5.5

Select a Mathematical Model

Student Text Pages
320–331

Suggested Timing
80 min

Tools
- masses, string, and supports for pendulums
- stopwatches
- graphing calculators

Related Resources
BLM 5-18 Section 5.5 Select a Mathematical Model
BLM 5-19 Section 5.5 Example 1 Use Technology
BLM 5-20 Section 5.5 Example 2 Use Technology
BLM 5-21 Section 5.5 Achievement Check Rubric

Link to Prerequisite Skills

Students should complete all the Prerequisite Skills questions before proceeding with this section.

Warm-Up

1. A newspaper article claims that the cost of entry-level DVD players has dropped exponentially over the last ten years. The table shows the data used to support the claim.

Year	Average Price ($)
0	400
2	250
4	160
6	100
8	70
10	40

a) Determine the first differences, second differences, and ratios.
b) Does the relation appear to be linear, quadratic, exponential, or none of these? Justify your answer.
c) Construct a scatter plot of the data. Does the graph support your answer in part b)?

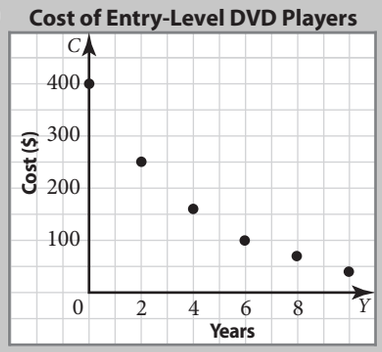
Warm-Up Answers

1. a)

First Differences	Second Differences	Ratios
−150		0.63
	60	
−90		0.64
	30	
−60		0.63
	30	
−30		0.70
	0	
−30		0.57

b) The first differences are not constant. The relation is not linear.
The second differences are not constant. The relation is not quadratic.
The ratios are almost constant. The relation could be exponential.

c)



The graph appears to show exponential decay. This supports my answer to part b).

Teaching Suggestions

Warm-Up

- Display the Warm-Up question. Have students complete the question independently. Then, discuss the solutions as a class.

Section Opener

- As a class, read the section opener.
- You can obtain a real tracking image of space debris by typing the keywords "space debris" into a search engine, and selecting "images."

Investigate

- The pendulum with the desired period of 1 s has an approximate length of 25 cm.
- Ensure that students measure the length from the support to the centre of the bob. A common error is to measure only the length of the string, or to the bottom of the bob.

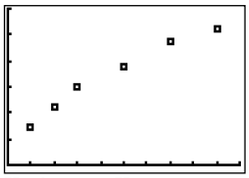
Investigate Answers (page 320–321)

Answers may vary. For example

3. to 5.

Time	Length of Pendulum (cm)	Period (s)
6	10	0.6
9	20	0.9
12	30	1.2
15	50	1.5
19	70	1.9
21	90	2.1

6. b)



Xmin = 0, Xmax = 100, Xscl = 10, Ymin = 0, Ymax = 2.4, Yscl = 0.4

7. Either a linear or a quadratic model appears to be appropriate for the data. The data appears to follow a straight line or perhaps a slight downward curve.

8. A quadratic model: $y = -0.0001x^2 + 0.0309x + 0.3259$, with $r^2 = 0.9952$.

9. 23.6 cm.

10. It took 10 s for the pendulum to make ten complete swings at a length of 23.6 cm. That is one swing per second, so it has a period of 1 s as predicted.

Examples

- For Example 1, remind students that most chemicals become toxic in large concentrations. The quadratic model for the fertilizer will only be valid for a restricted domain.
- For Example 2, explain that intervals between notes that are pleasing to the human ear are called *consonant*, while those that are not are called *dissonant*. Dissonance is used in audio soundtracks to produce feelings of disquiet or even anxiety in the audience.

Technology

- If you are using the TI-Nspire™ CAS graphing calculator, you can use **BLM 5-19 Section 5.5 Example 1 Use Technology** for **Example 1** and **BLM 5-20 Section 5.5 Example 2 Use Technology** for **Example 2**.

Key Concepts

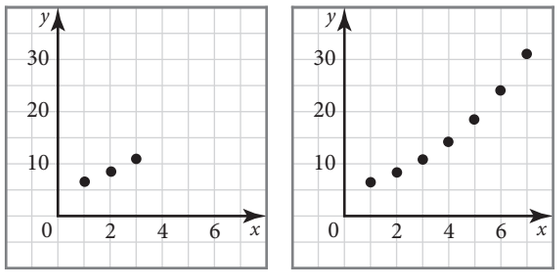
- There are many other possible graphical models. Ensure that students are not left with the impression that all situations are linear, quadratic, or exponential. You can use the list of possible regressions on a graphing calculator to point out other possible models.
- Although a scatter plot is useful in supporting a chosen model, it is often not obvious whether a given curve is quadratic, exponential, or some other curve.

Discuss the Concepts

- Emphasize the need for a large number of data points. Although a quadratic regression will work with a minimum of three points, the resulting model is not very reliable.
- Ensure that students understand most models only apply to a limited domain. For example, exponential bacterial growth is only valid as long as the nutrient supply is adequate. Most growth is limited by the resources available. You can sketch a simple logistics curve to illustrate this concept. **Question 12** explores this concept in more detail.

Discuss the Concepts Suggested Answers (page 325)

D1.



D2. No. There will be some practical or environmental considerations that will limit the growth of the bacteria, such as space, food, or disease. The size of the population cannot continue to grow forever.

D3. Exponential relation. The number of people who are told about the information at each stage is increasing as more people know, and so the rate of change is increasing. Since each person always tells three new people, the change is a constant percent increase.

Practise (A)

- You could assign small groups to each question and have each group display their answers on chart paper for a whole class discussion.
- Encourage students to refer to the Examples before asking for assistance.
- **Question 4** illustrates the dangers of modelling with a graph that has too few data points.

Apply (B)

- Discuss the usefulness of choosing a model based on the scatter plot in **question 5** as opposed to the finite differences or ratios in **question 6**.
- Before assigning **question 8**, review the meaning of *optimistic* and *pessimistic* in the **Literacy Connect**. Ensure students understand that the most optimistic model predicts the least amount of space debris and the most pessimistic predicts the greatest amount.
- **Question 9** is an Achievement Check question. It can be used for diagnostic or formative assessment, or assigned as a small summative assessment piece. You may wish to use **BLM 5-21 Section 5.5 Achievement Check Rubric** to assist you in assessing your students.

- **Question 10** links to the Chapter Problem. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the Apply questions.
- **Question 11** is a good example of applying different models to different parts of the data table.
- For **question 12**, the data follow a logistic curve, $y = \frac{3567}{1 + 55.56e^{-0.566x}}$, which can be modelled using the logistic regression on a graphing calculator. Logistic curves are covered in the Advanced Functions 12 course.

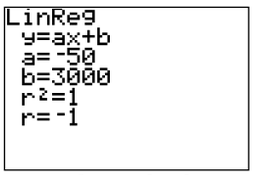
Achievement Check Answers (page 329)

9. a)

Xmin = 0, Xmax = 7, Xscl = 1, Ymin = 0, Ymax = 4000, Yscl = 500

b) Linear model. The points lie in a straight line.

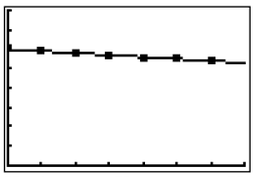
c) Check using linear regression.



Since $r^2 = 1$, the data matches the linear model perfectly. A linear model is appropriate.

d) Linear model: $y = -50x + 3000$, $r^2 = 1$
Quadratic model: $y = 0x^2 - 50x + 3000$, $r^2 = 1$
Exponential model: $y = 3002.3(0.9826)^x$, $r^2 = 0.99977$
The linear model is slightly better.

e) The equation for the linear model is $y = -50x + 3000$.



Xmin = 0, Xmax = 7, Xscl = 1, Ymin = 0, Ymax = 4000, Yscl = 500

f) 2000 psi

Literacy Connect

- Have students write a journal entry to explain how they can tell if a model is linear, quadratic, or exponential.

Common Errors

- Some students choose a model that seems to match some of the data instead of looking for a model that matches all the data.

R_x Have students test more than one model for a given data set, especially when the number of data points is not large. Remind students to use the coefficient of determination as part of their decision-making process.

Accommodations

Motor—have students tape the pendulum string to a desk edge or wall during the **Investigate**. Allow a partner to assist with measuring and recording the data.

Language—explain the terms *pivot*, *bob*, *complete swing*, and *period* using diagrams before starting the **Investigate**

Perceptual—provide a worked example of modelling data graphically and algebraically. In the margin, write an explanation for each step. Include graphing calculator screen captures where appropriate.

Memory—have students summarize the Key Concepts and Examples in their own words on index cards, a study sheet, or a mind map

ESL—ensure students understand how to distinguish between a linear model, a quadratic model, and an exponential model

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	8, 10–12
Reasoning and Proving	4–12
Reflecting	8–11
Selecting Tools and Computational Strategies	5–12
Connecting	5–12
Representing	5–12
Communicating	5–8, 10, 11

Ongoing Assessment

- You may wish to collect students' responses to the Discuss the Concepts questions.

Extra Practice

- Use **BLM 5-18 Section 5.5 Select a Mathematical Model** for extra practice or remediation.