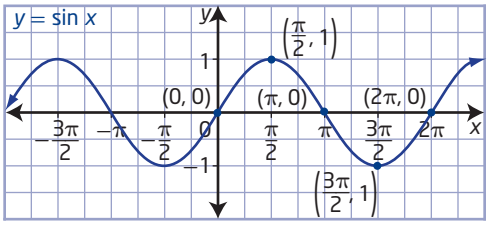
Chapter 5 Trigonometric Functions and Graphs

5.1 Graphing Sine and Cosine Functions, pages 233 to 237

1. a) $(0, 0), \left(\frac{\pi}{2}, 1\right), (\pi, 0), \left(\frac{3\pi}{2}, -1\right), (2\pi, 0)$

b)



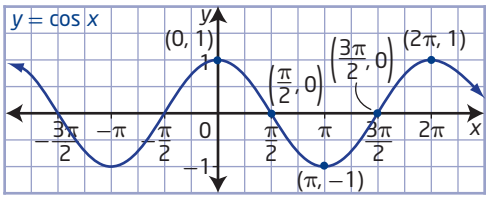
c) x-intercepts: $-2\pi, -\pi, 0, \pi, 2\pi$

d) y-intercept: 0

e) The maximum value is 1, and the minimum value is -1.

2. a) $(0, 1), \left(\frac{\pi}{2}, 0\right), (\pi, -1), \left(\frac{3\pi}{2}, 0\right), (2\pi, 1)$

b)



c) x-intercepts: $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

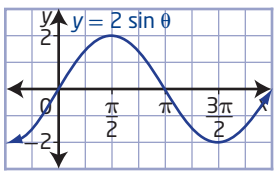
d) y-intercept: 1

e) The maximum value is 1, and the minimum value is -1.

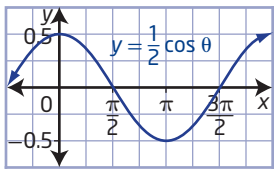
3.

Property	$y = \sin x$	$y = \cos x$
maximum	1	1
minimum	-1	-1
amplitude	1	1
period	2π	2π
domain	$\{x \mid x \in \mathrm{R}\}$	$\{x \mid x \in \mathrm{R}\}$
range	$\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$	$\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$
y-intercept	0	1
x-intercepts	$\pi n, n \in \mathrm{I}$	$\frac{\pi}{2} + \pi n, n \in \mathrm{I}$

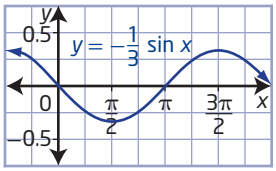
4. a) 2



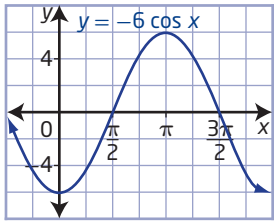
b) $\frac{1}{2}$



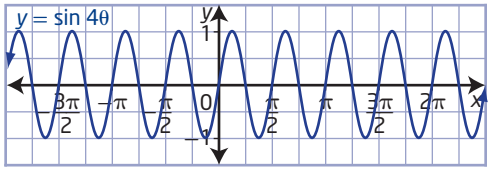
c) $\frac{1}{3}$



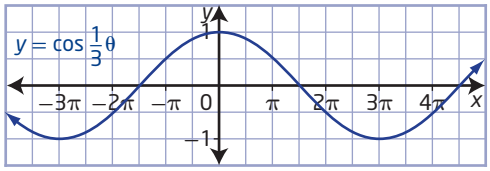
d) 6



5. a) $\frac{\pi}{2}$ or 90°

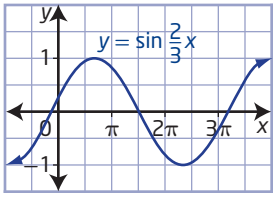


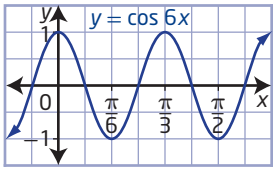
b) 6π or 1080°



c) 3π or 540°

d) $\frac{\pi}{3}$ or 60°





6. a) A **b)** D **c)** C **d)** B

7. a) Amplitude is 3; stretched vertically by a factor of 3 about the *x*-axis.

b) Amplitude is 5; stretched vertically by a factor of 5 about the *x*-axis and reflected in the *x*-axis.

c) Amplitude is 0.15; stretched vertically by a factor of 0.15 about the *x*-axis.

d) Amplitude is $\frac{2}{3}$; stretched vertically by a factor of $\frac{2}{3}$ about the *x*-axis and reflected in the *x*-axis.

8. a) Period is 180°; stretched horizontally by a factor of $\frac{1}{2}$ about the *y*-axis.

b) Period is 120°; stretched horizontally by a factor of $\frac{1}{3}$ about the *y*-axis and reflected in the *y*-axis.

c) Period is 1440°; stretched horizontally by a factor of 4 about the *y*-axis.

d) Period is 540°; stretched horizontally by a factor of $\frac{3}{2}$ about the *y*-axis.

9. a) Amplitude is 2; period is 360° or 2π.

b) Amplitude is 4; period is 180° or π.

c) Amplitude is $\frac{5}{3}$; period is 540° or 3π.

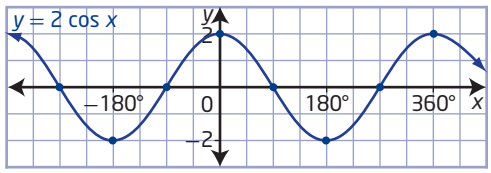
d) Amplitude is 3; period is 720° or 4π.

10. a) Graph A: Amplitude is 2 and period is 4π. Graph B: Amplitude is 0.5 and period is π.

b) Graph A: $y = 2 \sin \frac{1}{2}x$; Graph B: $y = 0.5 \cos 2x$

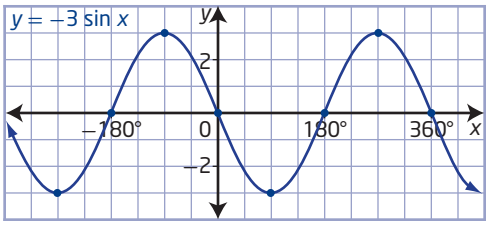
c) Graph A starts at 0, so the sine function is the obvious choice. Graph B starts at 1, so the cosine function is the obvious choice.

11. a)



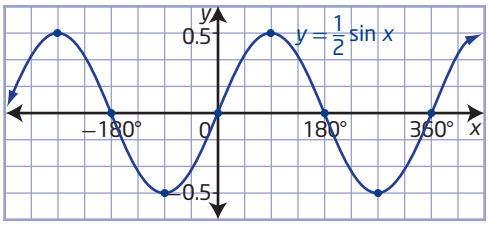
Property	Points on the Graph of $y = 2 \cos x$
maximum	(−360°, 2), (0°, 2), (360°, 2)
minimum	(−180°, −2), (180°, −2)
x-intercepts	(−270°, 0), (−90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, 2)
period	360°
range	$\{y \mid -2 \le y \le 2, y \in R\}$

b)



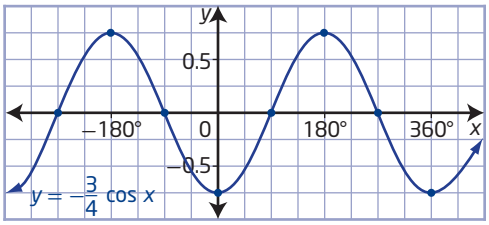
Property	Points on the Graph of $y = -3 \sin x$
maximum	(−90°, 3), (270°, 3)
minimum	(−270°, −3), (90°, −3)
x-intercepts	(−360°, 0), (−180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -3 \le y \le 3, y \in R\}$

c)



Property	Points on the Graph of $y = \frac{1}{2} \sin x$
maximum	(−270°, 0.5), (90°, 0.5)
minimum	(−90°, −0.5), (270°, −0.5)
x-intercepts	(−360°, 0), (−180°, 0), (0°, 0), (180°, 0), (360°, 0)
y-intercept	(0, 0)
period	360°
range	$\{y \mid -0.5 \le y \le 0.5, y \in R\}$

d)



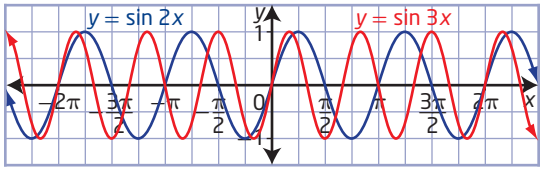
Property	Points on the Graph of $y = -\frac{3}{4} \cos x$
maximum	(−180°, 0.75), (180°, 0.75)
minimum	(−360°, −0.75), (0°, −0.75), (360°, −0.75)
x-intercepts	(−270°, 0), (−90°, 0), (90°, 0), (270°, 0)
y-intercept	(0, −0.75)
period	360°
range	$\{y \mid -0.75 \le y \le 0.75, y \in R\}$

12. a) $B\left(\frac{\pi}{4}, 3\right)$, $C\left(\frac{\pi}{2}, 0\right)$, $D\left(\frac{3\pi}{4}, -3\right)$, $E(\pi, 0)$

b) $C\left(\frac{\pi}{2}, 0\right)$, $D(\pi, -2)$, $E\left(\frac{3\pi}{2}, 0\right)$, $F(2\pi, 2)$

c) $B(-3\pi, 1)$, $C(-2\pi, 0)$, $D(-\pi, -1)$, $E(0, 0)$

13.



The amplitude, maximum, minimum, y-intercepts, domain, and range are the same for both graphs. The period and x-intercepts are different.

14. a) Amplitude is 5; period is $\frac{4\pi}{3}$.

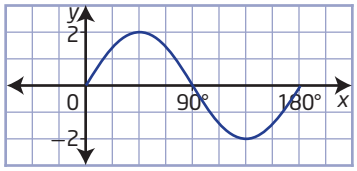
b) Amplitude is 4; Period is $\frac{2\pi}{3}$.

15. a) Amplitude is 20 mm Hg; Period is 0.8 s.

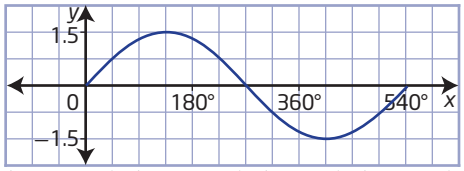
b) 75 bpm

16. Answers may vary.

17. a)



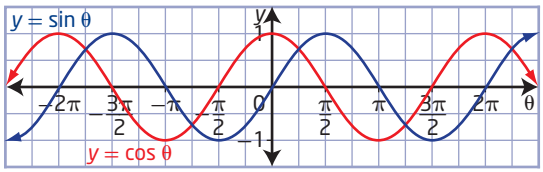
b)



18. a) $\left(-\frac{7\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(-\frac{5\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$, $\left(\frac{9\pi}{4}, \frac{\sqrt{2}}{2}\right)$;
Find the points of intersection of $y = \sin\theta$ and $y = \frac{\sqrt{2}}{2}$.

b) $\left(-\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(-\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{11\pi}{6}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{13\pi}{6}, \frac{\sqrt{3}}{2}\right)$;
Find the points of intersection of $y = \cos\theta$ and $y = \frac{\sqrt{3}}{2}$.

19.



a) The graphs have the same maximum and minimum values, the same period, and the same domain and range.

b) The graphs have different x- and y-intercepts.

c) A horizontal translation could make them the same graph.

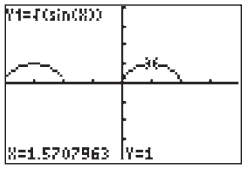
20. 12

21. a) $\frac{2\pi}{3}$ **b)** 12

22. 0.9

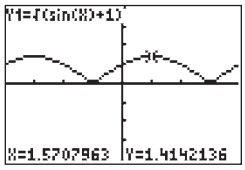
23. a) Example: The graph of $y = \sqrt{\sin x}$ will contain the portions of the graph of $y = \sin x$ that lie on or above the x-axis.

b)



c) Example: The function $y = \sqrt{\sin x + 1}$ is defined for all values of x, while the function $y = \sqrt{\sin x}$ is not.

d)



24. It is sinusoidal and the period is 2π.

C1 Step 5

a) The x-coordinate of each point on the unit circle represents $\cos\theta$. The y-coordinate of each point on the unit circle represents the $\sin\theta$.

b) The y-coordinates of the points on the sine graph are the same as the y-coordinates of the points on the unit circle. The y-coordinates of the points on the cosine graph are the same as the x-coordinates of the points on the unit circle.

C2 The constant is 1. The sum of the squares of the legs of each right triangle is equal to the radius of the unit circle, which is always 1.

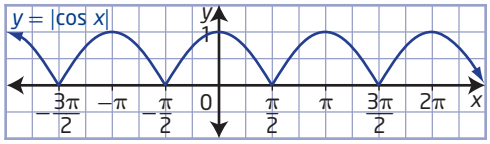
C3 a) Cannot determine because the amplitude is not given.

b) $f(4) = 0$; given in the question.

c) $f(84) = 0$; the period is 40° so it returns to 0 every 40°.

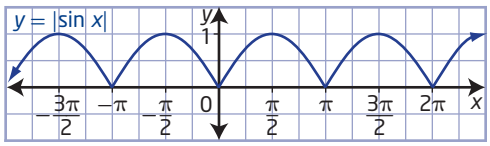
C4 a) Sine and Cosine **b)** Sine and Cosine
c) Sine and Cosine **d)** Sine and Cosine
e) Sine **f)** Cosine **g)** Cosine **h)** Sine
i) Cosine **j)** Sine **k)** Cosine **l)** Sine
m) Sine **n)** Cosine

C5 a)



The parts of the graph below the x-axis have been reflected across the x-axis.

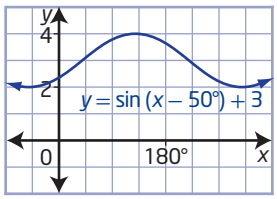
b)



The parts of the graph below the x-axis have been reflected across the x-axis.

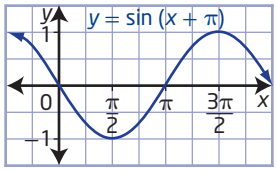
5.2 Transformations of Sinusoidal Functions, pages 250 to 255

1. a)

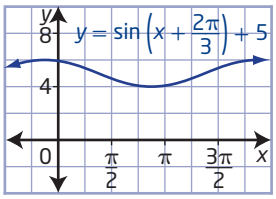


The phase shift is 50° right. The vertical displacement is 3 units up.

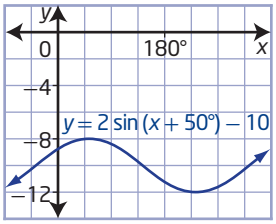
b)



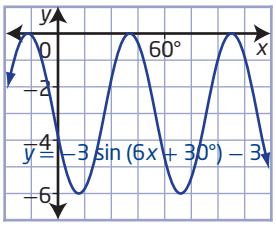
The phase shift is π units left. There is no vertical displacement.

c) 

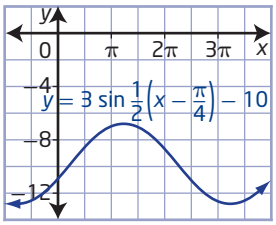
The phase shift is $\frac{2\pi}{3}$ units left. The vertical displacement is 5 units up.

d) 

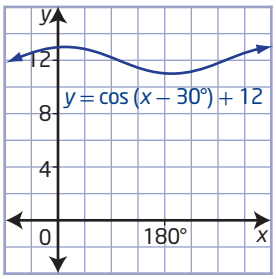
The phase shift is 50° left. The vertical displacement is 10 units down.

e) 

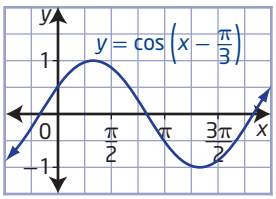
The phase shift is 5° left. The vertical displacement is 3 units down.

f) 

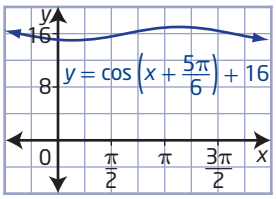
The phase shift is $\frac{\pi}{4}$ units right. The vertical displacement is 10 units down.

2. a) 

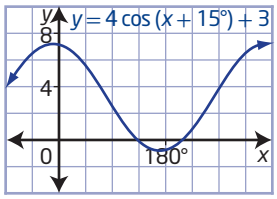
The phase shift is 30° right. The vertical displacement is 12 units up.

b) 

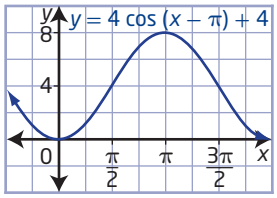
The phase shift is $\frac{\pi}{3}$ units right. There is no vertical displacement.

c) 

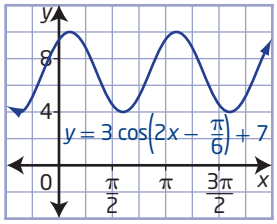
The phase shift is $\frac{5\pi}{6}$ units left. The vertical displacement is 16 units up.

d) 

The phase shift is 15° left. The vertical displacement is 3 units up.

e) 

The phase shift π units right. The vertical displacement is 4 units up.

f) 

The phase shift is $\frac{\pi}{12}$ units right. The vertical displacement is 7 units up.

3. a) **i)** $\{y \mid 2 \le y \le 8, y \in \mathrm{R}\}$
ii) $\{y \mid -5 \le y \le -1, y \in \mathrm{R}\}$
iii) $\{y \mid 2.5 \le y \le 5.5, y \in \mathrm{R}\}$
iv) $\left\{y \mid \frac{1}{12} \le y \le \frac{17}{12}, y \in \mathrm{R}\right\}$

b) Take the vertical displacement and add and subtract the amplitude to it. The region in between these points is the range.

4. a) D **b)** C **c)** B **d)** A **e)** E

5. a) D **b)** B **c)** C **d)** A

6. a) $y = 4\sin 2\left(x - \frac{\pi}{2}\right) - 6$

b) $y = 0.5\sin\frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$

c) $y = \frac{3}{4}\sin\frac{1}{2}x - 5$

7. a) $a = 3, b = \frac{1}{2}, c = -2, d = 3; y = 3\cos\frac{1}{2}(x + 2) + 3$

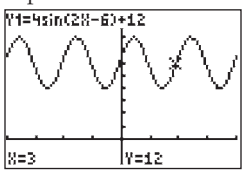
b) $a = \frac{1}{2}, b = 4, c = 3, d = -5;$
$y = \frac{1}{2}\cos 4(x - 3) - 5$

c) $a = -\frac{3}{2}, b = \frac{1}{3}, c = \frac{\pi}{4}, d = -1;$
$y = -\frac{3}{2}\cos\frac{1}{3}\left(x - \frac{\pi}{4}\right) - 1$

8. red, orange, yellow, green, blue, indigo, violet

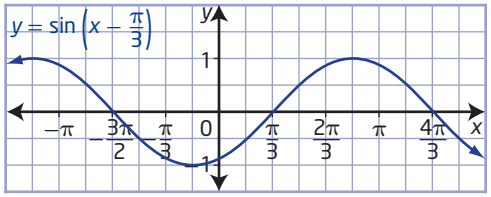
9. *b*

10. a) Stewart is correct. He remembered to factor the expression in brackets first.

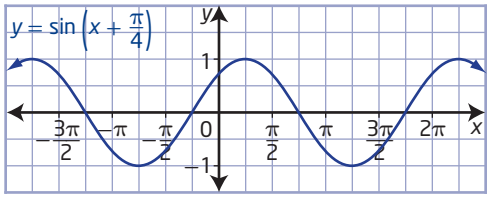
b) 

11. a) $\{y \mid -1 \le y \le 5, y \in \mathrm{R}\}$ **b)** $\{y \mid -6 \le y \le 0, y \in \mathrm{R}\}$
c) $\{y \mid -13 \le y \le -7, y \in \mathrm{R}\}$
d) $\{y \mid 5 \le y \le 11, y \in \mathrm{R}\}$

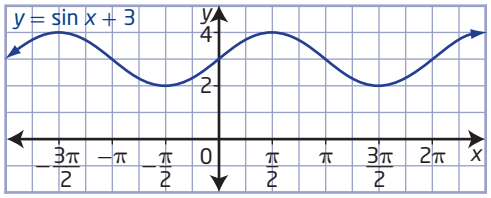
12. a)



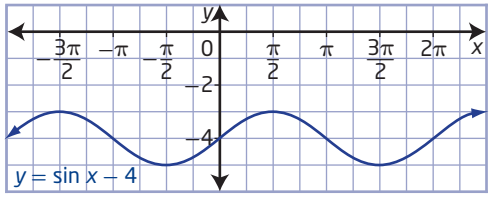
b)



c)



d)



13. $a = 9$, $d = -4$

14. a) **i)** 3 **ii)** 2π

iii) $\frac{\pi}{4}$ units right **iv)** none

v) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid -3 \le y \le 3, y \in \mathrm{R}\}$

vi) The maximum value of 3 occurs at $x = \frac{3\pi}{4}$.

vii) The minimum value of -3 occurs at $x = \frac{7\pi}{4}$.

b) **i)** 2 **ii)** 2π

iii) $\frac{\pi}{2}$ units right **iv)** 2 units down

v) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid -4 \le y \le 0, y \in \mathrm{R}\}$

vi) The maximum value of 0 occurs at $x = \frac{\pi}{2}$.

vii) The minimum value of -4 occurs at $x = \frac{3\pi}{2}$.

c) **i)** 2 **ii)** π

iii) $\frac{\pi}{4}$ units right **iv)** 1 unit up

v) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid -1 \le y \le 3, y \in \mathrm{R}\}$

vi) The maximum value of 3 occurs at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

vii) The minimum value of -1 occurs at $x = 0$, $x = \pi$, and $x = 2\pi$.

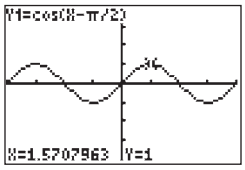
15. a) $y = 2 \sin x - 1$ **b)** $y = 3 \sin 2x + 1$

c) $y = 2 \sin 4\left(x - \frac{\pi}{4}\right) + 2$

16. a) $y = 2 \cos 2\left(x - \frac{\pi}{4}\right) + 1$

b) $y = 2 \cos \left(x + \frac{\pi}{2}\right) - 1$ **c)** $y = \cos (x - \pi) + 1$

17. a)



b) $y = \sin x$

c) The graph of the cosine function shifted $\frac{\pi}{2}$ units right is equivalent to the graph of the sine function.

18. phase shift of $\frac{\pi}{2}$ units left

19. a) **i)** Phase shift is 30° right; period is 360°; x-intercepts are at 120° and 300°.

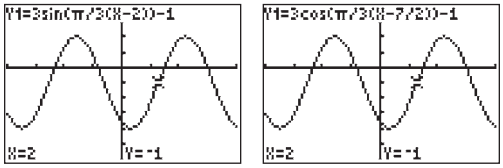
ii) Maximums occur at (30°, 3) and (390°, 3); minimum occurs at (210°, −3).

b) **i)** Phase shift is $\frac{\pi}{4}$ units right; period is π; x-intercepts are at $\frac{\pi}{2}$ and π

ii) Maximums occur at $\left(\frac{\pi}{4}, 3\right)$ and $\left(\frac{5\pi}{4}, 3\right)$; minimum occurs at $\left(\frac{3\pi}{4}, -3\right)$.

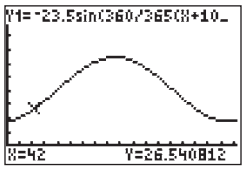
20. $y = 50 \cos \frac{\pi}{2640}(x - 9240) + 5050$

21. The graphs are equivalent.



22. $y = 4 \sin 4(x + \pi)$

23. a)

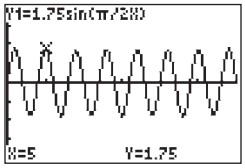


b) approximately 26.5°

c) day 171 or June 21

24. a) 4 s **b)** 15 cycles per minute

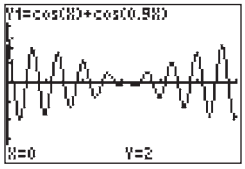
c)



d) The air flow velocity is 0 L/s. This corresponds to when the lungs are either completely full or completely empty.

e) The air flow velocity is −1.237 L/s. This corresponds to part of a cycle when the lungs are blowing out air.

25. a)



b) The amplitude is 2. The period is 20π.

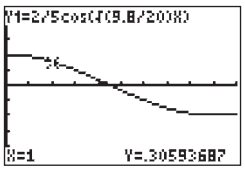
26. a) **i)** 120° **ii)** $\frac{3\pi}{4}$ **iii)** π **iv)** $\frac{\pi}{4}$

b) Example: When graphed, a cosine function is ahead of the graph of a sine function by 90°. So, adding 90° to the phase shift in part a) works.

27. a) $y = 3 \sin (x + \pi) + 2$ **b)** $y = 3 \sin 2\left(x - \frac{\pi}{2}\right) + 1$

c) $y = 2 \sin \left(x + \frac{\pi}{2}\right) + 5$ **d)** $y = 5 \sin 3(x - 120°) - 1$

28. a) $P = \frac{2}{5} \cos \sqrt{\frac{9.8}{20}} t$



b) approximately −0.20 radians or 3.9 cm along the arc to the left of the vertical

C1 a changes the amplitude, b changes the period, c changes the phase shift, d changes the vertical translation; Answers may vary.

C2 a) They are exactly same.

b) This is because the sine of a negative number is the same as the negative sine of the number.

c) They are mirror images reflected in the x-axis.

d) It is correct.

C3 $\frac{5\pi}{4}$ square units

C4 a) $0 < b < 1$ **b)** $a > 1$

c) Example: $c = 0$, $d = 0$ **d)** $d > a$

e) Example: $c = -\frac{\pi}{2}$, $b = 1$, $d = 0$ **f)** $b = 3$

5.3 The Tangent Function, pages 262 to 265

1. a) 1, 45° **b)** −1.7, 120.5°

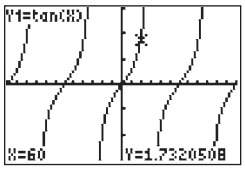
c) −1.7, 300.5° **d)** 1, 225°

2. a) undefined **b)** −1 **c)** 1

d) 0 **e)** 0 **f)** 1

3. No. The tangent function has no maximum or minimum, so there is no amplitude.

4.



−300°, −120°, 240°

5. $\frac{\tan \theta}{\sin \theta} = \frac{1}{\cos \theta}$; $\tan \theta = \frac{\sin \theta}{\cos \theta}$

6. a) slope $= \frac{y}{x}$

b) Since y is equal to $\sin \theta$ and x is equal to $\cos \theta$, then $\tan \theta = \frac{y}{x}$.

c) slope $= \frac{\sin \theta}{\cos \theta}$ **d)** $\tan \theta = \frac{y}{x}$

7. a) $\tan \theta = \frac{y}{x}$ **b)** $\tan \theta = \frac{\sin \theta}{\cos \theta}$

c) $\sin \theta$ and $\cos \theta$ are equal to y and x, respectively.

8. a)

θ	tan θ
89.5°	114.59
89.9°	572.96
89.999°	57 295.78
89.999 999°	57 295 779.51

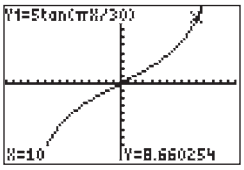
b) The value of $\tan \theta$ increases to infinity.

c)

θ	tan θ
90.5°	−114.59
90.01°	−5729.58
90.001°	−57 295.78
90.000 001°	−57 295 779.51

The value of $\tan \theta$ approaches negative infinity.

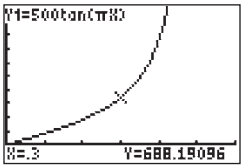
9. a) $d = 5 \tan \frac{\pi}{30} t$ **b)**



c) 8.7 m

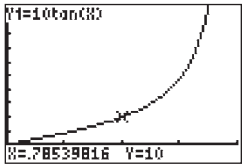
d) At $t = 15$ s, the camera is pointing along a line parallel to the wall and is turning away from the wall.

10. a) $d = 500 \tan \pi t$ **b)**



c) The asymptote represents the moment when the ray of light shines along a line that is parallel to the shore.

11. $d = 10 \tan x$



12. a) a tangent function

b) The slope would be undefined. It represents the place on the graph where the asymptote is.

13. Example:

a) (4, 3) **b)** 0.75

c) $\tan \theta$ is the slope of the graph.

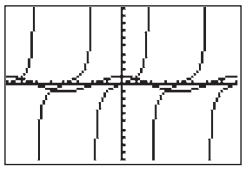
14. a) $\tan 0.5 \approx 0.5463$, power series ≈ 0.5463

b) $\sin 0.5 \approx 0.4794$, power series ≈ 0.4794

c) $\cos 0.5 \approx 0.8776$, power series ≈ 0.8776

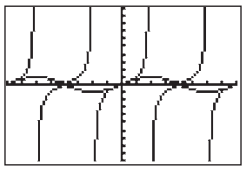
C1 The domain of $y = \sin x$ and $y = \cos x$ is all real numbers. The tangent function is not defined at $x = \frac{\pi}{2} + n\pi$, $n \in$ I. Thus, these numbers must be excluded from the domain of $y = \tan x$.

C2 a)



Example: The tangent function has asymptotes at the same x-values where zeros occur on the cosine function.

b)



Example: The tangent function has zeros at the same x-values where zeros occur on the sine function.

C3 Example: A circular or periodic function repeats its values over a specific period. In the case of $y = \tan x$, the period is π. So, the equation $\tan (x + \pi) = \tan x$ is true for all x in the domain of $\tan x$.

5.4 Equations and Graphs of Trigonometric Functions, pages 275 to 281

1. a) $x = 0, \pi, 2\pi$ **b)** $x = \pi n$ where n is an integer

c) $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi$

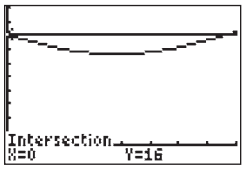
2. Examples:

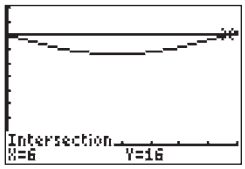
a) 1.25, 4.5

b) −3, −1.9, 0.1, 1.2, 3.2, 4.1, 6.3, 7.2

3. Examples: −50°, −10°, 130°, 170°, 310°, 350°

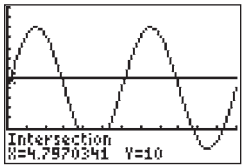
4. a)

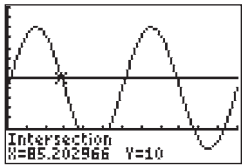


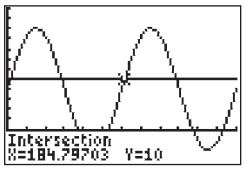


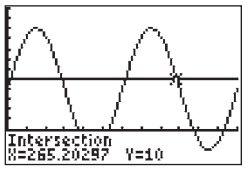
$x = 0$ and $x = 6$

b)



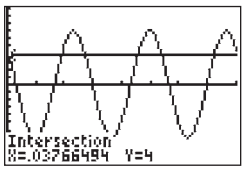


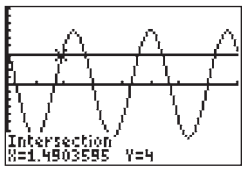


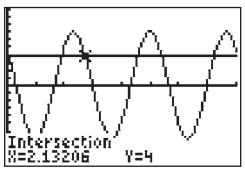


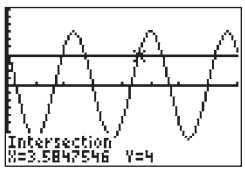
$x \approx 4.80°$, $x \approx 85.20°$, $x \approx 184.80°$, $x \approx 265.20°$

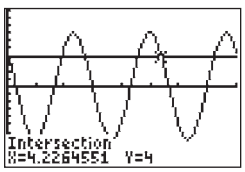
c)

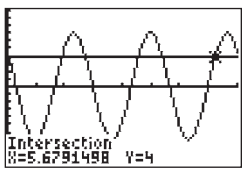






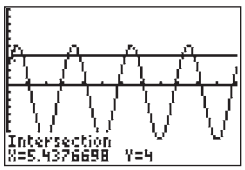


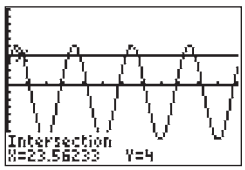


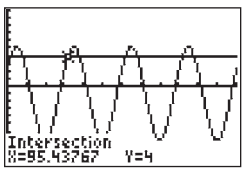


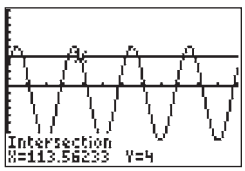
$x \approx 0.04$, $x \approx 1.49$, $x \approx 2.13$, $x \approx 3.58$, $x \approx 4.23$, and $x \approx 5.68$

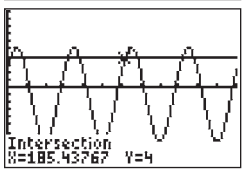
d)

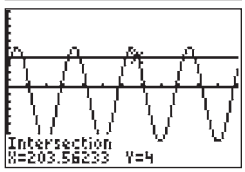


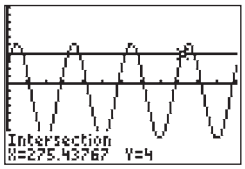


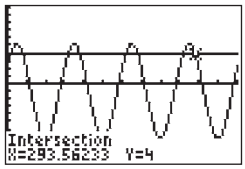












$x \approx 5.44°$, $x \approx 23.56°$, $x \approx 95.44°$, $x \approx 113.56°$, $x \approx 185.44°$, $x \approx 203.56°$, $x \approx 275.44°$, and $x \approx 293.56°$

5. a) $x \approx 1.33$

b) $x \approx 3.59°$ and $x \approx 86.41°$

c) $x \approx 1.91 + \pi n$ and $x \approx 3.09 + \pi n$, where n is an integer

d) $x \approx 4.50° + (8°)n$ and $x \approx 7.50° + (8°)n$, where n is an integer

6. a) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{\mathrm{P} \mid 2000 \leq \mathrm{P} \leq 14\,000, \mathrm{P} \in \mathrm{N}\}$

b) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{h \mid 1 \leq h \leq 13, h \in \mathrm{R}\}$

c) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{h \mid 6 \leq h \leq 18, h \in \mathrm{R}\}$

d) domain $\{t \mid t \geq 0, t \in \mathrm{R}\}$, range $\{h \mid 5 \leq h \leq 23, h \in \mathrm{R}\}$

7. $\frac{1}{200}$ s or 5 ms

8. a) Period is 100°; sinusoidal axis is at $y = 15$; amplitude is 9.

b) Period is $\frac{4\pi}{3}$; sinusoidal axis is at $y = -6$; amplitude is 10.

c) Period is $\frac{1}{50}$ s or 20 ms; sinusoidal axis is at $y = 0$; amplitude is 10.

9. a) 28 m **b)** 0 min, 0.7 min, 1.4 min, …

c) 2 m **d)** 0.35 min, 1.05 min, 1.75 min, …

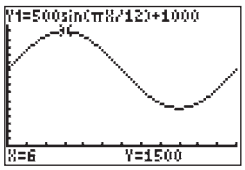
e) 0.18 min **f)** approximately 23.1 m

10. 78.5 cm

11. $V = 155 \sin 120\pi t$

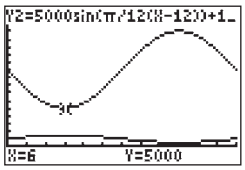
12. a) $\frac{1}{14}$ days **b)** 102.9 min **c)** 14 revolutions

13. a)



It takes approximately 15 months for the fox population to drop to 650.

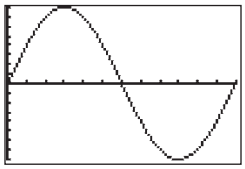
b)



c)

	Arctic Fox	Lemming
Maximum Population	1500	15 000
Month	6	18
Minimum Population	500	5000
Month	18	6

d) Example: The maximum for the predator occurs at a minimum for the prey and vice versa. The predators population depends on the prey, so every time the lemming's population changes the arctic fox population changes in accordance.

14. a) 

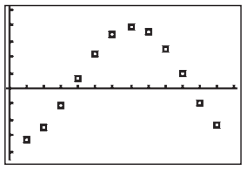
b) 35.1 cm

c) 1 s

15. a) Maximum is 7.5 Sun widths; minimum is 1 Sun width.

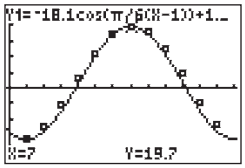
b) 24 h

c) $y = -3.25 \sin \frac{\pi}{12}x + 4.25$, where x represents the time, in hours, and y represents the number of Sun widths

16. a) 

b) 1.6 °C

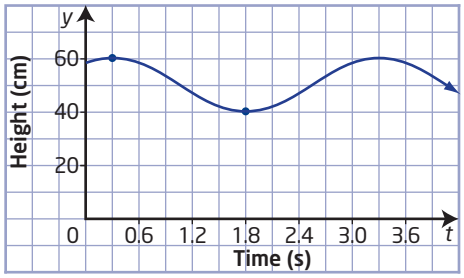
c) $y = -18.1 \cos \frac{\pi}{6}(x - 1) + 1.6$, where x represents the time, in months, and y represents the average monthly temperature, in degrees Celsius, for Winnipeg, Manitoba

d) 

e) about 2.5 months

17. a) $T = -4.5 \cos \frac{\pi}{30}t + 38.5$

b) 36.25 °C

18. a) 

b) $y = 10 \sin \frac{2\pi}{3}(t + 0.45) + 50$, where t represents the time, in seconds, and y represents the height of the mass, in centimetres, above the floor

c) 43.3 cm

d) 0.0847 s

19. a) $h = -10 \cos \frac{\pi}{30}t + 12$, where t represents the time, in seconds, and h represents the height of a passenger, in metres, above the ground

b) 15.1 m

c) approximately 21.1 s, 38.9 s

20. a) $h = 7 \sin \frac{2\pi}{5}(t + 1.75) + 15$ or

$h = 7 \cos \frac{2\pi}{5}(t + 0.5) + 15$, where t represents the time, in seconds, and h represents the height of the tip of the blade, in metres, above the ground

b) 20.66 m

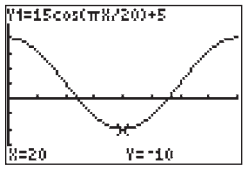
c) 4.078 s

21. a) $y = -9.7 \cos \frac{\pi}{183}(t - 26) + 13.9$, where t represents the time, in days, and y represents the average daily maximum temperature, in degrees Celsius

b) 18.6 °C

c) 88 days

22. a) $y = 15 \cos \frac{\pi}{20}t + 5$

b) 

c) approximately +9.6% of the total assets

d) Example: No, because it fluctuates too much.

23. a) $y = 1.2 \sin \frac{\pi}{2}t$, where t represents the time, in seconds, and y represents the distance for a turn, in metres, from the midline

b) $y = 1.2 \sin \frac{2\pi}{5}t$; The period increases.

C1 Examples:

a) Use a sine function as a model when the curve or data begins at or near the intersection of the vertical axis and the sinusoidal axis.

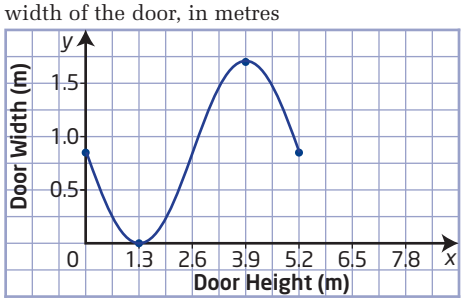
b) Use a cosine function as a model when the curve or data has a maximum or minimum near or at the vertical axis.

C2 Example:

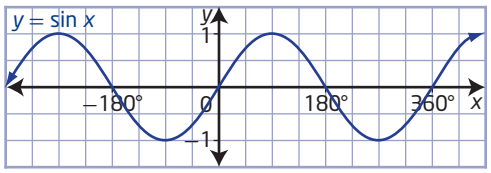
a)–b) The parameter b has the greatest influence on the graph of the function. It changes the period of the function. Parameters c and d change the location of the curve, but not the shape. Parameter a changes the maximum and minimum values.

C3 Examples:

a) $y = -0.85 \sin \frac{2\pi}{5.2}x + 0.85$, where x represents the height of the door, in metres, and y represents the width of the door, in metres

b) 

Chapter 5 Review, pages 282 to 285

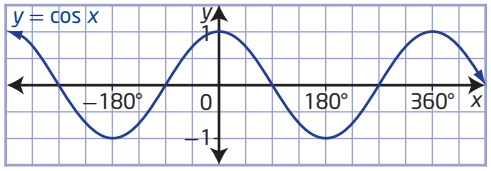
1. a) 

x-intercepts: $-360°$, $-180°$, $0°$, $180°$, $360°$

b) y-intercept: 0

c) domain $\{x \mid x \in \mathrm{R}\}$,
range $\{y \mid -1 \leq y \leq 1, y \in \mathrm{R}\}$, period is 2π

d) $y = 1$

2. a) 

x-intercepts: $-270°$, $-90°$, $90°$, $270°$

b) y-intercept: 1

c) domain $\{x \mid x \in R\}$,
range $\{y \mid -1 \le y \le 1, y \in R\}$, period is 2π

d) $y = 1$

3. a) A **b)** D **c)** B **d)** C

4. a) Amplitude is 3; period is π or 180°.

b) Amplitude is 4; period is 4π or 720°.

c) Amplitude is $\frac{1}{3}$; period is $\frac{12\pi}{5}$ or 432°.

d) Amplitude is 5; period is $\frac{4\pi}{3}$ or 240°.

5. a) Compared to the graph of $y = \sin x$, the graph of $y = \sin 2x$ completes two cycles in $0° \le x \le 360°$ and the graph of $y = 2 \sin x$ has an amplitude of 2.

b) Compared to the graph of $y = \sin x$, the graph of $y = -\sin x$ is reflected in the x-axis and the graph of $y = \sin(-x)$ is reflected in the y-axis. The graphs of $y = -\sin x$ and $y = \sin(-x)$ are the same.

c) Compared to the graph of $y = \cos x$, the graph of $y = -\cos x$ is reflected in the x-axis and the graph of $y = \cos(-x)$ is reflected in the y-axis. The graph of $y = \cos(-x)$ is the same as $y = \cos x$.

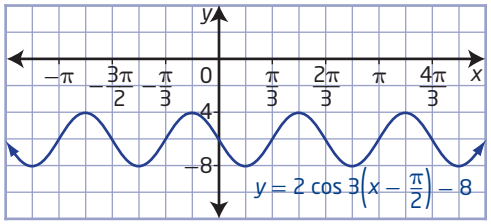
6. a) $y = 3 \cos 2x$ **b)** $y = 4 \cos \frac{12}{5}x$

c) $y = \frac{1}{2} \cos \frac{1}{2}x$ **d)** $y = \frac{3}{4} \cos 12x$

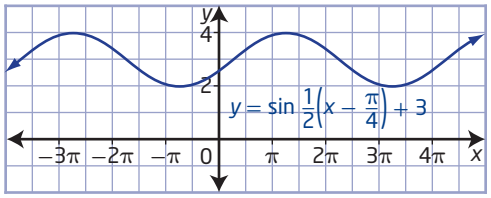
7. a) $y = 8 \sin 2x$ **b)** $y = 0.4 \sin 6x$

c) $y = \frac{3}{2} \sin \frac{1}{2}x$ **d)** $y = 2 \sin 3x$

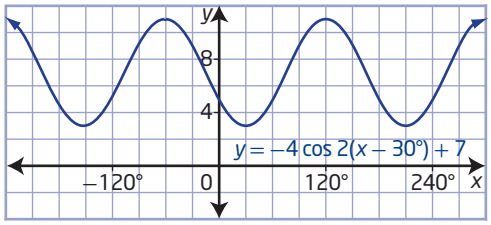
8. a) Amplitude is 2; period is $\frac{2\pi}{3}$; phase shift is $\frac{\pi}{2}$ units right; vertical displacement is 8 units down



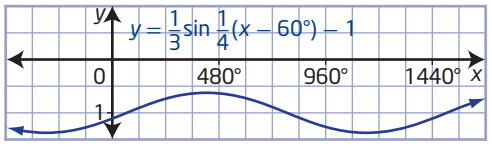
b) Amplitude is 1; period is 4π; phase shift is $\frac{\pi}{4}$ units right; vertical displacement is 3 units up



c) Amplitude is 4; period is 180°; phase shift is 30° right; vertical displacement is 7 units up



d) Amplitude is $\frac{1}{3}$; period is 1440°; phase shift is 60° right; vertical displacement is 1 unit down



9. a) They both have periods of π.

b) $f(x)$ has a phase shift of $\frac{\pi}{2}$ units right;
$g(x)$ has a phase shift of $\frac{\pi}{4}$ units right

c) π units right **d)** $\frac{\pi}{b}$ units right

10. a) $y = 3 \sin 2(x - 45°) + 1$, $y = -3 \cos 2x + 1$

b) $y = 2 \sin 2x - 1$, $y = 2 \cos 2(x - 45°) - 1$

c) $y = 2 \sin 2\left(x - \frac{\pi}{4}\right) - 1$, $y = -2 \cos 2x - 1$

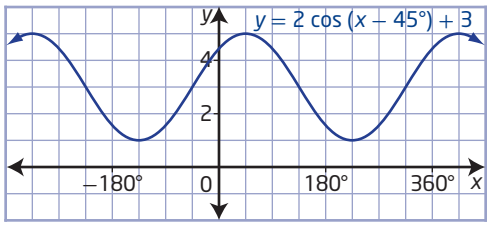
d) $y = 3 \sin \frac{1}{2}\left(x - \frac{\pi}{2}\right) + 1$, $y = 3 \cos \frac{1}{2}\left(x - \frac{3\pi}{2}\right) + 1$

11. a) $y = 4 \sin 2\left(x - \frac{\pi}{3}\right) - 5$

b) $y = \frac{1}{2} \cos \frac{1}{2}\left(x + \frac{\pi}{6}\right) + 1$

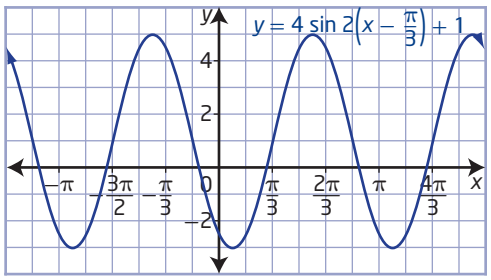
c) $y = \frac{2}{3} \sin \frac{2}{3}x - 5$

12. a)



domain $\{x \mid x \in R\}$, range $\{y \mid 1 \le y \le 5, y \in R\}$, maximum value is 5, minimum value is 1, no x-intercepts, y-intercept of approximately 4.41

b)



domain $\{x \mid x \in R\}$, range $\{y \mid -3 \le y \le 5, y \in R\}$, maximum value is 5, minimum value is -3, x-intercepts: approximately $0.92 + n\pi$, $2.74 + n\pi$, $n \in I$, y-intercept: approximately -2.5

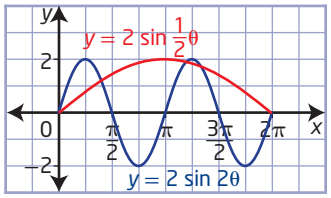
13. a) vertically stretched by a factor of 3 about the x-axis, horizontally stretched by a factor of $\frac{1}{2}$ about the y-axis, translated $\frac{\pi}{3}$ units right and 6 units up

b) vertically stretched by a factor of 2 about the x-axis, reflected in the x-axis, horizontally stretched by a factor of 2 about the y-axis, translated $\frac{\pi}{4}$ units left and 3 units down

c) vertically stretched by a factor of $\frac{3}{4}$ about the x-axis, horizontally stretched by a factor of $\frac{1}{2}$ about the y-axis, translated 30° right and 10 units up

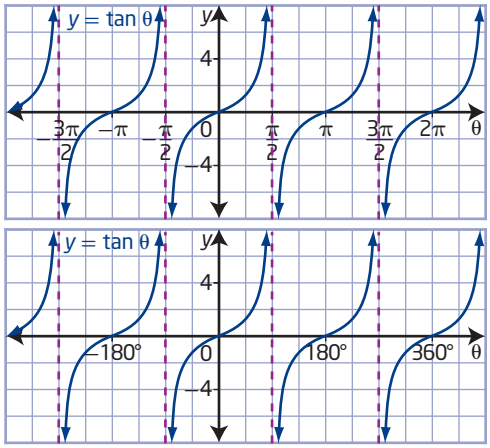
d) reflected in the x-axis, horizontally stretched by a factor of $\frac{1}{2}$ about the y-axis, translated 45° left and 8 units down

14. a)



b) Compared to the graph of $y = \sin\theta$, the graph of $y = 2\sin 2\theta$ is vertically stretched by a factor of 2 about the x-axis and half the period. Compared to the graph of $y = \sin\theta$, the graph of $y = 2\sin\frac{1}{2}\theta$ is vertically stretched by a factor of 2 about the x-axis and double the period.

15. a)



b) **i)** domain $\{x \mid -2\pi \le x \le 2\pi, x \ne -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, x \in \mathrm{R}\}$ or $\{x \mid -360° \le x \le 360°, x \ne -270°, -90°, 90°, 270°, x \in \mathrm{R}\}$
ii) range $\{y \mid y \in \mathrm{R}\}$ **iii)** y-intercept: 0
iv) x-intercepts: -2π, $-\pi$, 0, π, 2π or $-360°$, $-180°$, 0°, 180°, 360°
v) asymptotes: $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = -270°, -90°, 90°, 270°$

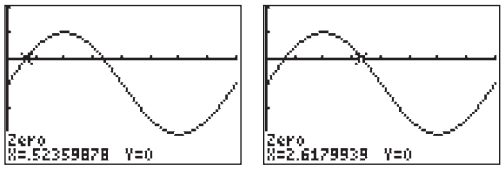
16. a) $\left(1, \frac{1}{\sqrt{3}}\right)$ **b)** $\tan\theta = \frac{\sin\theta}{\cos\theta}$
c) As θ approaches 90°, tan θ approaches infinity.
d) tan 90° is not defined.

17. a) Since cos θ is the denominator, when it is zero tan θ becomes undefined.
b) Since sin θ is the numerator, when it is zero tan θ becomes zero.

18. The shadow has no length which makes the slope infinite. This relates to the asymptotes on the graph of $y = \tan\theta$.

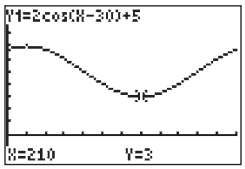
19. A vertical asymptote is an imaginary line that the graph comes very close to touching but in fact never does. If a trigonometric function is represented by a quotient, such as the tangent function, asymptotes generally occur at values for which the function is not defined; that is, when the function in the denominator is equal to zero.

20. a)



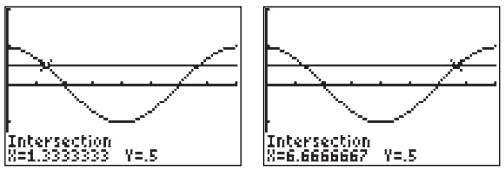
$x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$ or $x \approx 0.52$ and $x \approx 2.62$

b)



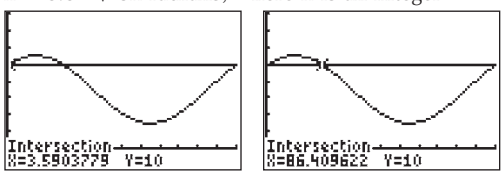
no solution

c)



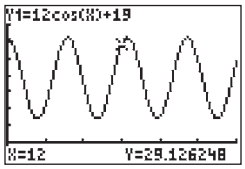
$x \approx 1.33 + 8n$ radians and
$x \approx 6.67 + 8n$ radians, where n is an integer

d)



$x \approx 3.59° + (360°)n$ and
$x \approx 86.41° + (360°)n$, where n is an integer

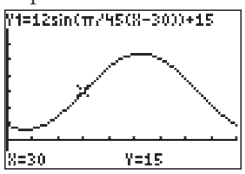
21. a)



b) 9.4 h

c) Example: A model for temperature variance is important for maintaining constant temperatures to preserve artifacts.

22. a)



b) maximum height: 27 m, minimum height: 3 m

c) 90 s **d)** approximately 25.4 m

23. a) $L = -3.7\cos\frac{2\pi}{365}(t + 10) + 12$
b) approximately 12.8 h of daylight

24. a) approximately 53 sunspots
b) around the year 2007
c) around the year 2003

Chapter 5 Practice Test, pages 286 to 287

1. A **2.** D **3.** C **4.** D **5.** B **6.** A **7.** C **8.** $\frac{\pi}{2}$

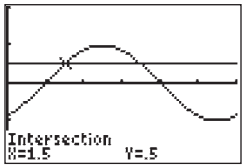
9. asymptotes: $x = \frac{\pi}{2} + n\pi$, $n \in \mathrm{I}$,
domain $\left\{x \mid x \ne \frac{\pi}{2} + n\pi, x \in \mathrm{R}, n \in \mathrm{I}\right\}$,
range $\{y \mid y \in \mathrm{R}\}$, period is π

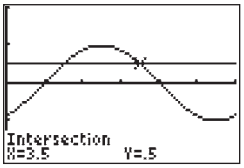
10. Example: They have the same maximum and minimum values. Neither function has a horizontal or vertical translation.

11. Amplitude is 120; period is 0.0025 s or 2.5 ms.

12. The minimum depth of 2 m occurs at 0 h, 12 h, and 24 hour. The maximum depth of 8 m occurs at 6 h and 18 h.

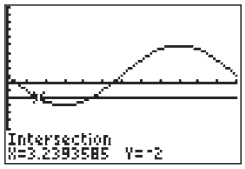
13. a)

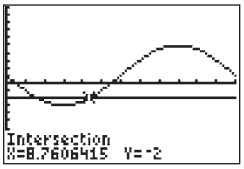




$x = 1.5 + 6n$ radians and $x = 3.5 + 6n$ radians, where n is an integer

b)



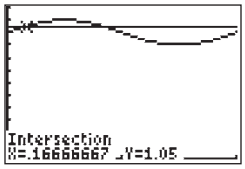


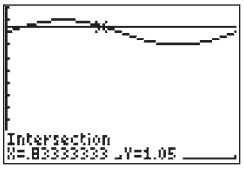
$x \approx 3.24° + (24°)n$ and
$x \approx 8.76° + (24°)n$, where n is an integer

14. Example: Graph II has half the period of graph I. Graph I represents a cosine curve with no phase shift. Graph II represents a sine curve with no phase shift. Graph I and II have the same amplitude and both graphs have no vertical translations.

15. a) $h = 0.1 \sin \pi t + 1$, where t represents the time, in seconds, and h represents the height of the mass, in metres, above the floor

b)





approximately 0.17 s and 0.83 s

c) $t = \frac{1}{6}$ or 0.1666… and $t = \frac{5}{6}$ or 0.8333

16. a) $y = 3 \sin 2\left(x - \frac{\pi}{4}\right) - 1$ **b)** $y = -3 \cos 2x - 1$

17. a) A, B **b)** A, B or C, D, E **c)** B