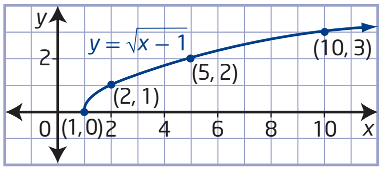
Chapter 2 Radical Functions

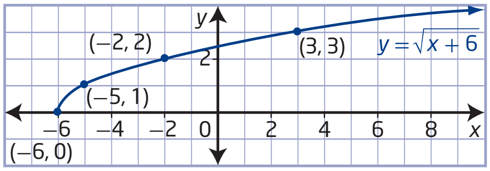
2.1 Radical Functions and Transformations, pages 72 to 77

1. a)



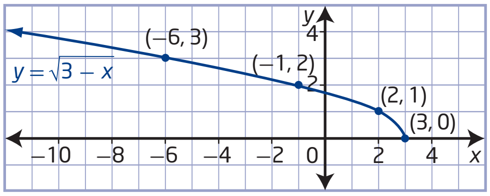
domain $\{x \mid x \geq 1, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b)



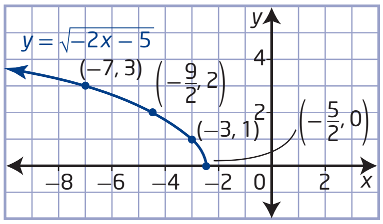
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

c)



domain $\{x \mid x \leq 3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d)



domain $\left\{x \mid x \leq -\frac{5}{2}, x \in \mathrm{R}\right\}$,
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

2. a) $a = 7 \rightarrow$ vertical stretch by a factor of 7
$h = 9 \rightarrow$ horizontal translation 9 units right
domain $\{x \mid x \geq 9, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) $b = -1 \rightarrow$ reflected in y-axis
$k = 8 \rightarrow$ vertical translation up 8 units
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 8, y \in \mathrm{R}\}$

c) $a = -1 \rightarrow$ reflected in x-axis
$b = \frac{1}{5} \rightarrow$ horizontal stretch factor of 5
domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

d) $a = \frac{1}{3} \rightarrow$ vertical stretch factor of $\frac{1}{3}$
$h = -6 \rightarrow$ horizontal translation 6 units left
$k = -4 \rightarrow$ vertical translation 4 units down
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$,
range $\{y \mid y \geq -4, y \in \mathrm{R}\}$

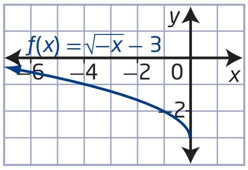
3. a) B **b)** A **c)** D **d)** C

4. a) $y = 4\sqrt{x + 6}$ **b)** $y = \sqrt{8x} - 5$

c) $y = \sqrt{-(x - 4)} + 11$ or $y = \sqrt{-x + 4} + 11$

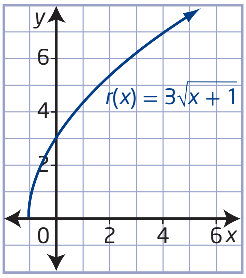
d) $y = -0.25\sqrt{0.1x}$ or $y = -\frac{1}{4}\sqrt{\frac{1}{10}x}$

5. a)



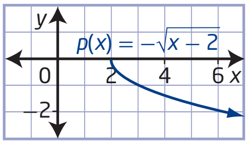
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq -3, y \in \mathrm{R}\}$

b)



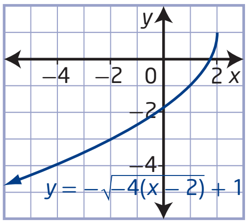
domain $\{x \mid x \geq -1, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

c)



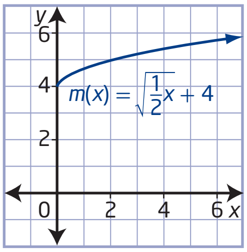
domain $\{x \mid x \geq 2, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

d)



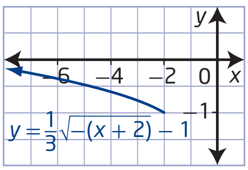
domain $\{x \mid x \leq 2, x \in \mathrm{R}\}$, range $\{y \mid y \leq 1, y \in \mathrm{R}\}$

e)



domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 4, y \in \mathrm{R}\}$

f)



domain $\{x \mid x \leq -2, x \in \mathrm{R}\}$, range $\{y \mid y \geq -1, y \in \mathrm{R}\}$

6. a) $a = \frac{1}{4} \rightarrow$ vertical stretch factor of $\frac{1}{4}$

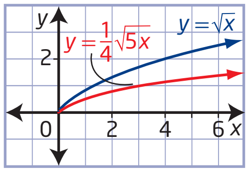
$b = 5 \rightarrow$ horizontal stretch factor of $\frac{1}{5}$

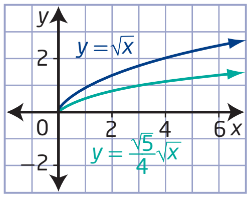
b) $y = \frac{\sqrt{5}}{4}\sqrt{x}$, $y = \sqrt{\frac{5}{16}x}$

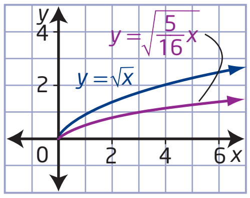
c) $a = \frac{\sqrt{5}}{4} \rightarrow$ vertical stretch factor of $\frac{\sqrt{5}}{4}$

$b = \frac{5}{16} \rightarrow$ horizontal stretch factor of $\frac{16}{5}$

d)





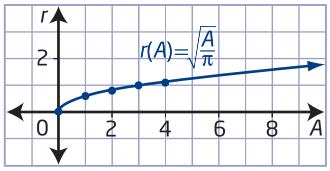


All graphs are the same.

7. a) $r(A) = \sqrt{\frac{A}{\pi}}$

b)

A	r
0	0
1	0.6
2	0.8
3	1.0
4	1.1



8. a) $b = 1.50 \rightarrow$ horizontal stretch factor of $\frac{1}{1.50}$ or $\frac{2}{3}$

b) $d \approx 1.22\sqrt{h}$ Example: I prefer the original function because the values are exact.

c) approximately 5.5 miles

9. a) domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq -13, y \in \mathrm{R}\}$

b) $h = 0 \rightarrow$ no horizontal translation

$k = 13 \rightarrow$ vertical translation down 13 units

10. a) $y = -\sqrt{x + 3} + 4$ **b)** $y = \frac{1}{2}\sqrt{x + 5} - 3$

c) $y = 2\sqrt{-(x - 5)} - 1$ or $y = 2\sqrt{-x + 5} - 1$

d) $y = -4\sqrt{-(x - 4)} + 5$ or $y = -4\sqrt{-x + 4} + 5$

11. Examples:

a) $y - 1 = \sqrt{x - 6}$ or $y = \sqrt{x - 6} + 1$

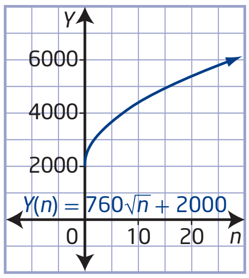
b) $y = -\sqrt{x + 7} - 9$ **c)** $y = 2\sqrt{-x + 4} - 3$

d) $y = -\sqrt{-(x + 5)} + 8$

12. a) $a = 760 \rightarrow$ vertical stretch factor of 760

$k = 2000 \rightarrow$ vertical translation up 2000

b)

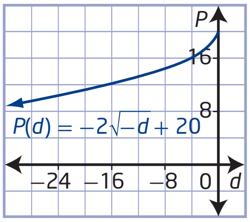


c) domain $\{n \mid n \geq 0, n \in \mathrm{R}\}$ range $\{Y \mid Y \geq 2000, Y \in \mathrm{R}\}$

d) The minimum yield is 2000 kg/hectare. Example: The domain and range imply that the more nitrogen added, the greater the yield without end. This is not realistic.

13. a) domain $\{d \mid -100 \leq d \leq 0, d \in \mathrm{R}\}$ range $\{P \mid 0 \leq P \leq 20, P \in \mathrm{R}\}$ The domain is negative indicating days remaining, and the maximum value of P is 20 million.

b) $a = -2 \rightarrow$ reflected in d-axis, vertical stretch factor of 2; $b = -1 \rightarrow$ reflected in P-axis; $k = 20 \rightarrow$ vertical translation up 20 units.

c) 

Since d is negative, then d represents the number of days remaining before release and the function has a maximum of 20 million pre-orders.

d) 9.05 million or 9 045 549 pre-orders.

14. a) Polling errors reduce as the election approaches.

b) $y = 0.49\sqrt{-x}$ There are no translations since the graph starts on the origin. The graph is reflected in the y-axis then $b = -1$. Develop the equation by using the point $(-150, 6)$ and substituting in the equation $y = a\sqrt{x}$, solving for a, then $a = 0.49$.

c) $a = 0.49 \rightarrow$ vertical stretch factor of 0.49
$b = -1 \rightarrow$ reflected in the y-axis

15. $y \approx 2.07\sqrt{-x}$

16. Examples

a) $y = -2\sqrt{x - 2} + 5$ **b)** $y = \frac{2}{3}\sqrt{3 - x} - 2$

17. a) China, India, and USA (The larger the country the more unfair the "one nation – one vote" system becomes.) Tuvalu, Nauru, Vatican City (The smaller the nation the more unfair the "one person – one vote" system becomes.)

b)

Nation	Percentage
China	18.6%
India	17.1%
US	4.5%
Canada	0.48%
Tuvalu	0.000 151%
Nauru	0.000 137%
Vatican City	0.000 014%

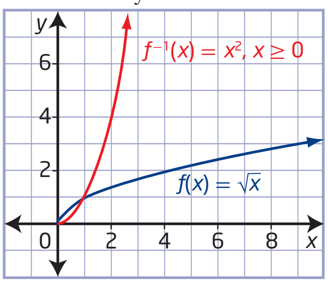
d)

Nation	Percentage
China	4.82%
India	4.62%
US	2.36%
Canada	0.77%
Tuvalu	0.014%
Nauru	0.013%
Vatican City	0.004%

c) $V(x) = \frac{1}{1000}\sqrt{x}$

e) The Penrose system gives larger nations votes based on population but also provides an opportunity for smaller nations to provide influence.

18. Answers will vary.

19. a) 

The positive domain of the inverse is the same as the range of the original function.

b) **i)** $g^{-1}(x) = x^2 + 5, x \leq 0$
ii) $h^{-1}(x) = -(x - 3)^2, x \geq 3$
iii) $j^{-1}(x) = \frac{1}{2}(x + 6)^2 + \frac{7}{2}, x \geq -6$

20. Vertical stretch by a factor of $\frac{16}{25}$. Horizontal stretch by a factor of $\frac{7}{72}$. Reflect in both the x and y axes. Horizontal translation of 3 units left. Vertical translation of 4 units down.

C1 The parameters b and h affect the domain. For example, $y = \sqrt{x}$ has domain $x \geq 0$ but $y = \sqrt{2(x - 3)}$ has domain $x \geq 3$. The parameters a and k affect the range. For example, $y = \sqrt{x}$ has range $y \geq 0$ but $y = \sqrt{x} - 4$ has range $y \geq -4$.

C2 Yes. For example, $y = \sqrt{9x}$ can be simplified to $y = 3\sqrt{x}$.

C3 The processes are similar because the parameters a, b, h, and k have the same effect on radical functions and quadratic functions. The processes are different because the base functions are different: one is the shape of a parabola and the other is the shape of half of a parabola.

C4 **Step 1** $\sqrt{2}$; **Step 2** $\sqrt{3}$

Step 4

Triangle Number, n	Length of Hypotenuse, L
First	$\sqrt{2} = 1.414...$
Second	$\sqrt{3} = 1.732...$
Third	$\sqrt{4} = 2$

Step 5 $L = \sqrt{n + 1}$ Yes, the equation involves a horizontal translation of 1 unit left.

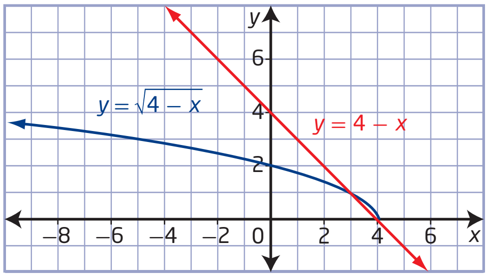
2.2 Square Root of a Function, pages 86 to 89

1.

$f(x)$	$\sqrt{f(x)}$
36	6
0.09	0.3
1	1
−9	undefined
2.56	1.6
0	0

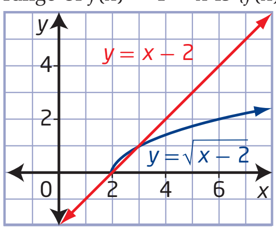
2. a) (4, 3.46) **b)** (−2, 0.63) **c)** does not exist
d) (0.09, 1) **e)** (−5, 0) **f)** $(m, \sqrt{n})$

3. a) C **b)** D **c)** A **d)** B

4. a) 

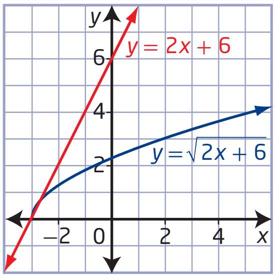
b) When $4 - x < 0$ then $\sqrt{4 - x}$ is undefined; when $0 < 4 - x < 1$ then $\sqrt{4 - x} > 4 - x$; when $4 - x > 1$ then $4 - x > \sqrt{4 - x}$; $4 - x = \sqrt{4 - x}$ when $y = 0$ and $y = 1$

c) The function $f(x) = \sqrt{4 - x}$ is undefined when $4 - x < 0$, therefore the domain is $\{x \mid x \leq 4, x \in R\}$ whereas the function $f(x) = 4 - x$ has a domain of $\{x \mid x \in R\}$. Since $\sqrt{f(x)}$ is undefined when $f(x) < 0$, the range of $\sqrt{f(x)}$ is $\{f(x) \mid f(x) \geq 0, f(x) \in R\}$, whereas the range of $f(x) = 4 - x$ is $\{f(x) \mid f(x) \in R\}$.

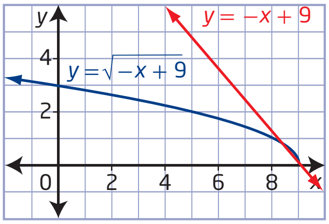
5. a) 

For $y = x - 2$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y = \sqrt{x - 2}$, domain $\{x \mid x \geq 2, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$.

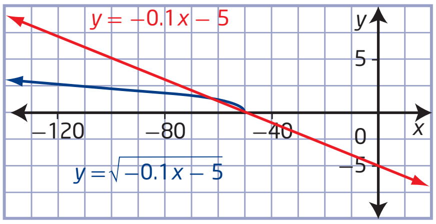
The domains differ since $\sqrt{x - 2}$ is undefined when $x < 2$. The range of $y = \sqrt{x - 2}$ is $y \geq 0$, when $x - 2 \geq 0$.

b) 

For $y = 2x + 6$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$. For $y = \sqrt{2x + 6}$, domain $\{x \mid x \geq -3, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$. $y = \sqrt{2x + 6}$ is undefined when $2x + 6 < 0$, therefore $x \geq -3$ and $y \geq 0$.

c) 

For $y = -x + 9$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y = \sqrt{-x + 9}$, domain $\{x \mid x \leq 9, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$. $y = \sqrt{-x + 9}$ is undefined when $-x + 9 < 0$, therefore $x \leq 9$ and $y \geq 0$.

d) 

For $y = -0.1x - 5$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y = \sqrt{-0.1x - 5}$, domain $\{x \mid x \leq -50, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$. $y = \sqrt{-0.1x - 5}$ is undefined when $-0.1x - 5 < 0$, therefore $x \leq -50$ and $y \geq 0$.

6. a) For $y = x^2 - 9$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq -9, y \in R\}$. For $y = \sqrt{x^2 - 9}$, domain $\{x \mid x \leq -3 \text{ and } x \geq 3, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$. $y = \sqrt{x^2 - 9}$ is undefined when $x^2 - 9 < 0$, therefore $x \leq -3$ and $x \geq 3$ and $y \geq 0$.

b) For $y = 2 - x^2$, domain $\{x \mid x \in R\}$, range $\{y \mid y \leq 2, y \in R\}$. For $y = \sqrt{2 - x^2}$, domain $\{x \mid -\sqrt{2} \leq x \leq \sqrt{2}, x \in R\}$, range $\{y \mid 0 \leq y \leq \sqrt{2}, y \in R\}$. $y = \sqrt{2 - x^2}$ is undefined when $2 - x^2 < 0$, therefore $x \leq \sqrt{2}$ and $x \geq -\sqrt{2}$ and $0 \leq y \leq \sqrt{2}$.

c) For $y = x^2 + 6$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 6, y \in R\}$. For $y = \sqrt{x^2 + 6}$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq \sqrt{6}, y \in R\}$. $y = \sqrt{x^2 + 6}$ is undefined when $x^2 + 6 < 0$, therefore $x \in R$ and $y \geq \sqrt{6}$.

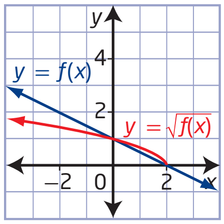
d) For $y = 0.5x^2 + 3$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 3, y \in R\}$. For $y = \sqrt{0.5x^2 + 3}$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq \sqrt{3}, y \in R\}$. $y = \sqrt{0.5x^2 + 3}$ is undefined when $0.5x^2 + 3 < 0$, therefore $x \in R$ and $y \geq \sqrt{3}$.

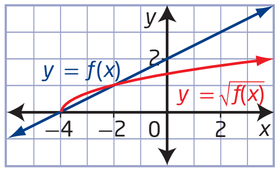
7. a) Since $y = \sqrt{x^2 - 25}$ is undefined when $x^2 - 25 < 0$, the domain changes from $\{x \mid x \in R\}$ to $\{x \mid x \leq -5 \text{ and } x \geq 5, x \in R\}$ and the range changes from $\{y \mid y \geq -25, y \in R\}$ to $\{y \mid y \geq 0, y \in R\}$.

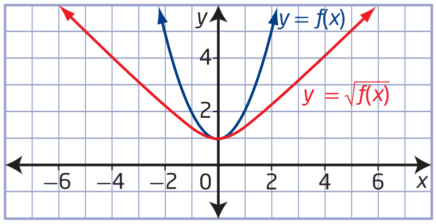
b) Since $y = \sqrt{x^2 + 3}$ is undefined when $x^2 + 3 < 0$, the range changes from $\{y \mid y \geq 3, y \in R\}$ to $\{y \mid y \geq \sqrt{3}, y \in R\}$.

c) Since $y = \sqrt{32 - 2x^2}$ is undefined when $32 - 2x^2 < 0$, the domain changes from $\{x \mid x \in R\}$ to $\{x \mid -4 \leq x \leq 4, x \in R\}$ and the range changes from $\{y \mid y \leq 32, y \in R\}$ to $\{y \mid 0 \leq y \leq \sqrt{32}, y \in R\}$ or $\{y \mid 0 \leq y \leq 4\sqrt{2}, y \in R\}$.

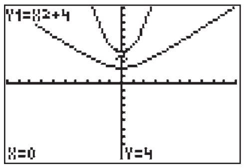
d) Since $y = \sqrt{5x^2 + 50}$ is undefined when $5x^2 + 50 < 0$, the range changes from $\{y \mid y \geq 50, y \in R\}$ to $\{y \mid y \geq \sqrt{50}, y \in R\}$ or $\{y \mid y \geq 5\sqrt{2}, y \in R\}$.

8. a) 

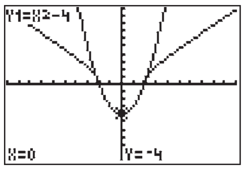
b) 

c) 

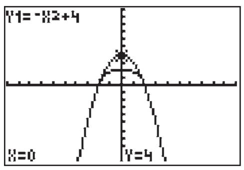
9. a) and **b)**

i) 

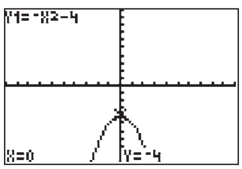
For $y = x^2 + 4$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 4, y \in R\}$

ii) 

For $y = x^2 - 4$, domain $\{x \mid x \in R\}$, range $\{y \mid y \geq -4, y \in R\}$

iii) 

For $y = -x^2 + 4$, domain $\{x \mid x \in R\}$, range $\{y \mid y \leq 4, y \in R\}$

iv) 

For $y = -x^2 - 4$, domain $\{x \mid x \in R\}$, range $\{y \mid y \leq -4, y \in R\}$.

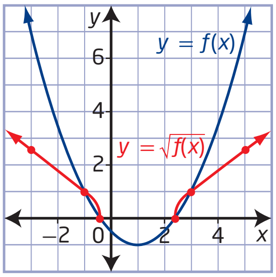
c) The graph of $y = \sqrt{j(x)}$ does not exist because all of the points on the graph $y = j(x)$ are below the x-axis. Since all values of $j(x) < 0$, then $\sqrt{j(x)}$ is undefined and produces no graph in the real number system.

d) The domains of the square root of a function are the same as the domains of the function when the value of the function ≥ 0. The domains of the square root of a function do not exist when the value of the function < 0. The ranges of the square root of a function are the square root of the range of the original function, except when the value of the function < 0 then the range is undefined.

10. a) For $y = x^2 - 4$, domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq -4, y \in \mathrm{R}\}$; for $y = \sqrt{x^2 - 4}$, domain $\{x \mid x \leq -2 \text{ and } x \geq 2, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

b) The value of y in the interval $(-2, 2)$ is negative therefore the domain of $y = \sqrt{x^2 - 4}$ is undefined and has no values in the interval $(-2, 2)$.

11. a)



I sketched the graph by locating key points, including invariant points, and determining the image points on the graph of the square root of the function.

b) For $y = f(x)$, domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq -1, y \in \mathrm{R}\}$; for $y = \sqrt{f(x)}$, domain $\{x \mid x \leq -0.4 \text{ and } x \geq 2.4, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

The domain of $y = \sqrt{f(x)}$, consists of all values in the domain of $f(x)$ for which $f(x) \geq 0$, and the range of $y = \sqrt{f(x)}$, consists of the square roots of all values in the range of $f(x)$ for which $f(x)$ is defined.

12. a) $d = \sqrt{h^2 + 12756h}$

b) domain $\{h \mid h \geq 0, h \in \mathrm{R}\}$, range $\{d \mid d \geq 0, d \in \mathrm{R}\}$

c) Find the point of intersection between the graph of the function and $h = 800$. The distance will be expressed as the d value of the ordered pair (h, d). In this case, d is approximately equal to 3293.

d) Yes, if h could be any real number then the domain is $\{h \mid h \leq -12\,756 \text{ or } h \geq 0, h \in \mathrm{R}\}$ and the range would remain the same- since all square root values must be greater than or equal to 0.

13. a) No, since $\sqrt{a}$, $a < 0$ is undefined, then $y = \sqrt{f(x)}$ will be undefined when $f(x) < 0$, but $f(x)$ represents values of the range not the domain as Chris stated.

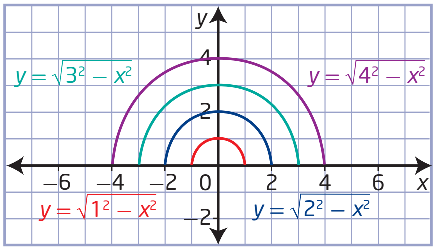
b) If the range consists of negative values, then you know that the graph represents $y = f(x)$ and not $y = \sqrt{f(x)}$.

14. a) $v = \sqrt{3.24 - h^2}$

b) domain $\{h \mid 0 \leq h \leq 1.8, h \in \mathrm{R}\}$, range $\{v \mid 0 \leq v \leq 1.8, v \in \mathrm{R}\}$ since both h and v represent distances.

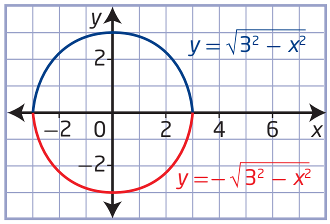
c) approximately 1.61 m

15. Step 1



Step 2 The parameter a determines the minimum value of the domain $(-a)$ and the maximum value of the domain (a); therefore the domain is $\{x \mid -a \leq x \leq a, x \in \mathrm{R}\}$. The parameter a also determines the maximum value of the range, where the minimum value of the range is 0; therefore the range is $\{y \mid 0 \leq y \leq a, y \in \mathrm{R}\}$.

Step 3 Example: $y = \sqrt{3^2 - x^2}$ the reflection of the graph in the x-axis is the equation $y = -\sqrt{3^2 - x^2}$.

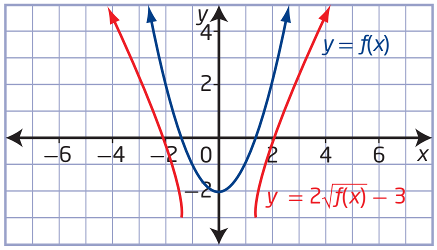


The graph forms a circle.

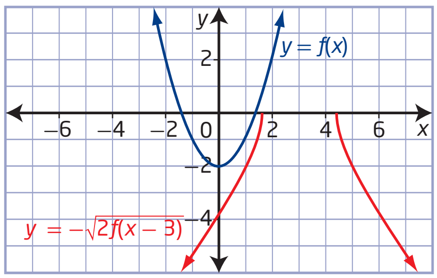
16. a) $(-27, 4\sqrt{3})$ **b)** $(-6, 12 - 2\sqrt{3})$

c) $(26, 6 - 4\sqrt{3})$

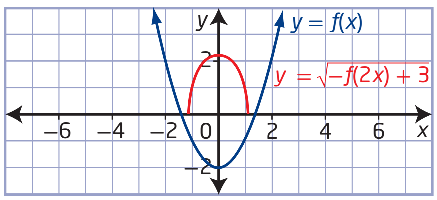
17. a)



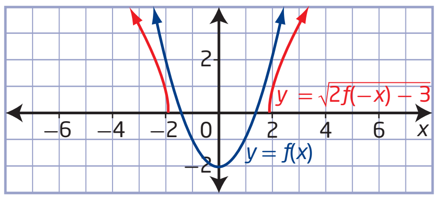
b)



c)



d)



18. Example: Sketch the graph in the following order:

1) $y = 2f(x)$ Stretch vertically by a factor of 2.
2) $y = 2f(x - 3)$ Translate horizontally 3 units right.
3) $y = \sqrt{2f(x - 3)}$ Plot invariant points and sketch a smooth curve above the x-axis.
4) $y = -\sqrt{2f(x - 3)}$ Reflect $y = \sqrt{2f(x - 3)}$ in the x-axis.

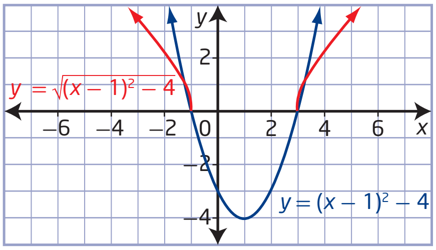
19. a) $r = \sqrt{\dfrac{A}{6\pi}}$ **b)** $r = \sqrt{\dfrac{A}{\pi(1 + \sqrt{37})}}$

C1 Example: Choose 4 to 5 key points on the graph of $y = f(x)$. Transform the points $(x, y) \rightarrow (x, \sqrt{y})$. Plot the new points and smooth out the graph. If you cannot get an idea of the general shape of the graph, choose more points to graph.

C2 The graph of $y = 16 - 4x$ is a linear function spanning from quadrant II to quadrant IV with an x-intercept of 4 and a y-intercept of 16. The graph of $y = \sqrt{16 - 4x}$ only exists when the graph of $y = 16 - 4x$ is on or above the x-axis. The y-intercept is at $\sqrt{16} = 4$ while the x-intercept stays the same. x-values for $x \leq 4$ are the same for both functions and the y-values for $y = \sqrt{16 - 4x}$ are the square root of y values for $y = 16 - 4x$.

C3 No, it is not possible, because the graph of $y = f(x)$ may exist when $y < 0$ but the graph of $y = \sqrt{f(x)}$ does not exist when $y < 0$.

C4 a)

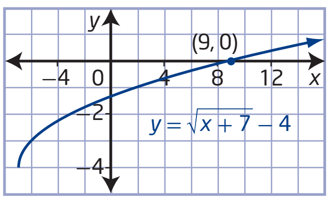


b) The graph of $y = (x - 1)^2 - 4$ is a quadratic function with a vertex of $(1, -4)$, y-intercept of -3, and x-intercepts of -1 and 3. It is above the x-axis when $x > 3$ and $x < -1$.
The graph of $y = \sqrt{(x - 1)^2 - 4}$ has the same x-intercepts but no y-intercept. The graph only exists when $x > 3$ and $x < -1$.

2.3 Solving Radical Equations Graphically, pages 96 to 98

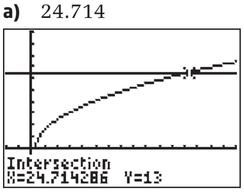
1. a) B **b)** A **c)** D **d)** C

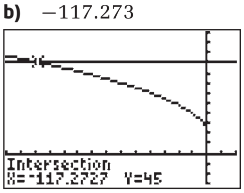
2. a) $x = 9$ **b)**



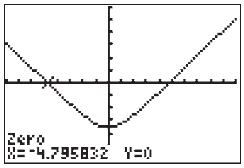
c) The roots of the equation are the same as the x-intercept on the graph.

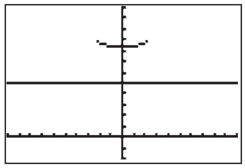
3. a) 24.714 **b)** −117.273



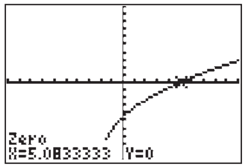


c) ± 4.796 **d)** no solution

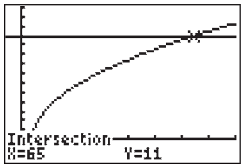


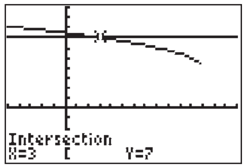


4. a) $x = 5.083$ **b)**

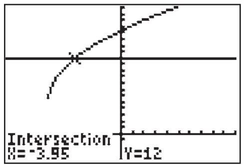


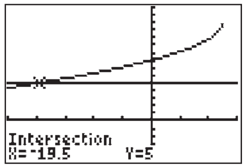
5. a) $x = 65$, $x \geq \dfrac{9}{2}$ **b)** $x = 3$, $x \leq 12$





c) $x = -3.95$, $x \geq -6.4$ **d)** $x = -19.5$, $x \leq 12.5$





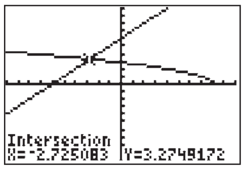
6. a) $x = \dfrac{7}{2}$, $x = -1$

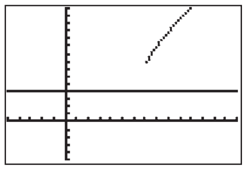
b) $x = 8$, $x = -2$, $x \leq -\dfrac{\sqrt{14}}{2}$ or $x \geq \dfrac{\sqrt{14}}{2}$

c) $x = 1.8$, $x = -1$, $-\dfrac{\sqrt{13}}{2} \leq x \leq \dfrac{\sqrt{13}}{2}$

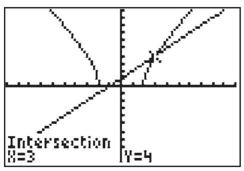
d) $x = 0$, $x = 2$, $\dfrac{-3\sqrt{2}}{2} \leq x \leq \dfrac{3\sqrt{2}}{2}$

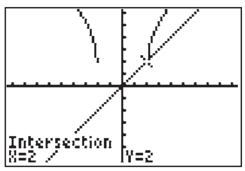
7. a) $x \approx -2.725$, $x \leq 8$ **b)** no real roots, $x \geq 7$



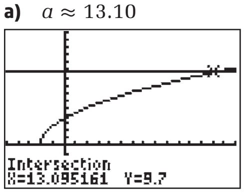


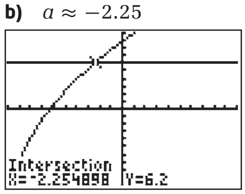
c) $x = 3$, $x \geq \dfrac{\sqrt{33}}{3}$ or $x \leq -\dfrac{\sqrt{33}}{3}$ **d)** $x = 2$, $x \geq 2$ or $x \leq -2$



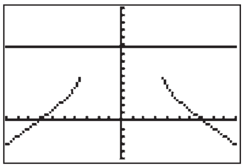


8. a) $a \approx 13.10$ **b)** $a \approx -2.25$

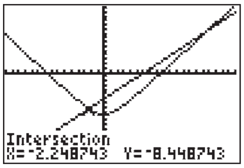


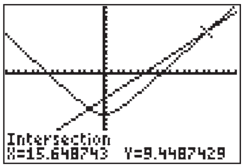


c) no solution

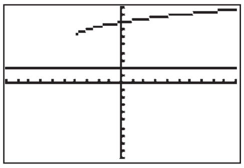


d) $a \approx -2.25$, $a \approx 15.65$





9. a) $6 + \sqrt{x + 4} = 2$
$\sqrt{x + 4} = -4$
$x + 4 = 16$
$x = 12$
Left Side $= 6 + \sqrt{12 + 4}$
$= 6 + \sqrt{16}$
$= 6 + 4$
$= 10$
Right Side $= 2$
Left Side $\neq$ Right Side
Since $10 \neq 2$, there is no solution.



b) Yes, if you isolate the radical expression like $\sqrt{x + 4} = -4$, if the radical is equal to a negative value then there is no solution.

10. Greg $\rightarrow N(t) = 1.3\sqrt{t} + 4.2 = 1.3\sqrt{6} + 4.2$
≈ 7.38 million,
Yolanda $\rightarrow N(t) = 1.3\sqrt{t} + 4.2 = 1.3\sqrt{1.5} + 4.2$
≈ 5.79 million
Greg is correct, it will take more than 6 years for the entire population to be affected.

11. approximately 99 cm

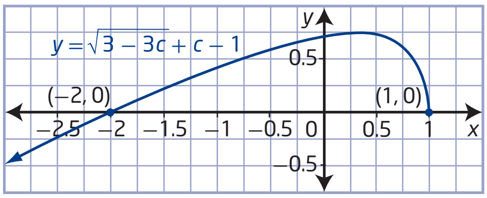
12. a) Yes **b)** 3000 kg

13. No, $\sqrt{x^2} = 9$ has two possible solutions ± 9, whereas $(\sqrt{x})^2 = 9$ has only one solution $+9$.

14. $x = \dfrac{3 + \sqrt{5}}{2}$

15. a) 5 m/s **b)** 75.2 kg

16. $c = -2$ or 1



If the function $y = \sqrt{-3(x + c)} + c$ passes through the point (0.25, 0.75), what is the value of c?

17. Lengths of sides are 55.3 cm, 60 cm, and 110.6 cm or 30.7 cm, 60 cm, and 61.4 cm.

C1 The x-intercepts of the graph of a function are the solutions to the corresponding equation. Example: A graph of the function $y = \sqrt{x - 1} - 2$ would show that the x-intercept has a value of 5. The equation that corresponds to this function is $0 = \sqrt{x - 1} - 2$ and the solution to the equation is 5.

C2 a) $s = \sqrt{9.8d}$ where s represents speed in metres per second and d represents depth in metres.

b) $s = \sqrt{9.8d}$
$s = \sqrt{(9.8 \text{ m/s}^2)(2500 \text{ m})}$
$s = \sqrt{24\,500 \text{ m}^2/\text{s}^2}$
$s \approx 156.5$ m/s

c) approximately 4081.6 m

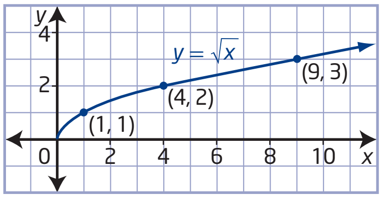
d) Example: I prefer the algebraic method because it is faster and I do not have to adjust window settings.

C3 Radical equations only have a solution in the real number system if the graph of the corresponding function has an x-intercept. For example, $y = \sqrt{x} + 4$ has no real solutions because there is no x-intercept.

C4 Extraneous roots occur when solving equations algebraically. Extraneous roots of a radical equation may occur anytime an expression is squared. For example, $x^2 = 1$ has two possible solutions, $x = \pm 1$. You can identify extraneous roots by graphing and by substituting into the original equation.

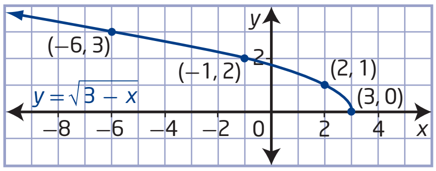
Chapter 2 Review, pages 99 to 101

1. a)



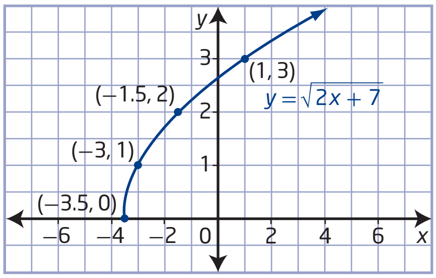
domain $\{x \mid x \geq 0, x \in R\}$
range $\{y \mid y \geq 0, y \in R\}$ All values in the table lie on the smooth curve graph of $y = \sqrt{x}$.

b)



domain $\{x \mid x \leq 3, x \in R\}$
range $\{y \mid y \geq 0, y \in R\}$ All points in the table lie on the graph of $y = \sqrt{3 - x}$.

c)



domain $\{x \mid x \geq -3.5, x \in R\}$
range $\{y \mid y \geq 0, y \in R\}$ All points in the table lie on the graph of $y = \sqrt{2x + 7}$.

2. Use $y = a\sqrt{b(x - h)} + k$ to describe transformations.

a) $a = 5 \rightarrow$ vertical stretch factor of 5
$h = -20 \rightarrow$ horizontal translation left 20 units;
domain $\{x \mid x \geq -20, x \in R\}$; range $\{y \mid y \geq 0, y \in R\}$

b) $b = -2 \rightarrow$ horizontal stretch factor of $\frac{1}{2}$, then reflected on y-axis: $k = -8 \rightarrow$ vertical translation of 8 units down.
domain $\{x \mid x \leq 0, x \in R\}$; range $\{y \mid y \geq -8, y \in R\}$

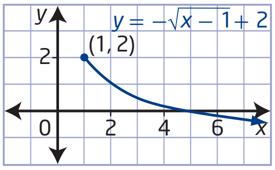
c) $a = -1 \rightarrow$ reflect in x-axis
$b = \frac{1}{6} \rightarrow$ horizontal stretch factor of 6
$h = 11 \rightarrow$ horizontal translation right 11 units;
domain $\{x \mid x \geq 11, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

3. a) $y = \sqrt{\frac{1}{10}x} + 12$, domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$,
range $\{y \mid y \geq 12, y \in \mathrm{R}\}$

b) $y = -2.5\sqrt{x + 9}$
domain $\{x \mid x \geq -9, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

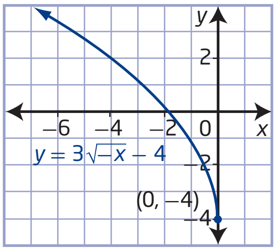
c) $y = \frac{1}{20}\sqrt{-\frac{2}{5}(x - 7)} - 3$,
domain $\{x \mid x \leq 7, x \in \mathrm{R}\}$, range $\{y \mid y \geq -3, y \in \mathrm{R}\}$

4. a)



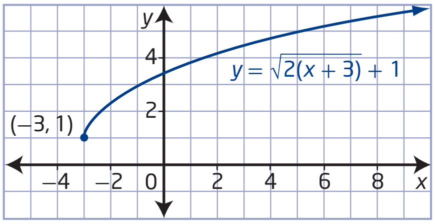
domain $\{x \mid x \geq 1, x \in \mathrm{R}\}$, range $\{y \mid y \leq 2, y \in \mathrm{R}\}$

b)



domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq -4, y \in \mathrm{R}\}$

c)

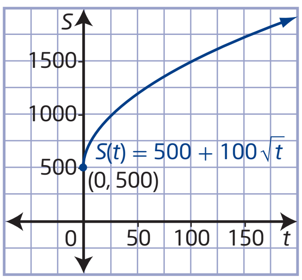


domain $\{x \mid x \geq -3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$

5. The domain is affected by a horizontal translation of 4 units right and by no reflection on the y-axis. The domain will have values of x greater than or equal to 4, due to a translation of the graph 4 units right. The range is affected by vertical translation of 9 units up and a reflection on the x-axis. The range will be less than or equal to 9, because the graph has been moved up 9 units and reflected on the x-axis, therefore the range is less than or equal to 9, instead of greater than or equal to 9.

6. a) Given the general equation $y = a\sqrt{b(x - h)} + k$ to describe transformations, $a = 100$ indicates a vertical stretch by a factor of 100, $k = 500$ indicates a vertical translation up 500 units.

b)



Since the minimum value of the graph is 500, the minimum estimated sales will be 500 units.

c) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$ The domain means that time is positive in this situation.
range $\{S(t) \mid S(t) \geq 500, S(t) \in \mathrm{W}\}$. The range means that the minimum sales are 500 units.

d) about 1274 units

7. a) $y = \sqrt{\frac{1}{4}(x + 3)} + 2$ **b)** $y = -2\sqrt{x + 4} + 3$

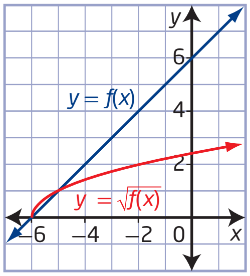
c) $y = 4\sqrt{-(x - 6)} - 4$

8. a) For $y = x - 2$, domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; for $y = \sqrt{x - 2}$, domain $\{x \mid x \geq 2, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$. The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the x-axis.

b) For $y = 10 - x$, domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; for $y = \sqrt{10 - x}$, domain $\{x \mid x \leq 10, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$ The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the x-axis.

c) For $y = 4x + 11$, domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$; for $y = \sqrt{4x + 11}$, domain $\left\{x \mid x \geq -\frac{11}{4}, x \in \mathrm{R}\right\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$. The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the x-axis.

9. a)



Plot invariant points at the intersection of the graph and lines $y = 0$ and $y = 1$. Plot any points $(x, \sqrt{y})$ where the value of y is a perfect square. Sketch a smooth curve through the invariant points and points satisfying $(x, \sqrt{y})$.

b) $y = \sqrt{f(x)}$ is positive when $f(x) > 0$,
$y = \sqrt{f(x)}$ does not exist when $f(x) < 0$.
$\sqrt{f(x)} > f(x)$ when $0 < f(x) < 1$ and
$f(x) > \sqrt{f(x)}$ when $f(x) > 1$

c) For $f(x)$: domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid y \in \mathrm{R}\}$; for $\sqrt{f(x)}$,
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$,
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$, since $\sqrt{f(x)}$ is undefined when $f(x) < 0$.

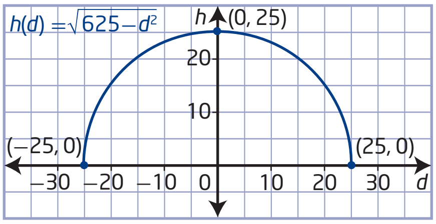
10. a) $y = 4 - x^2 \rightarrow$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \leq 4, y \in \mathrm{R}\}$ for $y = \sqrt{4 - x^2} \rightarrow$ domain $\{x \mid -2 \leq x \leq 2, x \in \mathrm{R}\}$, range $\{y \mid 0 \leq y \leq 2, y \in \mathrm{R}\}$, since $4 - x^2 > 0$ only between -2 and 2 then the domain of $y = \sqrt{4 - x^2}$ is $-2 \leq x \leq 2$. In the domain of $-2 \leq x \leq 2$ the maximum value of $y = 4 - x^2$ is 4, so the maximum value of $y = \sqrt{4 - x^2}$ is $\sqrt{4} = 2$ then the range of the function $y = \sqrt{4 - x^2}$ will be $0 \leq y \leq 2$.

b) $y = 2x^2 + 24 \rightarrow$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq 24, y \in \mathrm{R}\}$ for $y = \sqrt{2x^2 + 24} \rightarrow$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq \sqrt{24}, y \in \mathrm{R}\}$. The domain does not change since the entire graph of $y = 2x^2 + 24$ is above the x-axis. The range changes since the entire graph moves up 24 units and the graph itself opens up, so the range becomes $y \geq \sqrt{24}$.

c) $y = x^2 - 6x \rightarrow$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \geq -9, y \in \mathrm{R}\}$ for $y = \sqrt{x^2 - 6x} \rightarrow$ domain $\{x \mid x \leq 0 \text{ or } x \geq 6, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$, since $x^2 - 6x < 0$ between 0 and 6, then the domain is undefined in the interval (0, 6) and exists when $x \leq 0$ or $x \geq 6$. The range changes because the function only exists above the x-axis.

11. a) $h(d) = \sqrt{625 - d^2}$

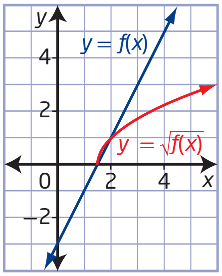
b)



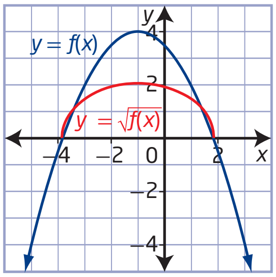
domain $\{d \mid -25 \leq d \leq 25, d \in \mathrm{R}\}$
range $\{h \mid 0 \leq h \leq 25, h \in \mathrm{R}\}$

c) In this situation, the values of h and d must be positive to express a positive distance. Therefore the domain changes to $\{d \mid 0 \leq d \leq 25, d \in \mathrm{R}\}$. Since the range of the original function $h(d) = \sqrt{625 - d^2}$ is always positive then the range does not change.

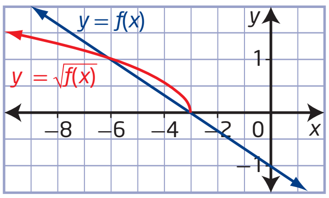
12. a)



b)

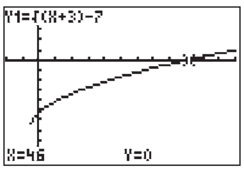


c)



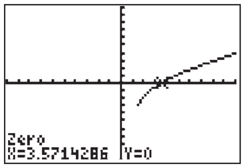
13. a) $x = 46$

b)

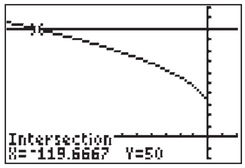


c) The root of the equation and the x value of the x-intercept are the same.

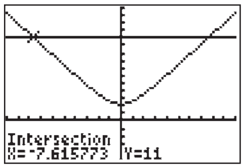
14. a) $x \approx 3.571$

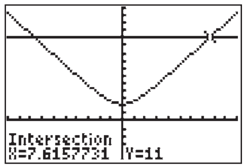


b) $x \approx -119.667$



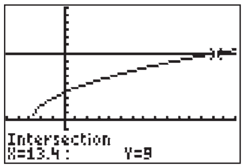
c) $x \approx -7.616$ and $x \approx 7.616$



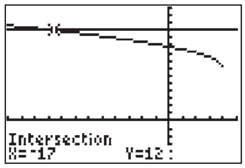


15. 4.13 m

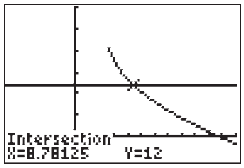
16. a) $x = 13.4$



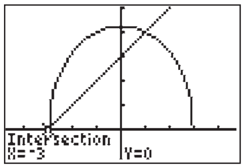
b) $x = -17$

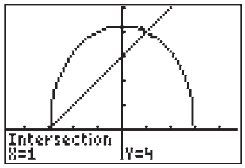


c) $x \approx 8.781$



d) $x = -3$ and 1





17. a) Jaime found two possible answers which are determined by solving a quadratic equation.

b) Carly found only one intersection at (5, 5) or x-intercept (5, 0) determined by possibly graphing.

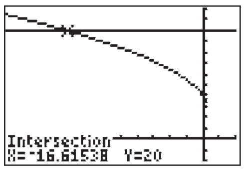
c) Atid found an extraneous root of $x = 2$.

18. a) 130 m^2 **b)** 6 m

Chapter 2 Practice Test, pages 102 to 103

1. B **2.** A **3.** A **4.** C **5.** D **6.** B

7. $x \approx -16.62$

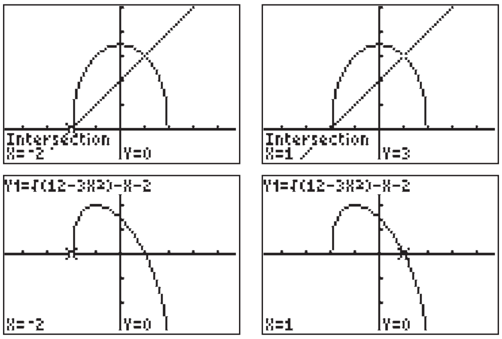


8. $y = 4\sqrt{x}$ or $y = \sqrt{16x}$

9. For $y = 7 - x \rightarrow$ domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y \in \mathrm{R}\}$. Since $y = \sqrt{7 - x}$ is the square root of the y-values for the function $y = 7 - x$, then the domain and ranges of $y = \sqrt{7 - x}$ will differ. Since $7 - x < 0$ when $x > 7$, then the domain of $y = \sqrt{7 - x}$ will be $\{x \mid x \leq 7, x \in \mathrm{R}\}$ and since $\sqrt{7 - x}$ indicates positive values only, then the range of $y = \sqrt{7 - x}$ is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

10. The domain of $y = f(x)$ is $\{x \mid x \in R\}$, and the range of $y = f(x)$ is $\{y \mid y \leq 8, y \in R\}$. The domain of $y = \sqrt{f(x)}$ is $\{x \mid -2 \leq x \leq 2, x \in R\}$ and the range of $y = \sqrt{f(x)}$ is $\{y \mid 0 \leq y \leq \sqrt{8}, y \in R\}$.

11.

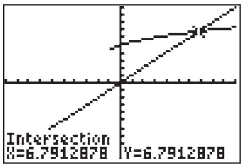


$x = -2, x = 1$

12.

$$4 + \sqrt{x + 1} = x$$
$$\sqrt{x + 1} = x - 4$$
$$x + 1 = (x - 4)^2$$
$$x + 1 = x^2 - 8x + 16$$
$$0 = x^2 - 9x + 15$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(15)}}{2(1)}$$
$$\approx 2.2 \text{ or } 6.8$$

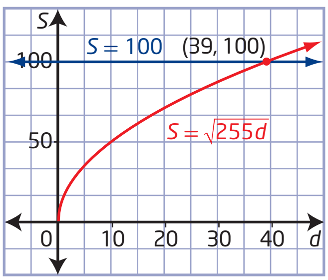
By checking, 2.2 is an extraneous root, therefore $x \approx 6.8$.



$x \approx 6.8$

13. a) Given the general equation $y = a\sqrt{b(x - h)} + k$ to describe transformations, $b = 255 \rightarrow$ indicating a horizontal stretch by a factor of $\frac{1}{255}$. To sketch the graph of $S = \sqrt{255d}$, graph the function $S = \sqrt{d}$ and apply a horizontal stretch of $\frac{1}{255}$, every point on the graph of $S = \sqrt{d}$ will become $\left(\frac{d}{255}, S\right)$.

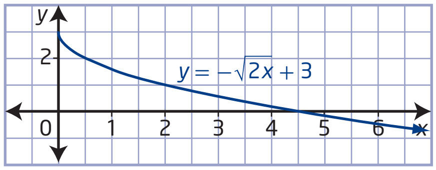
b)



$d \approx 39$ m The skid mark of the vehicle will be approximately 39 m.

14. a) Given the general equation $y = a\sqrt{b(x - h)} + k$ to describe transformations, $a = -1 \rightarrow$ reflection of the graph in the x-axis, $b = 2 \rightarrow$ horizontal stretch by a factor of $\frac{1}{2}$, $k = 3 \rightarrow$ vertical translation up 3 units.

b)

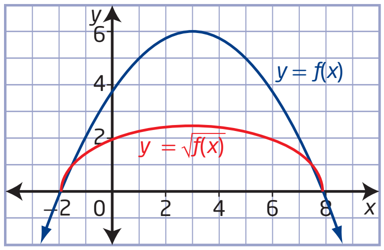


c) domain $\{x \mid x \geq 0, x \in R\}$, range $\{y \mid y \leq 3, y \in R\}$.

d) The domain remains the same because there was no horizontal translation or reflection on the y-axis. But since the graph was reflected on the x-axis and moved up 3 units and then the range becomes $y \leq 3$.

e) The equation $5 + \sqrt{2x} = 8$ can be rewritten as $0 = -\sqrt{2x} + 3$. Therefore the x-intercept of the graph $y = -\sqrt{2x} + 3$ is the solution of the equation $5 + \sqrt{2x} = 8$.

15.



Step 1 Plot invariant points at the intersection of $y = f(x)$ and functions $y = 0$ and $y = 1$.

Step 2 Plot points at $\sqrt{\text{max value}}$ and $\sqrt{\text{perfect square value of } y = f(x)}$

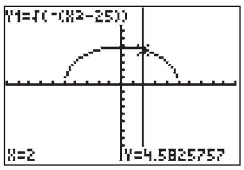
Step 3 Join all points with a smooth curve, remember that the graph of $y = \sqrt{f(x)}$ is above the original graph for the interval $0 \leq y \leq 1$. Note that for the interval where $f(x) < 0$, the function $y = \sqrt{f(x)}$ is undefined and has no graph.

16. a) $y = (\sqrt{5})\sqrt{-(x - 5)}$

b) domain $\{x \mid 0 \leq x \leq 5, x \in R\}$, range $\{y \mid 0 \leq y \leq 5, y \in R\}$

Domain: x cannot be negative nor greater than half the diameter of the base, or 5. Range: y cannot be negative nor greater than the height of the roof, or 5.

c)



The height of the roof 2 m from the centre is about 4.58 m.