Chapter 10 Function Operations

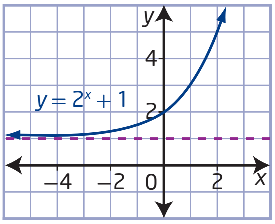
10.1 Sums and Differences of Functions, pages 483 to 487

1. a) $h(x) = |x - 3| + 4$ **b)** $h(x) = 2x - 3$
c) $h(x) = 2x^2 + 3x + 2$ **d)** $h(x) = x^2 + 5x + 4$

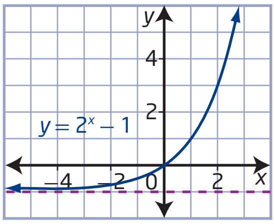
2. a) $h(x) = 5x + 2$ **b)** $h(x) = -3x^2 - 4x + 9$
c) $h(x) = -x^2 - 3x + 12$ **d)** $h(x) = \cos x - 4$

3. a) $h(x) = x^2 - 6x + 1$; $h(2) = -7$
b) $m(x) = -x^2 - 6x + 1$; $m(1) = -6$
c) $p(x) = x^2 + 6x - 1$; $p(1) = 6$

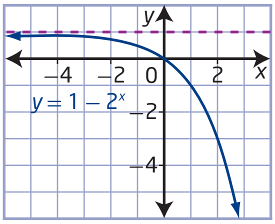
4. a) $y = 3x^2 + 2 + \sqrt{x + 4}$; domain $\{x \mid x \geq -4, x \in \mathrm{R}\}$
b) $y = 4x - 2 - \sqrt{x + 4}$; domain $\{x \mid x \geq -4, x \in \mathrm{R}\}$
c) $y = \sqrt{x + 4} - 4x + 2$; domain $\{x \mid x \geq -4, x \in \mathrm{R}\}$
d) $y = 3x^2 + 4x$; domain $\{x \mid x \in \mathrm{R}\}$

5. a) 

domain $\{x \mid x \in R\}$, range $\{y \mid y > 1, y \in R\}$

b) 

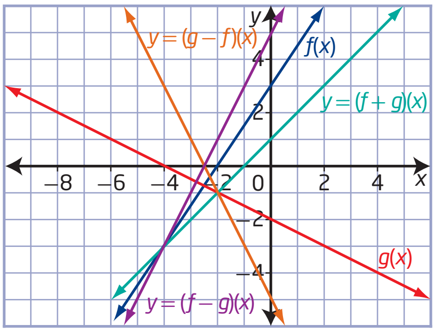
domain $\{x \mid x \in R\}$, range $\{y \mid y > -1, y \in R\}$

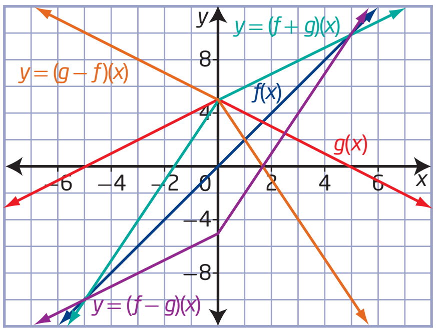
c) 

domain $\{x \mid x \in R\}$, range $\{y \mid y < 1, y \in R\}$

6. a) 8 **b)** 6 **c)** 7
d) not in the domain

7. a) B **b)** C **c)** A

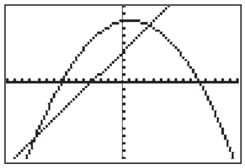
8. a) 

b) 

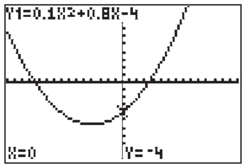
9. a) $y = 3x^2 + 11x + 1$ **b)** $y = 3x^2 - 3x + 3$
c) $y = 3x^2 + 3x + 1$ **d)** $y = 3x^2 - 11x + 3$

10. a) $g(x) = x^2$ **b)** $g(x) = \sqrt{x + 7}$
c) $g(x) = -3x + 1$ **d)** $g(x) = 3x^2 - x - 4$

11. a) $g(x) = x^2 - 1$ **b)** $g(x) = -\sqrt{x - 4}$
c) $g(x) = 8x - 9$ **d)** $g(x) = 2x^2 - 11x - 6$

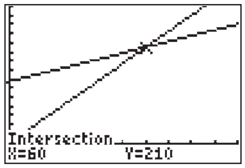
12. a) 

The points of intersection represent where the supply equals the demand. The intersection point in quadrant III should not be considered since the price cannot be negative.

b) 

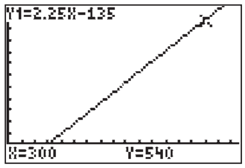
It represents the excess supply as a function of cost.

13. a) $C(n) = 1.25n + 135$, $R(n) = 3.5n$

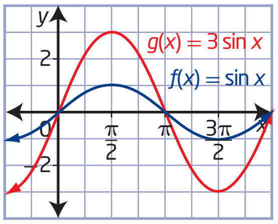
b) 

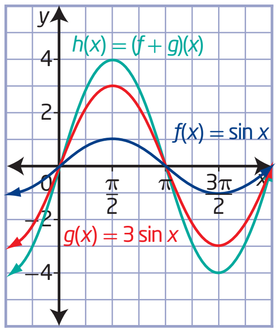
c) (60, 210)

d) $P(n) = 2.25n - 135$

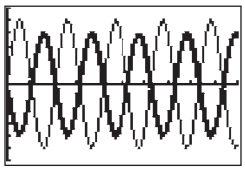


e) \$540

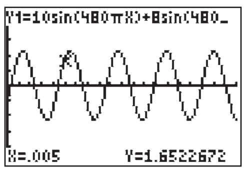
14. a) 

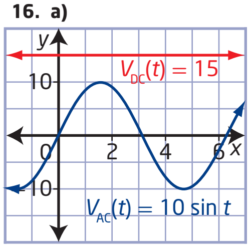
b) 

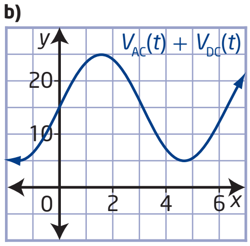
c) 4 cm

15. a) 

b) The maxima and minima are located at the same x-coordinates. This will result in destructive interference.

c) 

16. a) 

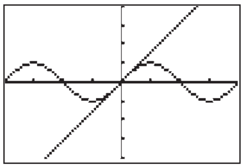
b) 

c) domain $\{t \mid t \in R\}$, range $\{V \mid 5 \leq V \leq 25, V \in R\}$

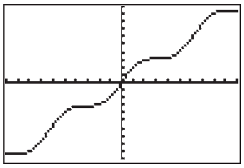
d) i) 5 V ii) 25 V

17. $h(t) = 5t^2 - 20t - 20$

18. a)



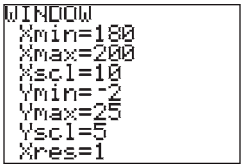
b) It will be a sinusoidal function on a diagonal according to $y = x$.

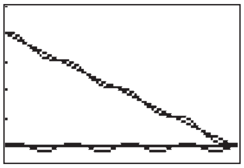


19. a) $h(t) = 200 - t$

b) $H(t) = 200 - t + 0.75 \sin 1.26t$

c)





20. Example: Replace all x with $-x$ and then simplify. If the new function is equal to the original, then it is even. If it is the negative of the original, then it is odd. Answers may vary.

21. The graph shows the sum of an exponential function and a constant function.

22. a) $f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \geq -9, y \in R\}$; $g(x)$: domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$

b) $h(x) = x^2 - 9 + \frac{1}{x}$

c) Example: The domain and range of $f(x)$ are different from the domain and range of $h(x)$. The domain of $g(x)$ is the same as that of $h(x)$, but the range of $g(x)$ is different from that of $h(x)$.

C1 a) Yes, addition is commutative.

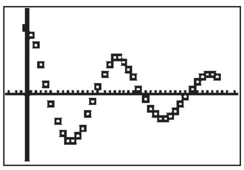
b) No, subtraction is not commutative.

C2 a) $y_3 = x^3 + 4$

b) domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

C3 Example:

Step 1:



The graph exhibits sinusoidal features in its shape and the fact that it is periodic.

Step 2: The graph exhibits exponential features in that it is decreasing and approaching 0 with asymptote $y = 0$.

Step 3: $h = \cos 0.35t$

Step 4: $h = 100(0.5)^{0.05t}$

Step 5: $h = (100 \cos 0.35t)((0.5)^{0.05t})$

Step 6: 15.5 m

10.2 Products and Quotients of Functions, pages 496 to 498

1. a) $h(x) = x^2 - 49$, $k(x) = \frac{x + 7}{x - 7}$, $x \neq 7$

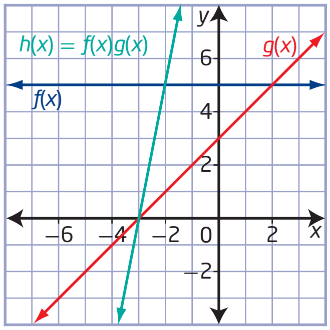
b) $h(x) = 6x^2 + 5x - 4$, $k(x) = \frac{2x - 1}{3x + 4}$, $x \neq -\frac{4}{3}$

c) $h(x) = (x + 2)\sqrt{x + 5}$, $k(x) = \frac{\sqrt{x + 5}}{x + 2}$, $x \geq -5$, $x \neq -2$

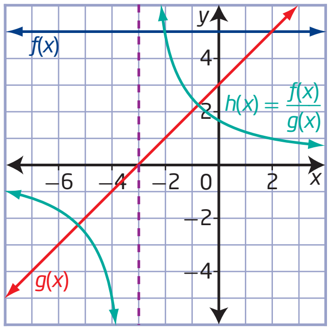
d) $h(x) = \sqrt{-x^2 + 7x - 6}$, $k(x) = \frac{\sqrt{x - 1}}{\sqrt{6 - x}}$, $1 \leq x < 6$

2. a) -3 b) 0 c) -1 d) 0

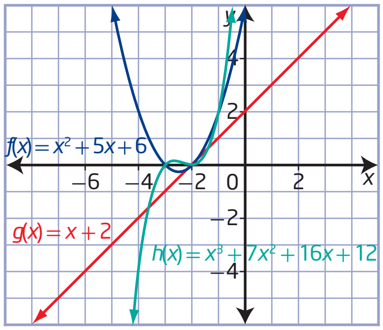
3. a)



b)

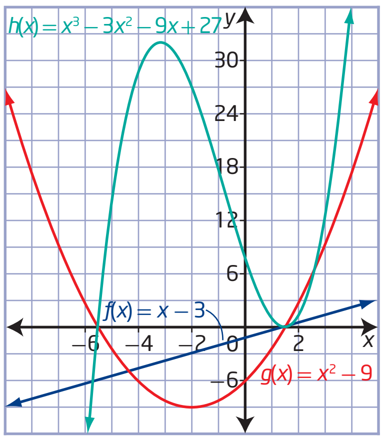


4. a) $h(x) = x^3 + 7x^2 + 16x + 12$



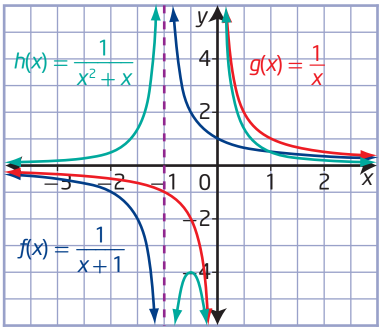
domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

b) $h(x) = x^3 - 3x^2 - 9x + 27$



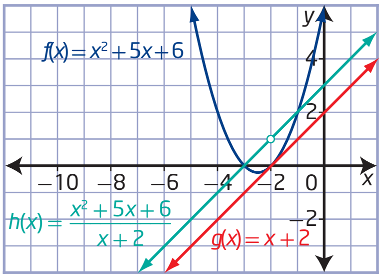
domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

c) $h(x) = \dfrac{1}{x^2 + x}$



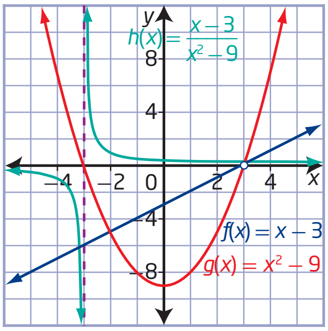
domain $\{x \mid x \neq 0, -1, x \in \mathrm{R}\}$,
range $\{y \mid y \leq -4 \text{ or } y > 0, y \in \mathrm{R}\}$

5. a) $h(x) = x + 3, x \neq -2$



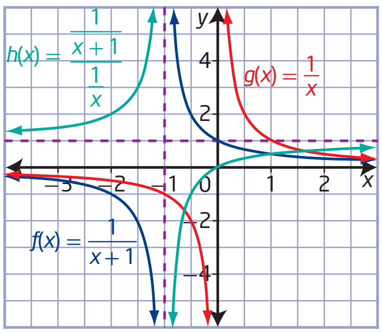
domain $\{x \mid x \neq -2, x \in \mathrm{R}\}$,
range $\{y \mid y \neq 1, y \in \mathrm{R}\}$

b) $h(x) = \dfrac{1}{x + 3}, x \neq \pm 3$



domain $\{x \mid x \neq \pm 3, x \in \mathrm{R}\}$,
range $\left\{y \mid y \neq 0, \dfrac{1}{6}, y \in \mathrm{R}\right\}$

c) $h(x) = \dfrac{x}{x + 1}, x \neq -1, 0$



domain $\{x \mid x \neq -1, 0, x \in \mathrm{R}\}$,
range $\{y \mid y \neq 0, 1, y \in \mathrm{R}\}$

6. a) $y = x^3 + 3x^2 - 10x - 24$ **b)** $y = \dfrac{x^2 - x - 6}{x + 4}$

c) $y = \dfrac{2x - 1}{x + 4}$ **d)** $y = \dfrac{x^2 - x - 6}{x^2 + 8x + 16}$

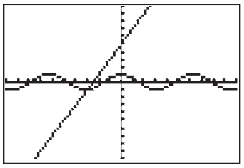
7. a) $g(x) = 3$ **b)** $g(x) = -x$

c) $g(x) = \sqrt{x}$ **d)** $g(x) = 5x - 6$

8. a) $g(x) = x + 7$ **b)** $g(x) = \sqrt{x + 6}$

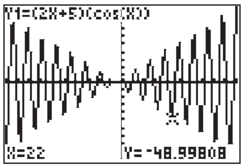
c) $g(x) = 2$ **d)** $g(x) = 3x^2 + 26x - 9$

9. a)



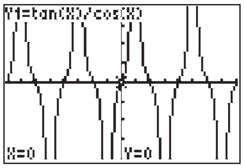
$f(x)$:
domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$
$g(x)$:
domain $\{x \mid x \in \mathrm{R}\}$,
range
$\{y \mid -1 \leq y \leq 1, y \in \mathrm{R}\}$

b)



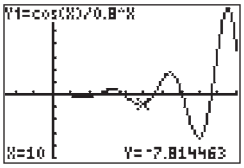
domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$

10. a)



domain
$\{x \mid x \neq (2n - 1)\dfrac{\pi}{2}$,
$n \in \mathrm{I}, x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$

b)

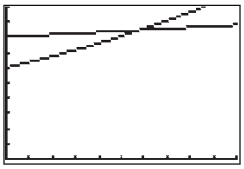


domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$

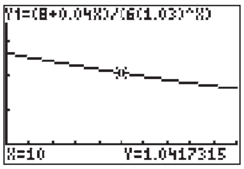
11. a) $y = \dfrac{f(x)}{g(x)}$ **b)** $y = f(x)f(x)$

c) The graphs of $y = \dfrac{\sin x}{\cos x}$ and $y = \tan x$ appear to be the same. The graphs of $y = 1 - \cos^2 x$ and $y = \sin^2 x$ appear to be the same.

12. a)



b)

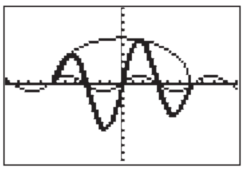


Yes; negative values of t should not be considered.

c) $t = 0$

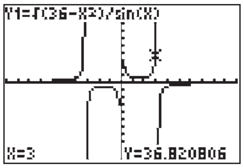
d) In approximately 11.6 years, there will be less than 1 unit of food per fish; determine the point of intersection for the graphs of $y = \dfrac{F(t)}{P(t)}$ and $y = 1$.

13. a)



b) domain $\{x \mid -6 \leq x \leq 6, x \in R\}$, range $\{y \mid -5.8 \leq y \leq 5.8, y \in R\}$

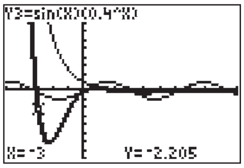
c)



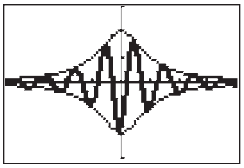
domain $\{x \mid -6 \leq x \leq 6, x \neq n\pi, n \in I, x \in R\}$, range $\{y \mid y \in R\}$

d) The domain and range in part b) are restricted to a small range of values, but inside this range there are no non-permissible values. In part c), the domain is restricted to a small set of numbers with non-permissible values in it but the range is not restricted.

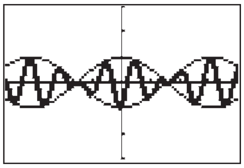
14. a), b) $f(t) = A \sin kt$, $g(t) = 0.4^{ct}$



15. a)



b) Yes



16. The price decreases per tonne.

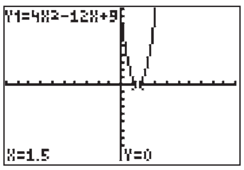
17. $A = 4x\sqrt{r^2 - x^2}$

C1 Yes; multiplication is commutative. Examples may vary.

C2 Example: Multiplication generally increases the range and domain, although this is not always true. Quotients generally produce asymptotes and points of discontinuity, although this is not always true.

C3 a) $A(x) = 4x^2 - 12x + 9$

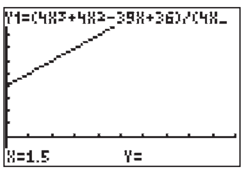
b)



domain $\{x \mid x \geq 1.5, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$

c) $h(x) = x + 4$, $x \neq \frac{3}{2}$; this represents the height of the box.

d)



domain $\{x \mid x > 1.5, x \in R\}$, range $\{y \mid y > 5.5, y \in R\}$

10.3 Composite Functions, pages 507 to 509

1. a) 3 **b)** 0 **c)** 2 **d)** −1

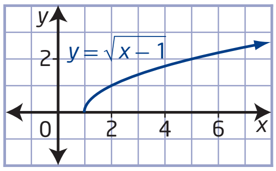
2. a) 2 **b)** 2 **c)** −4 **d)** −5

3. a) 10 **b)** −8 **c)** −2 **d)** 28

4. a) $f(g(a)) = 3a^2 + 1$ **b)** $g(f(a)) = 9a^2 + 24a + 15$
c) $f(g(x)) = 3x^2 + 1$ **d)** $g(f(x)) = 9x^2 + 24x + 15$
e) $f(f(x)) = 9x + 16$ **f)** $g(g(x)) = x^4 - 2x^2$

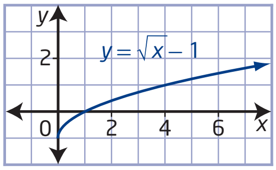
5. a) $f(g(x)) = x^4 + 2x^3 + 2x^2 + x$, $g(f(x)) = x^4 + 2x^3 + 2x^2 + x$
b) $f(g(x)) = \sqrt{x^4 + 2}$, $g(f(x)) = x^2 + 2$
c) $f(g(x)) = |x^2|$, $g(f(x)) = x^2$

6. a)



domain $\{x \mid x \geq 1, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$

b)



domain $\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \geq -1, y \in R\}$

7. a) $g(x) = 2x - 5$ **b)** $g(x) = 5x + 1$

8. Christine is right. Ron forgot to replace all x's with the other function in the first step.

9. Yes. $k(j(x)) = j(k(x)) = x^6$; using the power law: $2(3) = 6$ and $3(2) = 6$.

10. No. $s(t(x)) = x^2 - 6x + 10$ and $t(s(x)) = x^2 - 2$.

11. a) $W(C(t)) = 3\sqrt{100 + 35t}$
b) domain $\{t \mid t \geq 0, t \in R\}$, range $\{W \mid W \geq 30, W \in W\}$

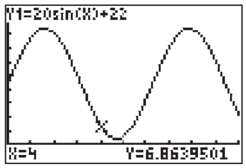
12. a) $s(p) = 0.75p$ **b)** $t(s) = 1.05s$
c) $t(s(p)) = 0.7875p$; \$70.87

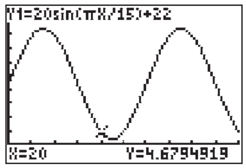
13. a) $g(d) = 0.06d$ **b)** $c(g) = 1.23g$
c) $c(g(d)) = 0.0738d$; \$14.76
d) $d(c) = 13.55c$; 542 km

14. a) $3x^2 - 21$ **b)** $3x^2 - 7$
c) $3x^2 - 42x + 147$ **d)** $9x^2 - 42x + 49$

15. a) $h(\theta(t)) = 20 \sin \frac{\pi t}{15} + 22$

b)





The period of the combined functions is much greater.

16. a) $C(P(t)) = 14.375(2)^{\frac{t}{10}} + 53.12$
b) approximately 17.1 years

17. a) $f(x) = 2x - 1$, $g(x) = x^2$
b) $f(x) = \frac{2}{3 - x}$, $g(x) = x^2$
c) $f(x) = |x|$, $g(x) = x^2 - 4x + 5$

18. a) $g(f(x)) = \frac{1 - x}{1 - 1 + x} = \frac{1 - x}{x} = \frac{1}{g(x)}$
b) $f(g(x)) = 1 - \frac{x}{1 - x} = \frac{1 - 2x}{1 - x} \neq \frac{1}{f(x)}$

No, they are not the same.

19. a) $m = \frac{m_0}{\sqrt{1 - \frac{t^6}{c^2}}}$ **b)** $\frac{2}{\sqrt{3}}m_0$

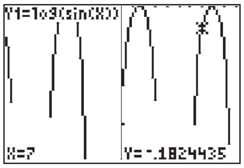
20. a) The functions $f(x) = 5x + 10$ and $g(x) = \frac{1}{5}x - 2$ are inverses of each other since $f(g(x)) = x$ and $g(f(x)) = x$.
b) The functions $f(x) = \frac{x - 1}{2}$ and $g(x) = 2x + 1$ are inverses of each other since $f(g(x)) = x$ and $g(f(x)) = x$.

c) The functions $f(x) = \sqrt[3]{x + 1}$ and $g(x) = x^3 - 1$ are inverses of each other since $f(g(x)) = x$ and $g(f(x)) = x$.

d) The functions $f(x) = 5^x$ and $g(x) = \log_5 x$ are inverses of each other since $f(g(x)) = x$ and $g(f(x)) = x$.

21. a) $\{x \mid x > 0, x \in R\}$ b) $f(g(x)) = \log(\sin x)$

c)



d) domain $\{x \mid 2n\pi < x < (2n + 1)\pi, n \in I, x \in R\}$, range $\{y \mid y \leq 0, y \in R\}$

22. $f(g(x)) = \dfrac{x + 2}{x + 3}$

23. a) i) $y = \dfrac{1}{1 - x}$ ii) $y = -\dfrac{x}{1 - x}$

iii) $y = \dfrac{1}{x}$ iv) $y = \dfrac{1}{x}$

b) $f_5(x)$

C1 No. One is a composite function, $f(g(x))$, and the other is the product of functions, $(f \cdot g)(x)$. Examples may vary.

C2 a) Example: Since $f(1) = 5$ and $g(5) = 10$, $g(f(1)) = 10$.

b) Example: Since $f(3) = 7$ and $g(7) = 0$, $g(f(3)) = 0$.

C3 Yes, the functions are inverses of each other.

C4 **Step 1:** a) $f(x + h) = 2x + 2h + 3$

b) $\dfrac{f(x + h) - f(x)}{h} = 2$

Step 2: a) $f(x + h) = -3x - 3h - 5$

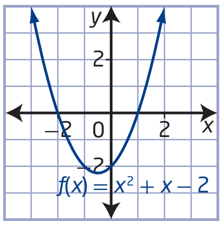
b) $\dfrac{f(x + h) - f(x)}{h} = -3$

Step 3: $\dfrac{f(x + h) - f(x)}{h} = \dfrac{3}{4}$; Each value is the slope of the linear function.

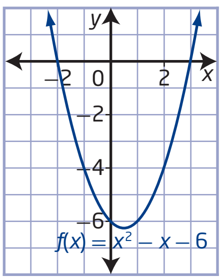
Chapter 10 Review, pages 510 to 511

1. a) 26 b) 1 c) −5 d) 13

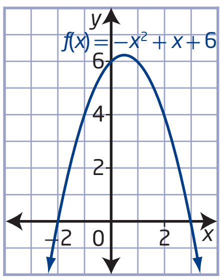
2. a) i) $f(x) = x^2 + x - 2$ domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$



ii) $f(x) = x^2 - x - 6$ domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

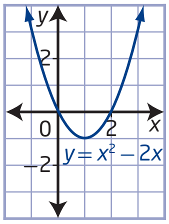


iii) $f(x) = -x^2 + x + 6$ domain $\{x \mid x \in R\}$ range $\{y \mid y \in R\}$



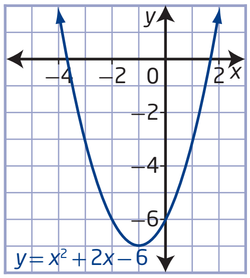
b) i) 4 ii) −4 iii) 4

3. a) $y = x^2 - 2x$



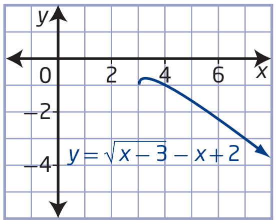
domain $\{x \mid x \in R\}$, range $\{y \mid y \geq -1, y \in R\}$

$y = x^2 + 2x - 6$



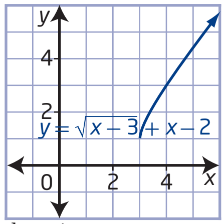
domain $\{x \mid x \in R\}$, range $\{y \mid y \geq -7, y \in R\}$

b) $y = \sqrt{x - 3} - x + 2$



domain $\{x \mid x \geq 3, x \in R\}$, range $\{y \mid y \leq -0.75, y \in R\}$

$y = \sqrt{x - 3} + x - 2$



domain $\{x \mid x \geq 3, x \in R\}$, range $\{y \mid y \geq 1, y \in R\}$

4. a) $y = \dfrac{1}{x - 1} + \sqrt{x}$; domain $\{x \mid x \geq 0, x \neq 1, x \in R\}$, range $\{y \mid y \leq -0.7886 \text{ or } y \geq 2.2287, y \in R\}$

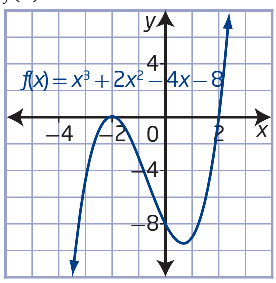
b) $y = \dfrac{1}{x - 1} - \sqrt{x}$; domain $\{x \mid x \geq 0, x \neq 1, x \in R\}$, range $\{y \mid y \in R\}$

5. a) $P = 2x - 6$

b) The net change will continue to increase, going from a negative value to a positive value in year 3.

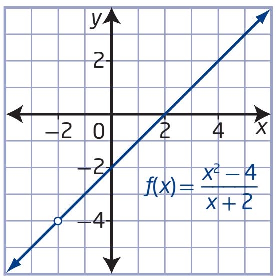
c) after year 3

6. a) $f(x) = x^3 + 2x^2 - 4x - 8$



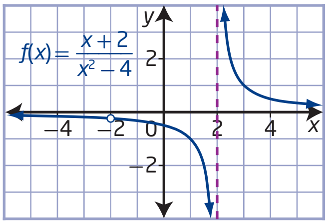
domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$, no asymptotes

b) $f(x) = x - 2, x \neq -2$



domain $\{x \mid x \neq -2, x \in R\}$, range $\{y \mid y \neq -4, y \in R\}$, no asymptotes

c) $f(x) = \frac{1}{x - 2}, x \neq -2, 2$



domain $\{x \mid x \neq -2, 2, x \in R\}$, range $\left\{y \mid y \neq -\frac{1}{4}, 0, y \in R\right\}$, horizontal asymptote $y = 0$, vertical asymptote $x = 2$

7. a) 0 **b)** does not exist

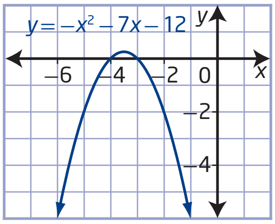
c) does not exist

8. a) $f(x) = \frac{1}{x^3 + 4x^2 - 16x - 64}, x \neq \pm 4$
domain $\{x \mid x \neq -4, 4, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$

b) $f(x) = x - 4, x \neq \pm 4$
domain $\{x \mid x \neq -4, 4, x \in R\}$, range $\{y \mid y \neq -8, 0, y \in R\}$

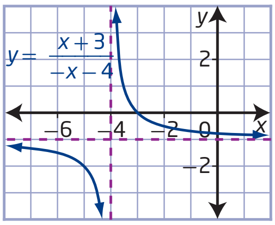
c) $f(x) = \frac{1}{x - 4}, x \neq \pm 4$
domain $\{x \mid x \neq -4, 4, x \in R\}$, range $\left\{y \mid y \neq -\frac{1}{8}, 0, y \in R\right\}$

9. a) $y = -x^2 - 7x - 12$



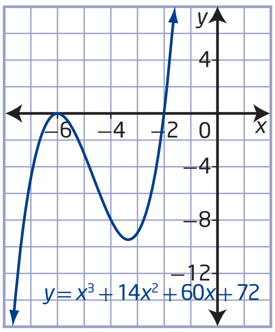
domain $\{x \mid x \in R\}$, range $\{y \mid y \leq 0.25, y \in R\}$

$y = \frac{x + 3}{-x - 4}, x \neq -4$



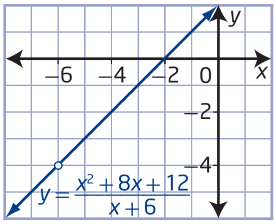
domain $\{x \mid x \neq -4, x \in R\}$, range $\{y \mid y \neq -1, y \in R\}$

b) $y = x^3 + 14x^2 + 60x + 72$



domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

$y = x + 2, x \neq -6$



domain $\{x \mid x \neq -6, x \in R\}$, range $\{y \mid y \neq -4, y \in R\}$

10. a) 1 **b)** 5

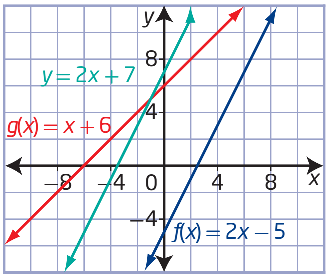
11. a) $y = \frac{32}{x^2}; x \neq 0$ **b)** $y = \frac{2}{x^2}; x \neq 0$

c) 0.5

12. a) $y = -\frac{2}{\sqrt{x}}, x > 0$

b) domain $\{x \mid x > 0, x \in R\}$, range $\{y \mid y < 0, y \in R\}$

13.



14. $T = 0.05t + 20$

15. a) $d(x) = 0.75x$; $c(x) = x - 10$

b) $c(d(x)) = 0.75x - 10$; this represents using the coupon after the discount.

c) $d(c(x)) = 0.75x - 7.5$; this represents applying the coupon before the discount.

d) Using the coupon after the discount results in a lower price of \$290.

Chapter 10 Practice Test, pages 512 to 513

1. B **2.** D **3.** A **4.** C **5.** A

6. a) $h(x) = \sin x + 2x^2$ **b)** $h(x) = \sin x - 2x^2$

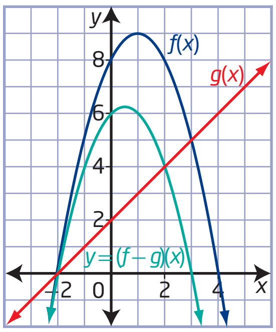
c) $h(x) = 2x^2 \sin x$ **d)** $h(x) = \frac{\sin x}{2x^2}, x \neq 0$

7.

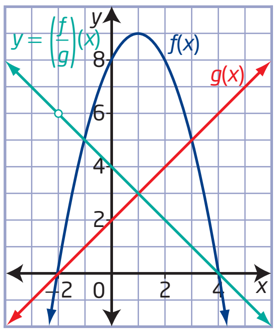
	$g(x)$	$f(x)$	$(f + g)(x)$	$(f \circ g)(x)$
a)	$x - 8$	$\sqrt{x}$	$\sqrt{x} + x - 8$	$\sqrt{x - 8}$
b)	$x + 3$	$4x$	$5x + 3$	$4x + 12$
c)	x^2	$\sqrt{x - 4}$	$\sqrt{x - 4} + x^2$	$\sqrt{x^2 - 4}$
d)	$\frac{1}{x}$	$\frac{1}{x}$	$\frac{2}{x}$	x

8. $y = \frac{1}{2x^2 + 5x + 3}, x \neq -\frac{3}{2}, -1$
domain $\left\{x \mid x \neq -\frac{3}{2}, -1, x \in R\right\}$

9. a)



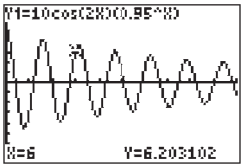
b)



10. a) $y = |6 - x|$; domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$
b) $y = 4^x + 1$; domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$
c) $y = x^2$; domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

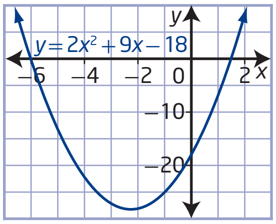
11. a) $r(x) = x - 200$; $t(x) = 0.72x$
b) $t(r(x)) = 0.72x - 144$; this represents applying federal taxes after deducting from her paycheque for her retirement.
c) \$1800
d) \$1744
e) The order changes the final amount. If you tax the income after subtracting \$200, you are left with more money.

12. a)

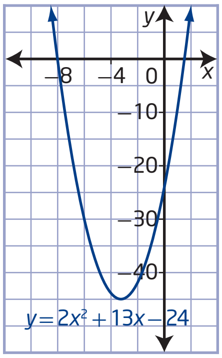


b) The function $f(t) = 10 \cos 2t$ is responsible for the periodic motion. The function $g(t) = 0.95^t$ is responsible for the exponential decay of the amplitude.

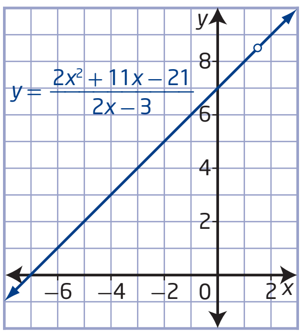
13. a) $y = 2x^2 + 9x - 18$



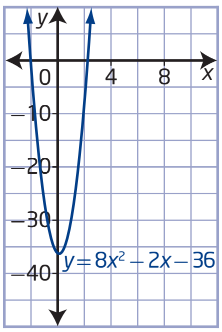
b) $y = 2x^2 + 13x - 24$



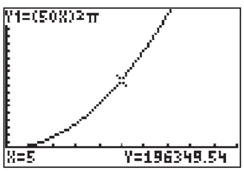
c) $y = x + 7, x \neq \frac{3}{2}$



d) $y = 8x^2 - 2x - 36$



14. a) $A(t) = 2500\pi t^2$
b)



c) approximately 196 350 cm^2
d) Example: No. In 30 s, the radius would be 1500 cm. Most likely the circular ripples would no longer be visible on the surface of the water due to turbulence.