Chapter 3 Polynomial Functions

3.1 Characteristics of Polynomial Functions, pages 114 to 117

1. a) No, this is a square root function.
   b) Yes, this is a polynomial function of degree 1.
   c) No, this is an exponential function.
   d) Yes, this is a polynomial function of degree 4.
   e) No, this function has a variable with a negative exponent.
   f) Yes, this is a polynomial function of degree 3.

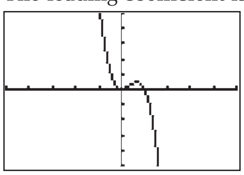
2. a) degree 1, linear, −1, 3
b) degree 2, quadratic, 9, 0
c) degree 4, quartic, 3, 1
d) degree 3, cubic, −3, 4
e) degree 5, quintic, −2, 9
f) degree 0, constant, 0, −6

3. a) odd degree, positive leading coefficient, 3 x-intercepts, domain $\{x \mid x \in R\}$ and range $\{y \mid y \in R\}$
b) odd degree, positive leading coefficient, 5 x-intercepts, domain $\{x \mid x \in R\}$ and range $\{y \mid y \in R\}$
c) even degree, negative leading coefficient, 3 x-intercepts, domain $\{x \mid x \in R\}$ and range $\{y \mid y \leq 16.9, y \in R\}$
d) even degree, negative leading coefficient, 0 x-intercepts, domain $\{x \mid x \in R\}$ and range $\{y \mid y \leq -3, y \in R\}$

4. a) degree 2 with positive leading coefficient, parabola opens upward, maximum of 2 x-intercepts, y-intercept of −1
b) degree 3 with negative leading coefficient, extends from quadrant II to IV, maximum of 3 x-intercept, y-intercept of 5
c) degree 4 with negative leading coefficient, opens downward, maximum of 4 x-intercepts, y-intercept of 4
d) degree 5 with positive leading coefficient, extends from quadrant III to I, maximum of 5 x-intercepts, y-intercept of 0
e) degree 1 with negative leading coefficient, extends from quadrant II to IV, 1 x-intercept, y-intercept of 4
f) degree 4 with positive leading coefficient, opens upward, maximum of 4 x-intercepts, y-intercept of 0

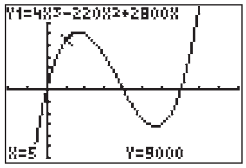
5. Example: Jake is right as long as the leading coefficient a is a positive integer. The simplest example would be a quadratic function with $a = 2$, $b = 2$, and $n = 2$.

6. a) degree 4
b) The leading coefficient is 1 and the constant is −3000. The constant represents the initial cost.
c) degree 4 with a positive leading coefficient, opens upward, 2 x-intercepts, y-intercept of −3000
d) The domain is $\{x \mid x \geq 0, x \in R\}$, since it is impossible to have negative snowboard sales.
e) The positive x-intercept is the breakeven point.
f) Let $x = 15$, then $P(x) = 62\ 625$.

7. a) cubic function
b) The leading coefficient is −3 and the constant is 0.
c) 
d) The domain is $\{d \mid 0 \leq d \leq 1, d \in R\}$ because you cannot give negative drug amounts and you must have positive reaction times.

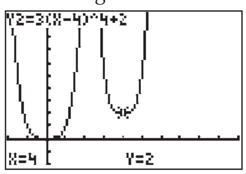
8. a) For 1 ring, the total number of hexagons is given by $f(1) = 1$. For 2 rings, the total number of hexagons is given by $f(2) = 7$. For 3 rings, the total number of hexagons is given by $f(3) = 19$.
b) 397 hexagons

9. a) End behaviour: the curve extends up in quadrants I and II; domain $\{t \mid t \in R\}$; range $\{P \mid P \geq 10\ 071, P \in R\}$; the range for the period $\{t \mid 0 \leq t \leq 20, t \in R\}$ that the population model can be used is $\{P \mid 15\ 000 \leq P \leq 37\ 000, P \in R\}$. t-intercepts: none; P-intercept: 15 000
b) 15 000 people c) 18 000 people d) 18 years

10. a) 

From the graph, the height of a single box must be greater than 0 and cannot be between 20 cm and 35 cm.
b) $V(x) = 4x(x - 20)(x - 35)$. The factored form clearly shows the three possible x-intercepts.

11. a) The graphs in each pair are the same. Let n represent a whole number, then $2n$ represents an even whole number.
$y = (-x)^{2n}$
$y = (-1)^{2n}x^{2n}$
$y = 1^{n}x^{2n}$
$y = x^{2n}$
b) The graphs in each pair are reflections of each other in the y-axis. Let n represent a whole number, then $2n + 1$ represents an odd whole number.
$y = (-x)^{2n+1}$
$y = (-1)^{2n+1}x^{2n+1}$
$y = (-1)^{2n}(-1)^{1}x^{2n+1}$
$y = -(1)^{n}x^{2n+1}$
$y = -x^{2n+1}$
c) For even whole numbers, the graph of the functions are unchanged. For odd whole numbers, the graph of the functions are reflected in the y-axis.

12. a) vertical stretch by a factor of 3 and translation of 4 units right and 2 units up
b) vertical stretch by a factor of 3 and translation of 4 units right and 2 units up
c) 

13. If there is only one root, $y = (x - a)^n$, then the function will only cross the x-axis once in the case of an odd-degree function and it will only touch the x-axis once if it is an even-degree function.

C1 Example: Odd degree: At least one x-intercept and up to a maximum of n x-intercepts, where n is the degree of the function. No maximum or minimum points. Domain is $\{x \mid x \in R\}$ and range is $\{y \mid y \in R\}$. Even degree: From zero to a maximum of n x-intercepts, where n is the degree of the function. Domain is $\{x \mid x \in R\}$ and the range depends on the maximum or minimum value of the function.

C2 a) Examples:
i) $y = x^3$ ii) $y = x^2$
iii) $y = -x^3$ iv) $y = -x^2$

b) Example: Parts i) and ii) have positive leading coefficients, while parts iii) and iv) have negative leading coefficients. Parts i) and iii) are odd-degree functions, while parts ii) and iv) are even-degree functions.

C3 Example: The line $y = x$ and polynomial functions with odd degree greater than one and positive leading coefficient extend from quadrant III to quadrant I. Both have no maximums or minimums. Both have the same domain and range. Odd degree polynomial functions have at least one x-intercept.

C4 **Step 1**

Function	Degree	End Behaviour
$y = x + 2$	1	extends from quadrant III to I
$y = -3x + 1$	1	extends from quadrant II to IV
$y = x^2 - 4$	2	opens upward
$y = -2x^2 - 2x + 4$	2	opens downward
$y = x^3 - 4x$	3	extends from quadrant III to I
$y = -x^3 + 3x - 2$	3	extends from quadrant II to IV
$y = 2x^3 + 16$	3	extends from quadrant III to I
$y = -x^3 - 4x$	3	extends from quadrant II to IV
$y = x^4 - 4x^2 + 5$	4	opens upward
$y = -x^4 + x^3 + 4x^2 - 4x$	4	opens downward
$y = x^4 + 2x^2 + 1$	4	opens upward
$y = x^5 - 2x^4 - 3x^3 + 5x^2 + 4x - 1$	5	extends from quadrant III to I
$y = x^5 - 1$	5	extends from quadrant III to I
$y = -x^5 + x^4 + 8x^3 + 8x^2 - 16x - 16$	5	extends from quadrant II to IV
$y = x(x + 1)^2(x + 4)^2$	5	extends from quadrant III to I

Step 2 The leading coefficient determines if it opens upward or downward; in the case of odd functions it determines if it is increasing or decreasing.
Step 3 Always have at least one minimum or maximum. Not all functions will have the same range. Either opens upward or downward.
Step 4. Always have the same domain and range. Either extends from quadrant III to I or from quadrant II to IV. No maximum or minimum.

3.2 Remainder Theorem, pages 124 to 125

1. a) $\dfrac{x^2 + 10x - 24}{x - 2} = x + 12$
b) $x \neq 2$ **c)** $(x - 2)(x + 12)$
d) Multiplying the statement in part c) yields $x^2 + 10x - 24$.

2. a) $\dfrac{3x^4 - 4x^3 - 6x^2 + 17x - 8}{x + 1} = 3x^3 - 7x^2 + x + 16 - \dfrac{24}{x + 1}$
b) $x \neq -1$
c) $(x + 1)(3x^3 - 7x^2 + x + 16) - 24$
d) Expanding the statement in part c) yields $3x^4 - 4x^3 - 6x^2 + 17x - 8$.

3. a) $Q(x) = x^2 + 4x + 1$ **b)** $Q(x) = x^2 + 4x + 1$
c) $Q(w) = 2w^2 - 3w + 4$ **d)** $Q(m) = 9m^2 + 3m + 6$
e) $Q(t) = t^3 + 5t^2 - 8t + 7$
f) $Q(y) = 2y^3 + 6y^2 + 15y + 45$

4. a) $Q(x) = x^2 - 3x + 12$ **b)** $Q(m) = m^3 + m + 14$
c) $Q(x) = -x^3 + x^2 - x + 1$ **d)** $Q(s) = 2s^2 + 7s + 5$
e) $Q(h) = h^2 - h$ **f)** $Q(x) = 2x^2 + 3x - 7$

5. a) $\dfrac{x^3 + 7x^2 - 3x + 4}{x + 2} = x^2 + 5x - 13 + \dfrac{30}{x + 2}, x \neq -2$
b) $\dfrac{11t - 4t^4 - 7}{t - 3} = -4t^3 - 12t^2 - 36t - 97 - \dfrac{298}{t - 3}, t \neq 3$
c) $\dfrac{x^3 + 3x^2 - 2x + 5}{x + 1} = x^2 + 2x - 4 + \dfrac{9}{x + 1}, x \neq -1$
d) $\dfrac{4n^3 + 7n - 5}{n + 3} = 4n - 5 + \dfrac{10}{n + 3}, n \neq -3$
e) $\dfrac{4n^3 - 15n + 2}{n - 3} = 4n^2 + 12n + 21 + \dfrac{65}{n - 3}, n \neq 3$
f) $\dfrac{x^3 + 6x^2 - 4x + 1}{x + 2} = x^2 + 4x - 12 + \dfrac{25}{x + 2}, x \neq -2$

6. a) 16 **b)** 38 **c)** -23
d) -67 **e)** -2 **f)** 8

7. a) 9 **b)** -40 **c)** 41 **d)** -4

8. a) -1 **b)** 3 **c)** 2 **d)** -1

9. 11

10. 4 and -2

11. a) $2x + 3$
b) 9, it represents the rest of the width that cannot be simplified any more.

12. a) $2n + 2 + \dfrac{9}{n - 3}$ **b)** -2 and -0.5

13. a) $9\pi x^2 + 24\pi x + 16\pi$, represents the area of the base
b) $\pi(3x + 4)^2(x + 3)$
c) $10 \text{ cm} \leq r \leq 28 \text{ cm}$ and $5 \leq h \leq 11$

14. $m = -\dfrac{11}{5}, n = \dfrac{59}{5}$

15. $a = -\dfrac{14}{3}, b = -\dfrac{2}{3}$

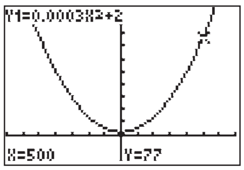
16. Divide using the binomial $x - \dfrac{3}{2}$.

17. Examples: **a)** $x^2 - 4x - 1$
b) $x^3 + 3x^2 + 3x + 6$ **c)** $2x^4 + x^3 + x^2 + x$

C1 Example: The process is the same. Long division of polynomials results in a restriction.

C2 a) $(x - a)$ is a factor of $bx^2 + cx + d$.
b) $d + ac + a^2b$

C3 a) 77 **b)** 77
c)



The remainder is the height of the cable at the given horizontal distance.

3.3 The Factor Theorem, pages 133 to 135

1. a) $x - 1$ **b)** $x + 3$ **c)** $x - 4$ **d)** $x - a$

2. a) Yes **b)** No **c)** No **d)** Yes
e) Yes **f)** No

3. a) No **b)** No **c)** No **d)** No
e) Yes **f)** No

4. a) $\pm1, \pm2, \pm4, \pm8$ **b)** $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$
c) $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$
d) $\pm1, \pm2, \pm4$ **e)** $\pm1, \pm3, \pm5, \pm15$
f) $\pm1, \pm2, \pm4$

5. a) $(x - 1)(x - 2)(x - 3)$ **b)** $(x - 1)(x + 1)(x + 2)$
c) $(v - 4)(v + 4)(v + 1)$
d) $(x + 4)(x + 2)(x - 3)(x + 1)$
e) $(k - 1)(k - 2)(k + 3)(k + 2)(k + 1)$

6. a) $(x + 3)(x - 2)(x - 3)$ **b)** $(t - 5)(t + 4)(t + 2)$
c) $(h - 5)(h^2 + 5h - 2)$ **d)** $x^5 + 8x^3 + 2x - 15$
e) $(q - 1)(q + 1)(q^2 + 2q + 3)$

7. a) $k = -2$ **b)** $k = 1, -7$
c) $k = -6$ **d)** $k = 6$

8. h, $h - 1$, and $h - 1$

9. $l - 5$ and $l + 3$

10. $x - 2$ cm, $x + 4$ cm, and $x + 3$ cm

11. $x + 5$ and $x + 3$

12. a) $x - 5$ is a possible factor because it is the corresponding factor for $x = 5$. Since $f(5) = 0$, $x - 5$ is a factor of the polynomial function.
b) 2-ft sections would be weak by the same principle applied in part a).

13. $x + 3$, $x + 2$, and $x + 1$

14. Synthetic division yields a remainder of $a + b + c + d + e$, which must equal 0 as given. Therefore, $x - 1$ is a possible divisor.

15. $m = -\frac{7}{10}$, $n = -\frac{51}{10}$

16. a) **i)** $(x - 1)(x^2 + x + 1)$ **ii)** $(x - 3)(x^2 + 3x + 9)$
iii) $(x + 1)(x^2 - x + 1)$ **iv)** $(x + 4)(x^2 - 4x + 16)$
b) $x + y$, $x^2 - xy + y^2$ **c)** $x - y$, $x^2 + xy + y^2$
d) $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$

C1 Example: Looking at the x-intercepts of the graph, you can determine at least one binomial factor, $x - 2$ or $x + 2$. The factored form of the polynomial is $(x - 2)(x + 2)(x^2 + 1)$.

C2 Example: Using the integral zero theorem, you have both ± 1 and ± 5 as possible integer values. The x-intercepts of the graph of the corresponding function will also give the factors.

C3 Example: Start by using the integral zero theorem to check for a first possible integer value. Apply the factor theorem using the value found from the integral zero theorem. Use synthetic division to confirm that the remainder is 0 and determine the remaining factor. Repeat the process until all factors are found.

3.4 Equations and Graphs of Polynomial Functions, pages 147 to 152

1. a) $x = -3, 0, 4$ **b)** $x = -1, 3, 5$ **c)** $x = -2, 3$

2. a) $x = -2, -1$ **b)** $x = 1$ **c)** $x = -4, -2$

3. a) $(x + 3)(x + 2)(x - 1) = 0$, roots are -3, -2 and 1
b) $-(x + 4)(x - 1)(x - 3) = 0$, roots are -4, 1 and 3
c) $-(x + 4)^2(x - 1)(x - 3) = 0$, roots are -4, 1 and 3

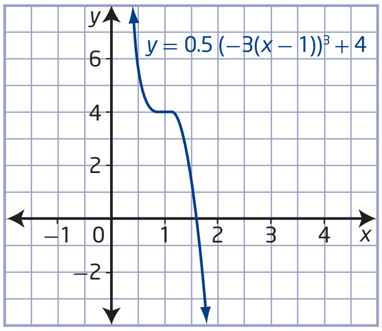
4. a) **i)** -4, -1, and 1
ii) positive for $-4 < x < -1$ and $x > 1$, negative for $x < -4$ and $-1 < x < 1$
iii) all three zeros are of multiplicity 1, the sign of the function changes
b) **i)** -1 and 4
ii) negative for all values of x, $x \neq -1, 4$
iii) both zeros are of multiplicity 2, the sign of the function does not change
c) **i)** -3 and 1
ii) positive for $x < -3$ and $x > 1$, negative for $-3 < x < 1$
iii) -3 (multiplicity 1) and 1 (multiplicity 3), at both the function changes sign but is flatter at $x = 1$
d) **i)** -1 and 3
ii) negative for $-1 < x < 3$ and $x > 3$, positive for $x < -1$
iii) -1 (multiplicity 3) and 3 (multiplicity 2), at $x = -1$ the function changes sign but not at $x = 3$

5. a) B **b)** D **c)** C **d)** A

6. a) $a = 0.5$ vertical stretch by a factor of 0.5, $b = -3$ horizontal stretch by a factor of $\frac{1}{3}$ and a reflection in the y-axis, $h = 1$ translation of 1 unit right, $k = 4$ translation of 4 units up
b)

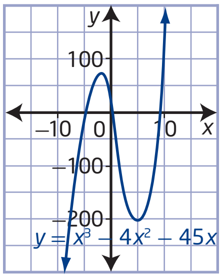
$y = x^3$	$y = (-3x)^3$	$y = 0.5(-3x)^3$	$y = 0.5(-3(x - 1))^3 + 4$
$(-2, -8)$	$\left(\frac{2}{3}, -8\right)$	$\left(\frac{2}{3}, -4\right)$	$\left(\frac{5}{3}, 0\right)$
$(-1, -1)$	$\left(\frac{1}{3}, -1\right)$	$\left(\frac{1}{3}, -\frac{1}{2}\right)$	$\left(\frac{4}{3}, \frac{7}{2}\right)$
$(0, 0)$	$(0, 0)$	$(0, 0)$	$(1, 4)$
$(1, 1)$	$\left(-\frac{1}{3}, 1\right)$	$\left(-\frac{1}{3}, \frac{1}{2}\right)$	$\left(\frac{2}{3}, \frac{9}{2}\right)$
$(2, 8)$	$\left(-\frac{2}{3}, 8\right)$	$\left(-\frac{2}{3}, 4\right)$	$\left(\frac{1}{3}, 8\right)$

c)



7. a) **i)** -5, 0, and 9
ii) degree 3 from quadrant III to I
iii) -5, 0, and 9 each of multiplicity 1
iv) 0
v) positive for $-5 < x < 0$ and $x > 9$, negative for $x < -5$ and $0 < x < 9$
b) **i)** -9, 0 and 9
ii) degree 4 opening upwards
iii) 0 (multiplicity 2), -9 and 9 each of multiplicity 1
iv) 0
v) positive for $x < -9$ and $x > 9$, negative for $-9 < x < 9$, $x \neq 0$
c) **i)** -3, -1, and 1
ii) degree 3 from quadrant III to I
iii) -3, -1, and 1 each of multiplicity 1
iv) -3
v) positive for $-3 < x < -1$ and $x > 1$, negative for $x < -3$ and $-1 < x < 1$
d) **i)** -3, -2, 1, and 2
ii) degree 4 opening downwards
iii) -3, -2, 1, and 2 each of multiplicity 1
iv) -12
v) positive for $-3 < x < -2$ and $1 < x < 2$, negative for $x < -3$ and $-2 < x < 1$ and $x > 2$

8. a)



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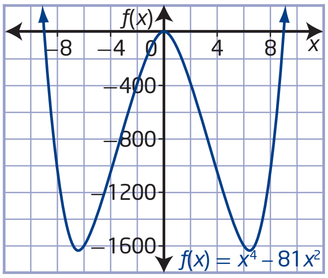

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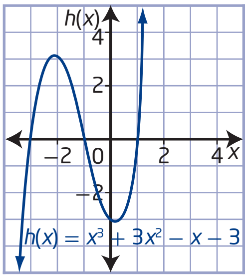

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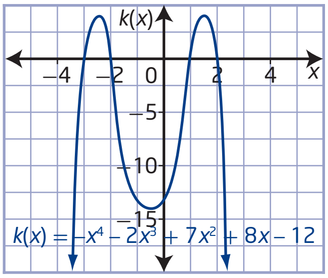
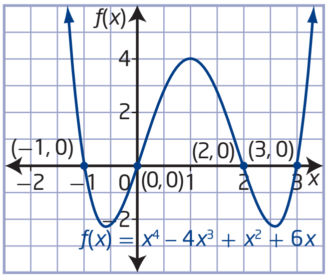


9. a)



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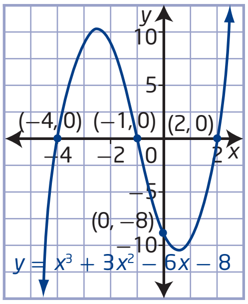

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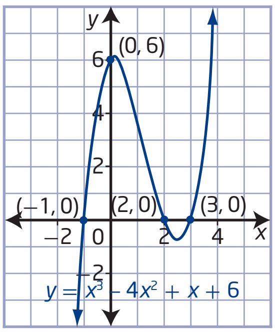

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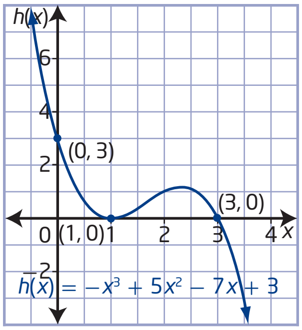

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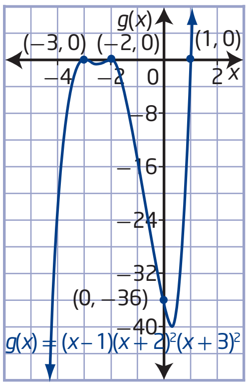

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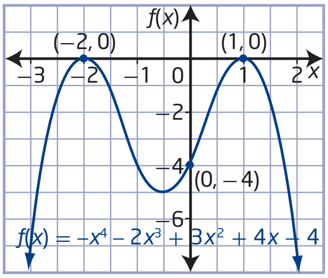


10. a) positive leading coefficient, x-intercepts: -2 and 3, positive for $-2 < x < 3$ and $x > 3$, negative for $x < -2$, $y = (x + 2)^3(x - 3)^2$

b) negative leading coefficient, x-intercepts: -4, -1, and 3, positive for $x < -4$ and $-1 < x < 3$, negative for $-4 < x < -1$ and $x > 3$, $y = -(x + 4)(x + 1)(x - 3)$

c) negative leading coefficient, x-intercepts: -2, -1, 2, and 3, positive for $-2 < x < -1$ and $2 < x < 3$, negative for $x < -2$ and $-1 < x < 2$ and $x > 3$, $y = -(x + 2)(x + 1)(x - 2)(x - 3)$

d) positive leading coefficient, x-intercepts: -1, 1, and 3, positive for $x < -1$ and $1 < x < 3$ and $x > 3$, negative for $-1 < x < 1$, $y = (x + 1)(x - 1)(x - 3)^2$

11. a) $a = 1$, $b = \frac{1}{2}$, $h = 2$, $k = -3$

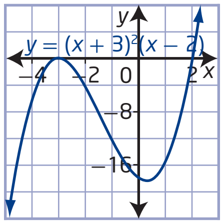
b) Horizontal stretch by a factor of 2, translation of 2 units right, and translation of 3 units down

c) domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

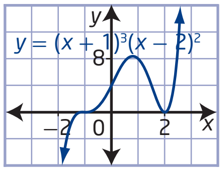
12. 2 m by 21 m by 50 m

13. 5 ft

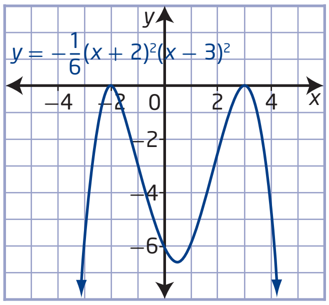
14. a) $y = (x + 3)^2(x - 2)$



b) $y = (x + 1)^3(x - 2)^2$



c) $y = -\frac{1}{6}(x + 2)^2(x - 3)^2$



15. 4 cm by 2 cm by 8 cm

16. -7, -5, and -3

17. The side lengths of the two cubes are 2 m and 3 m.

18. a) $(x^2 - 12)^2 - x^2$ **b)** 5 in. by 5 in. **c)** 13 in. by 13 in.

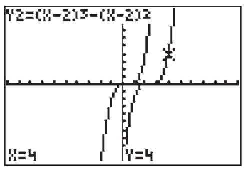
19. 4, 5, 6, and 7 or -7, -6, -5, and -4

20. $y = -\frac{1}{3}(x - \sqrt{3})(x + \sqrt{3})(x - 1)$

21. roots: -4.5, 8, and 2; $0 = (x + 4.5)(x - 8)(x - 2)$

22. a) translation of 2 units right

b)



c) $y = x^3 - x^2 = x^2(x - 1)$: 0 and 1,
$y = (x - 2)^3 - (x - 2)^2 = (x - 2)^2(x - 3)$: 2 and 3

23. When $x \approx 0.65$, or when the sphere is at a depth of approximately 0.65 m.

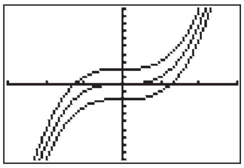
C1 Example: It is easier to identify the roots.

C2 Example: A root of an equation is a solution of the equation. A zero of a function is a value of x for which $f(x) = 0$. An x-intercept of a graph is the x-coordinate of the point where a line or curve crosses or touches the x-axis. They all represent the same thing.

C3 Example: If the multiplicity of a zero is 1, the function changes sign. If the multiplicity of a zero is even, the function does not change sign. The shape of a graph close to a zero of $x = a$ (order n) is similar to the shape of the graph of a function with degree equal to n of the form $y = (x - a)^n$.

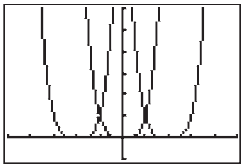
C4 **Step 1** Set A

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```



Set B

```
WINDOW
 Xmin=-5
 Xmax=5
 Xscl=1
 Ymin=-1
 Ymax=6
 Yscl=1
 Xres=1
```



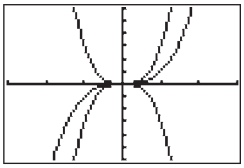
a) The graph of $y = x^3 + k$ is translated vertically k units compared to the graph of $y = x^3$.

b) The graph of $y = (x - h)^4$ is translated horizontally h units compared to the graph of $y = x^4$.

Step 2 h: horizontal translation; k: vertical translation

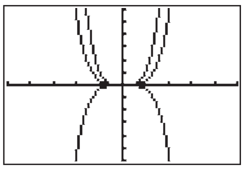
Step 3 Set C

```
WINDOW
 Xmin=-3
 Xmax=3
 Xscl=1
 Ymin=-6
 Ymax=6
 Yscl=1
 Xres=1
```



Set D

```
WINDOW
 Xmin=-5
 Xmax=5
 Xscl=1
 Ymin=-6
 Ymax=6
 Yscl=1
 Xres=1
```

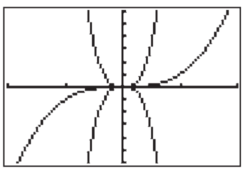


a) The graph of $y = ax^3$ is stretched vertically by a factor of $|a|$ relative to the graph of $y = x^3$. When a is negative, the graph is reflected in the x-axis.

b) When a is $-1 < a < 0$ or $0 < a < 1$, the graph of $y = ax^4$ is stretched vertically by a factor of $|a|$ relative to the graph of $y = x^4$. When a is negative, the graph is reflected in the x-axis.

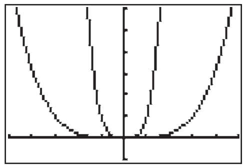
Step 4 Set E

```
WINDOW
 Xmin=-2
 Xmax=2
 Xscl=1
 Ymin=-6
 Ymax=6
 Yscl=1
 Xres=1
```



Set F

```
WINDOW
 Xmin=-5
 Xmax=5
 Xscl=1
 Ymin=-1
 Ymax=6
 Yscl=1
 Xres=1
```



a) The graph of $y = (bx)^3$ is stretched horizontally by a factor of $\frac{1}{|b|}$ relative to the graph of $y = x^3$. When b is negative, the graph is reflected in the y-axis.

b) When b is $-1 < b < 0$ or $0 < b < 1$, the graph of $y = (bx)^4$ is stretched horizontally by a factor of $\frac{1}{|b|}$ relative to the graph of $y = x^4$. When b is negative, the graph is reflected in the y-axis.

Step 5 a: vertical stretch; reflection in the x-axis; b: horizontal stretch; reflection in the y-axis

Chapter 3 Review, pages 153 to 154

1. a) No, this is a square root function.
b) Yes, this is a polynomial function of degree 4.
c) Yes, this is a polynomial function of degree 3.
d) Yes, this is a polynomial function of degree 1.

2. a) degree 4 with positive leading coefficient, opens upward, maximum of 4 x-intercepts, y-intercept of 0
b) degree 3 with negative leading coefficient, extends from quadrant II to quadrant IV, maximum of 3 x-intercepts, y-intercept of 4
c) degree 1 with positive leading coefficient, extends from quadrant III to quadrant I, 1 x-intercept, y-intercept of -2
d) degree 2 with positive leading coefficient, opens upward, maximum of 2 x-intercepts, y-intercept of -4
e) degree 5 with positive leading coefficient, extends from quadrant III to quadrant I, maximum of 5 x-intercepts, y-intercept of 1

3. a) quadratic function **b)** 9196 ft
c) 25 s **d)** 26.81 s

4. a) 37, $\frac{x^3 + 9x^2 - 5x + 3}{x - 2} = x^2 + 11x + 17 + \frac{37}{x - 2}, x \neq 2$
b) 2, $\frac{2x^3 + x^2 - 2x + 1}{x + 1} = 2x^2 - x - 1 + \frac{2}{x + 1}, x \neq -1$
c) 9, $\frac{12x^3 + 13x^2 - 23x + 7}{x - 1} = 12x^2 + 25x + 2 + \frac{9}{x - 1}, x \neq 1$
d) 1, $\frac{-8x^4 - 4x + 10x^3 + 15}{x + 1} = -8x^3 + 18x^2 - 18x + 14 + \frac{1}{x + 1}, x \neq -1$

5. a) -3 **b)** 166

6. -34

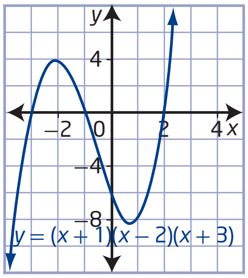
7. a) Yes, $P(1) = 0$. **b)** No, $P(-1) \neq 0$.
c) Yes, $P(-4) = 0$. **d)** Yes, $P(4) = 0$.

8. a) $(x - 2)(x - 3)(x + 1)$ **b)** $-4(x - 2)(x + 2)(x + 1)$
c) $(x - 1)(x - 2)(x - 3)(x + 2)$
d) $(x + 3)(x - 1)^2(x - 2)^2$

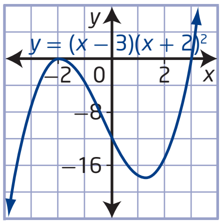
9. a) $x + 3$, $2x - 1$, and $x + 1$
b) 4 m by 1 m by 2 m

10. $k = -2$

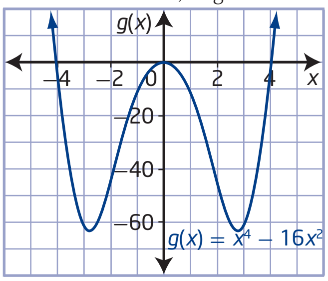
11. a) x-intercepts: -3, -1, and 2; degree 3 extending from quadrant III to quadrant I; -3, -1, and 2 each of multiplicity 1; y-intercept of -6; positive for $-3 < x < -1$ and $x > 2$, negative for $x < -3$ and $-1 < x < 2$



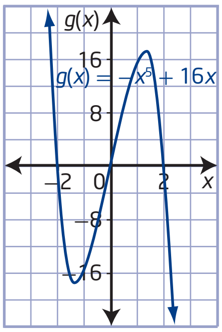
b) x-intercepts: -2 and 3; degree 3 extending from quadrant III to quadrant I; -2 (multiplicity 2) and 3 (multiplicity 1); y-intercept of -12; positive for $x > 3$, negative for $x < 3$, $x \neq -2$



c) x-intercepts: -4, 0, and 4; degree 4 opening upwards; 0 (multiplicity 2), -4 and 4 each of multiplicity 1; y-intercept of 0; positive for $x < -4$ and $x > 4$, negative for $-4 < x < 4$, $x \neq 0$



d) x-intercepts: -2, 0, and 2; degree 5 extending from quadrant II to quadrant IV; -2, 0, and 2 each of multiplicity 1; y-intercept of 0; positive for $x < -2$ and $0 < x < 2$, negative for $-2 < x < 0$ and $x > 2$

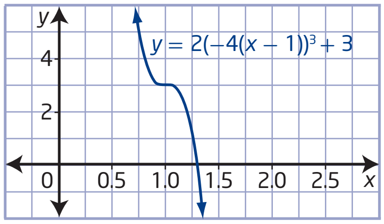


12. a) $a = 2$ vertical stretch by a factor of 2, $b = -4$ horizontal stretch by $\frac{1}{4}$ and reflection in the y-axis, $h = 1$ translation of 1 unit right, $k = 3$ translation of 3 units up

b)

Transformation	Parameter Value	Equation
horizontal stretch/ reflection in y-axis	-4	$y = (-4x)^3$
vertical stretch/ reflection in x-axis	2	$y = 2(-4x)^3$
translation right	1	$y = 2(-4(x - 1))^3$
translation up	3	$y = 2(-4(x - 1))^3 + 3$

c)



13. a) $y = (x + 1)(x + 3)^2$ **b)** $y = -(x + 1)(x - 2)^3$

14. a) Examples: $y = (x + 2)(x + 1)(x - 3)^2$ and $y = -(x + 2)(x + 1)(x - 3)^2$

b) $y = 2(x + 2)(x + 1)(x - 3)^2$

15. a) $V = 2l^2(l - 5)$ **b)** 8 cm by 3 cm by 16 cm

Chapter 3 Practice Test, pages 155 to 156

1. C **2.** B **3.** D **4.** B **5.** C

6. a) -4 and 3 **b)** -1 and 3

c) -2, 2, and 5 **d)** -3 and 3

7. a) $P(x) = (x + 2)(x + 1)^2$

b) $P(x) = (x - 1)(x^2 - 12x - 12)$

c) $P(x) = -x(x - 3)^2$

d) $P(x) = (x + 1)(x^2 - 4x + 5)$

8. a) B **b)** C **c)** A

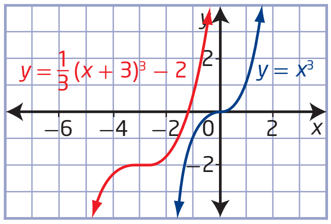
9. a) $V = x(20 - 2x)(18 - x)$

b) 2 cm by 16 cm by 16 cm

10. a) $a = \frac{1}{3}$, vertical stretch by a factor of $\frac{1}{3}$; $b = 1$, no horizontal stretch; $h = -3$, translation of 3 units left; $k = -2$, translation of 2 units down

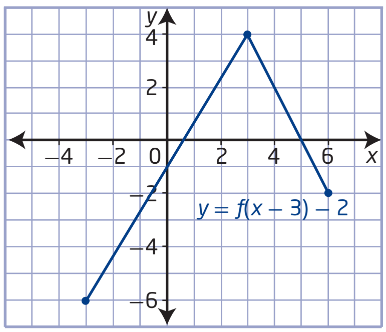
b) domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

c)

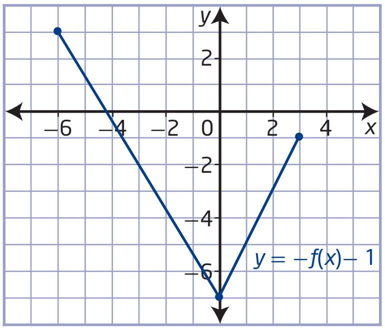


Cumulative Review, Chapters 1–3, pages 158 to 159

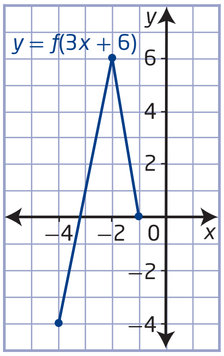
1. a)



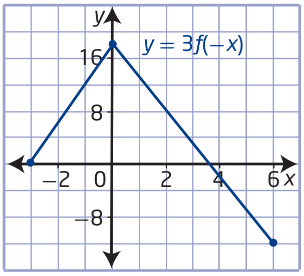
b)



c)



d)



2. $y + 4 = f(x - 3)$

3. a) translation of 1 unit left and 5 units down

b) vertical stretch by a factor of 3, reflection in the x-axis, and translation of 2 units right

c) reflection in the y-axis and translation of 1 unit right and 3 units up

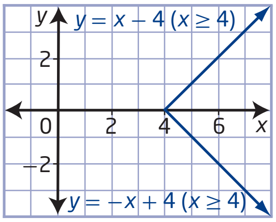
4. a) $(9, 10)$ **b)** $(6, -18)$ **c)** $(-2, 9)$

5. a) x-intercepts: $-\frac{4}{3}$ and 2, y-intercept: -3

b) x-intercepts: -4 and 6, y-intercept: 6

6. a) Yes

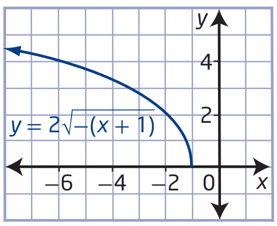
b)



c) Example: No; restrict domain of $y = |x| + 4$ to $\{x \mid x \geq 0, x \in R\}$.

7. $g(x) = \sqrt{2(x + 2)} - 3$

8. $y = 2\sqrt{-(x + 1)}$



domain $\{x \mid x \leq -1, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$

9. a) $g(x) = \sqrt{9x}$ **b)** $g(x) = 3\sqrt{x}$

c) $\sqrt{9x} = \sqrt{9}(\sqrt{x}) = 3\sqrt{x}$

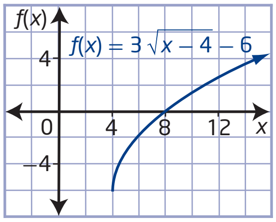
10. a) The x-intercepts are invariant points for square roots of functions, since $\sqrt{0} = 0$.

b) $f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \geq -1, y \in R\}$; $g(x)$: domain $\{x \mid x \leq -1 \text{ or } x \geq 1, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$; The square root function has a restricted domain.

11. a) No, substituting -2.75 back into the equation does not satisfy the equation.

b) Only one solution, $x = -2$.

12. a)



The x-intercept is 8.

b) $x = 8$

c) They are the same.

13. a) $\frac{x^4 + 3x + 4}{x + 1} = x^3 - x^2 + x + 2 + \frac{2}{x + 1}$; $P(-1) = 2$

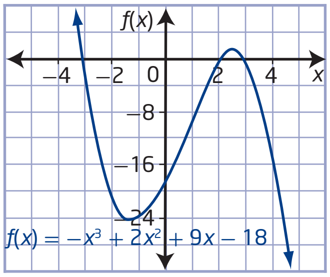
b) $\frac{x^3 + 5x^2 + x - 9}{x + 3} = x^2 + 2x - 5 + \frac{6}{x + 3}$; $P(-3) = 6$

14. $\pm1, \pm2, \pm3, \pm6$; $P(1) = 0$, $P(-1) = -16$, $P(2) = -4$, $P(-2) = 0$, $P(3) = 0$, $P(-3) = 96$, $P(6) = 600$, $P(-6) = 1764$

15. a) $(x + 5)(x - 1)(x - 4)$ **b)** $(x - 3)(x + 4)(x + 2)$

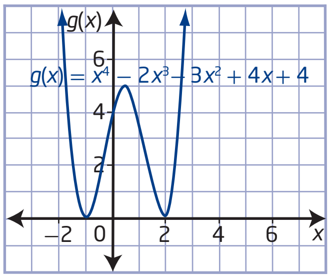
c) $-(x - 2)^2(x + 2)^2$

16. a)



x-intercepts: -3, 2, and 3; y-intercept: -18

b)



x-intercepts: -1 and 2, y-intercept: 4

17. a) $(x + 4)$ and $(x - 3)$ **b)** 4.5 m by 7.5 m by 0.5 m

18. $y = 3(-(x - 5))^3$

Unit 1 Test, pages 160 to 161

1. D **2.** C **3.** D **4.** A **5.** D **6.** C **7.** A

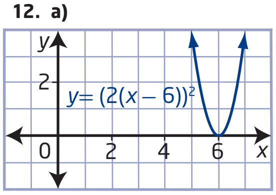
8. -13

9. $\{y \mid y \geq 8, y \in R\}$

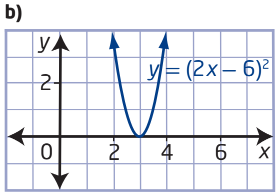
10. $g(x) = |x + 2| + 3$

11. 5

12. a)

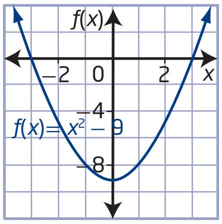


b)

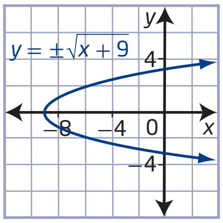


c) They both have the same shape but one of them is shifted right further.

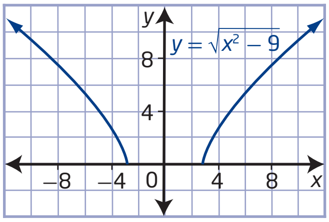
13. a)



b) $y = \pm\sqrt{x + 9}$



c) $y = \sqrt{x^2 - 9}$



d) for part a): domain $\{x \mid x \in R\}$, range $\{y \mid y \geq -9, y \in R\}$; for part *b*): domain $\{x \mid x \geq -9, x \in R\}$, range $\{y \mid y \in R\}$; for part c): domain $\{x \mid x \leq -3 \text{ or } x \geq 3, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$

14. Quadrant II: reflection in the y-axis, $y = f(-x)$; quadrant III: reflection in the y-axis and then the x-axis, $y = -f(-x)$; quadrant IV: reflection in the x-axis, $y = -f(x)$

15. a) Mary should have subtracted 4 from both sides in step 1. She also incorrectly squared the expression on the right side in step 2. The correct solution follows:

$2x = \sqrt{x + 1} + 4$

Step 1: $(2x - 4)^2 = (\sqrt{x + 1})^2$

Step 2: $4x^2 - 16x + 16 = x + 1$

Step 3: $4x^2 - 17x + 15 = 0$

Step 4: $(4x - 5)(x - 3) = 0$

Step 5: $4x - 5 = 0$ or $x - 3 = 0$

Step 6: $x = \frac{5}{4}$ $\quad x = 3$

Step 7: A check determines that $x = 3$ is the solution.

b) Yes, the point of intersection of the two graphs will yield the possible solution, $x = 3$.

16. $c = -3$; $P(x) = (x + 3)(x + 2)(x - 1)^2$

17. a) $\pm1, \pm2, \pm3, \pm6$

b) $P(x) = (x - 3)(x + 2)(x + 1)$

c) x-intercepts: -2, -1 and 3; y-intercept: -6

d) $-2 \leq x \leq -1$ and $x \geq 3$