5.4

Analyse Graphical Models

Student Text Pages
310–319

Suggested Timing
80 min

Tools
- graphing calculators

Related Resources
BLM 5-15 Section 5.4 Analyse Graphical Models
BLM 5-16 Section 5.4 Example 2 Use Technology
BLM 5-17 Section 5.4 Achievement Check Rubric
BLM A-8 Application General Scoring Rubric

Link to Prerequisite Skills

Students should complete all the Prerequisite Skills questions before proceeding with this section.

Warm-Up

1. Raquelle threw a stone into a calm pond, and made a video of the circular ripple as it expanded. She used the video to determine the area of the circle every 2 s.

Time (*s*)	Area (m^2)
2	13
4	49
6	112
8	202
10	315

a) Calculate the first differences, second differences, and ratios.
b) Does the relation appear to be linear, quadratic, exponential, or none of these? Justify your answer.
c) Construct a scatter plot of the data. Does the graph support your answer in part b)?

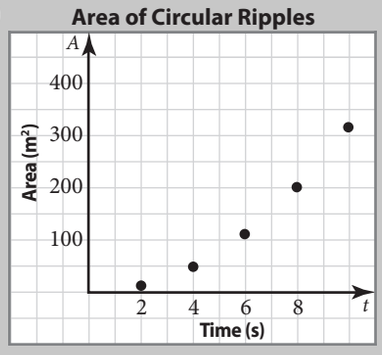
Warm-Up Answers

1. a)

First Differences	Second Differences	Ratios
36		3.8
	27	
63		2.3
	27	
90		1.8
	23	
113		1.6

b) The first differences are not constant. The relation is not linear.
The second differences are almost constant. The relation could be quadratic.
The ratios are not constant. The relation is not exponential.

c)



The graph appears to be an increasing quadratic function. This supports my answer to part b).

Teaching Suggestions

Warm-Up

- Display the Warm-Up questions. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Have students read the section opener. Point out that the extrapolations suggested in the opener appear to be attractive but could be dangerous. Have students consider the example of the antibiotic. Why is direct extrapolation hazardous in this case? Why might it result in unexpected results?

Investigate

- Students could use play money to simulate the effects of simple and compound interest on an investment. Have them lay out the amount for each investment at the end of each year.

Investigate Answers (page 310–311)

1. a)

Year	Amount of Investment ($)
0	1000.00
1	1100.00
2	1200.00
3	1300.00
4	1400.00
5	1500.00

b) All $100.00/year.

c) Constant. The units are dollars per year.

d) Linear. first differences are constant.

e) See graph for step 3.

2.

Year	Amount of Investment ($)
0	1000.00
1	1080.00
2	1166.40
3	1259.71
4	1360.49
5	1469.33

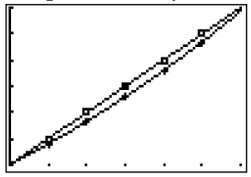
b) $80.00/year, $86.40/year, $93.31/year

c) increasing

d) Exponential. Ratios are constant.

e) See graph in step 3.

3. Simple interest: $y = 100x + 1000$. Compound interest: $y = 1000(1.08^x)$.



Xmin = 0, Xmax = 6, Xscl = 1, Ymin = 1000, Ymax = 1600, Yscl = 100

4. (6.54, 1654.00)

5. Answers may vary. For example: I would advise Sunita to invest the money with her brother if she wants to invest it for less than six years, but to invest in the GIC if she wants to invest it for more than six years.

6. Answers may vary. For example: The rate of change of the compound interest option is increasing while the rate of change of the simple interest option is constant. Initially, the rate of change of the compound interest option is less than the rate of change of the simple interest option but as time increases, the rate of change increases until it is greater than the constant rate of change of the simple interest option.

Examples

- The bacterial growth in Example 1 can be demonstrated using time-lapse photography. You can find examples on video sharing Web sites.

Technology

- If you are using the TI-Nspire™ CAS graphing calculator, you can use **BLM 5-16 Section 5.4 Example 2 Use Technology** for **Example 2**. Alternatively, if an interactive whiteboard is available, you can use *The Geometer's Sketchpad*® as a graphing tool. Either of these provides the higher resolution that emphasizes the divergence of the two models.

Key Concepts

- Emphasize the hazards of adopting a model based on a finite number of data points. Remind students that selecting the wrong model makes any prediction unreliable.

Discuss the Concepts

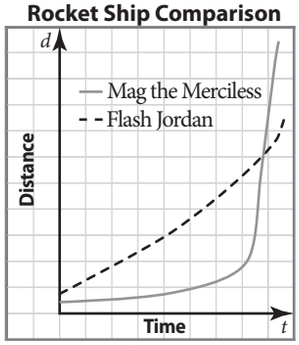
- **Question D1** provides students with an opportunity to revisit the concept of rate of change as applied to a graph, especially the difference between a graph that is decreasing and a rate of change that is decreasing.

Discuss the Concepts Suggested Answers (page 315)

D1. Suki. The rate of change is initially very negative and becomes less negative as the graph moves from left to right. Therefore, the rate of change is increasing. An example is the cup of hot water cooling over time.

D2. Yes. After one year, Achmed has $525.00 and Laila has $1050.00, so the ratio is 1:2. After two years, Achmed has $551.25 and Laila has $1102.50, which is also a ratio of 1:2.

D3. a)



b) Mag's ship. The rate of change is increasing faster than the rate of change for Flash Jordon's ship.

Common Errors

- Some students may have difficulty with the concept that more than one model appears to fit a data set.

R_x Use graphing software such as *The Geometer's Sketchpad®* to graph two models that appear to match over a given domain. Then, expand the graph area and drag each model to show where they diverge.

Accommodations

Visual—use the LCD projector to show ideal window settings for both graphs in the **Investigate** and to show the steps for finding the point of intersection

Perceptual—for the **Investigate**, have one partner model simple interest while the other models compound interest. Have pairs compare their results before merging their data to find the point of intersection. Repeat the think-pair-share process for questions that involve comparing two different models.

Memory—have the class create a story, poem, song, or picture that summarizes the key characteristics of linear, quadratic, and exponential models

ESL—have students work with a partner to assist with reading and interpreting the Investigate, Examples, and Practise questions

Gifted and Enrichment—have the students research the growth rate of common bacteria not mentioned in the section

Practise (A)

- Consider having students discuss these questions in small groups.
- Encourage students to refer to the Examples before asking for assistance.

Apply (B)

- Spectacular video clips to illustrate the jet-powered truck in **question 8** can be found on video sharing Web sites.
- **Question 9** is an Achievement Check question. It can be used for diagnostic or formative assessment, or assigned as a small summative assessment piece. You may wish to use **BLM 5-17 Section 5.4 Achievement Check Rubric** to assist you in assessing your students.
- **Question 10** links to the Chapter Problem. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up.
- **Question 10, part c)**, is a **Literacy Connect**. You may wish to assign this question as a journal entry or to discuss the question as a class. Literacy Connect questions offer the opportunity to explore literacy issues in the mathematics classroom and within the context of mathematics.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the Apply questions.

Achievement Check Answers (page 319)

9. a) to c)
First differences are all 20, so a linear model is appropriate.
Second differences are all zero so a quadratic model is appropriate.
The ratios are all approximately equal to 1.016, so an exponential model is appropriate.

d) The linear model is $y = 20x + 1200$.
The exponential model is $y = 1200.5(1.016)^x$.

e) Linear model: 1400 computers; exponential model: 1407 computers.

f) Linear model: 40 days; exponential model: 33 days.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	6, 8–12
Reasoning and Proving	1–4, 6, 9, 10
Reflecting	8
Selecting Tools and Computational Strategies	5–9, 11, 12
Connecting	4, 5, 7–12
Representing	4–6, 8
Communicating	1, 3–6, 10

Ongoing Assessment

- You may wish to use **BLM A-8 Application General Scoring Rubric** to assess students' responses to **question 8**.

Extra Practice

- Use **BLM 5-15 Section 5.4 Analyse Graphical Models** for extra practice or remediation.