7.1 Annuities

Student Text Pages
402–413

Suggested Timing
80–160 min

Tools
- graphing calculators
- computers with Internet access and spreadsheet software

Optional
- TI-Nspire™ CAS graphing calculators

Related Resources
BLM 7-3 Section 7.1 Annuities
BLM A-9 Communication General Scoring Rubric

Link to Prerequisite Skills

Students should complete all the Prerequisite Skills questions before proceeding with this section.

Warm-Up

1. How many months are in each time period?
a) 30 years **b)** 40 years **c)** 50 years
2. Calculate the future value of $1000 invested at 4% simple interest for two years.
3. Calculate the future value of $1000 invested at 4% per year, compounded annually, for ten years.
4. Calculate the future value of two $1000 investments earning no interest.

Warm-Up Answers

1. a) 360 months **b)** 480 months **c)** 600 months
2. $1080
3. $1081.60
4. $2000

Teaching Suggestions

Warm-Up

- Display the Warm-Up questions. Have students complete the questions independently. Then, discuss the solutions as a class.

Section Opener

- Use technology to calculate the annual investment needed to have $1 million in 40 years at 7% per year, compounded annually. Use these settings for the TVM Solver:
  N = 40
  I% = 7
  PV = 0
  PMT = 0
  FV = 1 000 000
  P/Y = 1
  C/Y = 1
  PMT: set to END

  Move the cursor to PMT and press ALPHA [SOLVE]. An annual payment of approximately $5000 for 40 years will yield the same future value as a one-time investment of $67 000 if both are invested at 7% per year, compounded annually.

Investigates

- Have students spend 5 min to 10 min working through Investigate 1. Discuss the results.
- Take 15 min to work through Investigate 2. Make sure students understand what each programmed cell is doing. Students could use a TVM Solver or an on-line calculator to reproduce the results

from Investigate 2. Ensure students understand that the calculator is performing the same operations as the spreadsheet.

- If a TVM Solver is not available, have students use the **future value of an annuity formula**:

$FV = \frac{PMT\,[(1 + i)^n - 1]}{i}$, where FV is the future value of the annuity, PMT is the annual payment, i is the interest rate per compounding period, as a decimal, and n is the number of payments.

Substitute the following values into the formula and solve for PMT.
$FV = 1\,000\,000$
$i = 0.07$
n = the difference between your age and 50
The formula can be another method for checking answers.

Investigate Answers (page 402–404)

Investigate 1, Method 1

Answers may vary. Sample answer is for a 17-year-old student.

5., 6.

Years Until Retirement	Annual Payment ($)
33	7858.00
38	5415.94
43	3771.86
48	2645.51

7. As the number of years until retirement increases, the amount of the annual payment decreases.

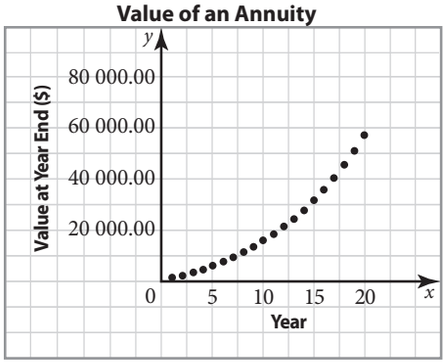
Investigate 2

1. to 7.

A	B	C	D	E	F	G
1	Year	Value at Start of Year ($)	Interest ($)	Value with Interest ($)	End of Year Investment ($)	Value at End of Year ($)
2	1	0.00	0.00	0.00	1 000.00	1 000.00
3	2	1 000.00	100.00	1 100.00	1 000.00	2 100.00
4	3	2 100.00	210.00	2 310.00	1 000.00	3 310.00
5	4	3 310.00	331.00	3 641.00	1 000.00	4 641.00
6	5	4 641.00	464.10	5 105.10	1 000.00	6 105.10
7	6	6 105.10	610.51	6 715.61	1 000.00	7 715.61
8	7	7 715.61	771.56	8 487.17	1 000.00	9 487.17
9	8	9 487.17	948.72	10 435.89	1 000.00	11 435.89
10	9	11 435.89	1 143.59	12 579.48	1 000.00	13 579.48
11	10	13 579.48	1 357.95	14 937.42	1 000.00	15 937.42
12	11	15 937.42	1 593.74	17 531.17	1 000.00	18 531.17
13	12	18 531.17	1 853.12	20 384.28	1 000.00	21 384.28
14	13	21 384.28	2 138.43	23 522.71	1 000.00	24 522.71
15	14	24 522.71	2 452.27	26 974.98	1 000.00	27 974.98
16	15	27 974.98	2 797.50	30 772.48	1 000.00	31 772.48
17	16	31 772.48	3 177.25	34 949.73	1 000.00	35 949.73
18	17	35 949.73	3 594.97	39 544.70	1 000.00	40 544.70
19	18	40 544.70	4 054.47	44 599.17	1 000.00	45 599.17
20	19	45 599.17	4 559.92	50 159.09	1 000.00	51 159.09
21	20	51 159.09	5 115.91	56 275.00	1 000.00	57 275.00

8. The future value of the investment.

10.



11. The value at the end of each year is increasing exponentially with the number of years. No. The value at the end of 40 years will be more than double the value at the end of 20 years because the interest is compounded.

Examples

- For Example 1, ensure students understand that a series of payments is being represented. Discuss the effect of time on each deposit. Point out that each successive deposit earns slightly less interest because it is invested for a shorter time period. Ensure that students understand that an annuity is merely the sum of a sequence of compound interest calculations. Have students compare the amount of money saved versus the future value at the end of the six months.
- For Example 2, ensure students understand that this is how most loan payment schedules are determined. A key concept here is that there is a difference between the amount borrowed and the amount repaid. This difference is the interest. Discuss how changing any one of the variables affects the others.
- Example 2, part a), could also be solved using the **present value of an annuity formula**:

a) $PV = \dfrac{PMT\,[1 - (1 + i)^{-n}]}{i}$

Substitute the following values into the formula and solve for PMT.

$n = 30$

$i = 0.0075$ $\qquad i = \dfrac{0.09}{12}$

$PV = 16\,000$

$$16\,000 = \frac{PMT\,[1 - (1 + 0.0075)^{-30}]}{0.0075}$$

$120 = PMT\,(0.2008\ldots)$ Multiply both sides by 0.0075.

$$PMT = \frac{120}{0.2008\ldots}$$

$PMT = 597.57$ Round your answers to two decimal places.

T.J.'s monthly payment will be $597.57.

This algebraic solution may be more difficult for some students so showing each step is important to reduce errors in calculations.

- For Example 3, when money is being borrowed, the present value is the amount borrowed. This is the amount that could be paid, in cash, on the day of purchase. By multiplying the payment amount by the number of payments, the total amount that is repaid (and, thus, the interest) can be determined.

- Example 3, part a), could also be solved using the **present value of an annuity formula:**

a) $PV = \dfrac{PMT\,[1 - (1 + i)^{-n}]}{i}$

Substitute the following values into the formula and solve for *PMT*.

$n = 12$

$i = 0.00\,875 \qquad i = \dfrac{0.105}{12}$

$PMT = 229.19$

$$PV = \frac{229.19\,[1 - (1 + 0.00\,875)^{-12}]}{0.00\,875}$$

$PV = 2600.04$ Round your answers to two decimal places.

This algebraic solution may be more difficult for some students, so showing each step is important to reduce errors in calculations.

Technology

- You may wish to use the Use Technology section on pages 412 to 413 as an alternative to Example 3. The section gives instructions on how to use the TI-Nspire™ CAS graphing calculator to find the present value of an annuity. Students need to become familiar with their calculators' functions.

Key Concepts

- Review the Key Concepts as a class. To assess their understanding, have students provide an example of each concept.

Discuss the Concepts

- Have students provide examples to support their answers to **questions D1 and D2**. For question D2, time has two effects: more money is invested and the money that is invested has more time to earn interest. Earning more interest is the more obvious answer. More money being invested is a significant difference from the one-time investments that were the focus of grade 11.

Discuss the Concepts Suggested Answers (page 408)

D1. An annuity is a series of investments, separated by the same period of time and subject to the same interest rates. A one-time investment is a single investment.

D2. As time increases, the future value also increases. More money is invested in the annuity and the money that is invested earns more interest.

D3. If you know the present value of an annuity, you can determine the future value using a TVM Solver. Then you will know how much money you will have in the future and can make a financial plan.

Practise (A)

- For **question 1**, students may benefit from checking their answers using a TVM Solver or an on-line calculator. Ensure that students enter the values into the correct fields.

Apply (B)

- Understanding how to use the TVM Solver is key to students' success with these questions. Have struggling students use the Examples as models when filling in the fields in the TVM Solver. Not all questions need to be assigned to all students.
- **Question 17** links to the Chapter Problem. Remind students to keep the solution to this question handy as the methods they used may help them with the Chapter Problem Wrap-Up.

Common Errors

- Some students enter the given values in the wrong fields in the TVM Solver.

R_x Have students estimate their answers before using the TVM Solver. Some students may benefit from drawing a timeline.

- Some students enter the number of years rather than the number of payments for **N**. This error is examined in **question 8**.

R_x Have students estimate their answers before using the TVM Solver

Accommodations

Motor—for **Investigate 1**, have students work with a partner who can read the instructions and values during data entry

Perceptual—for **Investigate 2**, use an LCD projector to complete the spreadsheet and graph with the class

Spatial—for the **Examples**, construct a timeline along a wall to follow the path of each deposit or payment

ESL—ask students to record unfamiliar words and terms in their personal math dictionaries. Encourage students to use diagrams, symbols, their first language, or other means of recording and understanding the meaning of the unfamiliar word. Pair them with a classmate who can help them understand the meanings of new terms.

Language—start a Word Wall of key terms and definitions

Memory—create a handout or a poster of the TVM Solver screen and provide a written description of each field. For example, **N** = total number of payments or deposits, **I%** = annual interest rate.

Visual—post graphing calculator posters around the classroom for quick reference

Extend (C)

- Assign the Extend question to students who are not being challenged by the questions in Apply. When discussing the Extend question with students, emphasize the importance of goal setting and planning, and the effects of time with respect to meeting one's goals.

Literacy Connect

- Allow students to work in pairs when completing the Investigate and Practise questions.
- Discuss the meaning of the new financial terms in this section: annuity, ordinary annuity, annuity due, timeline, future value, present value, Registered Retirement Savings Plan (RRSP), and depreciation rate. Encourage students to record the terms and definitions in their personal math dictionaries and give an example for each term.
- Students should also explain the meaning of each variable in the TVM Solver, with examples.

Mathematical Process Expectations

Process Expectation	Questions
Problem Solving	11, 13, 17, 18
Reasoning and Proving	7, 10, 11, 13, 14, 18
Reflecting	7, 8
Selecting Tools and Computational Strategies	2, 9–13, 15–18
Connecting	6, 7, 9, 11–14, 16–18
Representing	1, 13
Communicating	7, 10, 11, 13, 14

Ongoing Assessment

- Assess students' ability to communicate mathematically and to justify their thinking. You may wish to use **BLM A-9 Communication General Scoring Rubric** to assist you in assessing students' responses for **question 11**.

Extra Practice

- Use **BLM 7-3 Section 7.1 Annuities** for extra practice or remediation.