Blueprint for *Pre-Calculus 12* Final Exam

Trigonometry **General Outcome:** Develop trigonometric reasoning.	
Specific Outcome: Demonstrate an understanding of angles in standard position, expressed in degrees and radians. [CN, ME, R, V]	
1.1 Sketch, in standard position, an angle (positive or negative) when the measure is given in degrees.	
1.2 Describe the relationship among different systems of angle measurement, with emphasis on radians and degrees.	
1.3 Sketch, in standard position, an angle with a measure of 1 radian.	MC #28 Conceptual
1.4 Sketch, in standard position, an angle with a measure expressed in the form $k\pi$ radians, where $k \in Q$.	
1.5 Express the measure of an angle in radians (exact value or decimal approximation), given its measure in degrees.	
1.6 Express the measure of an angle in degrees, given its measure in radians (exact value or decimal approximation).	
1.7 Determine the measures, in degrees or radians, of all angles in a given domain that are coterminal with a given angle in standard position.	WR #17 Procedural WR #18 Procedural
1.8 Determine the general form of the measures, in degrees or radians, of all angles that are coterminal with a given angle in standard position.	MC #42 Conceptual
1.9 Explain the relationship between the radian measure of an angle in standard position and the length of the arc cut on a circle of radius *r*, and solve problems based upon that relationship.	
Specific Outcome: Develop and apply the equation of the unit circle. [CN, R, V]	
2.1 Derive the equation of the unit circle from the Pythagorean theorem.	
2.2 Describe the six trigonometric ratios, using a point P (x, y) that is the intersection of the terminal arm of an angle and the unit circle.	MC #35 Procedural
2.3 Generalize the equation of a circle with centre (0, 0) and radius *r*.	
Specific Outcome: Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees. [ME, PS, R, T, V]	
3.1 Determine, with technology, the approximate value of a trigonometric ratio for any angle with a measure expressed in either degrees or radians.	MC #36 Procedural
3.2 Determine, using a unit circle or reference triangle, the exact value of a trigonometric ratio for angles expressed in degrees that are multiples of 0º, 30º, 45º, 60º or 90º, or for angles expressed in radians that are multiples of 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, or $\frac{\pi}{2}$, and explain the strategy.	MC #37 Procedural
3.3 Determine, with or without technology, the measures, in degrees or radians, of the angles in a specified domain, given the value of a trigonometric ratio.	
3.4 Explain how to determine the exact values of the six trigonometric ratios, given the coordinates of a point on the terminal arm of an angle in standard position.	
3.5 Determine the measures of the angles in a specified domain in degrees or radians, given a point on the terminal arm of an angle in standard position.	MC #38 Procedural MC #39 Procedural
3.6 Determine the exact values of the other trigonometric ratios, given the value of one trigonometric ratio in a specified domain.	MC #40 Procedural

3.7 Sketch a diagram to represent a problem that involves trigonometric ratios.	
3.8 Solve a problem, using trigonometric ratios.	MC #41Problem Solving
Specific Outcome: Graph and analyze the trigonometric functions sine, cosine and tangent to solve problems. [CN, PS, T, V]	
4.1 Sketch, with or without technology, the graph of $y = \sin x$, $y = \cos x$ or $y = \tan x$.	
4.2 Determine the characteristics (amplitude, asymptotes, domain, period, range and zeros) of the graph of $y = \sin x$, $y = \cos x$ or $y = \tan x$.	MC #25 Conceptual, Procedural
4.3 Determine how varying the value of a affects the graphs of $y = a \sin x$ and $y = a \cos x$.	MC #26 Conceptual, Procedural
4.4 Determine how varying the value of d affects the graphs of $y = \sin x + d$ and $y = \cos x + d$.	MC #27 Conceptual
4.5 Determine how varying the value of c affects the graphs of $y = \sin (x + c)$ and $y = \cos (x + c)$.	
4.6 Determine how varying the value of b affects the graphs of $y = \sin bx$ and $y = \cos bx$.	
4.7 Sketch, without technology, graphs of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$, using transformations, and explain the strategies.	WR #16 Procedural
4.8 Determine the characteristics (amplitude, asymptotes, domain, period, phase shift, range and zeros) of the graph of a trigonometric function of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.	
4.9 Determine the values of a, b, c and d for functions of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$ that correspond to a given graph, and write the equation of the function.	
4.10 Determine a trigonometric function that models a situation to solve a problem.	
4.11 Explain how the characteristics of the graph of a trigonometric function relate to the conditions in a problem situation.	
4.12 Solve a problem by analyzing the graph of a trigonometric function.	
Specific Outcome: Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians. [CN, PS, R, T, V]	
5.1 Verify, with or without technology, that a given value is a solution to a trigonometric equation.	MC #29 Procedural
5.2 Determine, algebraically, the solution of a trigonometric equation, stating the solution in exact form when possible.	MC #30 Procedural MC #31 Procedural
5.3 Determine, using technology, the approximate solution of a trigonometric equation in a restricted domain.	
5.4 Relate the general solution of a trigonometric equation to the zeros of the corresponding trigonometric function (restricted to sine and cosine functions).	
5.5 Determine, using technology, the general solution of a given trigonometric equation.	
5.6 Identify and correct errors in a solution for a trigonometric equation.	
Specific Outcome: Prove trigonometric identities, using: · reciprocal identities · quotient identities · Pythagorean identities · sum or difference identities (restricted to sine, cosine and tangent) · double-angle identities (restricted to sine, cosine and tangent). [R, T, V]	
6.1 Explain the difference between a trigonometric identity and a trigonometric equation.	MC #32 Conceptual

6.2 Verify a trigonometric identity numerically for a given value in either degrees or radians.	
6.3 Explain why verifying that the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid.	
6.4 Determine, graphically, the potential validity of a trigonometric identity, using technology.	MC #33 Problem Solving
6.5 Determine the non-permissible values of a trigonometric identity.	MC #34 Procedural
6.6 Prove, algebraically, that a trigonometric identity is valid.	
6.7 Determine, using the sum, difference and double-angle identities, the exact value of a trigonometric ratio.	
Relations and Functions **General Outcome:** Develop algebraic and graphical reasoning through the study of relations.	
Specific Outcome: Demonstrate an understanding of operations on, and compositions of, functions. [CN, R, T, V]	
1.1 Sketch the graph of a function that is the sum, difference, product or quotient of two functions, given their graphs.	WR #9 Procedural
1.2 Write the equation of a function that is the sum, difference, product or quotient of two or more functions, given their equations.	
1.3 Determine the domain and range of a function that is the sum, difference, product or quotient of two functions.	WR #9 Procedural
1.4 Write a function $h(x)$ as the sum, difference, product or quotient of two or more functions.	WR #9 Procedural
1.5 Determine the value of the composition of functions when evaluated at a point, including: • $f(f(a))$ • $f(g(a))$ • $g(f(a))$.	WR #12 Procedural
1.6 Determine, given the equations of two functions $f(x)$ and $g(x)$, the equation of the composite function: • $f(f(x))$ • $f(g(x))$ • $g(f(x))$ and explain any restrictions.	WR #12 Procedural
1.7 Sketch, given the equations of two functions $f(x)$ and $g(x)$, the graph of the composite function: • $f(f(x))$ • $f(g(x))$ • $g(f(x))$.	
1.8 Write a function $h(x)$ as the composition of two or more functions.	MC #14 Procedural
1.9 Write a function $h(x)$ by combining two or more functions through operations on, and compositions of, functions.	
Specific Outcome: Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations. [C, CN, R, V]	
2.1 Compare the graphs of a set of functions of the form $y – k = f(x)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of k.	MC #1 Conceptual
2.2 Compare the graphs of a set of functions of the form $y = f(x – h)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of h.	MC #2 Conceptual
2.3 Compare the graphs of a set of functions of the form $y − k = f(x – h)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effects of h and k.	WR #2 Procedural

2.4 Sketch the graph of $y - k = f(x)$, $y = f(x - h)$ or $y - k = f(x - h)$ for given values of h and k, given a sketch of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.	
2.5 Write the equation of a function whose graph is a vertical and/or horizontal translation of the graph of the function $y = f(x)$.	MC #15 Conceptual
Specific Outcome: Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations. [C, CN, R, V]	
3.1 Compare the graphs of a set of functions of the form $y = af(x)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of a.	
3.2 Compare the graphs of a set of functions of the form $y = f(bx)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of b.	
3.3 Compare the graphs of a set of functions of the form $y = af(bx)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effects of a and b.	MC #3 Procedural
3.4 Sketch the graph of $y = af(x)$, $y = f(bx)$ or $y = af(bx)$ for given values of a and b, given a sketch of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.	
3.5 Write the equation of a function, given its graph which is a vertical and/or horizontal stretch of the graph of the function $y = f(x)$.	
Specific Outcome: Apply translations and stretches to the graphs and equations of functions. [C, CN, R, V]	
4.1 Sketch the graph of the function $y - k = af(b(x - h))$ for given values of a, b, h and k, given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.	WR #3 Procedural
4.2 Write the equation of a function, given its graph which is a translation and/or stretch of the graph of the function $y = f(x)$.	WR #4 Procedural
Specific Outcome: Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the: · x-axis · y-axis · line $y = x$. [C, CN, R, V]	
5.1 Generalize the relationship between the coordinates of an ordered pair and the coordinates of the corresponding ordered pair that results from a reflection through the x-axis, the y-axis or the line $y = x$.	MC #6 Conceptual
5.2 Sketch the reflection of the graph of a function $y = f(x)$ through the x-axis, the y-axis or the line $y = x$, given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.	WR #5 Procedural
5.3 Generalize, using inductive reasoning, and explain rules for the reflection of the graph of the function $y = f(x)$through the x-axis, the y-axis or the line $y = x$.	
5.4 Sketch the graphs of the functions $y = -f(x)$, $y = f(-x)$ and $x = -f(y)$, given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.	
5.5 Write the equation of a function, given its graph which is a reflection of the graph of the function $y = f(x)$ through the x-axis, the y-axis or the line $y = x$.	
Specific Outcome: Demonstrate an understanding of inverses of relations. [C, CN, R, V]	
6.1 Explain how the graph of the line $y = x$ can be used to sketch the inverse of a relation.	
6.2 Explain how the transformation $(x, y) => (y, x)$ can be used to sketch the inverse of a relation.	
6.3 Sketch the graph of the inverse relation, given the graph of a relation.	
6.4 Determine if a relation and its inverse are functions.	MC #4 Conceptual

6.5 Determine restrictions on the domain of a function in order for its inverse to be a function.	
6.6 Determine the equation and sketch the graph of the inverse relation, given the equation of a linear or quadratic relation.	
6.7 Explain the relationship between the domains and ranges of a relation and its inverse.	
6.8 Determine, algebraically or graphically, if two functions are inverses of each other.	MC #5 Conceptual
Specific Outcome: Demonstrate an understanding of logarithms. [CN, ME, R]	
7.1 Explain the relationship between logarithms and exponents.	MC #7 Conceptual
7.2 Express a logarithmic expression as an exponential expression and vice versa.	
7.3 Determine, without technology, the exact value of a logarithm, such as $\log_2 8$.	MC #8 Conceptual
7.4 Estimate the value of a logarithm, using benchmarks, and explain the reasoning; e.g., since $\log_2 8 = 3$ and $\log_2 16 = 4$, $\log_2 9$ is approximately equal to 3.1.	
Specific Outcome: Demonstrate an understanding of the product, quotient and power laws of logarithms. [C, CN, R, T]	
8.1 Develop and generalize the laws for logarithms, using numeric examples and exponent laws.	
8.2 Derive each law of logarithms.	
8.3 Determine, using the laws of logarithms, an equivalent expression for a logarithmic expression.	
8.4 Determine, with technology, the approximate value of a logarithmic expression, such as $\log_2 9$.	
Specific Outcome: Graph and analyze exponential and logarithmic functions. [C, CN, T, V]	
9.1 Sketch, with or without technology, a graph of an exponential function of the form $y = a^x$, $a > 0$.	MC #9 Procedural
9.2 Identify the characteristics of the graph of an exponential function of the form $y = a^x$, $a > 0$, including the domain, range, horizontal asymptote and intercepts, and explain the significance of the horizontal asymptote.	
9.3 Sketch the graph of an exponential function by applying a set of transformations to the graph of $y = a^x$, $a > 0$, and state the characteristics of the graph.	
9.4 Sketch, with or without technology, the graph of a logarithmic function of the form $y = \log b\, x$, $b > 1$.	WR #6 Procedural
9.5 Identify the characteristics of the graph of a logarithmic function of the form $y = \log_b x$, $b > 1$, including the domain, range, vertical asymptote and intercepts, and explain the significance of the vertical asymptote.	WR #7 Procedural
9.6 Sketch the graph of a logarithmic function by applying a set of transformations to the graph of $y = \log_b x$, $b > 1$, and state the characteristics of the graph.	
9.7 Demonstrate, graphically, that a logarithmic function and an exponential function with the same base are inverses of each other.	
Specific Outcome: Solve problems that involve exponential and logarithmic equations. [C, CN, PS, R]	
10.1 Determine the solution of an exponential equation in which the bases are powers of one another.	
10.2 Determine the solution of an exponential equation in which the bases are not powers of one another, using a variety of strategies.	
10.3 Determine the solution of a logarithmic equation, and verify the solution.	WR #1 Procedural

10.4 Explain why a value obtained in solving a logarithmic equation may be extraneous.	
10.5 Solve a problem that involves exponential growth or decay.	
10.6 Solve a problem that involves the application of exponential equations to loans, mortgages and investments.	WR #8 Problem Solving
10.7 Solve a problem that involves logarithmic scales, such as the Richter scale and the pH scale.	
10.8 Solve a problem by modelling a situation with an exponential or a logarithmic equation.	
Specific Outcome: Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree ≤ 5 with integral coefficients). [C, CN, ME]	
11.1 Explain how long division of a polynomial expression by a binomial expression of the form $x - a$, $a \in I$, is related to synthetic division.	
11.2 Divide a polynomial expression by a binomial expression of the form $x - a$, $a \in I$, using long division or synthetic division.	WR #10 Procedural
11.3 Explain the relationship between the linear factors of a polynomial expression and the zeros of the corresponding polynomial function.	
11.4 Explain the relationship between the remainder when a polynomial expression is divided by $x - a$, $a \in I$, and the value of the polynomial expression at $x = a$ (remainder theorem).	WR #10 Procedural
11.5 Explain and apply the factor theorem to express a polynomial expression as a product of factors.	
Specific Outcome: Graph and analyze polynomial functions (limited to polynomial functions of degree ≤ 5). [C, CN, T, V]	
12.1 Identify the polynomial functions in a set of functions, and explain the reasoning.	
12.2 Explain the role of the constant term and leading coefficient in the equation of a polynomial function with respect to the graph of the function.	
12.3 Generalize rules for graphing polynomial functions of odd or even degree.	WR #11 Conceptual
12.4 Explain the relationship between: • the zeros of a polynomial function • the roots of the corresponding polynomial equation • the *x*-intercepts of the graph of the polynomial function.	
12.5 Explain how the multiplicity of a zero of a polynomial function affects the graph.	
12.6 Sketch, with or without technology, the graph of a polynomial function.	MC #16 Conceptual
12.7 Solve a problem by modelling a given situation with a polynomial function and analyzing the graph of the function.	
Specific Outcome: Graph and analyze radical functions (limited to functions involving one radical). [CN, R, T, V]	
13.1 Sketch the graph of the function $y = \sqrt{x}$, using a table of values, and state the domain and range.	MC #10 Procedural MC #11 Procedural
13.2 Sketch the graph of the function $y - k = a\sqrt{b(x-h)}$ by applying transformations to the graph of the function $y = \sqrt{x}$, and state the domain and range.	MC #12 Procedural
13.3 Sketch the graph of the function $y = \sqrt{f(x)}$, given the graph of the function $y = f(x)$, and explain the strategies used.	MC #17 Problem Solving

13.4 Compare the domain and range of the function $y = \sqrt{f(x)}$, to the domain and range of the function $y = f(x)$, and explain why the domains and ranges may differ.	MC #18 Problem Solving
13.5 Describe the relationship between the roots of a radical equation and the *x*-intercepts of the graph of the corresponding radical function.	
13.6 Determine, graphically, an approximate solution of a radical equation.	
Specific Outcome: Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials or trinomials). [CN, R, T, V]	
14.1 Graph, with or without technology, a rational function.	MC #13 Procedural
14.2 Analyze the graphs of a set of rational functions to identify common characteristics.	
14.3 Explain the behaviour of the graph of a rational function for values of the variable near a non-permissible value.	
14.4 Determine if the graph of a rational function will have an asymptote or a hole for a non-permissible value.	MC #19 Procedural, Problem Solving
14.5 Match a set of rational functions to their graphs, and explain the reasoning.	
14.6 Describe the relationship between the roots of a rational equation and the *x*-intercepts of the graph of the corresponding rational function.	
14.7 Determine, graphically, an approximate solution of a rational equation.	MC #20 Procedural
Permutations, Combinations and Binomial Theorem **General Outcome:** Develop algebraic and numeric reasoning that involves combinatorics.	
Specific Outcome: Apply the fundamental counting principle to solve problems. [C, PS, R, V]	
1.1 Count the total number of possible choices that can be made, using graphic organizers such as lists and tree diagrams.	MC #43 Problem Solving MC #44 Conceptual
1.2 Explain, using examples, why the total number of possible choices is found by multiplying rather than adding the number of ways the individual choices can be made.	
1.3 Solve a simple counting problem by applying the fundamental counting principle.	MC #21 Problem Solving
Specific Outcome: Determine the number of permutations of *n* elements taken *r* at a time to solve problems. [C, PS, R, V]	
2.1 Count, using graphic organizers such as lists and tree diagrams, the number of ways of arranging the elements of a set in a row.	
2.2 Determine, in factorial notation, the number of permutations of *n* different elements taken *n* at a time to solve a problem.	MC #22 Problem Solving
2.3 Determine, using a variety of strategies, the number of permutations of *n* different elements taken *r* at a time to solve a problem.	
2.4 Explain why *n* must be greater than or equal to *r* in the notation ${}_nP_r$.	
2.5 Solve an equation that involves ${}_nP_r$ notation, such as ${}_nP_2 = 30$.	MC #22 Problem Solving
2.6 Explain, using examples, the effect on the total number of permutations when two or more elements are identical.	
Specific Outcome: Determine the number of combinations of *n* different elements taken *r* at a time to solve problems. [C, PS, R, V]	
3.1 Explain, using examples, the difference between a permutation and a combination.	MC #23 Procedural
3.2 Determine the number of ways that a subset of *k* elements can be selected from a set of *n* different elements.	
3.3 Determine the number of combinations of *n* different elements taken *r* at a time to solve a problem.	

3.4 Explain why n must be greater than or equal to r in the notation ${}_nC_r$ or $\binom{n}{r}$.	
3.5 Explain, using examples, why ${}_nC_r = {}_nC_{n-r}$ or $\binom{n}{r} = \binom{n}{n-r}$.	WR #13 Procedural
3.6 Solve an equation that involves ${}_nC_r$ or $\binom{n}{r}$ notation, such as ${}_nC_2 = 15$ or $\binom{n}{2} = 15$.	
Specific Outcome: Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers). [CN, R, V]	
4.1 Explain the patterns found in the expanded form of $(x + y)^n$, $n \leq 4$ and $n \in N$, by multiplying n factors of $(x + y)$.	
4.2 Explain how to determine the subsequent row in Pascal's triangle, given any row.	WR #14 Procedural WR #13 Procedural
4.3 Relate the coefficients of the terms in the expansion of $(x + y)^n$ to the $(n + 1)$ row in Pascal's triangle.	
4.4 Explain, using examples, how the coefficients of the terms in the expansion of $(x + y)^n$ are determined by combinations.	
4.5 Expand, using the binomial theorem, $(x + y)^n$.	
4.6 Determine a specific term in the expansion of $(x + y)^n$.	MC #24 Procedural