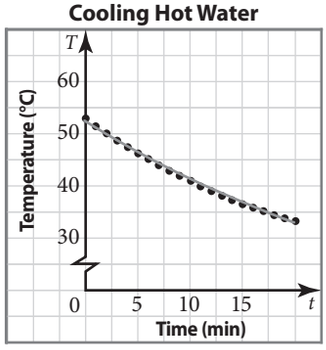
5. a) As time increases, the temperature decreases.

b) Increasing. The temperatures decrease at a slower rate as time goes on. The rate of change is increasing.

6. The relationship between temperature and time is not linear or quadratic, since the first and second differences are not constant.

8. The numbers in the ratio column are all equal.

9.



A curve that starts off steep and then becomes less steep as time increases. The curve goes down from left to right.

10. room temperature

Examples

- Consider having students use a think-pair-share strategy to work through Example 1. Students should be able to reason why thawing frozen meats is more safely done in a refrigerator than on the kitchen counter.
- For Example 2, if you can obtain a smoke detector and a Geiger counter (possibly from the science department), remove the cover from the smoke detector and show students the box that holds the radioactive Am-241. Use the Geiger counter to let students hear evidence that it is emitting radioactive particles.
- You can generate your own data for Example 3 in a few minutes. You will need a pack of cards for each student. Five repetitions of the experiment per student in an average class will provide enough data for analysis.

Technology

- You may wish to use the Use Technology section on pages 308 to 309 as an alternative to **Example 2**. The section gives instructions on how to generate multiple models using a spreadsheet.
- You may wish to use the Use Technology section on pages 306 to 307 as an alternative to **Example 3**. The section gives instructions on how to use the TI-Nspire™ CAS graphing calculator to perform exponential regression.
- Students should be comfortable performing linear and quadratic regressions using a calculator. Point out the similarities, and differences, in performing an exponential regression.
- Using a Smartboard® or projector would be an efficient way to demonstrate the CAS and spreadsheet features as part of the lesson.

Key Concepts

- Ensure that students understand why a constant ratio means a constant percent increase over equal intervals.

Discuss the Concepts

- For **question D1**, note that claims of exponential growth are frequent in the media. It might be interesting to ask students to write a letter to the editor referring to such a claim, and asking for evidence that the growth really is exponential.
- After discussing **question D3**, ensure students understand the intent of the question: that exponential functions are either always increasing or always decreasing.

Discuss the Concepts Suggested Answers (page 301)

D1. Look at two pairs of consecutive data values and divide the smaller value in each pair by the greater value. If the ratios are equal then the data is exponential.

D2. Yes. An example is the temperature of a cooling cup of hot water.

D3. Quadratic. The graph is initially decreasing, then there is a minimum value and then it begins increasing again. This shape is characteristic of a quadratic function.

Practise (A)

- You may wish to have students work in pairs or small groups to complete the Practise questions.
- Encourage students to refer to the Examples before asking for assistance.

Apply (B)

- For **question 5**, ask students to explain in their own words why such a process should result in an exponential decay. (The number of pennies that turn up heads will decrease as the number of pennies thrown decreases.)
- Students could obtain their own data for **question 7**. Consider asking each student to select a make and model that has been in production for at least five years. Then, have them search for the selling price and age using the Internet or newspaper ads. Remind students to exclude vehicles with unreasonable odometer readings.
- Usually only top-end synthesizers will have a built-in waveform display, such as the one mentioned in **question 8**. You may wish to borrow an oscilloscope from the physics department, which will display the waveform from any sound source.
- **Question 9** is an Achievement Check question. You may wish to use **BLM 5-14 Section 5.3 Achievement Check Rubric** to assist you in assessing your students.
- **Question 10** links to the Chapter Problem. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up. Wait time relations such as this occur in many places. Ask students to give other examples. (Waiting in line at a checkout counter, waiting to cross the street at an intersection with no traffic signals, waiting for a desired prize to turn up in a box of cereal, etc.).
- **Question 10, part e)** is a **Literacy Connect**. You may wish to assign this question as a journal entry or to discuss the question as a class. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the Apply questions.
- For **question 11**, you could show a video clip of yeast cells dividing. Such clips are available on video sharing Web sites.

Achievement Check Answers (page 304)

9. a) to c) Use a differences table.

Year	Cost Index	First Differences	Second Differences	Ratios
1999	143.88			
		4.43		1.031
2000	148.31		−0.03	
		4.40		1.030
2001	152.71		−0.19	
		4.21		1.028
2002	156.92		1.07	
		5.28		1.034
2003	162.20			

First differences are not constant so the relationship is not linear. Second differences are not constant so the relationship is not quadratic. Ratios of successive values are all approximately 1.03. The index can be modelled by an exponential model.

d) Use exponential regression on a graphing calculator.
The equation $y = 143.9(1.03)^x$ where y is the cost index value for $x = 1999$.

e) 199.19

Common Errors

- Some students may have difficulty calculating common ratios or may confuse them with finite differences.

R_x Each time you take up a problem, calculate one or two of the entries, making it clear whether finite differences are being calculated (subtraction) or common ratios are being calculated (division).

Accommodations

Visual—once the **Investigate** data is collected, use the overhead to show students how to set up their data table and calculate the ratios

Motor—have students work in pairs for the **Investigate** to assist with recording data

ESL—allow students to use point form in their **Investigate** responses

Perceptual—complete **Example 3** as a class using an LCD projector to review the calculator steps involved in modelling an exponential function

Language—have students take turns reading the **Examples** and solutions to a partner to increase their familiarity with the language and steps necessary for completing additional questions

Spatial—allow technology for drawing graphs when completing the questions at the end of the section. Assist students with finding optimal viewing windows for the graphs.

Memory—have students record calculator steps or note the page in their textbook where they can find help on using graphing calculators to graph and model functions

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	7, 8, 10, 12
Reasoning and Proving	2–10, 12
Reflecting	
Selecting Tools and Computational Strategies	7–12
Connecting	5–12
Representing	6–8, 10–12
Communicating	2–10

Ongoing Assessment

- Assess students' ability to communicate mathematically, and to justify their thinking. You may wish to use **BLM A-9 Communication General Scoring Rubric** to assist you in assessing your students.

Extra Practice

- Use **BLM 5-12 Section 5.3 Exponential Models** for extra practice or remediation.