Answers

Chapter 1 Function Transformations

1.1 Horizontal and Vertical Translations, pages 12 to 15



- **5.** a) h = -5, k = 4; y 4 = f(x + 5)
 - **b)** h = 8, k = 6; y 6 = f(x 8)

c)
$$h = 10, k = -8; y + 8 = f(x - 10)$$

- **d)** h = -7, k = -12; y + 12 = f(x + 7)
- **6.** It has been translated 3 units up.
- **7.** It has been translated 1 unit right.

8.			
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Translation	Transformed Function	Transformation of Points
vertical	y = f(x) + 5	$(x, y) \rightarrow (x, y + 5)$
horizontal	y = f(x + 7)	$(x, y) \rightarrow (x - 7, y)$
horizontal	y = f(x - 3)	$(x, y) \rightarrow (x + \exists, y)$
vertical	y = f(x) - 6	$(x, y) \rightarrow (x, y - 6)$
horizontal and vertical	y + 9 = f(x + 4)	$(x, y) \rightarrow (x - 4, y - 9)$
horizontal and vertical	y = f(x - 4) - 6	$(x, y) \rightarrow (x + 4, y - 6)$
horizontal and vertical	y = f(x + 2) + 3	$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	y = f(x - h) + k	$(x, y) \rightarrow (x + h, y + k)$

- **9.** a) $y = (x + 4)^2 + 5$ b) $\{x \mid x \in R\}, \{y \mid y \ge 5, y \in R\}$
- c) To determine the image function's domain and range, add the horizontal and vertical translations to the domain and range of the base function. Since the domain is the set of real numbers, nothing changes, but the range does change.
- **10. a)** g(x) = |x 9| + 5
 - **b)** The new graph is a vertical and horizontal translation of the original by 5 units up and 9 units right.
 - c) Example: $(0, 0), (1, 1), (2, 2) \rightarrow (9, 5), (10, 6), (11, 7)$
 - d) Example: $(0, 0), (1, 1), (2, 2) \rightarrow (9, 5), (10, 6), (11, 7)$
 - e) The coordinates of the image points from partsc) and d) are the same. The order that the translations are made does not matter.
- **11.** a) y = f(x 3) b) y + 5 = f(x 6)
- 12. a) Example: It takes her 2 h to cycle to the lake,25 km away. She rests at the lake for 2 h and then returns home in 3 h.
 - **b)** This translation shows what would happen if she left the house at a later time.
 - c) y = f(x 3)
- **13. a)** Example: Translated 8 units right.
 - **b)** Example: y = f(x 8), y = f(x 4) + 3.5, y = f(x + 4) + 3.5
- **14.** a) Example: A repeating X by using two linear equations $y = \pm x$.
 - **b)** Example: y = f(x 3). The translation is horizontal by 3 units right.
- **15.** a) The transformed function starts with a higher number of trout in 1970. y = f(t) + 2
 - **b)** The transformed function starts in 1974 instead of 1971. y = f(t 3)
- **16.** The first case, n = f(A) + 10, represents the number of gallons he needs for a given area plus 10 more gallons. The second case, n = f(A + 10), represents how many gallons he needs to cover an area A less 10 units of area.
- **17.** a) y = (x 7)(x 1) or $y = (x 4)^2 9$
 - **b)** Horizontal translation of 4 units right and vertical translation of 9 units down.
 - c) y-intercept 7

- **18. a)** The original function is 4 units lower.
 - **b)** The original function is 2 units to the right.
 - c) The original function is 3 units lower and 5 units left.
 - d) The original function is 4 units higher and 3 units right.
- **19.** a) The new graph will be translated 2 units right and 3 units down.



- **C1 a)** $y = f(x) \rightarrow y = f(x h) \rightarrow y = f(x h) + k$. Looking at the problem in small steps, it is easy to see that it does not matter which way the translations are done since they do not affect the other translation.
 - **b)** The domain is shifted by *h* and the range is shifted by *k*.
- **C2 a)** $f(x) = (x + 1)^2$; horizontal translation of 1 unit left **b)** $g(x) = (x - 2)^2 - 1$; horizontal translation of
- 2 units right and 1 unit down
- **C3** The roots are 2 and 9.
- **C4** The 4 can be taken as h or k in this problem. If it is h then it is -4, which makes it in the left direction.

1.2 Reflections and Stretches, pages 28 to 31

x	f(x) = 2x + 1	g(x) = -f(x)	h(x) = f(-x)
-4	-7	7	9
-2	-3	3	5
0	1	-1	1
2	5	-5	-3
4	9	-9	-7
	x -4 -2 0 2 4	x $f(x) = 2x + 1$ -4-7-2-3012549	x $f(x) = 2x + 1$ $g(x) = -f(x)$ -4-77-2-3301-125-549-9



- d) The graph of g(x) is the reflection of the graph of f(x) in the x-axis, while the graph of h(x) is the reflection of the graph of f(x) in the y-axis.
- 2. a

)	x	$f(x)=x^2$	$g(x) = \exists f(x)$	$h(x) = \frac{1}{3}f(x)$
	-6	36	108	12
	-3	9	27	З
	0	0	0	0
	З	9	27	3
	6	36	108	12



- c) The y-coordinates of g(x) are three times larger. The invariant point is (0, 0). The y-coordinates of h(x) are three times smaller. The invariant point is (0, 0).
- d) The graph of g(x) is a vertical stretch by a factor of 3 of the graph of f(x), while the graph of h(x) is a

vertical stretch by a factor of $\frac{1}{3}$ of the graph of f(x).



Answers • MHR 555

- **5.** a) The graph of y = 4f(x) is a vertical stretch by a factor of 4 of the graph of y = f(x). $(x, y) \rightarrow (x, 4y)$
 - **b)** The graph of y = f(3x) is a horizontal stretch by a factor of $\frac{1}{3}$ of the graph of y = f(x). $(x, y) \to \left(\frac{x}{3}, y\right)$
 - The graph of y = -f(x) is a reflection in the x-axis C) of the graph of y = f(x). $(x, y) \rightarrow (x, -y)$
 - d) The graph of y = f(-x) is a reflection in the y-axis of the graph of y = f(x). $(x, y) \rightarrow (-x, y)$
- **6.** a) domain $\{x \mid -6 \le x \le 6, x \in R\}$, range $\{y \mid -8 \le y \le 8, y \in R\}$
 - **b)** The vertical stretch affects the range by increasing it by the stretch factor of 2.
- The graph of g(x) is a vertical stretch by a factor of 7.a) 4 of the graph of f(x). y = 4f(x)
 - **b)** The graph of g(x) is a reflection in the x-axis of the graph of f(x). y = -f(x)
 - c) The graph of g(x) is a horizontal stretch by a factor of $\frac{1}{3}$ of the graph of f(x). y = f(3x)
 - d) The graph of g(x) is a reflection in the y-axis of the graph of f(x). y = f(-x)



- 9. a) horizontally stretched by a factor of $\frac{1}{4}$
- horizontally stretched by a factor of 4 b)
- vertically stretched by a factor of $\frac{1}{2}$ C)
- vertically stretched by a factor of 4 d)
- horizontally stretched by a factor of $\frac{1}{3}$ and e) reflected in the y-axis
- vertically stretched by a factor of 3 and reflected f) in the *x*-axis

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C) They are both incorrect. It does not matter in which order you proceed.



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- **b)** Both the functions are reflections of the base function in the *t*-axis. The object falling on Earth is stretched vertically more than the object falling on the moon.
- **12.** Example: When the graph of y = f(x) is transformed to the graph of y = f(bx), it undergoes a horizontal stretch about the *y*-axis by a factor of $\frac{1}{|b|}$ and only the

x-coordinates are affected. When the graph of y = f(x)is transformed to the graph of y = af(x), it undergoes a vertical stretch about the x-axis by a factor of |a| and only the y-coordinates are affected.



- **C1** Example: When the input values for g(x) are b times the input values for f(x), the scale factor must be $\frac{1}{h}$ for the same output values. $g(x) = f(\frac{1}{b}(bx)) = f(x)^{T}$
- C2 Examples:
 - a) a vertical stretch or a reflection in the x-axis
 - **b**) a horizontal stretch or a reflection in the *y*-axis

C3	<i>f</i> (<i>x</i>)	g(x)	Transformation
	(5, 6)	(5, –6)	reflection in the <i>x</i> -axis
	(4, 8)	(-4, 8)	reflection in the y-axis
	(2, 3)	(2, 12)	vertical stretch by a factor of 4
	(4, -12)	(2, -6)	horizontal stretch by a factor of $\frac{1}{2}$ and vertical stretch by a factor of $\frac{1}{2}$



1.3 Combining Transformations, pages 38 to 43

1. a)
$$y = -f(\frac{1}{2}x)$$
 or $y = -\frac{1}{4}x^{2}$
b) $y = \frac{1}{4}f(-4x)$ or $y = 4x^{2}$

2. The function f(x) is transformed to the function g(x) by a horizontal stretch about the *y*-axis by a factor of $\frac{1}{4}$. It is vertically stretched about the *x*-axis by a factor of 3. It is reflected in the *x*-axis, and then translated 4 units right and 10 units down.





- 7. a) vertical stretch by a factor of 2 and translation of 3 units right and 4 units up; $(x, y) \rightarrow (x + 3, 2y + 4)$
 - **b)** horizontal stretch by a factor of $\frac{1}{3}$, reflection in the *x*-axis, and translation of 2 units down; (*x*, *y*) $\rightarrow \left(\frac{1}{3}x, -y - 2\right)$
 - c) reflection in the *y*-axis, reflection in the *x*-axis, vertical stretch by a factor of $\frac{1}{4}$, and translation of

2 units left; $(x, y) \rightarrow \left(-x - 2, -\frac{1}{4}y\right)$

- d) horizontal stretch by a factor of ¹/₄, reflection in the *x*-axis, and translation of 2 units right and 3 units up; (*x*, *y*) → (¹/₄*x* + 2, -*y* + 3)
 e) reflection in the *y*-axis, horizontal stretch by a
- e) reflection in the *y*-axis, horizontal stretch by a factor of ⁴/₃, reflection in the *x*-axis, and vertical stretch by a factor of ²/₃; (x, y) → (-⁴/₃x, -²/₃y)
 f) reflection in the *y*-axis, horizontal stretch by a
- f) reflection in the *y*-axis, horizontal stretch by a factor of $\frac{1}{2}$, vertical stretch by a factor of $\frac{1}{3}$, and translation of 6 units right and 2 units up; $(x, y) \rightarrow \left(-\frac{1}{2}x + 6, \frac{1}{2}y + 2\right)$

8. a)
$$y + 5 = -3f(x + 4)$$
 b) $y - 2 = -\frac{3}{4}f(-3(x - 6))$





- b) $y = -f(\frac{1}{2}(x+3)) + 4$ **13.** a) The graphs are in two locations because the
- transformations performed to obtain Graph 2 do not match those in y = |2x - 6| + 2. Gil forgot to factor out the coefficient of the *x*-term, 2, from -6. The horizontal translation should have been 3 units right, not 6 units.
 - **b)** He should have rewritten the function as y = |2(x 3)| + 2.



- 15. a) (-a, 0), (0, -b) b) (2a, 0), (0, 2b)
 c) and d) There is not enough information to determine the locations of the new intercepts. When a transformation involves translations, the locations of the new intercepts will vary with different base functions.
- **16. a)** $A = -2x^3 + 18x$
- Bx **b)** $A = -\frac{1}{8}x^3 + 18x$
 - c) For (2, 5), the area of the rectangle in part a) is 20 square units. $A = -2x^3 + 18x$ A = 20For (8, 5), the area of the rectangle in part b) is 80 square units. $A = -\frac{1}{8}x^3 + 18x$ $A = -\frac{1}{8}(8)^3 + 18(8)$ A = 80
- **17.** $y = 36(x-2)^2 + 6(x-2) 2^{-1}$
- **18.** Example: vertical stretches and horizontal stretches followed by reflections
- **C1** Step 1 They are reflections in the axes. 1: y = x + 3, 2: y = -x - 3, 3: y = x - 3Step 2 They are vertical translations coupled with reflections. 1: $y = x^2 + 1$, 2: $y = x^2 - 1$, 3: $y = -x^2$, 4: $y = -x^2 - 1$
- **C2 a)** The cost of making b + 12 bracelets, and it is a horizontal translation.
 - **b)** The cost of making *b* bracelets plus 12 more dollars, and it is a vertical translation.
 - c) Triple the cost of making *b* bracelets, and it is a vertical stretch.
 - d) The cost of making $\frac{b}{2}$ bracelets, and it is a horizontal stretch.
- **C3** $y = 2(x 3)^2 + 1$; a vertical stretch by a factor of 2 and a translation of 3 units right and 1 unit up
- $\label{eq:c4-a} {\bf H} \mbox{ is repeated}; \mbox{ J is transposed}; \mbox{ K is repeated and transposed}$
 - **b)** H is in retrograde; J is inverted; K is in retrograde and inverted
 - c) H is inverted, repeated, and transposed; J is in retrograde inversion and repeated; K is in retrograde and transposed

1.4 Inverse of a Relation, pages 51 to 55















- **16. a)** approximately 32.22 °C
 - b) y = ⁹/₅x + 32; x represents temperatures in degrees Celsius and y represents temperatures in degrees Fahrenheit
 c) c) c) Fahrenheit
 - c) 89.6 °Fd)



The temperature is the same in both scales (-40 °C = -40 °F).

- 17. a) male height = 171.02 cm, female height = 166.44 cm
 b) i) male femur = 52.75 cm
 - ii) female femur = 49.04 cm
- **18. a)** 5
 - **b)** y = 2.55x + 36.5; y is finger circumference and x is ring size
 - c) 51.8 mm, 54.35 mm, 59.45 mm

19. Examples:





b) Example: The graph of the original linear function is perpendicular to y = x, thus after a reflection the graph of the inverse is the same.

c) They are perpendicular to the line.

- **C3** Example: If the original function passes the vertical line test, then it is a function. If the original function passes the horizontal line test, then the inverse is a function.
- C4 Step 1

$$f(x)$$
: (1, 2), (4, 3), (-8, -1), and $\left(a, \frac{a+5}{3}\right)$
 $g(x)$: (2, 1), (3, 4), (-1, -8), and $\left(\frac{a+5}{3}, a\right)$

The output values for g(x) are the same as the input values for f(x).

Example: Since the functions are inverses of each other, giving one of them a value and then taking the inverse will always return the initial value. A good way to determine if functions are inverses is to see if this effect takes place.

Step 2 The order in which you apply the functions does not change the final result.

Step 4 The statement is saying that if you have a function that when given a outputs b and another that when given b outputs a, then the functions are inverses of each other.

Chapter 1 Review, pages 56 to 57





- **b)** If the coefficient is greater than 1, then the function moves closer to the *y*-axis. The opposite is true for when the coefficient is between 0 and 1.
- **8.** a) In this case, it could be either. It could be a vertical stretch by a factor of $\frac{1}{2}$ or a horizontal stretch by a factor of $\sqrt{2}$.
 - **b)** Example: $g(x) = \frac{1}{2}f(x)$



10. They are both horizontal stretches by a factor of $\frac{1}{4}$. The difference is in the horizontal translation, the first being 1 unit left and the second being $\frac{1}{4}$ unit left.











Chapter 1 Practice Test, pages 58 to 59



c) $g(x) = f(\frac{1}{2}x)$; a horizontal stretch by a factor of 2

d)
$$\frac{1}{4}f(x) = \frac{1}{4}x^2; f(\frac{1}{2}x) = (\frac{1}{2}x)^2 = \frac{1}{4}x^2$$

15. a) Using the horizontal line test, if a horizontal line passes through the function more than once the inverse is not a function.

b)
$$y = \pm \sqrt{-x - 5} - 3$$

c) Example: restricted domain $\{x \mid x \ge -3, x \in \mathbb{R}\}$