Chapter 6 Trigonometric Identities

6.1 Reciprocal, Quotient, and Pythagorean Identities, pages 296 to 298

1. a) $x \neq \pi n;\ n \in \mathrm{I}$ **b)** $x \neq \left(\frac{\pi}{2}\right)n,\ n \in \mathrm{I}$

c) $x \neq \frac{\pi}{2} + 2\pi n$ and $x \neq \pi n,\ n \in \mathrm{I}$

d) $x \neq \frac{\pi}{2} + \pi n$ and $x \neq \pi + 2\pi n,\ n \in \mathrm{I}$

2. Some identities will have non-permissible values because they involve trigonometric functions that have non-permissible values themselves or a function occurs in a denominator. For example, an identity involving sec θ has non-permissible values $\theta \neq 90° + 180°n$, where $n \in \mathrm{I}$, because these are the non-permissible values for the function.

3. a) tan x **b)** sin x **c)** sin x

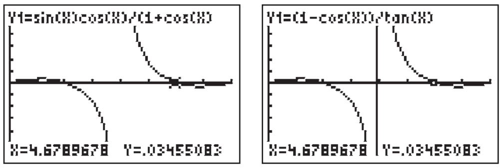
4. a) cot x **b)** csc x **c)** sec x

5. a) When substituted, both values satisfied the equation.

b) $x \neq 0°, 90°, 180°, 270°$

6. a) $x \neq \pi + 2\pi n,\ n \in \mathrm{I};\ x \neq \frac{\pi}{2} + \pi n,\ n \in \mathrm{I}$

b)



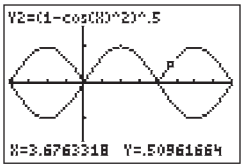
Yes, it appears to be an identity.

c) The equation is verified for $x = \frac{\pi}{4}$.

7. a) $\cos^2 \theta$ **b)** 0.75 **c)** 25%

8. a) All three values check when substituted.

b)



c) The equation is not an identity since taking the square then the square root removes the negative sign and sin x is negative from π to 2π.

9. a) $E = \frac{I\cos\theta}{R^2}$ **b)** $E = \frac{I\cot\theta}{R^2\csc\theta}$

$$E = \frac{I\left(\frac{\cos\theta}{\sin\theta}\right)}{R^2\left(\frac{1}{\sin\theta}\right)}$$

$$E = \left(\frac{I\cos\theta}{\sin\theta}\right)\left(\frac{\sin\theta}{R^2}\right)$$

$$E = \frac{I\cos\theta}{R^2}$$

10. $\cos x,\ x \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

11. a) It appears to be equivalent to sec x.

b) $x \neq \frac{\pi}{2} + \pi n,\ n \in \mathrm{I}$

c)
$$\frac{\csc^2 x - \cot^2 x}{\cos x} = \frac{\frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}}{\cos x}$$
$$= \frac{\frac{1 - \cos^2 x}{\sin^2 x}}{\cos x}$$
$$= \frac{\frac{\sin^2 x}{\sin^2 x}}{\cos x}$$
$$= \frac{1}{\cos x}$$
$$= \sec x$$

12. a) Yes, it could be an identity.

b)
$$\frac{\cot x}{\sec x} + \sin x = \frac{\cos x}{\sin x} \div \frac{1}{\cos x} + \sin x$$
$$= \frac{\cos^2 x}{\sin x} + \sin x$$
$$= \frac{\cos^2 x + \sin^2 x}{\sin x}$$
$$= \csc x$$

13. a) $1 = 1$

b) The left side $= 1$, but the right side is undefined.

c) The chosen value is not permissible for the tan x function.

d) The left side $= \frac{2}{\sqrt{2}}$, but the right side $= 2$.

e) Giselle has found a permissible value for which the equation is not true, so they can conclude that it is not an identity.

14. 2

15. 7.89

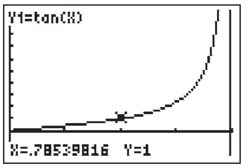
16. $\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta}=\dfrac{1-\sin\theta+1+\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$
$=\dfrac{2}{(1-\sin^2\theta)}$
$=2\sec^2\theta$

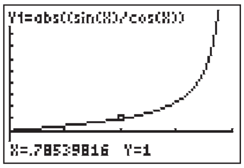
17. $m=\csc x$

C1 $\cot^2 x+1$
$=\dfrac{\cos^2 x}{\sin^2 x}+\dfrac{\sin^2 x}{\sin^2 x}$
$=\dfrac{\cos^2 x+\sin^2 x}{\sin^2 x}$
$=\dfrac{1}{\sin^2 x}$
$=\csc^2 x$

C2 $\left(\dfrac{\sin\theta}{1+\cos\theta}\right)\left(\dfrac{1-\cos\theta}{1-\cos\theta}\right)$
$=\dfrac{\sin\theta-\sin\theta\cos\theta}{1-\cos^2\theta}$
$=\dfrac{\sin\theta-\sin\theta\cos\theta}{\sin^2\theta}$
$=\dfrac{1-\cos\theta}{\sin\theta}$
It helps to simplify by creating an opportunity to use the Pythagorean identity.

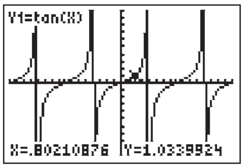
C3 **Step 1**

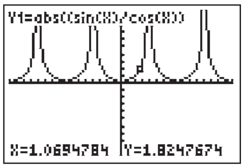




Yes, over this domain it is an identity.

Step 2





The equation is not an identity since the graphs of the two sides are not the same.

Step 3 Example: $y=\cot\theta$ and $y=\left|\dfrac{\cos\theta}{\sin\theta}\right|$ are identities over the domain $0\le\theta\le\dfrac{\pi}{2}$ but not over the domain $-2\pi\le\theta\le 2\pi$

Step 4 The weakness with this approach is that for some more complicated identities you may think it is an identity when really it is only an identity over that domain.

6.2 Sum, Difference, and Double-angle Identities, pages 306 to 308

1. a) $\cos 70°$ **b)** $\sin 35°$ **c)** $\cos 38°$
d) $\sin\dfrac{\pi}{4}$ **e)** $4\sin\dfrac{2\pi}{3}$

2. a) $\cos 60°=0.5$ **b)** $\sin 45°=\dfrac{1}{\sqrt{2}}$ or $\dfrac{\sqrt{2}}{2}$
c) $\cos\dfrac{\pi}{3}=0.5$ **d)** $\cos\dfrac{5\pi}{6}=-\dfrac{\sqrt{3}}{2}$

3. $\cos 2x=1-2\sin^2 x$;
$1-\cos 2x=1-1+2\sin^2 x=2\sin^2 x$

4. a) $\sin\dfrac{\pi}{2}$ **b)** $6\sin 48°$ **c)** $\tan 152°$ **d)** $\cos\dfrac{\pi}{3}$
e) $-\cos\dfrac{\pi}{6}$

5. a) $\sin\theta$ **b)** $\cos x$ **c)** $\cos\theta$ **d)** $\cos x$

6. Example: When $x=60°$ and $y=30°$, then left side $=0.5$, but right side ≈ 0.366.

7. $\cos(90°-x)=\cos 90°\cos x+\sin 90°\sin x$
$=\sin x$

8. a) $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$ or $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ **b)** $\dfrac{-\sqrt{3}+1}{\sqrt{3}+1}$ or $\sqrt{3}-2$
c) $\dfrac{1+\sqrt{3}}{2\sqrt{2}}$ or $\dfrac{\sqrt{2}+\sqrt{6}}{4}$ **d)** $\dfrac{-\sqrt{3}-1}{2\sqrt{2}}$ or $\dfrac{-\sqrt{6}-\sqrt{2}}{4}$
e) $\sqrt{2}(1+\sqrt{3})$ **f)** $\dfrac{1-\sqrt{3}}{2\sqrt{2}}$ or $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

9. a) $P=1000\sin(x+113.5°)$
b) **i)** 101.056 W/m^2 **ii)** 310.676 W/m^2
iii) -50.593 W/m^2
c) The answer in part iii) is negative which means that there is no sunlight reaching Igloolik. At latitude 66.5°, the power received is 0 W/m^2.

10. $-2\cos x$

11. a) $\dfrac{119}{169}$ **b)** $-\dfrac{120}{169}$ **c)** $-\dfrac{12}{13}$

12. a) Both sides are equal for this value.
b) Both sides are equal for this value.
c) $\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$
$=\dfrac{2\tan x}{1-\tan^2 x}\left(\dfrac{\cos^2 x}{\cos^2 x}\right)$
$=\dfrac{2\left(\dfrac{\sin x}{\cos x}\right)(\cos^2 x)}{\left(1-\dfrac{\sin^2 x}{\cos^2 x}\right)\cos^2 x}$
$=\dfrac{2\sin x\cos x}{\cos^2 x-\sin^2 x}$

13. a) $d=\dfrac{v_o^2\sin 2\theta}{g}$ **b)** 45°
c) It is easier after applying the double-angle identity since there is only one trigonometric function whose value has to be found.

14. $k-1$

15. a) $\cos^4 x-\sin^4 x=(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)$
$=\cos^2 x-\sin^2 x$
$=\cos 2x$
b) $\dfrac{\csc^2 x-2}{\csc^2 x}=1-\dfrac{2}{\csc^2 x}$
$=1-2\sin^2 x$
$=\cos 2x$

16. a) $\dfrac{1-\cos 2x}{2}=\dfrac{1-1+2\sin^2 x}{2}=\sin^2 x$
b) $\dfrac{4-8\sin^2 x}{2\sin x\cos x}=\dfrac{4\cos 2x}{\sin 2x}=\dfrac{4}{\tan 2x}$

17. $-\dfrac{2}{\sqrt{29}}$

18. $k=3$

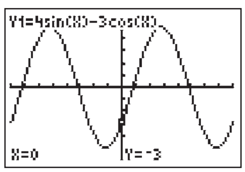
19. a) $0.9928, -0.392\ 82$ or $\dfrac{\pm 4\sqrt{3}+3}{10}$
b) 0.9500 or $\dfrac{\sqrt{5}+2\sqrt{3}}{6}$

20. a) $\dfrac{56}{65}$ **b)** $\dfrac{63}{65}$ **c)** $\dfrac{-7}{25}$ **d)** $\dfrac{24}{25}$

21. a) $\sin x$ **b)** $\tan x$

22. $\cos x=2\cos^2\left(\dfrac{x}{2}\right)-1$
$\dfrac{\cos x+1}{2}=\cos^2\left(\dfrac{x}{2}\right)$
$\pm\sqrt{\dfrac{\cos x+1}{2}}=\cos\dfrac{x}{2}$

23. a)



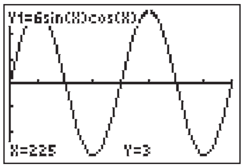
b) $a=5$, $c=37°$
c) $y=5\sin(x-36.87°)$

24. $y = 3 \sin 2x - 3$

C1 a) **i)** $\frac{120}{169}$ or 0.7101 **ii)** $\frac{120}{169}$ or 0.7101

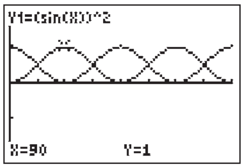
b) Using identities is more straightforward.

C2 a)

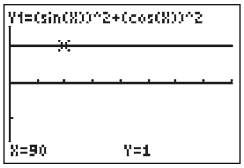


b) To find the sine function from the graph, compare the amplitude and the period to that of a base sine curve. The alternative equation is $y = 3 \sin 2x$.

C3 a)

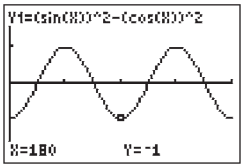


b)



The graph will be the horizontal line $y = 1$.

c)



The resultant graph is a cosine function reflected over the x-axis and the period becomes π.

d) $f(x) = -\cos 2x$. Using trigonometric identities,

$$\begin{aligned}\sin^2 x - \cos^2 x &= 1 - \cos^2 x - \cos^2 x.\\ &= 1 - 2\cos^2 x\\ &= -\cos 2x\end{aligned}$$

6.3 Proving Identities, pages 314 to 315

1. a) $\sin x$ **b)** $\frac{\cos x + 1}{6}$

c) $\frac{\sin x}{\cos x + 1}$ **d)** $\sec x - 4 \csc x$

2. a)
$$\begin{aligned}\cos x + \cos x \tan^2 x &= \cos x + \frac{\sin^2 x}{\cos x}\\ &= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x}\\ &= \frac{1}{\cos x}\\ &= \sec x\end{aligned}$$

b)
$$\begin{aligned}\frac{\sin^2 x - \cos^2 x}{\sin x + \cos x} &= \frac{(\sin x - \cos x)(\sin x + \cos x)}{\sin x + \cos x}\\ &= \sin x - \cos x\end{aligned}$$

c)
$$\begin{aligned}\frac{\sin x \cos x - \sin x}{\cos^2 x - 1} &= \frac{\sin x \cos x - \sin x}{-\sin^2 x}\\ &= \frac{-\sin x(1 - \cos x)}{-\sin^2 x}\\ &= \frac{1 - \cos x}{\sin x}\end{aligned}$$

d)
$$\begin{aligned}\frac{1 - \sin^2 x}{1 + 2\sin x - 3\sin^2 x} &= \frac{(1 - \sin x)(1 + \sin x)}{(1 - \sin x)(1 + 3\sin x)}\\ &= \frac{1 + \sin x}{1 + 3\sin x}\end{aligned}$$

3. a) $\frac{\sin x + 1}{\cos x}$ **b)** $\frac{-2\tan x}{\cos x}$

c) $\csc x$ **d)** $2\cot^2 x$

4. a) $\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$ **b)** $\sin x$

5. $\frac{\sin 2x}{2\sin x} = \frac{2\sin x \cos}{2\sin x} = \cos x,\ x \neq \pi n;\ n \in \mathrm{I}$

6. $\cos x$

7. a)
$$\begin{aligned}\frac{\csc x}{2\cos x} &= \frac{1}{2\sin x \cos x}\\ &= \frac{1}{\sin 2x}\\ &= \csc 2x\end{aligned}$$

b)
$$\begin{aligned}\sin x + \cos x \cot x &= \sin x + \frac{\cos^2 x}{\sin x}\\ &= \frac{1}{\sin x}\\ &= \csc x\end{aligned}$$

8. Hannah's choice takes fewer steps.

9. a) 42.3 m

b)
$$\begin{aligned}\frac{v_o^2 \sin 2\theta}{g} &= \frac{v_o^2 2\sin\theta\cos\theta}{g}\\ &= \frac{2v_o^2 \sin^2\theta\cos\theta}{g\sin\theta}\\ &= \frac{2v_o^2\sin^2\theta}{g\tan\theta}\\ &= \frac{2v_o^2(1 - \cos^2\theta)}{g\tan\theta}\end{aligned}$$

10. a) Left Side
$$\begin{aligned}&= \frac{\csc x}{2\cos x}\\ &= \frac{1}{2\sin x\cos x}\\ &= \frac{1}{\sin 2x}\\ &= \csc 2x\\ &= \text{Right Side}\end{aligned}$$

b) Left Side
$$\begin{aligned}&= \frac{\sin x\cos x}{1 + \cos x}\\ &= \frac{(\sin x\cos x)(1 - \cos x)}{(1 + \cos x)(1 - \cos x)}\\ &= \frac{\sin x\cos x - \sin x\cos^2 x}{\sin^2 x}\\ &= \frac{\cos x - \cos^2 x}{\sin x}\\ &= \frac{1 - \cos x}{\tan x}\\ &= \text{Right Side}\end{aligned}$$

c)
$$\begin{aligned}\text{Left Side} &= \frac{\sin x + \tan x}{1 + \cos x}\\ &= \left(\frac{\sin x}{1} + \frac{\sin x}{\cos x}\right) \div (1 + \cos x)\\ &= \left(\frac{\sin x\cos x + \sin x}{\cos x}\right) \times \frac{1}{1 + \cos x}\\ &= \left(\frac{\sin x(1 + \cos x)}{\cos x}\right) \times \frac{1}{1 + \cos x}\\ &= \frac{\sin x}{\cos x}\end{aligned}$$

$$\begin{aligned}\text{Right Side} &= \frac{\sin 2x}{2\cos^2 x}\\ &= \frac{2\sin x\cos x}{2\cos^2 x}\\ &= \frac{\sin x}{\cos x}\end{aligned}$$

Left Side = Right Side

11. a)
$$\begin{aligned}\text{Left Side} &= \frac{\sin 2x}{\cos x} + \frac{\cos 2x}{\sin x}\\ &= \frac{2\sin x\cos x}{\cos x} + \frac{1 - 2\sin^2 x}{\sin x}\\ &= 2\sin x + \csc x - 2\sin x\\ &= \csc x\\ &= \text{Right Side}\end{aligned}$$

b) Left Side

$= \csc^2 x + \sec^2 x$

$= \dfrac{1}{\sin^2 x} + \dfrac{1}{\cos^2 x}$

$= \dfrac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$

$= \dfrac{1}{\sin^2 x \cos^2 x}$

$= \csc^2 x \sec^2 x$

= Right Side

c) Left Side

$= \dfrac{\cot x - 1}{1 - \tan x}$

$= \dfrac{\dfrac{1 - \tan x}{\tan x}}{1 - \tan x}$

$= \dfrac{1 - \tan x}{\tan x(1 - \tan x)}$

$= \dfrac{1}{\tan x}$

$= \dfrac{\csc x}{\sec x}$

= Right Side

12. a) Left Side $= \sin (90° + \theta)$

$= \sin 90° \cos \theta + \cos 90° \sin \theta$

$= \cos \theta$

Right Side $= \sin (90° - \theta)$

$= \sin 90° \cos \theta - \cos 90° \sin \theta$

$= \cos \theta$

b) Left Side $= \sin (2\pi - \theta)$

$= \sin (2\pi) \cos (\theta) - \cos (2\pi) \sin (\theta)$

$= -\sin \theta$

= Right Side

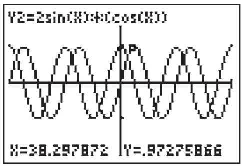
13. Left Side $= 2 \cos x \cos y$

Right Side $= \cos(x + y) + \cos(x - y)$

$= \cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y$

$= 2 \cos x \cos y$

14. a)



No, this is not an identity.

b) Replacing the variable with 0 is a counter example.

15. a) $x \neq \pi n;\ n \in I$

b) Left Side $= \dfrac{\sin 2x}{1 - \cos 2x}$

$= \dfrac{2 \sin x \cos x}{1 - 1 + 2 \sin^2 x}$

$= \dfrac{\cos x}{\sin x}$

$= \cot x$

= Right Side

16. Right Side

$= \dfrac{\sin 4x - \sin 2x}{\cos 4x + \cos 2x}$

$= \dfrac{2 \sin 2x \cos 2x - 2 \sin x \cos x}{\cos 4x + 2 \cos^2 x - 1}$

$= \dfrac{2(2 \sin x \cos x)(2 \cos^2 x - 1) - 2 \sin x \cos x}{2 \cos^2 2x - 1 + 2 \cos^2 x - 1}$

$= \dfrac{(2 \sin x \cos x)(2(2 \cos^2 x - 1) - 1)}{2(2 \cos^2 x - 1)^2 + 2 \cos^2 x - 2}$

$= \dfrac{(2 \sin x \cos x)(4 \cos^2 x - 3)}{2(4 \cos^4 x - 4 \cos^2 x + 1) + 2 \cos^2 x - 2}$

$= \dfrac{(2 \sin x \cos x)(4 \cos^2 x - 3)}{8 \cos^4 x - 6 \cos^2 x}$

$= \dfrac{(2 \sin x \cos x)(4 \cos^2 x - 3)}{2 \cos^2 x(4 \cos^2 x - 3)}$

$= \dfrac{2 \sin x \cos x}{2 \cos^2 x}$

$= \tan x$

= Left Side

17. Left Side $= \dfrac{\sin 2x}{1 - \cos 2x}$

$= \dfrac{\sin 2x}{1 - \cos 2x}\left(\dfrac{1 + \cos 2x}{1 + \cos 2x}\right)$

$= \dfrac{\sin 2x + \sin 2x \cos 2x}{1 - \cos^2 2x}$

$= \dfrac{\sin 2x + \sin 2x \cos 2x}{\sin^2 2x}$

$= \dfrac{1}{\sin 2x} + \dfrac{\cos 2x}{\sin 2x}$

$= \dfrac{1}{\sin 2x} + \dfrac{1 - 2 \sin^2 x}{\sin 2x}$

$= \dfrac{2}{\sin 2x} - \dfrac{2 \sin^2 x}{\sin 2x}$

$= 2 \csc 2x - \dfrac{2 \sin^2 x}{2 \sin x \cos x}$

$= 2 \csc 2x - \tan x$

= Right Side

18. Left Side $= \dfrac{1 - \sin^2 x - 2 \cos x}{\cos^2 x - \cos x - 2}$

$= \dfrac{\cos^2 x - 2 \cos x}{\cos^2 x - \cos x - 2}$

$= \dfrac{\cos x(\cos x - 2)}{(\cos x - 2)(\cos x + 1)}$

$= \dfrac{\cos x}{\cos x + 1}$

$= \dfrac{\dfrac{\cos x}{\cos x}}{\dfrac{\cos x + 1}{\cos x}}$

$= \dfrac{1}{1 + \sec x}$

= Right Side

19. a) $\sin \theta_t = \dfrac{n_1 \sin \theta_i}{n_2}$

b) Using $\sin^2 x + \cos^2 x = 1$, $\cos x = \sqrt{1 - \sin^2 x}$
Then, replace this in the equation.

c) Substitute $\sin \theta_t = \dfrac{n_1 \sin \theta_i}{n_2}$.

C1 Graphing gives a visual approximation, so some functions may look the same but actually are not. Verifying numerically is not enough since it may not hold for other values.

C2 Left Side $= \cos\left(\dfrac{\pi}{2} - x\right)$

$= \cos\left(\dfrac{\pi}{2}\right) \cos x + \sin\left(\dfrac{\pi}{2}\right) \sin x$

$= \sin x$

= Right Side

C3 a) $\cos x \geq 0$, $\dfrac{\pi}{2} + 2\pi n < x < \dfrac{3\pi}{2} + 2\pi n,\ n \in I$

b) $x = 1$

c) $x = \pi$, $\cos x$ will give a negative answer and radical functions always give a positive answer, so the equation is not an identity.

d) An identity is always true whereas an equation is true for certain values or a restricted domain.

6.4 Solving Trigonometric Equations Using Identities, pages 320 to 321

1. a) $0, \dfrac{\pi}{4}, \pi, \dfrac{5\pi}{4}$

b) $0, \dfrac{\pi}{3}, \pi, \dfrac{5\pi}{3}$

c) $\dfrac{3\pi}{2}$

d) $\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$

2. a) 0°, 120°, 240°

b) 270°

c) no solution

d) 0°, 120°, 180°, 300°

3. a) $2\sin^2 x + 3\sin x + 1 = 0; \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

b) $2\sin^2 x + 3\sin x + 1 = 0; \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

c) $\sin^2 x + 2\sin x - 3 = 0, \frac{\pi}{2}$

d) $2 - \sin^2 x = 0$; no solution

4. $-150°, -30°, 30°, 150°$

5. 0.464, 2.034, 3.605, 5.176

6. There are two more solutions that Sanesh did not find since she divided by $\cos(x)$. The extra solutions are $x = 90° + 360°n$ and $x = 270° + 360°n$.

7. a) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ b) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

8. $x = \frac{\pi}{2} + \pi n, n \in I$

9. $x = \frac{\pi}{2} + 2\pi n, n \in I$

10. 7. Inspection of each factor shows that there are 2 + 1 + 4 solutions, which gives a total of 7 solutions over the interval $0° < x \le 360°$.

11. $\frac{\pi}{2}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}$

12. $B = -3, C = -2$

13. Example: $\sin 2x - \sin 2x\cos^2 x = 0; x = \left(\frac{\pi}{2}\right)n, n \in I$

14. $x = \left(\frac{\pi}{2}\right)(2n + 1), n \in I, x = \frac{\pi}{6} + 2\pi n, n \in I,$
$x = \frac{5\pi}{6} + 2\pi n, n \in I$

15. 12 solutions

16. $x = \pi + 2\pi n, n \in I, x = \pm 0.955\,32 + n\pi, n \in I$

17. $x = \frac{\pi}{4} + \pi n, n \in I, x = -\frac{\pi}{4} + \pi n, n \in I$

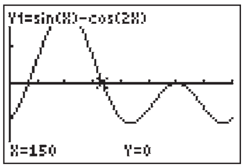
18. $-1.8235, 1.8235$

19. $x = 2\pi n, n \in I, x = \pm\frac{\pi}{3} + 2\pi n, n \in I$

20. 1 and -2

C1 a) $\cos 2x = 1 - 2\sin^2 x$ b) $(2\sin x - 1)(\sin x + 1)$

c) 30°, 150°, 270° d)



C2 a) You cannot factor the left side of the equation because there are no two integers whose product is -3 and whose sum is 1.

b) $-0.7676, 0.4343$

c) 64.26°, 140.14°, 219.86°, 295.74°, 424.26°, 500.14°, 579.86°, 655.74°

C3 Example: $\sin 2x\cos x + \cos x = 0$; The reason this is not an identity is that it is not true for all replacement values of the variable. For example, if $x = 30°$, the two sides are not equal. The solutions are $90° + 180°n, n \in I$ and $135° + 180°n, n \in I$.

Chapter 6 Review, pages 322 to 323

1. a) $x \ne \frac{\pi}{2} + n\pi, n \in I$ b) $x \ne \left(\frac{\pi}{2}\right)n, n \in I$

c) $x = \pm\frac{\pi}{3} + 2\pi n, n \in I$ d) $x \ne \frac{\pi}{2} + n\pi, n \in I$

2. a) $\cos x$ b) $\tan x$ c) $\tan x$ d) $\cos x$

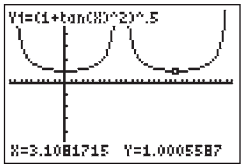
3. a) 1 b) 1 c) 1

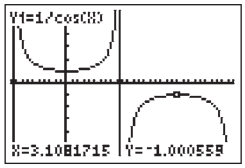
4. a) Both sides have the same value so the equation is true for those values.

b) $x \ne 90°, 270°$

5. a) Example: $x = 0, 1$

b)





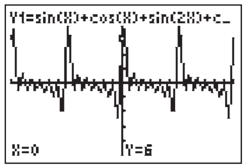
c) The graphs are the same for part of the domain. Outside of this interval they are not the same.

6. a) $f(0) = 2, f\left(\frac{\pi}{6}\right) = 1 + \sqrt{3}$

b) $\sin x + \cos x + \sin 2x + \cos 2x$
$= \sin x + \cos x + 2\sin x\cos x + 1 - \sin^2 x$

c) No, because you cannot write the first two terms as anything but the way they are.

d) You cannot get a perfect saw tooth graph but the approximation gets closer as you increase the amount of iterations. Six terms give a reasonable approximation.



7. a) $\sin 90° = 1$ b) $\sin 30° = 0.5$

c) $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ d) $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

8. a) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} - \sqrt{2}}{4}$ b) $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} + \sqrt{2}}{4}$

c) $\sqrt{3} - 2$ d) $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} + \sqrt{2}}{4}$

9. a) $\frac{7}{13\sqrt{2}}$ or $\frac{7\sqrt{2}}{26}$ b) $\frac{12 - 5\sqrt{3}}{26}$

c) $-\frac{120}{169}$

10. $1 + \frac{1}{\sqrt{2}}$

11. $\tan x$

12. a) $\frac{\cos x}{\sin x - 1}$ or $\frac{-1 - \sin x}{\cos x}$

b) $\tan^2 x\sin^2 x$

13. a) Left Side
$= 1 + \cot^2 x$
$= 1 + \frac{\cos^2 x}{\sin^2 x}$
$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x}$
$= \frac{1}{\sin^2 x}$
$= \csc^2 x$
= Right Side

b) Right Side
$= \csc 2x - \cot 2x$
$= \frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}$
$= \frac{1 - (2\cos^2 x - 1)}{2\sin x\cos x}$
$= \frac{2\sin^2 x}{2\sin x\cos x}$
$= \tan x$
= Left Side

c) Left Side
$= \sec x + \tan x$
$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$
$= \frac{1 + \sin x}{\cos x}$
$= \frac{1 - \sin^2 x}{(1 - \sin x)\cos x}$
$= \frac{\cos x}{1 - \sin x}$
= Right Side

d) Left Side
$= \frac{1}{1 + \cos x} + \frac{1}{1 - \cos x}$
$= \frac{1 - \cos x}{1 - \cos^2 x} + \frac{1 + \cos x}{1 - \cos^2 x}$
$= \frac{2}{\sin^2 x}$
$= 2\csc^2 x$
= Right Side

14. a) It is true when $x = \frac{\pi}{4}$. The equation is not necessarily an identity. Sometimes equations can be true for a small domain of x.

b) $x = \frac{\pi}{2} + n\pi, n \in I$

c) Left Side $= \sin 2x$
$= 2 \sin x \cos x$
$= \dfrac{2 \sin x \cos^2 x}{\cos x}$
$= \dfrac{2 \tan x}{\sec^2 x}$
$= \dfrac{2 \tan x}{1 + \tan^2 x}$
$=$ Right Side

15. a) Left Side
$= \dfrac{\cos x + \cot x}{\sec x + \tan x}$
$= \dfrac{\cos x + \dfrac{\cos x}{\sin x}}{\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}}$
$= \dfrac{\dfrac{\sin x \cos^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x}}{1 + \sin x}$
$= \dfrac{\dfrac{(\sin x + 1)\cos^2 x}{\sin x}}{1 + \sin x}$
$= \dfrac{\cos x \cos x}{\sin x}$
$= \cos x \cot x$
$=$ Right Side

b) Left Side
$= \sec x + \tan x$
$= \dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}$
$= \dfrac{1 + \sin x}{\cos x}$
$= \dfrac{1 - \sin^2 x}{(1 - \sin x)\cos x}$
$= \dfrac{\cos x}{1 - \sin x}$
$=$ Right Side

16. a) You can disprove it by trying a value of x or by graphing.
b) Substituting $x = 0$ makes the equation fail.

17. a) $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ **b)** $x = \frac{5\pi}{6}, \frac{11\pi}{6}$
c) $x = \frac{7\pi}{6}, \frac{11\pi}{6}$ **d)** $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$

18. a) $x = 15°, 75°, 195°, 255°$ **b)** $x = 90°, 270°$
c) $x = 30°, 150°, 270°$ **d)** $x = 0°, 180°$

19. $x = \pm\frac{\pi}{3} + n\pi, n \in I$

20. $\cos x = \pm\frac{4}{5}$

21. $x = -2\pi, -\pi, 0, \pi, 2\pi$

Chapter 6 Practice Test, page 324

1. A **2.** A **3.** D **4.** D **5.** A **6.** D

7. a) $\dfrac{1 - \sqrt{3}}{2\sqrt{2}}$ or $\dfrac{\sqrt{2} - \sqrt{6}}{4}$

b) $\dfrac{\sqrt{3} + 1}{2\sqrt{2}}$ or $\dfrac{\sqrt{6} + \sqrt{2}}{4}$

8. Left Side $= \cot \theta - \tan \theta$
$= \dfrac{1}{\tan \theta} - \tan \theta$
$= \dfrac{1 - \tan^2 \theta}{\tan \theta}$
$= 2\left(\dfrac{1 - \tan^2 \theta}{2 \tan \theta}\right)$
$= 2 \cot 2\theta$
$=$ Right Side
$\theta = \left(\dfrac{\pi}{2}\right)n, n \in I$

9. Theo's Formula $= I_0 \cos^2 \theta$
$= I_0 - I_0 \sin^2 \theta$
$= I_0 - \dfrac{I_0}{\csc^2 \theta}$
$=$ Sany's Formula

10. a) $A = \frac{2\pi}{3} + 2\pi n, n \in I, A = \frac{4\pi}{3} + 2\pi n, n \in I$
b) $B = \pi n, n \in I, B = \frac{\pi}{6} + 2\pi n, n \in I,$
$B = \frac{5\pi}{6} + 2\pi n, n \in I$
c) $\theta = \pi n, n \in I, \theta = \pm\frac{\pi}{3} + 2\pi n, n \in I$

11. $x = \frac{\pi}{2} + n\pi, n \in I$

12. $\dfrac{-4 - 3\sqrt{3}}{10}$

13. $x = \frac{\pi}{4}, \frac{5\pi}{4}$

14. $x = 0°, 90°, 270°$

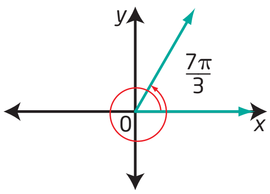
15. a) Left Side $= \dfrac{\cot x}{\csc x - 1}$
$= \dfrac{\cot x(\csc x + 1)}{\csc^2 x - 1}$
$= \dfrac{\cot x(\csc x + 1)}{1 + \cot^2 x - 1}$
$= \dfrac{(\csc x + 1)}{\cot x}$
$=$ Right Side

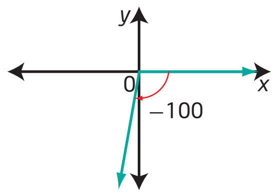
b) Left Side $= \sin (x + y) \sin (x - y)$
$= (\sin x \cos y + \sin y \cos x) \times (\sin x \cos y - \sin y \cos x)$
$= \sin^2 x \cos^2 y - \sin^2 y \cos^2 x$
$= \sin^2 x(1 - \sin^2 y) - \sin^2 y (1 - \sin^2 x)$
$= \sin^2 x - \sin^2 y$
$=$ Right Side

16. $x = \frac{\pi}{2} + 2\pi n, n \in I, x = \frac{\pi}{6} + 2\pi n, n \in I,$
$x = \frac{5\pi}{6} + 2\pi n, n \in I$

Cumulative Review, Chapters 4–6, pages 326 to 327

1. a) $\frac{7\pi}{3} \pm 2\pi n, n \in N$ **b)** $-100° \pm (360°)n, n \in N$





2. a) 229° **b)** −300°

3. a) $\frac{7\pi}{6}$ **b)** $-\frac{25\pi}{9}$

4. a) 13.1 ft **b)** 106.9 ft

5. a) $x^2 + y^2 = 25$ **b)** $x^2 + y^2 = 16$

6. a) quadrant III **b)** $-\frac{2\pi}{3}, \frac{4\pi}{3}$

c) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$; when the given quadrant III angle is rotated through $\frac{\pi}{2}$, its terminal arm is in quadrant IV and its coordinates are switched and the signs adjusted.

d) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; when the given quadrant III angle is rotated through $-\pi$, its terminal arm is in quadrant I and its coordinates are the same but the signs adjusted.

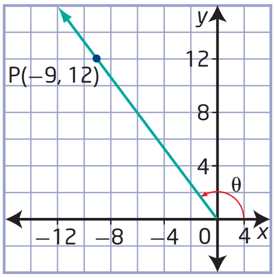
7. a) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; the points have the same x-coordinates but opposite y-coordinates.

b) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$; the points have the same x-coordinates but opposite y-coordinates.

8. a) $-\frac{\sqrt{3}}{2}$ **b)** $\frac{1}{2}$ **c)** $-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

d) $\sqrt{2}$ **e)** undefined **f)** $-\sqrt{3}$

9. a)



b) $\sin \theta = \frac{4}{5}$, $\cos \theta = -\frac{3}{5}$, $\tan \theta = -\frac{4}{3}$,

$\csc \theta = \frac{5}{4}$, $\sec \theta = -\frac{5}{3}$, $\cot \theta = -\frac{3}{4}$

c) $\theta = 126.87° + (360°)n, n \in I$

10. a) $-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $-30°, 30°$

c) $\frac{3\pi}{4}, \frac{7\pi}{4}$

11. a) $\theta = \frac{3\pi}{4} + 2\pi n, n \in I; \frac{5\pi}{4} + 2\pi n, n \in I$

b) $\theta = \frac{\pi}{2} + 2\pi n, n \in I$ **c)** $\theta = \frac{\pi}{2} + \pi n, n \in I$

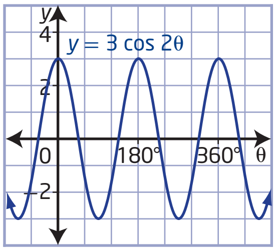
12. a) $\theta = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$ **b)** $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

13. a) $\theta = 27°, 153°, 207°, 333°$

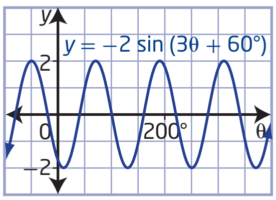
b) $\theta = 90°, 199°, 341°$

14. $y = 3 \sin \frac{1}{2}\left(x + \frac{\pi}{4}\right)$

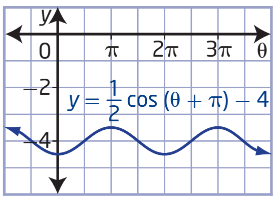
15. a) amplitude 3, period 180°, phase shift 0, vertical displacement 0



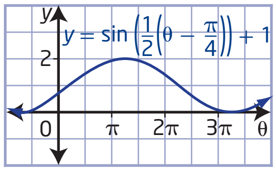
b) amplitude 2, period 120°, phase shift 20° left, vertical displacement 0



c) amplitude $\frac{1}{2}$, period 2π, phase shift π units left, vertical displacement 4 units down



d) amplitude 1, period 4π, phase shift $\frac{\pi}{4}$ units right, vertical displacement 1 unit up

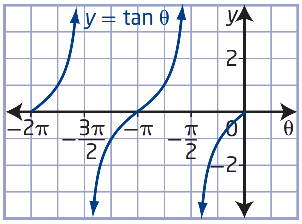


16. a) $y = 2 \sin (x - 30°) + 3$, $y = 2 \cos (x - 120°) + 3$

b) $y = \sin 2\left(x + \frac{\pi}{3}\right) - 1$, $y = \cos 2\left(x + \frac{\pi}{12}\right) - 1$

17. $y = 4 \cos 1.2(x + 30°) - 3$

18. a)



b) $x = -\frac{\pi}{2}$, $x = -\frac{3\pi}{2}$

19. a) $h(x) = -25 \cos \frac{2\pi}{11}x + 26$ **b)** $x = 3.0$ min

20. a) $\theta \neq \frac{\pi}{2} + \pi n, n \in I, \tan^2 \theta$

b) $x \neq \left(\frac{\pi}{2}\right)n, n \in I, \sec^2 x$

21. a) $-\frac{\sqrt{3} - 1}{2\sqrt{2}}$ or $-\frac{\sqrt{6} - \sqrt{2}}{4}$

b) $\frac{\sqrt{3} - 1}{2\sqrt{2}}$ or $\frac{\sqrt{6} - \sqrt{2}}{4}$

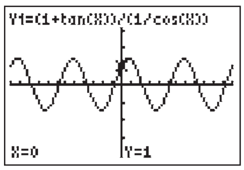
22. a) $\cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$ **b)** $\sin 90° = 1$

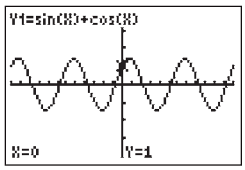
c) $\tan \frac{7\pi}{3} = \sqrt{3}$

23. a) Both sides have the same value for A = 30°.

b) Left Side $= \sin^2 A + \cos^2 A + \tan^2 A$
$= 1 + \tan^2 A$
$= \sec^2 A$
= Right Side

24. a) It could be an identity as the graphs look the same.





b) Left Side $= \frac{1 + \tan x}{\sec x}$
$= \frac{1}{\sec x} + \frac{\tan x}{\sec x}$
$= \cos x + \frac{\sin x}{\cos x} \div \frac{1}{\cos x}$
$= \cos x + \sin x$
= Right Side

25. Right Side $= \frac{\cos 2\theta}{1 + \sin 2\theta}$
$= \frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta}$
$= \frac{(\cos \theta - \sin \theta)(\cos \theta + \sin \theta)}{(\cos \theta + \sin \theta)(\cos \theta + \sin \theta)}$
$= \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}$
= Left Side

26. a) $x = \frac{5\pi}{6} + \pi n, n \in I, x = \frac{\pi}{6} + \pi n, n \in I$

b) $x = \frac{\pi}{2} + \pi n, n \in \text{I}, x = \frac{7\pi}{6} + 2\pi n, n \in \text{I},$
$x = \frac{11\pi}{6} + 2\pi n, n \in \text{I}$

27. a) This is an identity so all θ are a solution.
b) Yes, because the left side can be simplified to 1.

Unit 2 Test, pages 328 to 329

1. B **2.** D **3.** C **4.** C **5.** B **6.** D **7.** C **8.** A

9. $-\frac{\sqrt{3}}{2}$

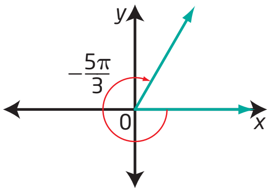
10. $-\frac{2}{3}, \frac{2}{3}$

11. $\frac{7}{13\sqrt{2}}$ or $\frac{7\sqrt{2}}{26}$

12. 1.5, 85.9°

13. $-\frac{11\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}$

14. a)

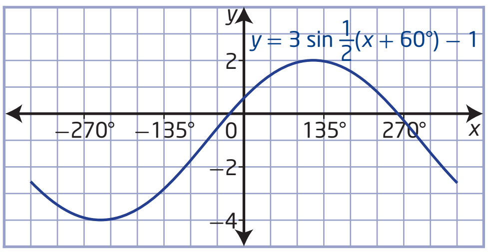


b) −300°
c) $-\frac{5\pi}{3} \pm 2\pi n, n \in \text{N}$
d) No, following the equation above it is impossible to obtain $\frac{10\pi}{3}$.

15. $x = 0.412, 2.730, 4.712$

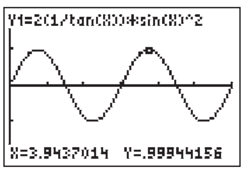
16. Sam is correct, there are four solutions in the given domain. Pat made an error when finding the square root. Pat forgot to solve for the positive and negative solutions.

17. a)



b) $-4 \leq y \leq 2$
c) amplitude 3, period 720°, phase shift 60° left, vertical displacement 1 unit down
d) $x \approx -21°, 261°$

18. a)



b) $g(\theta) = \sin 2\theta$
c) $f(\theta) = 2 \cot \theta \sin^2 \theta$
$= \frac{2 \cos \theta \sin^2 \theta}{\sin \theta}$
$= 2 \cos \theta \sin \theta$
$= \sin 2\theta$
$= g(\theta)$

19. a) It is true: both sides have the same value.
b) $x \neq \frac{\pi n}{2}, n \in \text{I}$
c) Left Side
$= \tan x + \frac{1}{\tan x}$
$= \frac{\tan^2 x + 1}{\tan x}$
$= \frac{\sec^2 x}{\tan x}$
$= \sec x\left(\frac{1}{\cos x}\right)\left(\frac{\cos x}{\sin x}\right)$
$= \frac{\sec x}{\sin x}$
= Right Side

20. a) 6.838 m **b)** 12.37 h **c)** 3.017 m