Chapter 7 Exponential Functions

7.1 Characteristics of Exponential Functions, pages 342 to 345

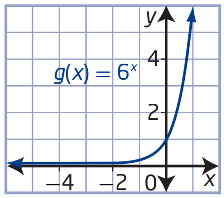
1. a) No, the variable is not the exponent.
b) Yes, the base is greater than 0 and the variable is the exponent.
c) No, the variable is not the exponent.
d) Yes, the base is greater than 0 and the variable is the exponent.

2. a) $f(x) = 4^x$ **b)** $g(x) = \left(\frac{1}{4}\right)^x$
c) $x = 0$, which is the y-intercept

3. a) B **b)** C **c)** A

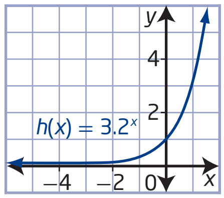
4. a) $f(x) = 3^x$ **b)** $f(x) = \left(\frac{1}{5}\right)^x$

5. a)



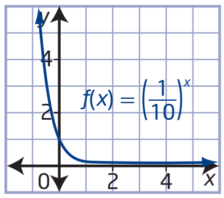
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, function increasing, horizontal asymptote $y = 0$

b)



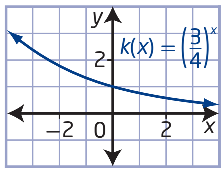
domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, function increasing, horizontal asymptote $y = 0$

c)



domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, function decreasing, horizontal asymptote $y = 0$

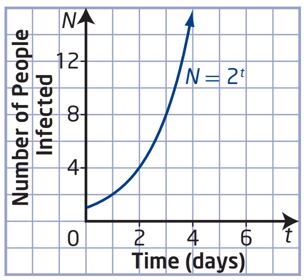
d)



domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, function decreasing, horizontal asymptote $y = 0$

6. a) $c > 1$; number of bacteria increases over time
b) $0 < c < 1$; amount of actinium-225 decreases over time
c) $0 < c < 1$; amount of light decreases with depth
d) $c > 1$; number of insects increases over time

7. a)

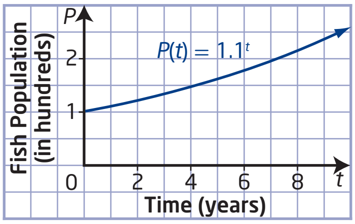


The function $N = 2^t$ is exponential since the base is greater than zero and the variable t is an exponent.

b) **i)** 1 person **ii)** 2 people
iii) 16 people **iv)** 1024 people

8. a) If the population increases by 10% each year, the population becomes 110% of the previous year's population. So, the growth rate is 110% or 1.1 written as a decimal.

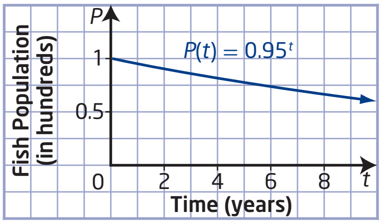
b)



domain $\{t \mid t \geq 0, t \in R\}$ and range $\{P \mid P \geq 100, P \in R\}$

c) The base of the exponent would become 100% − 5% or 95%, written as 0.95 in decimal form.

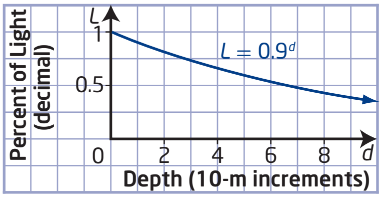
d)



domain $\{t \mid t \geq 0, t \in R\}$ and range $\{P \mid 0 < P \leq 100, P \in R\}$

9. a) $L = 0.9^d$

b)

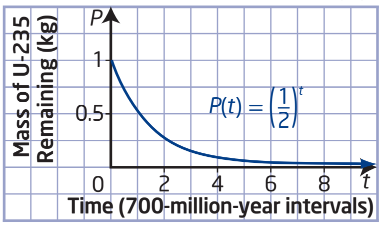


c) domain $\{d \mid d \geq 0, d \in R\}$ and range $\{L \mid 0 < L \leq 1, L \in R\}$

d) 76.8%

10. a) Let P represent the percent, as a decimal, of U-235 remaining. Let t represent time, in 700-million-year intervals. $P(t) = \left(\frac{1}{2}\right)^t$

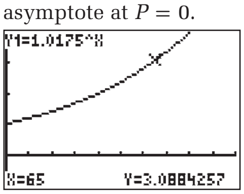
b)



c) 2.1×10^9 years

d) No, the sample of U-235 will never decay to 0 kg, since the graph of $P(t) = \left(\frac{1}{2}\right)^t$ has a horizontal asymptote at $P = 0$.

11. a)



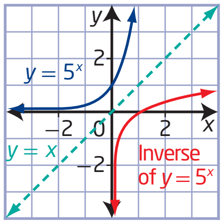
b) 64 years

c) No; since the amount invested triples, it does not matter what initial investment is made.

d) graph: 40 years; rule of 72: 41 years

12. 19.9 years

13. a)



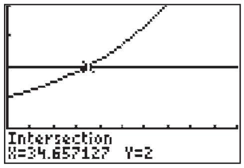
b) The x- and y-coordinates of any point and the domains and ranges are interchanged. The horizontal asymptote becomes a vertical asymptote.

c) $x = 5^y$

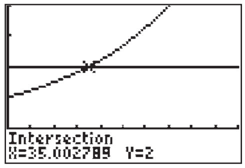
14. a) Another way to express $D = 2^{-\varphi}$ is as $D = \left(\frac{1}{2}\right)^{\varphi}$, which indicates a decreasing exponential function. Therefore, a negative value of φ represents a greater value of D.

b) The diameter of fine sand (0.125 mm) is $\frac{1}{256}$ the diameter of course gravel (32 mm).

15. a) 34.7 years

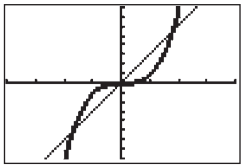


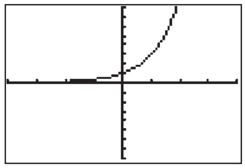
b) 35 years



c) The results are similar, but the continuous compounding function gives a shorter doubling period by approximately 0.3 years.

C1 a)





b)

Feature	$f(x) = 3x$	$g(x) = x^3$	$h(x) = 3^x$
domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
range	$\{y \mid y \in R\}$	$\{y \mid y \in R\}$	$\{y \mid y > 0, y \in R\}$
intercepts	x-intercept 0, y-intercept 0	x-intercept 0, y-intercept 0	no x-intercept, y-intercept 1
equations of asymptotes	none	none	$y = 0$

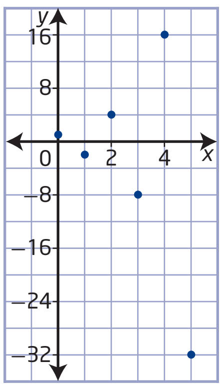
c) Example: All three functions have the same domain, and each of their graphs has a y-intercept. The functions $f(x)$ and $g(x)$ have all key features in common.

d) Example: The function $h(x)$ is the only function with an asymptote, which restricts its range and results in no x-intercept.

C2 a)

x	$f(x)$
0	1
1	−2
2	4
3	−8
4	16
5	−32

b)



c) No, the points do not form a smooth curve. The locations of the points alternate between above the x-axis and below the x-axis.

d) The values are undefined because they result in the square root of a negative number.

$f(x) = (-2)^x$ $\quad$ $f(x) = (-2)^x$

$f\left(\frac{1}{2}\right) = (-2)^{\frac{1}{2}}$ $\quad$ $f\left(\frac{5}{2}\right) = (-2)^{\frac{5}{2}}$

$f\left(\frac{1}{2}\right) = \sqrt{-2}$ $\quad$ $f\left(\frac{5}{2}\right) = \sqrt{(-2)^5}$

e) Example: Exponential functions with positive bases result in smooth curves.

7.2 Transformations of Exponential Functions, pages 354 to 357

1. a) C **b)** D **c)** A **d)** B

2. a) D **b)** A **c)** B **d)** C

3. a) $a = 2$: vertical stretch by a factor of 2; $b = 1$: no horizontal stretch; $h = 0$: no horizontal translation; $k = -4$: vertical translation of 4 units down

b) $a = 1$: no vertical stretch; $b = 1$: no horizontal stretch; $h = 2$: horizontal translation of 2 units right; $k = 3$: vertical translation of 3 units up

c) $a = -4$: vertical stretch by a factor of 4 and a reflection in the x-axis; $b = 1$: no horizontal stretch; $h = -5$: horizontal translation of 5 units left; $k = 0$: no vertical translation

d) $a = 1$: no vertical stretch; $b = 3$: horizontal stretch by a factor of $\frac{1}{3}$; $h = 1$: horizontal translation of 1 unit right; $k = 0$: no vertical translation

e) $a = -\frac{1}{2}$: vertical stretch by a factor of $\frac{1}{2}$ and a reflection in the x-axis; $b = 2$: horizontal stretch by a factor of $\frac{1}{2}$; $h = 4$: horizontal translation of 4 units right; $k = 3$: vertical translation of 3 units up

f) $a = -1$: reflection in the x-axis; $b = 2$: horizontal stretch by a factor of $\frac{1}{2}$; $h = 1$: horizontal translation of 1 unit right; $k = 0$: no vertical translation

g) $a = 1.5$: vertical stretch by a factor of 1.5; $b = \frac{1}{2}$: horizontal stretch by a factor of 2; $h = 4$: horizontal translation of 4 units right; $k = -\frac{5}{2}$: vertical translation of $\frac{5}{2}$ units down

4. a) C: reflection in the x-axis, $a < 0$ and $0 < c < 1$, and vertical translation of 2 units up, $k = 2$

b) A: horizontal translation of 1 unit right, $h = 1$, and vertical translation of 2 units down, $k = -2$

c) D: reflection in the x-axis, $a < 0$ and $c > 1$, and vertical translation of 2 units up, $k = 2$

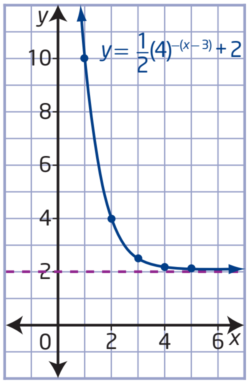
d) B: horizontal translation of 2 units right, $h = 2$, and vertical translation of 1 unit up, $k = 1$

5. a) $a = \frac{1}{2}$: vertical stretch by a factor of $\frac{1}{2}$; $b = -1$: reflection in the y-axis; $h = 3$: horizontal translation of 3 units right 3; $k = 2$: vertical translation of 2 units up

b)

$y = 4^x$	$y = 4^{-x}$	$y = \frac{1}{2}(4)^{-x}$	$y = \frac{1}{2}(4)^{-(x-3)} + 2$
$\left(-2, \frac{1}{16}\right)$	$\left(2, \frac{1}{16}\right)$	$\left(2, \frac{1}{32}\right)$	$\left(5, \frac{65}{32}\right)$
$\left(-1, \frac{1}{4}\right)$	$\left(1, \frac{1}{4}\right)$	$\left(1, \frac{1}{8}\right)$	$\left(4, \frac{17}{8}\right)$
(0, 1)	(0, 1)	$\left(0, \frac{1}{2}\right)$	$\left(3, \frac{5}{2}\right)$
(1, 4)	(−1, 4)	(−1,2)	(2, 4)
(2, 16)	(−2, 16)	(−2, 8)	(1, 10)

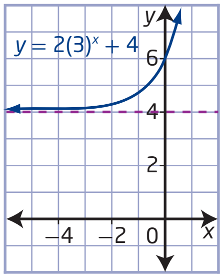
c)



d) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 2, y \in \mathrm{R}\}$, horizontal asymptote $y = 2$, y-intercept 34

6. a) i), ii) $a = 2$: vertical stretch by a factor of 2; $b = 1$: no horizontal stretch; $h = 0$: no horizontal translation; $k = 4$: vertical translation of 4 units up

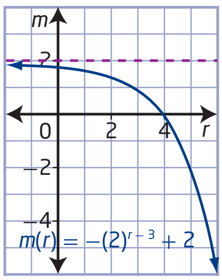
iii)



iv) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 4, y \in \mathrm{R}\}$, horizontal asymptote $y = 4$, y-intercept 6

b) i), ii) $a = -1$: reflection in the x-axis; $b = 1$: no horizontal stretch; $h = 3$: horizontal translation of 3 units right; $k = 2$: vertical translation of 2 units up

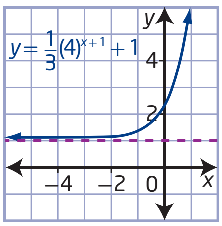
iii)



iv) domain $\{r \mid r \in \mathrm{R}\}$, range $\{m \mid m < 2, m \in \mathrm{R}\}$, horizontal asymptote $m = 2$, m-intercept $\frac{15}{8}$, r-intercept 4

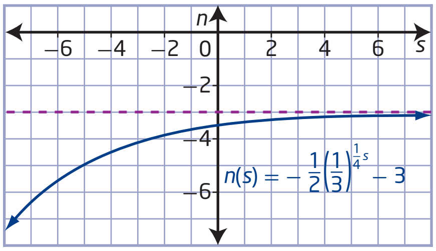
c) i), ii) $a = \frac{1}{3}$: vertical stretch by a factor of $\frac{1}{3}$; $b = 1$: no horizontal stretch; $h = -1$: horizontal translation of 1 unit left; $k = 1$: vertical translation of 1 unit up

iii)



iv) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 1, y \in \mathrm{R}\}$, horizontal asymptote $y = 1$, y-intercept $\frac{7}{3}$

d) i), ii) $a = -\frac{1}{2}$: vertical stretch by a factor of $\frac{1}{2}$ and a reflection in the x-axis; $b = \frac{1}{4}$: horizontal stretch by a factor of 4; $h = 0$: no horizontal translation; $k = -3$: vertical translation of 3 units down

iii) 

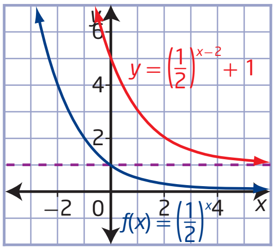
iv) domain $\{s \mid s \in R\}$, range $\{n \mid n < -3, n \in R\}$, horizontal asymptote $n = -3$, n-intercept $-\frac{7}{2}$

7. a) horizontal translation of 2 units right and vertical translation of 1 unit up; $y = \left(\frac{1}{2}\right)^{x-2} + 1$

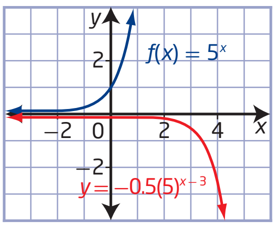
b) reflection in the x-axis, vertical stretch by a factor of 0.5, and horizontal translation of 3 units right; $y = -0.5(5)^{x-3}$

c) reflection in the x-axis, horizontal stretch by a factor of $\frac{1}{3}$, and vertical translation of 1 unit up; $y = -\left(\frac{1}{4}\right)^{3x} + 1$

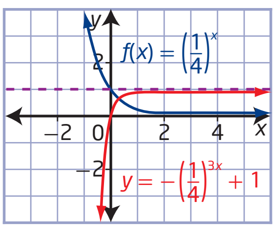
d) vertical stretch by a factor of 2, reflection in the y-axis, horizontal stretch by a factor of 3, horizontal translation of 1 unit right, and vertical translation of 5 units down; $y = 2(4)^{-\frac{1}{3}(x-1)} - 5$

8. a) 

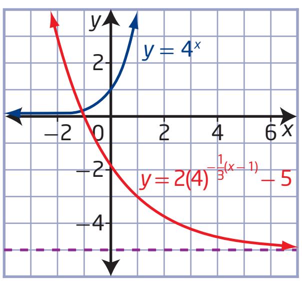
Map all points (x, y) on the graph of $f(x)$ to $(x + 2, y + 1)$.

b) 

Map all points (x, y) on the graph of $f(x)$ to $(x + 3, -0.5y)$.

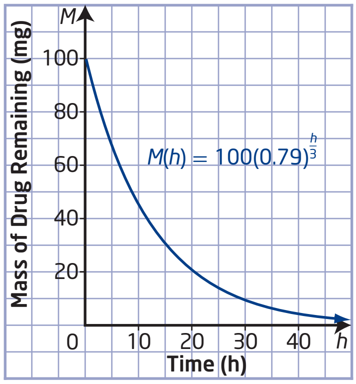
c) 

Map all points (x, y) on the graph of $f(x)$ to $\left(\frac{1}{3}x, -y + 1\right)$.

d) 

Map all points (x, y) on the graph of $f(x)$ to $(-3x + 1, 2y - 5)$.

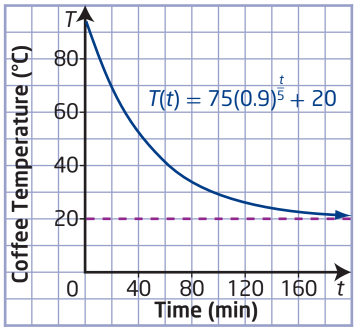
9. a) 0.79 represents the 79% of the drug remaining in exponential decay after $\frac{1}{3}$ h.

b) 

c) The M-intercept represents the drug dose taken.

d) domain $\{h \mid h \geq 0, h \in R\}$, range $\{M \mid 0 < M \leq 100, M \in R\}$

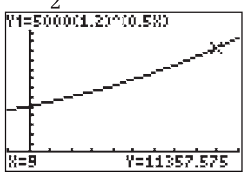
10. a) $a = 75$: vertical stretch by a factor of 75; $b = \frac{1}{5}$: horizontal stretch by a factor of 5; $h = 0$: no horizontal translation; $k = 20$: vertical translation of 20 units up

b) 

c) 29.1 °C **d)** final temperature of the coffee

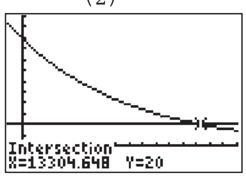
11. a) $P = 5000(1.2)^{\frac{1}{2}x}$

b) $a = 5000$: vertical stretch by a factor of 5000; $b = \frac{1}{2}$: horizontal stretch by a factor of 2

c) 

approximately 11 357 bacteria

12. a) $P = 100\left(\frac{1}{2}\right)^{\frac{t}{5730}}$

b) 

approximately 13 305 years old

13. a) 527.8 cm^2 **b)** 555 h

14. a) 1637 foxes

b) Example: Disease or lack of food can change the rate of growth of the foxes. Exponential growth suggests that the population will grow without bound, and therefore the fox population will grow beyond the possible food sources, which is not good if not controlled.

C1 Example: The graph of an exponential function of the form $y = c^x$ has a horizontal asymptote at $y = 0$. Since $y \neq 0$, the graph cannot have an x-intercept.

C2 a) Example: For a function of the form $y = a(c)^{b(x-h)} + k$, the parameters a and k can affect the x-intercept. If $a > 0$ and $k < 0$ or $a < 0$ and $k > 0$, then the graph of the exponential function will have an x-intercept.

b) Example: For a function of the form $y = a(c)^{b(x-h)} + k$, the parameters a, h, and k can affect the y-intercept. The point $(0, y)$ on the graph of $y = c^x$ gets mapped to $(h, ay + k)$.

7.3 Solving Exponential Equations, pages 364 to 365

1. a) 2^{12} **b)** 2^9 **c)** 2^{-6} **d)** 2^4

2. a) 2^3 and 2^4 **b)** 3^{2x} and 3^3

c) $\left(\frac{1}{2}\right)^{2x}$ and $\left(\frac{1}{2}\right)^{2x-2}$ **d)** 2^{-3x+6} and 2^{4x}

3. a) 4^2 **b)** $4^{\frac{2}{3}}$ **c)** 4^3 **d)** 4^3

4. a) $x = 3$ **b)** $x = -2$ **c)** $w = 3$ **d)** $m = \frac{7}{4}$

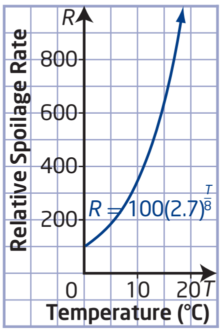
5. a) $x = -3$ **b)** $x = -4$ **c)** $y = \frac{11}{4}$ **d)** $k = 9$

6. a) 10.2 **b)** 11.5 **c)** −2.8 **d)** 18.9

7. a) 58.71 **b)** −1.66 **c)** −5.38 **d)** −8

e) 2.71 **f)** 14.43 **g)** −3.24 **h)** −1.88

8. a)



b) approximately 5.6 °C
c) approximately 643
d) approximately 13.0 °C

9. 3 h

10. 4 years

11. a) $A = 1000(1.02)^n$ **b)** \$1372.79 **c)** 9 years

12. a) $C = \left(\frac{1}{2}\right)^{\frac{t}{5.3}}$ **b)** $\frac{1}{32}$ of the original amount

c) 47.7 years

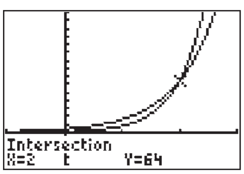
13. a) $A = 500(1.033)^n$ **b)** \$691.79

c) approximately 17 years

14. \$5796.65

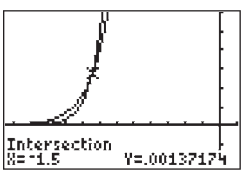
15. a) i) $x > 2$ **ii)** $x > -\frac{3}{2}$

b) i)



Since the graph of $y = 2^{3x}$ is greater than (above) the graph of $y = 4^{x+1}$ when $x > 2$, the solution is $x > 2$.

ii)



Since the graph of $y = 81^x$ is less than (below) the graph of $y = 27^{2x+1}$ when $x > -\frac{3}{2}$, the solution is $x > -\frac{3}{2}$.

c) Example: Solve the inequality $\left(\frac{1}{2}\right)^{x+3} > 2^{x-1}$.
Answer: $x < -1$

16. Yes. Rewrite the equation as $(4^x)^2 + 2(4^x) - 3 = 0$ and factor as $(4^x + 3)(4^x - 1) = 0$; $x = 0$

17. $(2^x)^x = \left(2^{\frac{5}{2}}\right)^{\frac{5}{2}} \approx 76.1$

18. 20 years

C1 a) You can express 16^2 with a base of 4 by writing 16 as 4^2 and simplifying.
$16^2 = (4^2)^2$
$16^2 = 4^4$

b) Example: You can express 16^2 with a base of 2 by writing 16 as 2^4 and simplifying.
$16^2 = (2^4)^2$
$16^2 = 2^8$
Or, you can express 16^2 with a base of $\frac{1}{4}$ by writing 16 as $\left(\frac{1}{4}\right)^{-2}$ and simplifying.
$16^2 = \left(\left(\frac{1}{4}\right)^{-2}\right)^2$
$16^2 = \left(\frac{1}{4}\right)^{-4}$

C2 a)
$16^{2x} = 8^{x-3}$
$(2^4)^{2x} = (2^3)^{x-3}$
$2^{8x} = 2^{3x-9}$
$8x = 3x - 9$
$5x = -9$
$x = -\frac{9}{5}$

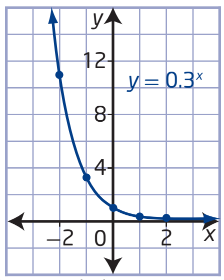
b) **Step 1:** Express the bases on both sides as powers of 2.
Step 2: Apply the power of a power law.
Step 3: Equate the exponents.
Step 4: Isolate the term containing x.
Step 5: Solve for x.

Chapter 7 Review, pages 366 to 367

1. a) B **b)** D **c)** A **d)** C

2. a)

x	y
−2	$11.\overline{1}$
−1	$3.\overline{3}$
0	1
1	0.3
2	0.09



b) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, y-intercept 1, function decreasing, horizontal asymptote $y = 0$

3. $y = \left(\frac{1}{4}\right)^x$

4. a) Since the interest rate is 3.25% per year, each year the investment grows by a factor of 103.25%, which, written as a decimal, is 1.0325.

b) \$1.38 **c)** 21.7 years

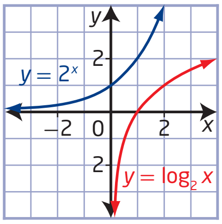
5. a) $a = -2$: vertical stretch by a factor of 2 and reflection in the x-axis; $b = 3$: horizontal stretch by a factor of $\frac{1}{3}$; $h = 1$: horizontal translation of 1 unit right; $k = 2$: vertical translation of 2 units up

b)

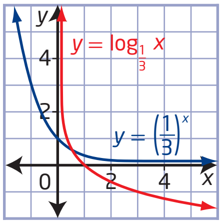
Transformation	Parameter Value	Function Equation
horizontal stretch	$b = 3$	$y = 4^{3x}$
vertical stretch	$a = -2$	$y = -2(4)^x$
translation left/right	$h = 1$	$y = (4)^{x-1}$
translation up/down	$k = 2$	$y = 4^x + 2$

Chapter 8 Logarithmic Functions

8.1 Understanding Logarithms, pages 380 to 382

1. a) i)


ii) $y = \log_2 x$
iii) domain $\{x \mid x > 0, x \in R\}$, range $\{y \mid y \in R\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

b) i)


ii) $y = \log_{\frac{1}{3}} x$
iii) domain $\{x \mid x > 0, x \in R\}$, range $\{y \mid y \in R\}$, x-intercept 1, no y-intercept, vertical asymptote $x = 0$

2. a) $\log_{12} 144 = 2$ b) $\log_8 2 = \frac{1}{3}$
c) $\log_{10} 0.000\ 01 = -5$ d) $\log_7 (y + 3) = 2x$

3. a) $5^2 = 25$ b) $8^{\frac{2}{3}} = 4$
c) $10^6 = 1\ 000\ 000$ d) $11^y = x + 3$

4. a) 3 b) 0 c) $\frac{1}{3}$ d) −3

5. $a = 4$; $b = 5$

6. a) $x > 1$ b) $0 < x < 1$ c) $x = 1$
d) Example: $x = 9$

7. a) 0 raised to any non-zero power is 0.
b) 1 raised to any power is 1.
c) Exponential functions with a negative base are not continuous.