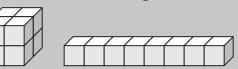# 1.5 Investigate Optimum Volume and Surface Area

Link to Prerequisite Skills

Students should complete all the Prerequisite Skills questions before proceeding with this section.

Warm-Up

Consider these two linking cube models.

1. How are these models similar? How are they different?
2. What is the volume of each object, in cubic units?
3. What is the surface area of each object, in square units?

Warm-Up Answers

1. Similar: They are both square-based prisms. Different: They are not congruent.
2. Both have a volume of eight cubic units.
3. cube: 24 square units; prism: 34 square units

Teaching Suggestions

Warm-Up
• Use linking cubes to build the models and show them to students. Discuss the questions as a class.

Investigates
• This entire section is an Investigate activity. Section 1.6 has the corresponding Examples, Discuss the Concepts questions, and Practise questions.
• Consider scheduling the class in a computer lab and organizing activity stations for Investigate 1 and the four methods of Investigate 2, five stations in all. Have small groups of students work through the stations, taking approximately 15 to 20 min per station. Depending on their progress, each group may only work through three or four stations. All students should complete Investigate 1 and at least two methods of Investigate 2.
• Investigate 1 has students explore the minimum surface area of a square-based prism with a given volume. Students should discover that this occurs when the prism is a cube. This is important in architecture and engineering. It relates to minimizing expenses related to heat loss through walls, and amount of building materials.
• From Investigate 2, students should discover that the maximum volume of a square-based prism with given surface area occurs when the prism is a cube.
• For Method 1, students use *The Geometer's Sketchpad*® with a pre-made sketch to model different possible prisms. This allows students to visualize the model in two dimensions and alter it dynamically to quickly explore several cases.

Visual—have students draw the sketches for Investigate 1, step 1, on poster-sized grid paper, with dimensions and surface area given, to compare the various dimensions of the 64-unit building before proceeding to step 2

Perceptual—for Investigate 2, provide various 1 L bottles or a set of containers with the same volume. Have students measure and compare the dimensions of each container.

Motor—use an LCD projector to demonstrate the technology steps for Investigate 2. Provide a partner to assist with giving instructions and operating the technology.

ESL—provide a partner to assist with reading and interpreting the Investigates. Discuss the meanings of any special terms in this lesson.

Gifted and Enrichment—have students collect and examine a variety of containers at home. Have them prepare a presentation on the optical illusions often used in marketing to create the appearance that certain containers hold a larger volume than others.

- For Method 2, students use TABS+ to create their own model and examine it in both two and three dimensions. Students can also click and drag this model to examine several cases dynamically. A spreadsheet or graphing calculator is recommended to perform the required surface area calculations for the several cases being examined.
- Method 3 involves a more abstract, algebraic approach to the problem. Students who exhibit strength in algebraic reasoning should have the opportunity to work at this station. A graphing calculator or spreadsheet is recommended to perform the required surface area calculations for the several cases being examined.
- In Method 4, the TI-Nspire™ CAS graphing calculator is used to take an algebraic approach to the problem. This method may be suitable for students who would benefit from seeing an algebraic method but whose algebra skills are weak.
- After students work through several stations, provide an opportunity for the groups to share their methods and results with the class, and to compare the relative strengths and shortcomings of each tool and strategy.

Investigate Answers (pages 46–53)

Investigate 1

1. Answers may vary.
2. a) A cube with a side length of four units.
   b) 96 units^2. This is the minimum surface area because the maximum number of units is inside the building.
3. Answers may vary. For example; Minimizing surface area is important because fewer windows will be needed (and windows are expensive), the building temperature will be easier to regulate, and the cost of exterior building materials will be minimized.

Investigate 2

Method 3

1. a) 19.9 cm
   b) 600.5 cm^2
2.

Radius (cm)	Diameter (cm)	Height (cm)	Surface Area (cm^2)
4	8	19.9	600.5
5	10	12.7	556
6	12	8.8	558
7	14	6.5	594

3. a) The surface area decreases as the radius increases from 4 cm to 5 cm, then it increases.
   b) The models with radius 5 cm or 6 cm have the least surface area.
   c) radius: 5.4 cm, height: 10.8 cm
4. The radius is half the height of the cylinder, or the diameter and the height are equal. Diagrams may vary.

Literacy Connect

- Ask individual students to read the steps of each Investigate and clarify the instructions if necessary.
- Ensure students are provided with support when using technology. Have students refer to the Technology Tips and the Technology Appendix in the back of their textbook.

Ongoing Assessment

- While students are working on the Investigates, circulate to see how well each student works in a group. This may be an opportunity to observe and record individual students' group work. Use **BLM A-12 Group Work Assessment General Scoring Rubric** to assist you in assessing your students.