6.1

Exponent Laws

Student Text Pages
342–351

Suggested Timing
80–160 min

Tools
- scientific or graphing calculators

Related Resources
BLM 6-3 Section 6.1 Investigate
BLM 6-4 Section 6.1 Exponents Laws
BLM 6-5 Section 6.1 Achievement Check Rubric
BLM T-5 The Computer Algebra System

Link to Prerequisite Skills

Students should complete Powers and Square Roots, Exponent Laws, and Zero and Negative Exponents in the Prerequisite Skills before proceeding with this section.

Warm-Up

1. a) What is the product law of exponents?
b) Give an example.
2. a) What is the quotient law of exponents?
b) Give an example.
3. a) What is the power of a power law of exponents?
b) Give an example.

Warm-Up Answers

Examples may vary.

1. a) When multiplying powers with the same base, add the exponents.
b) $(3^2)(3^4) = 3^6$
2. a) When dividing powers with the same base, subtract the exponents.
b) $4^5 \div 4^3 = 4^2$
3. a) When raising a power to a power, multiply the exponents.
b) $(2^3)^2 = 2^6$

Teaching Suggestions

- Depending on the needs of the class, this lesson could be divided into two periods. Monitor students' progress as they work through the Investigate. If necessary, do the Examples and questions the next day.

Warm-Up

- Display the Warm-Up questions. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Ask students what they think rabbits have to do with exponents. (Many populations of animals grow exponentially.)

Investigate

- Students use patterning to discover that the exponent laws hold true for powers with zero and negative exponents. Students can use the tables in **BLM 6-3 Section 6.1 Investigate** to organize their information.
- Since repeated calculations are required, students might find a spreadsheet or Computer Algebra System (CAS) useful. Provide **BLM T-5 The Computer Algebra System** to students who need assistance using CAS graphing calculators.

Investigate Answers (pages 342–344)

1., 2. c)

Product of Powers	Method 1		Method 2		
	Evaluate Each Power	Multiply	Apply the Product Law	Single Power	Evaluate
$2^2 \times 2^3$	4×8	32	2^{2+3}	2^5	32
$2^2 \times 2^2$	4×4	16	2^{2+2}	2^4	16
$2^2 \times 2^1$	4×2	8	2^{2+1}	2^3	8
$2^2 \times 2^0$	4×1	4	2^{2+0}	2^2	4
$2^2 \times 2^{-1}$	$4 \times \frac{1}{2}$	2	$2^{2+(-1)}$	2^1	2
$2^2 \times 2^{-2}$	$4 \times \frac{1}{4}$	1	$2^{2+(-2)}$	2^0	1
$2^2 \times 2^{-3}$	$4 \times \frac{1}{8}$	$\frac{1}{2}$	$2^{2+(-3)}$	2^{-1}	$\frac{1}{2}$

2. a) The numbers in the third and sixth columns are equal. Each number in the third or sixth column is half the value of the number in the row above it.

b) Predictions may vary.

3. a) Predictions may vary.

b) $5, \frac{1}{64}, 0, \frac{1}{27}$

4. Yes. Powers with negative exponents are fractions and multiplying by a fraction is equal to dividing by an integer.

5., 6. c)

Quotient of Powers	Method 1		Method 2		
	Evaluate Each Power	Divide	Apply the Quotient Law	Single Power	Evaluate
$2^6 \div 2^3$	$64 \div 8$	8	2^{6-3}	2^3	8
$2^6 \div 2^2$	$64 \div 4$	16	2^{6-2}	2^4	16
$2^6 \div 2^1$	$64 \div 2$	32	2^{6-1}	2^5	32
$2^6 \div 2^0$	$64 \div 1$	64	2^{6-0}	2^6	64
$2^6 \div 2^{-1}$	$64 \div \frac{1}{2}$	128	$2^{6-(-1)}$	2^7	128
$2^6 \div 2^{-2}$	$64 \div \frac{1}{4}$	256	$2^{6-(-2)}$	2^8	256
$2^6 \div 2^{-3}$	$64 \div \frac{1}{8}$	512	$2^{6-(-3)}$	2^9	512

6. a) The numbers in the third and sixth columns are equal. Each number in the third or sixth column is twice the value of the number in the row above it.

b) Predictions may vary.

7. a) Predictions may vary.

b) $125, 2187, 0, \frac{1}{8}$

8. Yes. Powers with negative exponents are fractions and dividing by a fraction is the same as multiplying by an integer.

9., 10. c)

Power of a Power	Method 1		Method 2		
	Evaluate the Power in the Brackets	Evaluate the Second Power	Apply the Power of a Power Law	Single Power	Evaluate
$(2^2)^3$	$(4)^3$	64	$2^{2 \times 3}$	2^6	64
$(2^2)^2$	$(4)^2$	16	$2^{2 \times 2}$	2^4	16
$(2^2)^1$	$(4)^1$	4	$2^{2 \times 1}$	2^2	4
$(2^2)^0$	$(4)^0$	1	$2^{2 \times 0}$	2^0	1
$(2^2)^{-1}$	$(4)^{-1}$	$\frac{1}{4}$	$2^{2 \times (-1)}$	2^{-2}	$\frac{1}{4}$
$(2^2)^{-2}$	$(4)^{-2}$	$\frac{1}{16}$	$2^{2 \times (-2)}$	2^{-4}	$\frac{1}{16}$
$(2^2)^{-3}$	$(4)^{-3}$	$\frac{1}{64}$	$2^{2 \times (-3)}$	2^{-6}	$\frac{1}{64}$

10. a) The numbers in the third and sixth columns are equal. Each number in the third or sixth column is one quarter the value of the number in the row above it.

b) Predictions may vary.

11. a) Predictions may vary.

b) $\frac{1}{729}$, $\frac{1}{125}$, 256, 1

12. Yes. If there is a negative exponent, it will turn the expression into a fraction. If there are two negative exponents, then the expression is turned into a fraction and then turned back into an integer. Using a zero exponent makes the expression equal to 1.

Examples

- Examples 1 to 3 illustrate and consolidate understanding of the exponent laws involving powers with integer exponents. Some students may need to review operations with integers.
- Discuss the importance of brackets as described in the **Literacy Connect** beside Example 2. Remind students to be careful when writing and evaluating expressions with brackets, especially when substituting numerical values into the expressions. They should also check the signs of their answers when evaluating expressions with negative bases or exponents.
- In Example 4, exponent laws are applied to simplify algebraic expressions with integer exponents. As an extension, students could explore expressions involving a combination of numerical and variable expressions, such as $(2k^{-3}m^2)^{-4}$.
- Example 5 illustrates how a calculator can be used to evaluate a power with integer exponents after it has been simplified. Provide calculator keystroke support, as required.

Key Concepts

- Review the Key Concepts as a class. Have students write the exponent laws in their notebooks and provide an example of how to use each law.

Discuss the Concepts

- Consider having students discuss **questions D1 and D2** in pairs before writing individual responses in their notebooks.
- It may be beneficial to have students discuss **questions D3 and D4** in pairs or small groups and then discuss as a class.

Discuss the Concepts Suggested Answers (page 348)

D1. a) None. There is no law for addition of exponents.

b) Quotient law. The common base is being divided.

c) Product law for the expression in the brackets and then the zero exponent law because the exponents add to zero.

d) Negative exponent law for the first power because the power is negative and the zero exponent law for the second power.

D2. Product Law: $5^3 \times 5^2 = 5^6$; $7^{-2} \times 7^{-1} = 7^{-3}$
Quotient Law: $3^9 \div 3^6 = 3^3$; $10^{-4} \div 10^{-2} = 10^{-2}$
Power of a Power Law: $(4^3)^4 = 4^{12}$; $(5^{-2})^{-5} = 5^{10}$
Power of a Product Law: $(4 \times 8)^5 = 4^5 \times 8^5$; $(3 \times 9)^{-4} = 3^{-4} \times 9^{-4}$
Zero Exponent Law: $16^0 = 1$; $(-5)^0 = 1$
Negative Exponent Law: $5^{-4} = \frac{1}{5^4}$; $3^{-(-3)} = 3^3$

D3. a) $2 \times (-3) = -6$, so:

$$3^{2(-3)} = 3^{-6}$$
$$= \frac{1}{3^6}$$

b) $(10^{-2})(1) = 10^{-2}$, so:

$$10^{-2} = \frac{1}{10^2}$$
$$= \frac{1}{100}$$

D4. The negative sign in the base makes the entire expression negative and the negative sign in the exponent makes the expression a fraction: $-\frac{1}{9}$.

Practise (A)

- You may wish to have students work in pairs or small groups to complete the Practise questions.
- Encourage students to refer to the Examples before asking for assistance.
- For **questions 1 to 7**, students might find it useful to check their answers with a CAS graphing calculator.
- **Questions 8 and 11** repeat Parts A and C of the Investigate using a negative base. Students should note that a negative exponent and a negative base do not affect each other.

Apply (B)

- For **question 14**, some students may benefit from a review of the use of nested brackets. This is an Achievement Check question. You may wish to use **BLM 6-5 Section 6.1 Achievement Check Rubric** to assist you in assessing your students' responses.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the Apply questions.
- **Questions 16 and 17** challenge students to make connections between algebraic and graphical reasoning.
- In **question 18**, students might be tempted to cancel out the common expressions in the numerator and denominator. It might be useful to have students discuss whether or not this can be done.

Common Errors

- Students confuse the mathematical meaning of a negative base and a negative exponent.

R$_x$ Have students perform patterning activities such as **questions 8 and 11**, as needed. Have students use a CAS graphing calculator to check their answers.

Accommodations

Language—encourage students to use an on-line dictionary to research new terms found in this chapter

Visual—have students work in groups to create posters describing the exponent laws and related examples for reference throughout the chapter

Motor—encourage students to check calculator keystrokes with a partner if their answers are incorrect

ESL—provide a partner to assist students when completing the Investigate, and to help students read and understand the Examples and Practise questions. Have students add new terms to their personal math dictionaries.

Achievement Check Answers (page 351)

14. a) i) Substitute, then evaluate.

$$\begin{aligned}\left((-2)^2\,3^{-1}\right)\left((-2)^{-2}\,3^2\right) &= 4\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)9\\ &= 36\left(\frac{1}{12}\right)\\ &= 3\end{aligned}$$

ii) Evaluate, then substitute.

$$\begin{aligned}(a^2b^{-1})(a^{-2}b^2) &= a^{2\,+\,(-2)}b^{-1\,+\,2}\\ &= a^0b^1\\ &= 1 \times b\\ &= 3\end{aligned}$$

b) Answers may vary. For example: I prefer the second method because it is easier to evaluate the expression once it is simplified.

Literacy Connect

- Have one or two students read the section opener out loud and discuss as a class.
- Have students record the six exponent laws in their notebooks.
- Encourage students to continue adding new terms to their personal math dictionaries.
- Have students discuss the meaning of each term in pairs or as a class.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	6
Reasoning and Proving	14, 16, 17
Reflecting	14
Selecting Tools and Computational Strategies	6, 15, 17, 18
Connecting	6, 16, 18
Representing	6, 16
Communicating	14

Extra Practice

- Use **BLM 6-4 Section 6.1 Exponent Laws** for extra practice or remediation.