6.2 Rational Exponents

Student Text Pages
352–361

Suggested Timing
80–160 min

Tools
- graphing calculators

Optional
- computers with *The Geometer's Sketchpad®*

Related Resources
BLM 6-6 Section 6.2 Rational Exponents
BLM 6-7 Section 6.2 Achievement Check Rubric

Link to Prerequisite Skills

Students should complete Powers and Square Roots, Exponent Laws, and Zero and Negative Exponents sections of the Prerequisite Skills before proceeding with this section.

Warm-Up

1. Evaluate.
a) $\sqrt{49}$ **b)** $\sqrt{144}$

2. Does the expression $\sqrt{-25}$ have any meaning? Explain.

Warm-Up Answers

1. a) 7 **b)** 12

2. No. The expression has no meaning because there is no real number that, when multiplied by itself, is -25.

Teaching Suggestions

- Depending on the needs of the class, this lesson could be divided into two periods. Monitor students' progress as they work through the first half of the lesson. If necessary, teach the second half the next day.

Warm-Up

- Display the Warm-Up questions. Have students complete the questions independently. Then, discuss the solutions as a class.

Investigate 1

- This Investigate reviews the concepts and terminology related to square roots, and extends this understanding to cube roots and *n*th roots. Use the **Literacy Connect** and key terms definitions to clarify the terminology and conventions.

Investigate 1 Answers (pages 352–353)

1. a) 6 **b)** 11

2. a) i) 8 **ii)** 27 **iii)** 216 **iv)** 1000

b) The cube root of a number can be determined by finding a value that when multiplied by itself three times gives the number.

c) 5; $5 \times 5 \times 5 = 125$

3. The fourth root of a number can be determined by finding a value that when multiplied by itself four times gives the number.

4. The fifth root of 32 could be written as $\sqrt[5]{32}$ and is evaluated by determining a value that when multiplied by itself five times is equal to 32. The fifth root of 32 is 2.

Examples 1 and 2

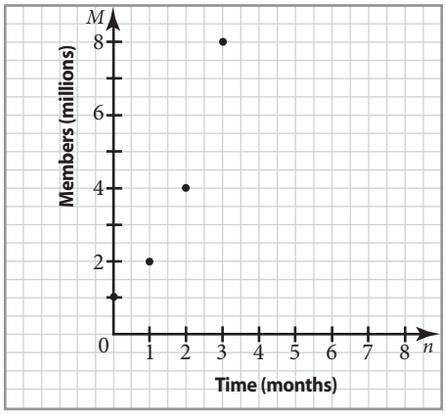
- Example 1 models the process of evaluating non-square roots by inspection. Students can use a calculator to verify their results by multiplying the base by itself the appropriate number of times.
- Example 2 shows how approximate values of non-square roots can be determined using a calculator when they cannot be easily evaluated by inspection. Calculator keystroke support may be needed. Remind students that not all calculators operate the same way, particularly when evaluating radicals. Have students use systematic trial or consult their user's manual to clarify the appropriate keystroke sequences for their calculators.

Investigate 2

- This Investigate has students discover the meaning of a power having a rational (i.e., fractional) exponent. The technique requires students to analyse the graph of an exponential relation and apply interpolation to make a connection between the graphic and the algebraic representation.
- Use the summary on page 356 to help consolidate student understanding of powers involving rational exponents.

Investigate 2 Answers (pages 354–356)

1. **Social Network Web Site Membership**



Each month, the number of members doubles.

2.

Time (months)	Members (millions)	First Differences	Second Differences	Ratio
0	1			
		1		2
1	2		1	
		2		2
2	4		2	
		4		2
3	8			

3. Yes. The ratios are equal.

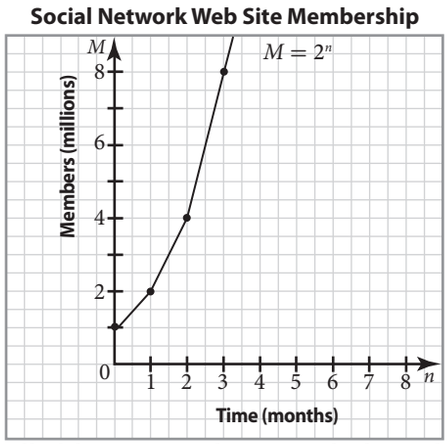
4. Estimates may vary. For example:

a) i) 1.5 **ii)** 1.25

b) The number of members doubled after each month. I assumed a constant increase in membership, so the number of members in the first half month would be $1 + (1 \div 2)$ and after the first quarter month would be $1 + (1 \div 4)$.

5. a) If $n = 0$, then $M = 2^0 = 1$, as given. If $n = 1$, then $M = 2^1 = 2$, as given. If $n = 2$, then $M = 2^2 = 4$, as given. If $n = 3$, then $M = 2^3 = 8$, as given. Thus, the table of values can be modelled by $M = 2^n$.

b)



All of the points on the scatter plot lie on the curve of the equation.

6. a) 1.41

b) 1.19

7. a) The values in step 6 represent the number of members of the Web site after one half month and after one quarter month.

b) Answers may vary. For example: These values are different from my predictions in step 4. They are lower than my predicted values.

8. a) 1.414 213 562

b) 1.189 207 115

c) These values are equal to those from step 6.

Examples 3 to 6

- These Examples show how powers with rational exponents can be evaluated.
- Example 3 focuses on powers with rational exponents of the form $\frac{1}{n}$.

  Students should become comfortable with the relationship between this type of a power and its corresponding representation in radical form, which suggests a more intuitive interpretation of the expression's value.
- Example 4 extends the previous concept to powers with rational exponents of the form $\frac{m}{n}$. The power of a power law is used to split the exponent into two parts, $\frac{1}{n}$ and m. The radical is evaluated first, then the power.
- Example 5 illustrates how to approximate values of powers involving rational exponents using a calculator. Calculator keystroke support may be needed.
- Example 6 poses a contextual problem where students need to apply the skills learned in previous Examples. Point out that a calculator is used to obtain a decimal answer.

Key Concepts

- Review the Key Concepts as a class. Have students write the concepts in their notebooks and provide an example for each.

Discuss the Concepts

- Students can complete these questions individually. Keystroke instructions may vary if students have different calculators.

Discuss the Concepts Suggested Answers (page 358)

D1. a) The cube root of a number can be found by determining a value that when multiplied by itself three times gives the number. For example, $4 \times 4 \times 4 = 64$, so the cube root of 64 is 4.

b) The cube root of a number can be found using the cube root button on a scientific calculator. For example, the cube root of 50 is approximately 3.684.

D2. The fourth root of a number can be found by determining a value that when multiplied by itself four times gives the number. For example, $2 \times 2 \times 2 \times 2 = 16$, so the fourth root of 16 is 2. The fourth root of a number can be found using the "*n*th-root" button on a scientific calculator. For example, the fourth root of 80 is approximately 2.991.

D3. First step: Apply the power law in reverse and write the power in radical form. Second step: Evaluate the radical. Third step: Evaluate the power.

Practise (A)

- You may wish to have students work in pairs or small groups to complete the Practise questions.
- Encourage students to refer to the Examples before asking for assistance.
- For **questions 1, 2,** and **4 to 7,** have students check their answers using a calculator.

Apply (B)

- Students might need a review of the mathematical terminology in **questions 8 to 10**. Ensure that students understand that an astronomical unit (AU) is a very large unit of length.
- **Question 11** provides a connection to physics. This question is an Achievement Check question. You may wish to use **BLM 6-7 Section 6.2 Achievement Check Rubric** to assist you in assessing your students' responses.
- **Question 12** provides an opportunity to assess students' ability to apply reasoning and communication.
- **Question 13** links to the Chapter Problem. Remind students to keep the solution to this question handy as it may help them with the Chapter Problem Wrap-Up.
- Students could use a spreadsheet or a CAS graphing calculator to examine additional cases quickly in **question 14**.
- Students should recognise the implied multiplication between the radicals in **question 15**.

Extend (C)

- Assign the Extend questions to students who are not being challenged by the Apply questions.
- **Question 17** provides an opportunity to assess students' ability to apply reasoning and communication.
- **Question 18** guides students to make connections between the powers involving negative rational exponents and related graphical representations. Graphing technology is recommended.

Common Errors

- Some students confuse the numerator (power) and denominator (radical) of a rational exponent.

R$_x$ Use **Example 4, part a)** as a model to show students how to apply the power law to separate the rational exponent into two parts. Explain which part is the power and which part is the radical.

- Some students confuse the mathematical meaning of a negative base and a negative exponent.

R$_x$ Have students perform patterning activities similar to **questions 8 and 11** in section 6.1, as needed. Have students use a CAS graphing calculator to check their answers.

Accommodations

ESL—discuss the importance of brackets as described in the **Literacy Connect** beside **Investigate 1**. Allow students to work in pairs when using the graphing calculator or computer in **Investigate 2**. Have students add new terms to their personal math dictionaries.

Language—Post the definition and an example of a rational exponent. Note the difference between the terms *rational* and *radical*.

Motor—for **Investigate 2**, have students work in pairs and use graphing calculators to graph the data

Perceptual—have students write the steps beside their examples, as in **Example 4, part a),** to enhance their understanding of converting a power with a rational exponent into radical form

Gifted and Enrichment—have students use the link beside **question 16** to research the solar system and present their findings to the class

Visual—encourage students to use colour to highlight the denominator of a rational exponent and the index of an equivalent radical

Achievement Check Answers (page 360)

11. a) Initially, $K = 10$ and $m = 5$.

$$\begin{aligned} v &= \left(\frac{2K}{m}\right)^{\frac{1}{2}} \\ &= \left(\frac{2 \times 10}{5}\right)^{\frac{1}{2}} \\ &= 4^{\frac{1}{2}} \\ &= \sqrt{4} \\ &= 2 \end{aligned}$$

The rabbit's velocity is 2 m/s.

b) The kinetic energy doubles; therefore $K = 20$.

$$\begin{aligned} v &= \left(\frac{2K}{m}\right)^{\frac{1}{2}} \\ &= \left(\frac{2 \times 20}{5}\right)^{\frac{1}{2}} \\ &= 8^{\frac{1}{2}} \\ &= \sqrt{8} \\ &\doteq 2.8 \end{aligned}$$

The rabbit's new velocity is approximately 2.8 m/s, which is not double its original velocity.

c) Use algebra.

$$v = \left(\frac{2K}{m}\right)^{\frac{1}{2}}$$

$v^2 = \frac{2K}{m}$ Square both sides.

$mv^2 = 2K$ Multiply both sides by m.

$K = \frac{mv^2}{2}$ Divide both sides by 2.

Literacy Connect

- Allow students to work in pairs to provide support when completing the Investigate and the Practise questions.
- Encourage students to continue adding new terms to their personal math dictionaries.
- Have students discuss the meaning of each term in pairs or as a class.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	14, 16, 17
Reasoning and Proving	2, 5, 10–14, 18
Reflecting	10, 12, 18
Selecting Tools and Computational Strategies	6, 10e), 17
Connecting	8–11, 13, 16
Representing	10, 13, 18
Communicating	2, 5, 10–14, 18

Extra Practice

- Use **BLM 6-6 Section 6.2 Rational Exponents** for extra practice or remediation.