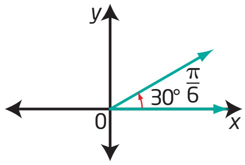
Chapter 4 Trigonometry and the Unit Circle

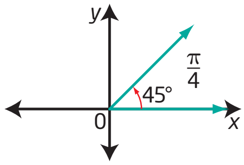
4.1 Angles and Angle Measure, pages 175 to 179

1. **a)** clockwise **b)** counterclockwise
c) clockwise **d)** counterclockwise

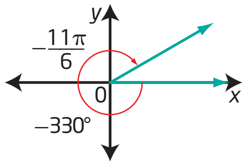
2. **a)** $30° = \frac{\pi}{6}$



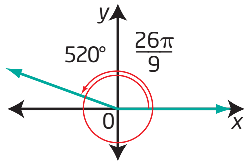
b) $45° = \frac{\pi}{4}$



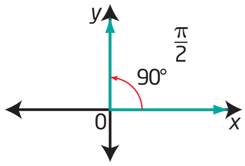
c) $-330° = -\frac{11\pi}{6}$



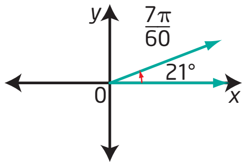
d) $520° = \frac{26\pi}{9}$



e) $90° = \frac{\pi}{2}$



f) $21° = \frac{7\pi}{60}$

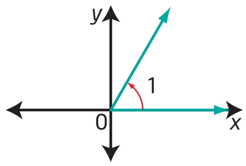


3. **a)** $\frac{\pi}{3}$ or 1.05 **b)** $\frac{5\pi}{6}$ or 2.62
c) $-\frac{3\pi}{2}$ or −4.71 **d)** $\frac{2\pi}{5}$ or 1.26
e) $-\frac{37\pi}{450}$ or −0.26 **f)** 3π or 9.42

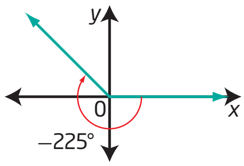
4. **a)** 30° **b)** 120°
c) −67.5° **d)** −450°
e) $\frac{180°}{\pi}$ or 57.3° **f)** $\frac{495°}{\pi}$ or 157.6°

5. **a)** $\frac{360°}{7}$ or 51.429° **b)** $\frac{1260°}{13}$ or 96.923°
c) $\frac{120°}{\pi}$ or 38.197° **d)** $\frac{3294°}{5\pi}$ or 209.703°
e) $\frac{-1105.2°}{\pi}$ or −351.796° **f)** $\frac{-3600°}{\pi}$ or −1145.916°

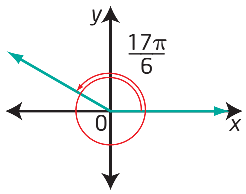
6. **a)** quadrant I



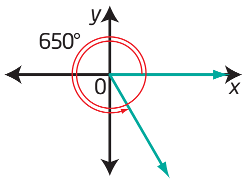
b) quadrant II



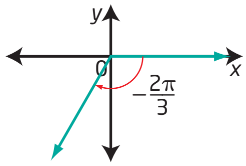
c) quadrant II



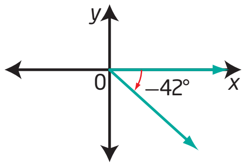
d) quadrant IV



e) quadrant III



f) quadrant IV



7. Examples:

a) 432°, −288° **b)** $\frac{11\pi}{4}$, $-\frac{5\pi}{4}$

c) 240°, −480° **d)** $\frac{7\pi}{2}$, $-\frac{\pi}{2}$

e) 155°, −565° **f)** 1.5, −4.8

8. a) coterminal, $\frac{17\pi}{6} = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{5\pi}{6} + 2\pi$

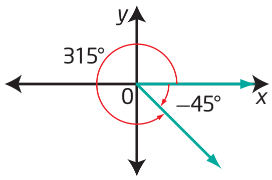
b) not coterminal **c)** not coterminal

d) coterminal, $-493° = 227° - 2(360°)$

9. a) $135° \pm (360°)n, n \in N$ **b)** $-\frac{\pi}{2} \pm 2\pi n, n \in N$

c) $-200° \pm (360°)n, n \in N$ **d)** $10 \pm 2\pi n, n \in N$

10. Example:



$-45° + 360° = 315°$, $-45° \pm (360°)n, n \in N$

11. a) 425° **b)** 320°

c) −400°, 320°, 680° **d)** $-\frac{5\pi}{4}$

e) $-\frac{23\pi}{6}, \frac{\pi}{6}, \frac{13\pi}{6}$ **f)** $-\frac{5\pi}{3}, \frac{\pi}{3}$

g) −3.9 **h)** −0.9, 5.4

12. a) 13.30 cm **b)** 4.80 m

c) 15.88 cm **d)** 30.76 in.

13. a) 2.25 radians **b)** 10.98 ft

c) 3.82 cm **d)** 17.10 m

14. a) $\frac{25\pi}{3}$ or 26.18 m

b) $\frac{A_{\text{sector}}}{A_{\text{circle}}} = \frac{\text{sector angle}}{2\pi}$

$A_{\text{sector}} = \frac{\pi r^2\left(\frac{5\pi}{3}\right)}{2\pi}$

$A_{\text{sector}} = \frac{5\pi(5)^2}{6}$

$A_{\text{sector}} = \frac{125\pi}{6}$

The area watered is approximately 65.45 m².

c) 16π radians or 2880°

15. a) Examples: $\frac{\pi}{12}$ radians/h, 1 revolution per day, 15°/h

b) $\frac{100\pi}{3}$ or 104.72 radians/s

c) 54 000°/min

16. a) 2.36 **b)** 135.3°

17.

	Revolutions	Degrees	Radians
a)	1 rev	360°	2π
b)	0.75 rev	270°	$\frac{3\pi}{2}$ or 4.7
c)	0.4 rev	150°	$\frac{5\pi}{6}$
d)	−0.3 rev	−97.4°	−1.7
e)	−0.1 rev	−40°	$-\frac{2\pi}{9}$ or −0.7
f)	0.7 rev	252°	$\frac{7\pi}{5}$ or 4.4
g)	−3.25 rev	−1170°	$-\frac{13\pi}{2}$ or −20.4
h)	$\frac{23}{18}$ or 1.3 rev	460°	$\frac{23\pi}{9}$ or 8.0
i)	$-\frac{3}{16}$ or −0.2 rev	−67.5°	$-\frac{3\pi}{8}$

18. Jasmine is correct. Joran's answer includes the solution when $k = 0$, which is the reference angle 78°.

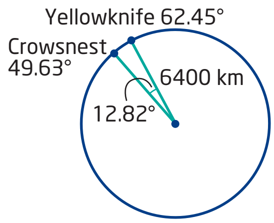
19. a) 55.6 grad

b) Use a proportion: $\frac{\text{gradians}}{\text{degrees}} = \frac{400 \text{ grad}}{360°}$.

So, measure in gradians $= \frac{10(\text{number of degrees})}{9}$.

c) The gradian was developed to express a right angle as a metric measure. A right angle is equivalent to 100 grads.

20. a)



b) 1432.01 km

c) Example: Bowden (51.93° N, 114.03° W) and Airdrie (51.29° N, 114.01° W) are 71.49 km apart.

21. a) 2221.4 m/min **b)** 7404.7 radians/min

22. 8.5 km/h

23. 66 705.05 mph

24. a) $69.375° = 69° + 0.375(60')$

$= 69°\ 22.5'$

$= 69°\ 22'\ 30''$

b) i) 40° 52′ 30″ **ii)** 100° 7′ 33.6″

iii) 14° 33′ 54″ **iv)** 80° 23′ 6″

25. a) $69°\ 22'\ 30'' = 69°22.5'$

$= 69° + \left(\frac{22.5}{60}\right)^{\circ}$

$= 69.375°$

b) i) 45.508° **ii)** 72.263°

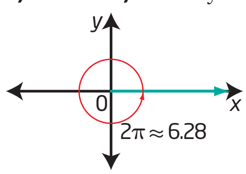
iii) 105.671° **iv)** 28.167°

26. $A_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin \theta)$

27. a) 120° **b)** 65° **c)** Examples: 3:00 and 9:00

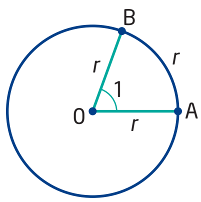
d) 2 **e)** shortly after 4:05

C1



π is 180° and 2π is 360°. 2(3.14) = 6.282 which is more than 6. Therefore, 6 radians must be less than 360°.

C2

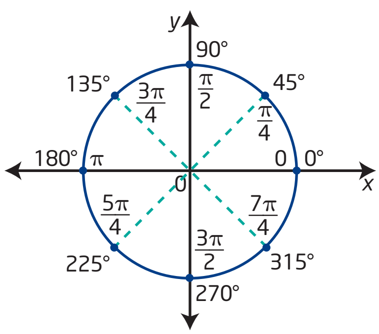


1° is a very small angle, it is $\frac{1}{360}$ of one rotation. One radian is much larger than 1°; 1 radian is the angle whose arc is the same as the radius, it is nearly $\frac{1}{6}$ of one rotation.

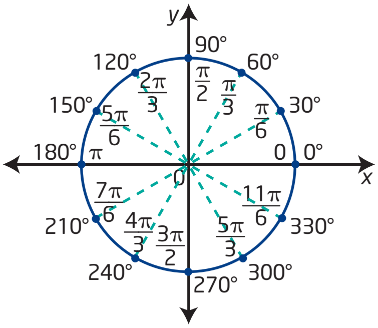
C3 a) 40°; $140° \pm (360°)n, n \in N$

b) 0.72; $0.72 \pm 2\pi n, n \in N$

C4 a)



b)



C5 a) $x = 3$ **b)** $y = x - 3$

4.2 The Unit Circle, pages 186 to 190

1. a) $x^2 + y^2 = 16$ **b)** $x^2 + y^2 = 9$
c) $x^2 + y^2 = 144$ **d)** $x^2 + y^2 = 6.76$

2. a) No; $\left(-\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{5}{8} \neq 1$

b) No; $\left(\frac{\sqrt{5}}{8}\right)^2 + \left(\frac{7}{8}\right)^2 = \frac{27}{32} \neq 1$

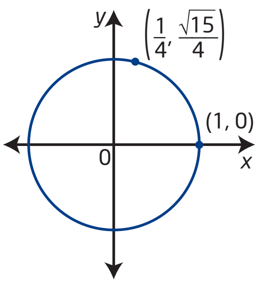
c) Yes; $\left(-\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = 1$

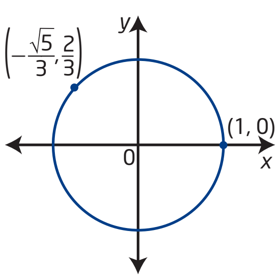
d) Yes; $\left(\frac{4}{5}\right)^2 + \left(-\frac{3}{5}\right)^2 = 1$

e) Yes; $\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 = 1$

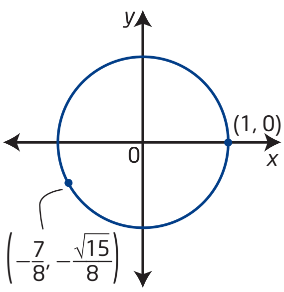
f) Yes; $\left(\frac{\sqrt{7}}{4}\right)^2 + \left(\frac{3}{4}\right)^2 = 1$

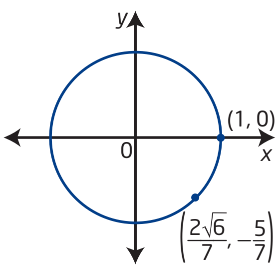
3. a) $y = \frac{\sqrt{15}}{4}$ **b)** $x = -\frac{\sqrt{5}}{3}$



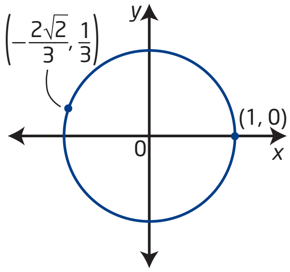


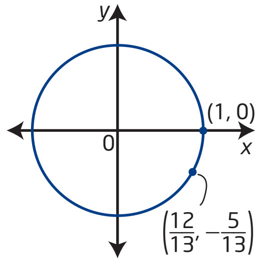
c) $y = -\frac{\sqrt{15}}{8}$ **d)** $x = \frac{2\sqrt{6}}{7}$





e) $x = -\frac{2\sqrt{2}}{3}$ **f)** $y = -\frac{5}{13}$



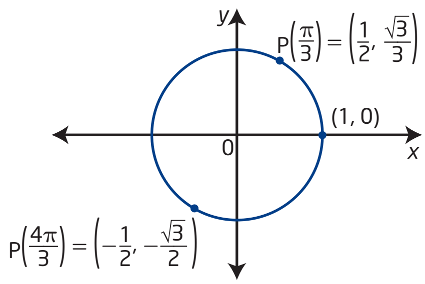


4. a) $(-1, 0)$ **b)** $(0, -1)$
c) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **d)** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
e) $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **f)** $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
g) $(1, 0)$ **h)** $(0, 1)$
i) $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ **j)** $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

5. a) $\frac{3\pi}{2}$ **b)** 0 **c)** $\frac{\pi}{4}$ **d)** $\frac{3\pi}{4}$
e) $\frac{\pi}{3}$ **f)** $\frac{5\pi}{3}$ **g)** $\frac{5\pi}{6}$ **h)** $\frac{7\pi}{6}$
i) $\frac{5\pi}{4}$ **j)** π

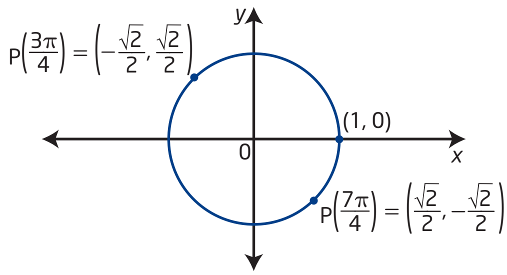
6. $\frac{5\pi}{6}$ and $-\frac{7\pi}{6}$

7. a)



If $\theta = \frac{\pi}{3}$ then $\theta + \pi = \frac{\pi}{3} + \pi$ or $\frac{4\pi}{3}$ since $P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $P\left(\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

b)



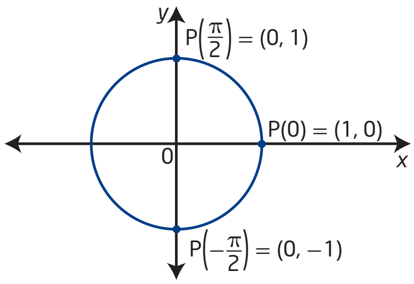
If $\theta = \frac{3\pi}{4}$ then $\theta + \pi = \frac{3\pi}{4} + \pi$ or $\frac{7\pi}{4}$ since $P\left(\frac{3\pi}{4}\right) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ and $P\left(\frac{7\pi}{4}\right) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

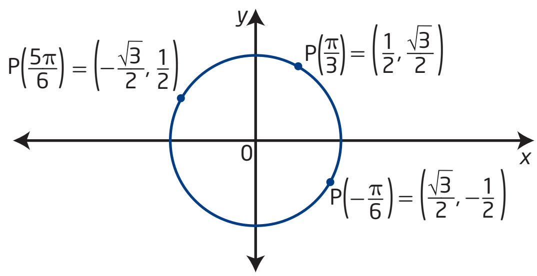
8.

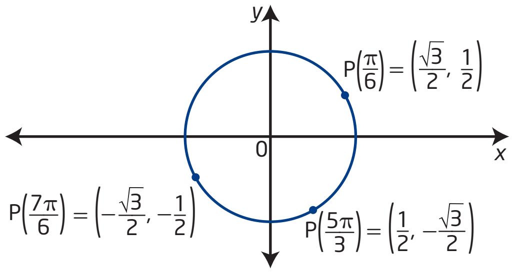
Point	$+\frac{1}{4}$ rotation	$-\frac{1}{4}$ rotation	Step 4: Description
$P(0) = (1, 0)$	$P\left(\frac{\pi}{2}\right) = (0, 1)$	$P\left(-\frac{\pi}{2}\right) = (0, -1)$	x- and y-values change places and take signs of new quadrant
$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$P\left(\frac{\pi}{3} + \frac{\pi}{2}\right) = P\left(\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$P\left(\frac{\pi}{3} - \frac{\pi}{2}\right) = P\left(-\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	x- and y-values change places and take signs of new quadrant
$P\left(\frac{5\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	$P\left(\frac{5\pi}{3} + \frac{\pi}{2}\right) = P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$P\left(\frac{5\pi}{3} - \frac{\pi}{2}\right) = P\left(\frac{7\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	x- and y-values change places and take signs of new quadrant

Diagrams:

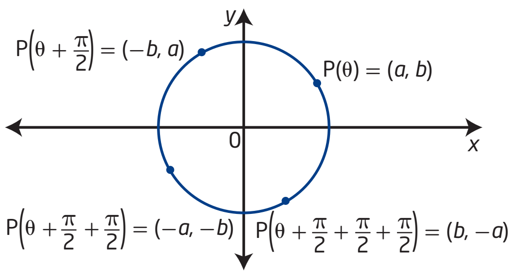
Steps 1–3







Step 4



9. a) $x^2 + y^2 = 1$ **b)** $\left(\frac{\sqrt{5}}{3}, \frac{2}{3}\right)$

c) $\theta + \frac{\pi}{2}$ **d)** quadrant IV

e) maximum value is +1, minimum value is − 1

10. a) Yes. In quadrant I the values of cos θ decrease from 1 at θ = 0° to 0 at θ = 90°, since the x-coordinate on the unit circle represents cos θ, in the first quadrant the values of x will range from 1 to 0.

b) Substitute the values of x and y into the equation $x^2 + y^2 = 1$, Mya was not correct, the correct answer is $y = \sqrt{1 - (0.807)^2}$

$= \sqrt{0.348\ 751}$

$\approx 0.590\ 551$

c) $x = 0.9664$

11. b) All denominators are 2.

c) The numerators of the x-coordinates decrease from $\sqrt{3}$, $\sqrt{2}$, $\sqrt{1} = 1$, the numerators of the y-coordinates increase from $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$. The x-coordinates are moving closer to the y-axis and therefore decrease in value, whereas the y-coordinates are moving further away from the x-axis and therefore increase in value.

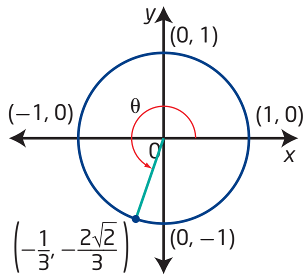
d) Since $x^2 + y^2 = 1$ then $x = \sqrt{1 - y^2}$ and $y = \sqrt{1 - x^2}$, all solutions involve taking square roots.

12. a) $-2\pi \le \theta < 4\pi$ represents three rotations around the unit circle and includes three coterminal angles for each point on the unit circle.

b) If $P(\theta) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, then $\theta = -\frac{4\pi}{3}$ when $-2\pi \le \theta \le 0$, $\theta = \frac{2\pi}{3}$ when $0 \le \theta \le 2\pi$, and $\theta = \frac{8\pi}{3}$ when $2\pi \le \theta < 4\pi$.

c) All these angles are coterminal since they are all 2π radians apart.

13. a)

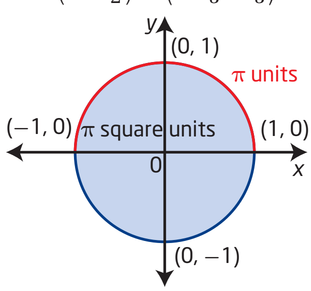


This point represents the terminal point of an angular rotation on the unit circle.

b) quadrant III **c)** $P\left(\theta + \frac{\pi}{2}\right) = \left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right)$

d) $P\left(\theta - \frac{\pi}{2}\right) = \left(-\frac{2\sqrt{2}}{3}, \frac{1}{3}\right)$

14.



π units is the perimeter of half of a unit circle since $a = r\theta = (1)\pi = \pi$ units. π square units is the area of a unit circle since $A = \pi r^2 = \pi(1)^2$
$= \pi$ square units.

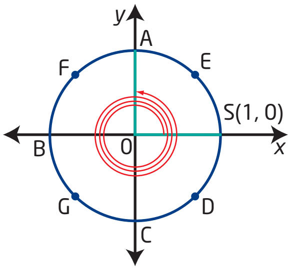
15. a) B($-a$, b), C($-a$, $-b$), D(a, $-b$)

b) **i)** $\theta + \pi = \text{C}(-a, -b)$ **ii)** $\theta - \pi = \text{C}(-a, -b)$
iii) $-\theta + \pi = \text{B}(-a, b)$ **iv)** $-\theta - \pi = \text{B}(-a, b)$

c) They do not differ.

16. a) $\theta = \frac{5\pi}{4}$; $a = r\theta = (1)\left(\frac{5\pi}{4}\right) = \frac{5\pi}{4}$

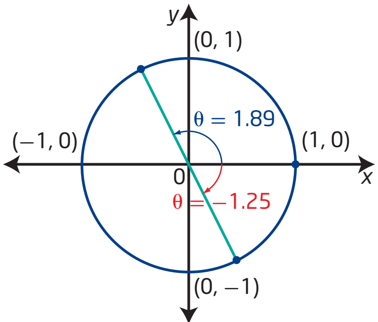
b) $\text{P}\left(\frac{13\pi}{2}\right)$ represents the ordered pair of the point where the terminal arm of the angle $\frac{13\pi}{2}$ intersects the unit circle. Since one rotation of the unit circle is 2π, then $\frac{13\pi}{2}$ represents three complete rotations with an extra $\frac{\pi}{2}$ or quarter rotation, therefore ending at point A.



c) Point C = $\text{P}\left(\frac{3\pi}{2}\right) \approx \text{P}(4.71)$ and point D = $\text{P}\left(\frac{7\pi}{4}\right) \approx \text{P}(5.50)$. Therefore P(5), lies between points C and D.

17. a) $\left(-\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$ and $\left(\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$

b)

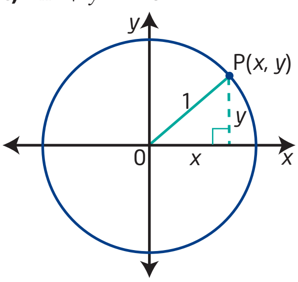


θ represents the angle in standard position.

18. a) $\left(\frac{5}{\sqrt{29}}, \frac{2}{\sqrt{29}}\right)$ **b)** $\sqrt{29}$

c) $x^2 + y^2 = 29$

19.

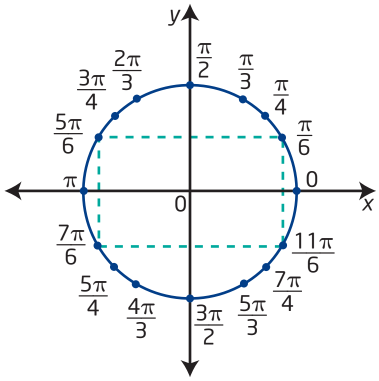


From the diagram: opposite side = y, adjacent side = x and hypotenuse = 1.
Since $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ then $\sin\theta = \frac{y}{1} = y$ or $y = \sin\theta$. Similarly, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, so $\cos\theta = \frac{x}{1} = x$ or $x = \cos\theta$. Therefore any point on the unit circle can be represented by the coordinates ($\cos\theta$, $\sin\theta$).

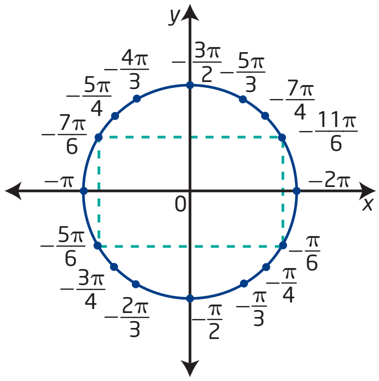
20. a) $\left(1, \frac{\pi}{4}\right)$ **b)** $\left(\frac{\sqrt{31}}{6}, 3.509\right)$

c) $\left(2\sqrt{2}, \frac{\pi}{4}\right)$ **d)** (5, 5.640)

C1 a)



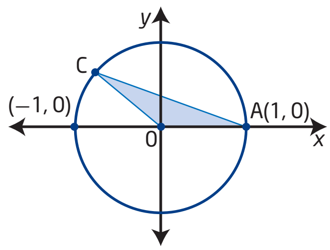
b)



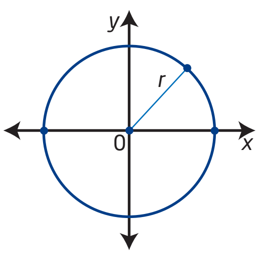
c) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$, $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

d) Example: The circumference is divided into eighths by successive quarter rotations, each eighth of the circumference measures $\frac{\pi}{4}$. The exact coordinates of the points can be determined using the special right triangles ($1 : 1 : \sqrt{2}$ and $1 : \sqrt{3} : 2$) with signs adjusted according to the quadrant.

C2 a) $\frac{\pi}{5}$ **b)** $\frac{3\pi}{20}$



C3 a) $x^2 + y^2 = r^2$



b) Compare with a quadratic function. When $y = x^2$ is translated so its vertex moves from (0, 0) to (h, k), its equation becomes $y = (x - h)^2 + k$. So, a reasonable conjecture for the circle centre (0, 0) moving its centre to (h, k) is $(x - h)^2 + (y - k)^2 = r^2$. Test some key points on the circle centre (0, 0) such as $(r, 0)$. When the centre moves to (h, k) the test point moves to $(r + h, k)$. Substitute into the left side of the equation.
$(r + h - h)^2 + (k - k)^2 = r^2 + 0 =$ right side.

C4 a) 21.5% **b)** $\pi : 4$

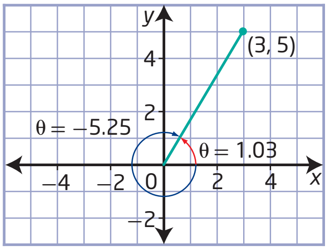
4.3 Trigonometric Ratios, pages 201 to 205

1. a) $\frac{\sqrt{2}}{2}$ **b)** $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ **c)** $-\frac{\sqrt{2}}{2}$
d) $\sqrt{3}$ **e)** -2 **f)** -2
g) undefined **h)** -1 **i)** $\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
j) $\frac{\sqrt{3}}{2}$ **k)** $-\frac{\sqrt{3}}{2}$ **l)** $\sqrt{2}$

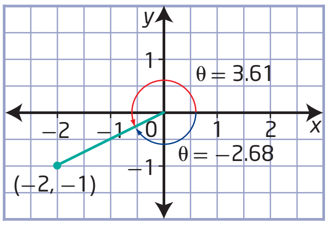
2. a) 0.68 **b)** −2.75 **c)** 1.04
d) −1.00 **e)** −0.96 **f)** 1.37
g) 0.78 **h)** 0.71 **i)** 0.53
j) −0.97 **k)** −3.44 **l)** undefined

3. a) I or IV **b)** II or IV **c)** III or IV
d) II **e)** II **f)** I

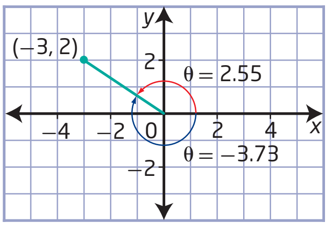
4. a) sin 250° = −sin 70° **b)** tan 290° = −tan 70°
c) sec 135° = −sec 45° **d)** cos 4 = −cos (4 − π)
e) csc 3 = csc (π − 3) **f)** cot 4.95 = cot (4.95 − π)

5. a)


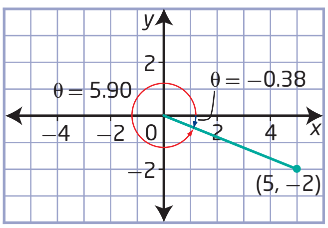
1.03, −5.25

b)


3.61, −2.68

c)


2.55, −3.73

d)


5.90, −0.38

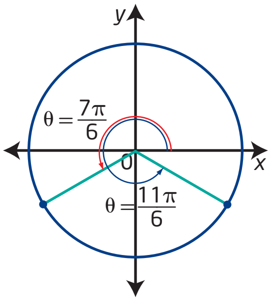
6. a) positive **b)** negative **c)** negative
d) positive **e)** positive **f)** positive

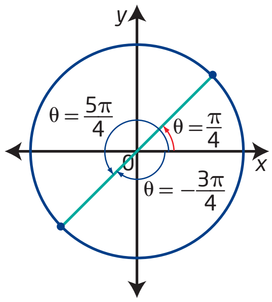
7. a) $\sin^{-1} 0.2014 = 0.2$; an angle of 0.2 radians has a sine ratio of 0.2014
b) $\tan^{-1} 1.429 = 7$; an angle of 7 radians has a tangent ratio of 1.429
c) sec 450° is undefined; an angle of 450° has a secant ratio that is undefined
d) cot (−180°) is undefined; an angle of −180° has a cotangent ratio that is undefined

8. a) $-\frac{4}{5}$ **b)** $-\frac{4}{3}$ **c)** −1.25

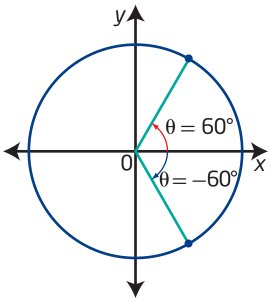
9. a) 1 **b)** 2 **c)** 1
d) −1 **e)** 1 **f)** 3

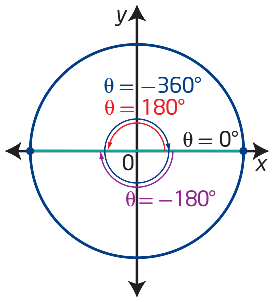
10. a) $\frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $-\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$



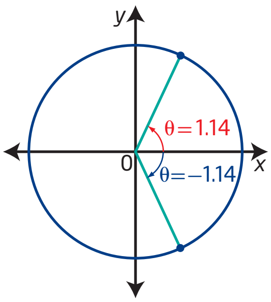


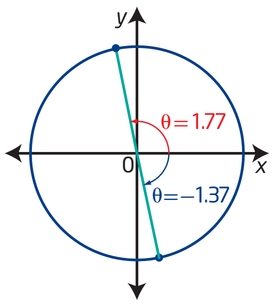
c) −60°, 60° **d)** −360°, −180°, 0°, 180°



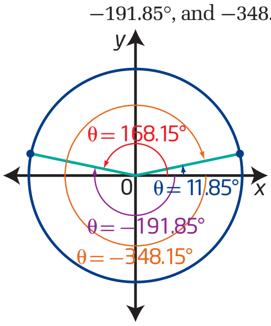


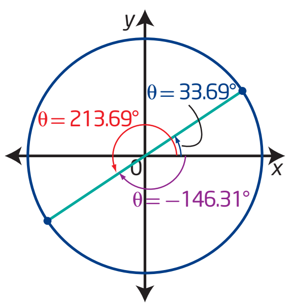
11. a) 1.14 or −1.14 **b)** −1.37 or 1.77





c) 11.85°, 168.15°, −191.85°, and −348.15° **d)** 33.69°, 213.69° and −146.31°





12. a) $\cos \theta = -\frac{4}{5}$, $\tan \theta = -\frac{3}{4}$, $\csc \theta = \frac{5}{3}$, $\sec \theta = -\frac{5}{4}$, $\cot \theta = -\frac{4}{3}$

b) $\sin \theta = \pm\frac{1}{3}$, $\tan \theta = \pm\frac{\sqrt{2}}{4}$, $\csc \theta = \pm 3$, $\sec \theta = -\frac{3\sqrt{2}}{4}$, $\cot \theta = \pm 2\sqrt{2}$

c) $\sin \theta = \pm\frac{2}{\sqrt{13}}$, $\cos \theta = \pm\frac{3}{\sqrt{13}}$, $\csc \theta = \pm\frac{\sqrt{13}}{2}$, $\sec \theta = \pm\frac{\sqrt{13}}{3}$, $\cot \theta = \frac{3}{2}$

d) $\sin \theta = \pm\frac{\sqrt{39}}{4\sqrt{3}}$ or $\pm\frac{\sqrt{13}}{4}$, $\cos \theta = \frac{3}{4\sqrt{3}}$ or $\frac{\sqrt{3}}{4}$, $\csc \theta = \pm\frac{4\sqrt{3}}{\sqrt{39}}$ or $\pm\frac{4\sqrt{13}}{13}$, $\tan \theta = \pm\frac{\sqrt{39}}{3}$, $\cot \theta = \pm\frac{3}{\sqrt{39}}$ or $\pm\frac{\sqrt{39}}{13}$

13. Sketch the point and angle in standard position. Draw the reference triangle. Find the missing value of the hypotenuse by using the equation $x^2 + y^2 = r^2$. Use $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ to find the exact value. Therefore, $\cos \theta = -\frac{2}{\sqrt{13}}$ or $-\frac{2\sqrt{13}}{13}$.

14. a) $\frac{4900°}{360°} = 13\frac{11}{18}$ revolutions counterclockwise

b) quadrant III **c)** 40°

d) $\sin 4900° = -0.643$, $\cos 4900° = -0.766$, $\tan 4900° = 0.839$, $\csc 4900° = -1.556$, $\sec 4900° = -1.305$, $\cot 4900° = 1.192$

15. a) 0.8; For an angle whose cosine is 0.6, think of a 3-4-5 right triangle, or in this case a 0.6-0.8-1 right triangle. The *x*-coordinate is the same as the cosine or 0.6, the sine is the *y*-coordinate which will be 0.8.

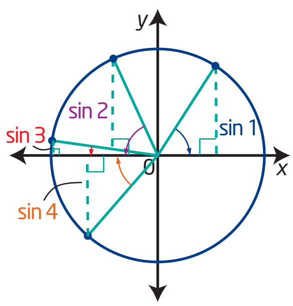
b) 0.8; Since $\cos^{-1} 0.6 = 90° - \sin^{-1} 0.6$ and $\sin^{-1} 0.6 = 90° - \cos^{-1} 0.6$, then $\cos (\sin^{-1} 0.6) = \sin (\cos^{-1} 0.6)$. Alternatively use similar reasoning as in part a) except the *x*- and *y*-coordinates are switched.

16. a) He is not correct. His calculator was in degree measure but the angle is expressed in radians.

b) Set calculator to radian mode and find the value of $\cos\left(\frac{40\pi}{7}\right)$. Since $\sec \theta = \frac{1}{\cos \theta}$, take the reciprocal of $\cos\left(\frac{40\pi}{7}\right)$ to get $\sec\left(\frac{40\pi}{7}\right) \approx 1.603\ 875\ 472$.

17. a) sin 4, sin 3, sin 1, sin 2

b)

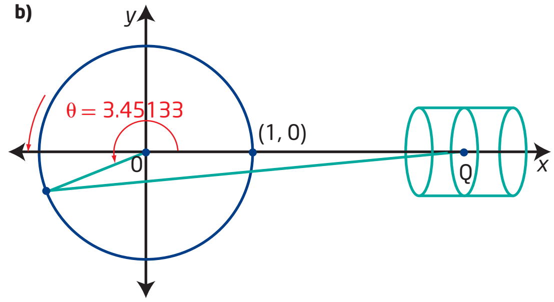


Sin 4 is in quadrant III and has a negative value, therefore it has the least value. Sin 3 is in quadrant II but has the smallest reference angle and is therefore the second smallest. Sin 1 has a smaller reference angle than sin 2.

c) cos 3, cos 4, cos 2, cos 1

18. a) 2 units

b)

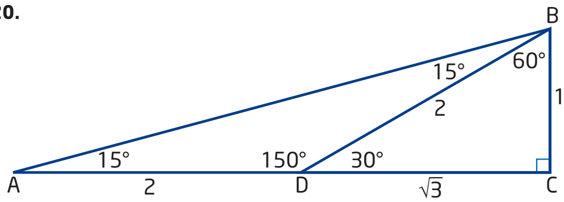


c) 0.46 units

19. a) 2.21, 8.50 **b)** −11.31°, 348.69°

c) −2.16, 4.12, 10.41

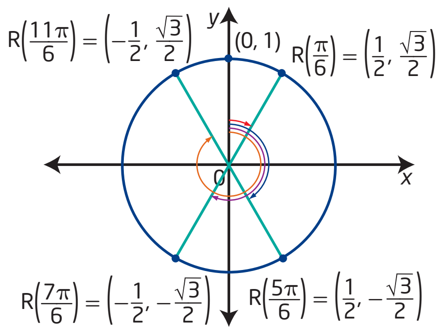
20.



△BCD is a 30°-60°-90° triangle, so DC = $\sqrt{3}$ units and BD = 2 units. △ABD has two equal angles of 15°, so AD = BD = 2. Then $\tan 15° = \frac{BC}{AC} = \frac{BC}{CD + DA} = \frac{1}{\sqrt{3} + 2}$.

21. Since $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{2.5}{5.0} = \frac{1}{2}$ then $\theta = 60°$. Since 60° is $\frac{2}{3}$ of 90° then the point is $\frac{1}{3}$ the distance on the arc from (0, 5) to (5, 0).

22. a)



b) $R\left(\frac{\pi}{6}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $R\left(\frac{5\pi}{6}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

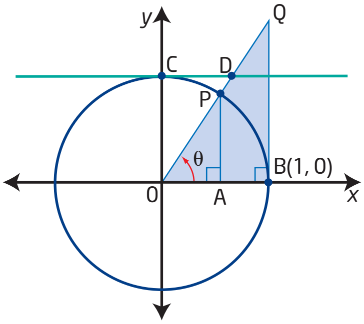
c) $R\left(\frac{\pi}{6}\right) = P\left(\frac{\pi}{3}\right)$, $R\left(\frac{5\pi}{6}\right) = P\left(\frac{5\pi}{3}\right)$, $R\left(\frac{7\pi}{6}\right) = P\left(\frac{4\pi}{3}\right)$, $R\left(\frac{11\pi}{6}\right) = P\left(\frac{2\pi}{3}\right)$, where R(θ) represents the new angle and P(θ) represents the conventional angle in standard position.

d) The new system is the same as bearings in navigation, except bearings are measured in degrees, not radians.

23. a) In △OBQ, $\cos \theta = \frac{OB}{OQ} = \frac{1}{OQ}$.

So, $\sec \theta = \frac{1}{\cos \theta} = OQ$.

b)



In △OCD, ∠ODC = θ (alternate angles). Then, $\sin \theta = \frac{OC}{OD} = \frac{1}{OD}$. So, $\csc \theta = \frac{1}{\sin \theta} = OD$. Similarly, $\cot \theta = CD$.

C1 a) Paula is correct. Examples: $\sin 0° = 0$, $\sin 10° \approx 0.1736$, $\sin 25° \approx 0.4226$, $\sin 30° = 0.5$, $\sin 45° \approx 0.7071$, $\sin 60° \approx 0.8660$, $\sin 90° = 1$.

b) In quadrant II, sine decreases from $\sin 90° = 1$ to $\sin 180° = 0$. This happens because the y-value of points on the unit circle are decreasing toward the horizontal axis as the value of the angle moves from 90° to 180°.

c) Yes, the sine ratio increases in quadrant IV, from its minimum value of −1 at 270° up to 0 at 0°.

C2 When you draw its diagonals, the hexagon is composed of six equilateral triangles. On the diagram shown, each vertex will be 60° from the previous one. So, the coordinates, going in a positive direction from (1, 0) are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, $(-1, 0)$, $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$, and $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

C3 a) $\text{slope}_{OP} = \frac{\sin \theta}{\cos \theta}$ or $\tan \theta$

b) Yes, this formula applies in each quadrant. In quadrant II, $\sin \theta$ is negative, which makes the slope negative, as expected. Similar reasoning applies in the other quadrants.

c) $y = \left(\frac{\sin \theta}{\cos \theta}\right)x$ or $y = (\tan \theta)x$

d) Any line whose slope is defined can be translated vertically by adding the value of the y-intercept b. The equation will be $y = \left(\frac{\sin \theta}{\cos \theta}\right)x + b$ or $y = (\tan \theta)x + b$.

C4 a) $\frac{4}{5}$ **b)** $\frac{3}{5}$ **c)** $\frac{5}{4}$ **d)** $-\frac{4}{5}$

4.4 Introduction to Trigonometric Equations, pages 211 to 214

1. a) two solutions; $\sin \theta$ is positive in quadrants I and II

b) four solutions; $\cos \theta$ is positive in quadrants I and IV, giving two solutions for each of the two complete rotations

c) three solutions; $\tan \theta$ is negative in quadrants II and IV, and the angle rotates through these quadrants three times from −360° to 180°

d) two solutions; $\sec \theta$ is positive in quadrants I and IV and the angle is in each quadrant once from −180° to 180°

2. a) $\theta = \frac{\pi}{3} + 2\pi n, n \in I$ **b)** $\theta = \frac{5\pi}{3} + 2\pi n, n \in I$

3. a) $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$ **b)** $\theta = 0°, 180°$

c) $\theta = -135°, -45°, 45°, 135°, 225°, 315°$

d) $\theta = -\frac{3\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

4. a) $\theta = 1.35, 4.49$ **b)** $\theta = 1.76, 4.52$

c) $\theta = 1.14, 2.00$ **d)** $\theta = 0.08, 3.22$

e) 1.20 and 5.08 **f)** 3.83 and 5.59

5. a) $\theta = \pi$ **b)** $\theta = -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

c) $x = -315°, -225°, 45°, 135°$

d) $x = -150°, -30°$

e) $x = -45°, 135°, 315°$

f) $\theta = -\frac{5\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}$

6. a) $\theta \in [-2\pi, 2\pi]$ **b)** $\theta \in \left[-\frac{\pi}{3}, \frac{7\pi}{3}\right]$

c) $\theta \in [0°, 270°]$ **d)** $0 \le \theta < \pi$

e) $0° < \theta < 450°$ **f)** $-2\pi < \theta \le 4\pi$

7. a) $\theta = 0, \frac{\pi}{3}, \frac{5\pi}{3}$

b) $\theta = 63.435°, 243.435°, 135°, 315°$

c) $\theta = 0, \frac{\pi}{2}, \pi$

d) $\theta = -180°, -70.529°, 70.529°$

8. Check for $\theta = 180°$.
Left Side $= 5(\cos 180°)^2 = 5(-1)^2 = 5$
Right Side $= -4 \cos 180° = -4(-1) = 4$
Since Left Side ≠ Right Side, $\theta = 180°$ is not a solution.
Check for $\theta = 270°$.
Left Side $= 5(\cos 270°)^2 = 5(0)^2 = 0$
Right Side $= -4 \cos 270° = -4(0) = 0$
Since Left Side = Right Side, $\theta = 270°$ is a solution.

9. a) They should not have divided both sides of the equation by $\sin \theta$. This will eliminate one of the possible solutions.

b)
$$2 \sin^2 \theta = \sin \theta$$
$$2 \sin^2 \theta - \sin \theta = 0$$
$$\sin \theta(2 \sin \theta - 1) = 0$$
$$\sin \theta = 0 \quad \text{and} \quad 2 \sin \theta - 1 = 0$$
$$\sin \theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \pi$$

10. $\sin \theta = 0$ when $\theta = 0$, π, and 2π but none of these values are in the interval $(\pi, 2\pi)$.

11. $\sin \theta$ is only defined for the values $-1 \le \sin \theta \le 1$, and 2 is outside this range, so $\sin \theta = 2$ has no solution.

12. Yes, the general solutions are $\theta = \frac{\pi}{3} + 2\pi n, n \in I$ and $\theta = \frac{5\pi}{3} + 2\pi n, n \in I$. Since there are an infinite number of integers, there will be an infinite number of solutions coterminal with $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

13. a) Helene can check her work by substituting π for θ in the original equation.
Left Side $= 3(\sin \pi)^2 - 2 \sin \pi$
$= 3(0)^2 - 2(0)$
$= 0$
$=$ Right Side

b) $\theta = 0, 0.7297, 2.4119, \pi$

14. 25.56°

15. a) June **b)** December

c) Yes. Greatest sales of air conditioners be expected to happen before the hottest months (June) and the least sales before the coldest months (December).

16. The solution is correct as far as the statement "Sine is negative in quadrants II and III." Sine is actually negative in quadrants III and IV. Quadrant III solution is $180° + 41.8° = 221.8°$ and quadrant IV solution is $360° - 41.8° = 318.2°$.

17. Examples: Tan 90° has no solution since division by 0 is undefined. $\sin \theta = 2$ does not have a solution. The range of $y = \sin \theta$ is $-1 \leq y \leq 1$ and 2 is beyond this range.

18. $\sec \theta = -\frac{5}{3}$

19. **a)** 0 s, 3 s, 6 s, 9 s **b)** 1.5 s, $1.5 + 6n, n \in W$
c) 1.4 m below sea level

20. **a)** Substitute $I = 0$, then $0 = 4.3 \sin 120\pi t$
$0 = \sin 120\pi t$
$\sin \theta = 0$ at $\theta = 0, \pi, 2\pi, \ldots$
$0 = 120\pi t \rightarrow t = 0$
$\pi = 120\pi t \rightarrow t = \frac{1}{120}$
$2\pi = 120\pi t \rightarrow t = \frac{1}{60}$
Since the current must alternate from 0 to positive back to 0 and then negative back to 0, it will take $\frac{1}{60}$ s for one complete cycle or 60 cycles in one second.
b) $t = 0.004\ 167 + \frac{1}{60}n, n \in W$ seconds
c) $t = 0.0125 + \frac{1}{60}n, n \in W$ seconds
d) 4.3 amps

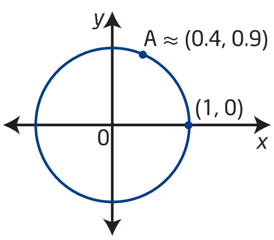
21. $x = \frac{\pi}{3}, \frac{2\pi}{3}$

22. **a)** No. **b)** $\sin \theta = \frac{-1 + \sqrt{5}}{2}$ and $\frac{-1 - \sqrt{5}}{2}$
c) 0.67, 2.48

23. **a)** The height of the trapezoid is $4 \sin \theta$ and its base is $4 + 2(4 \cos \theta)$. Use the formula for the area of a trapezoid:
$A = \frac{\text{sum of parallel sides}}{2} \times \text{height}$
$A = \left(\frac{4 + 4 + 8 \cos \theta}{2}\right)(4 \sin \theta)$
$A = 8(1 + \cos \theta)(2 \sin \theta)$
$A = 16 \sin \theta(1 + \cos \theta)$
b) $\frac{\pi}{3}$
c) Example: Graph $y = 16 \sin \theta(1 + \cos \theta)$ and find the maximum for domain in the first quadrant.

C1 The principles involved are the same up to the point where you need to solve for a trigonometric ratio.

C2 **a)** Check if $x^2 + y^2 = 1$. Yes, A is on the unit circle.
b) $\cos \theta = 0.385$, $\tan \theta = 2.400$, $\csc \theta = 1.083$
c) 67.4°; this angle measure seems reasonable as shown on the diagram.



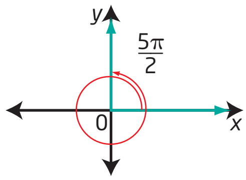
C3 **a)** Non-permissible values are values that the variable can never be because the expression is not defined in that case. For a rational expression, this occurs when the denominator is zero.
Example: $\frac{3}{x}, x \neq 0$
b) Example: $\tan \frac{\pi}{2}$ **c)** $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$
d) $\frac{\pi}{2} + \pi n, n \in I$

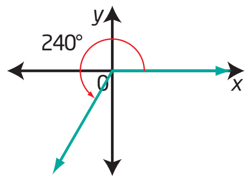
C4 **a)** 30°, 150°, 270°
b) Exact, because $\sin^{-1}(0.5)$ and $\sin^{-1}(-1)$ correspond to exact angle measures.
c) Example: Substitute $\theta = 30°$ in each side. Left side $= 2 \sin^2 30° = 2(0.5)^2 = 0.5$. Right side $= 1 - \sin 30° = 1 - 0.5 = 0.5$. The value checks.

Chapter 4 Review, pages 215 to 217

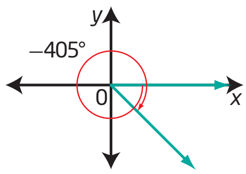
1. **a)** quadrant II **b)** quadrant II
c) quadrant III **d)** quadrant II

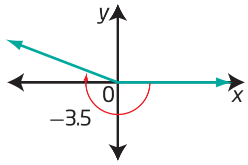
2. **a)** 450° **b)** $\frac{4\pi}{3}$





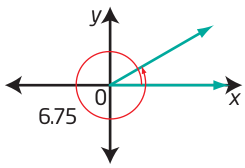
c) $-\frac{9\pi}{4}$ **d)** $-\frac{630°}{\pi}$

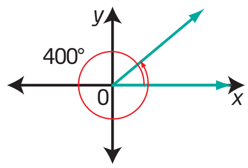




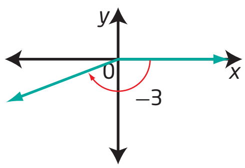
3. **a)** 0.35 **b)** −3.23 **c)** −100.27° **d)** 75°

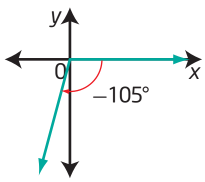
4. **a)** 0.467 **b)** 40°





c) 3.28 **d)** 255°





5. **a)** $250° \pm (360°)n, n \in N$ **b)** $\frac{5\pi}{2} \pm 2\pi n, n \in N$
c) $-300° \pm (360°)n, n \in N$ **d)** $6 \pm 2\pi n, n \in N$

6. **a)** $160\ 000\pi$ radians/minute **b)** $480\ 000°$/s

7. **a)** $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$ **b)** $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ **c)** $(0, 1)$
d) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ **e)** $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **f)** $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

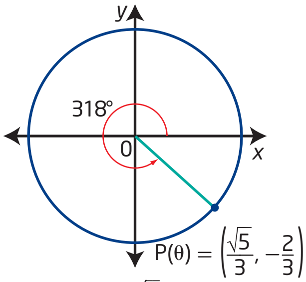
8. **a)** Reflect $P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ in the y-axis to give $P\left(\frac{2\pi}{3}\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; then reflect this point in the x-axis to give $P\left(\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$. Reflect about the original point in the x-axis to give $P\left(\frac{5\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.
b) $\left(-\frac{1}{3}, -\frac{2\sqrt{2}}{3}\right)$

c) quadrant IV; $P\left(\frac{5\pi}{6}\right)$ lies in quadrant II and $P\left(\frac{5\pi}{6} + \pi\right)$ is a half circle away, so it lies in quadrant IV. $\theta = \frac{11\pi}{6}$ $P\left(\frac{11\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

9. a) $P\left(\frac{\pi}{2}\right)$ and $P\left(-\frac{3\pi}{2}\right)$ **b)** $P\left(\frac{11\pi}{6}\right)$ and $P\left(-\frac{\pi}{6}\right)$
c) $P\left(\frac{3\pi}{4}\right)$ and $P\left(-\frac{5\pi}{4}\right)$ **d)** $P\left(\frac{2\pi}{3}\right)$ and $P\left(-\frac{4\pi}{3}\right)$

10. a) P(–150°) and P(210°) **b)** P(180°)
c) P(135°) **d)** P(–60°) and P(300°)

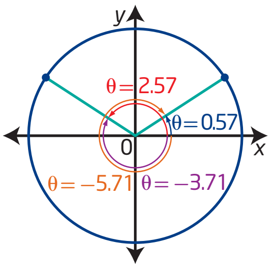
11. a) $\theta = 318°$ or 5.55 **b)** IV
c) $\left(-\frac{\sqrt{5}}{3}, \frac{2}{3}\right)$
d) $\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$
e) $\left(-\frac{2}{3}, -\frac{\sqrt{5}}{3}\right)$

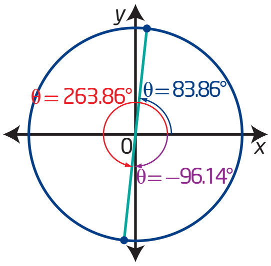


12. $\sin\theta = \frac{2\sqrt{2}}{3}$, $\tan\theta = 2\sqrt{2}$, $\sec\theta = 3$,
$\csc\theta = \frac{3}{2\sqrt{2}}$ or $\frac{3\sqrt{2}}{4}$, $\cot\theta = \frac{1}{2\sqrt{2}}$ or $\frac{\sqrt{2}}{4}$

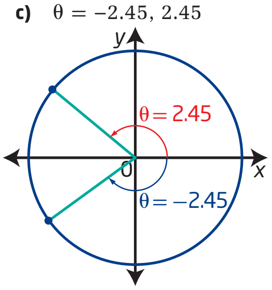
13. a) 1 **b)** $-\frac{\sqrt{2}}{2}$ **c)** $\sqrt{3}$
d) $-\frac{2\sqrt{3}}{3}$ **e)** 0 **f)** $-\frac{2\sqrt{3}}{3}$

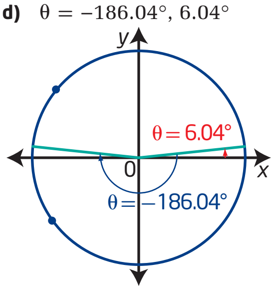
14. a) $\theta = -5.71, -3.71,$ 0.57, and 2.57 **b)** $\theta = -96.14°, 83.86°,$ 263.86°





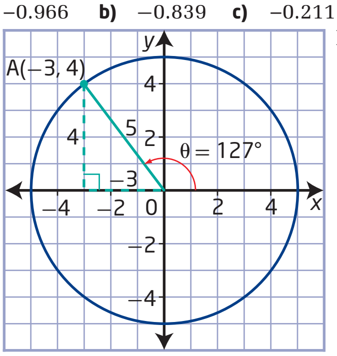
c) $\theta = -2.45, 2.45$ **d)** $\theta = -186.04°, 6.04°$





15. a) –0.966 **b)** –0.839 **c)** –0.211 **d)** 2.191

16. a) Example: 127°



b) $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = -\frac{3}{5}$ **c)** $-\frac{1}{12}$
d) 126.9° or 2.2

17. a) $\cos\theta(\cos\theta + 1)$ **b)** $(\sin\theta - 4)(\sin\theta + 1)$
c) $(\cot\theta + 3)(\cot\theta - 3)$ **d)** $(2\tan\theta - 5)(\tan\theta - 2)$

18. a) 2 is not a possible value for $\sin\theta$, $|\sin\theta| \le 1$
b) $\tan 90° = \frac{\sin 90°}{\cos 90°} = \frac{1}{0}$, but division by 0 is undefined, so tan 90° has no solutions

19. a) 2 solutions **b)** 2 solutions
c) 1 solution **d)** 6 solutions

20. a) $\theta = 45°, 135°$ **b)** $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$
c) $\theta = -150°, 30°, 210°$ **d)** $\theta = -\frac{\pi}{4}, \frac{3\pi}{4}$

21. a) $\theta = \frac{\pi}{2}$
b) $\theta = 108.435°, 180°, 288.435°, 360°$
c) $\theta = 70.529°, 120°, 240°$, and 289.471°
d) $\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

22. Examples:
a) $0 \le \theta < 2\pi$ **b)** $-2\pi \le \theta < \frac{\pi}{2}$
c) $-720° \le \theta < 0°$ **d)** $-270° \le \theta < 450°$

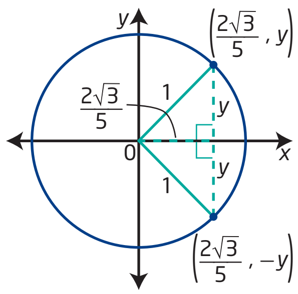
23. a) $x = \frac{7\pi}{6} + 2\pi n, n \in I$ and $x = \frac{11\pi}{6} + 2\pi n, n \in I$
b) $x = 90° + (360°)n, n \in I$ and $x = (180°)n, n \in I$
c) $x = 120° + (360°)n, n \in I$ and $x = 240° + (360°)n, n \in I$
d) $x = \frac{\pi}{4} + \pi n, n \in I$ and $x = \frac{\pi}{3} + \pi n, n \in I$

Chapter 4 Practice Test, pages 218 to 219

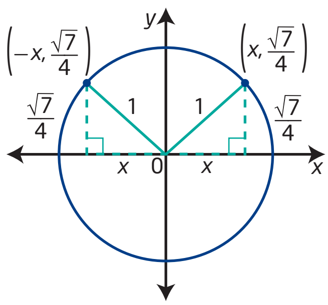
1. D **2.** C **3.** A **4.** B **5.** B

6. a) 4668.5° or 81.5
b) 92.6 Yes; a smaller tire requires more rotations to travel the same distance so it will experience greater tire wear.

7. a) $x^2 + y^2 = 1$
b) i) $y = \pm\frac{\sqrt{13}}{5}$



ii) $x = -\frac{3}{4}$



c) In the expression $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, substitute the y-value for the opposite side and 1 for the hypotenuse. Since $x^2 + y^2 = 1$ then $\cos^2\theta + \sin^2\theta = 1$. Substitute the value you determined for $\sin\theta$ into $\cos^2\theta + \sin^2\theta = 1$ and solve for $\cos\theta$.

8. a) Cosine is negative in quadrants II and III. Find the reference angle by subtracting π from the given angle in quadrant III. To find the solution in quadrant II, subtract the reference angle from π.

b) Given each solution θ, add $2\pi n$, $n \in I$ to obtain each general solution $\theta + 2\pi n$, $n \in I$.

9. $\theta = \frac{3\pi}{4} + 2\pi n, n \in I$ or $\theta = \frac{5\pi}{4} + 2\pi n, n \in I$

10. Since $1° = \frac{\pi}{180}$, then $3° = \frac{3\pi}{180}$ or $\frac{\pi}{60}$.

$3 = \frac{3(180°)}{\pi} \approx 172°$.

11. a) quadrant III **b)** 40°

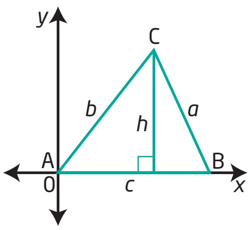
c) $\sin(-500°) = -0.6$, $\cos(-500°) = -0.8$, $\tan(-500°) = 0.8$, $\csc(-500°) = -1.6$, $\sec(-500°) = -1.3$, $\cot(-500°) = 1.2$

12. a) $\frac{5\pi}{4}, -\frac{3\pi}{4}; \frac{5\pi}{4} \pm 2\pi n, n \in N$

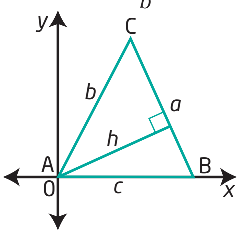
b) 145°, −215°, 145° ± (360°)n, $n \in N$

13. 7.7 km

14.



Given $A = \frac{1}{2}bh$, b = side c, since $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ then $\sin A = \frac{h}{b}$ or $h = b\sin A$ and $A = \frac{1}{2}bc\sin A$ or



Given $A = \frac{1}{2}bh$, b = side a, since $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ then $\sin B = \frac{h}{c}$ or $h = c\sin B$, therefore $A = \frac{1}{2}ac\sin B$.

15. a) $\theta = -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$, −2.21, 0.93, 4.07

b) 0.67, 2.48 **c)** 0, π, 2π, 4.47, 1.33

16. $\frac{28\pi}{3}$ m or 29.32 m