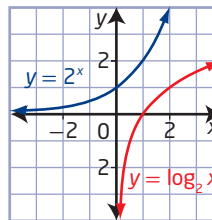


Chapter 8 Logarithmic Functions

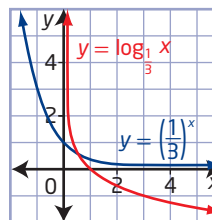
8.1 Understanding Logarithms, pages 380 to 382

1. a) i)



ii) $y = \log_2 x$
 iii) domain
 $\{x \mid x > 0, x \in \mathbb{R}\}$,
 range $\{y \mid y \in \mathbb{R}\}$,
 x-intercept 1, no
 y-intercept,
 vertical asymptote
 $x = 0$

b) i)



ii) $y = \log_{\frac{1}{3}} x$
 iii) domain
 $\{x \mid x > 0, x \in \mathbb{R}\}$,
 range $\{y \mid y \in \mathbb{R}\}$,
 x-intercept 1,
 no y-intercept,
 vertical asymptote
 $x = 0$

2. a) $\log_{12} 144 = 2$

b) $\log_8 2 = \frac{1}{3}$

c) $\log_{10} 0.000\ 01 = -5$

d) $\log_7 (y + 3) = 2x$

3. a) $5^2 = 25$

b) $8^{\frac{2}{3}} = 4$

c) $10^6 = 1\ 000\ 000$

d) $11^y = x + 3$

4. a) 3

b) 0

c) $\frac{1}{3}$

d) -3

5. $a = 4$; $b = 5$

6. a) $x > 1$

b) $0 < x < 1$

c) $x = 1$

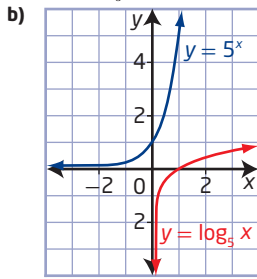
d) Example: $x = 9$

7. a) 0 raised to any non-zero power is 0.

b) 1 raised to any power is 1.

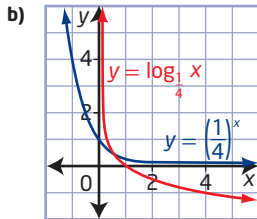
c) Exponential functions with a negative base are not continuous.

8. a) $y = \log_5 x$



domain $\{x \mid x > 0, x \in \mathbb{R}\}$,
range $\{y \mid y \in \mathbb{R}\}$,
x-intercept 1,
no y-intercept,
vertical asymptote $x = 0$

9. a) $g^{-1}(x) = \left(\frac{1}{4}\right)^x$



domain $\{x \mid x \in \mathbb{R}\}$,
range $\{y \mid y > 0, y \in \mathbb{R}\}$,
no x-intercept,
y-intercept 1,
horizontal asymptote $y = 0$

10. They are reflections of each other in the line $y = x$.

11. a) They have the exact same shape.

b) One of them is increasing and the other is decreasing.

12. a) 216 b) 81 c) 64 d) 8

13. a) 7 b) 6

14. a) 0 b) 1

15. -1

16. 16

17. a) $t = \log_{1.1} N$ b) 145 days

18. The larger asteroid had a relative risk that was 1479 times as dangerous.

19. 1000 times as great

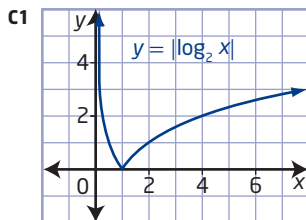
20. 5

21. $m = 14, n = 13$

22. $4n$

23. $y = 3^{2x}$

24. $n = 8; m = 3$



The function has the same general shape, but instead of decreasing, after $x = 1$ the function increases without limit.

C2 Answers will vary.

C3 Step 1: a) $e = 2.718\ 281\ 828$ b) 10^{10}

Step 2: a) domain $\{x \mid x > 0, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$,
x-intercept 1, no y-intercept,
vertical asymptote $x = 0$

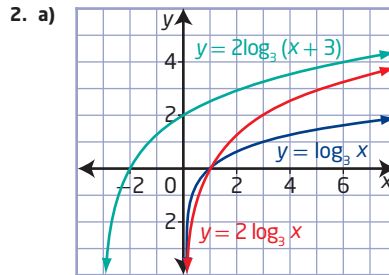
b) $y = \ln x$

Step 3: a) $r = 2.41$

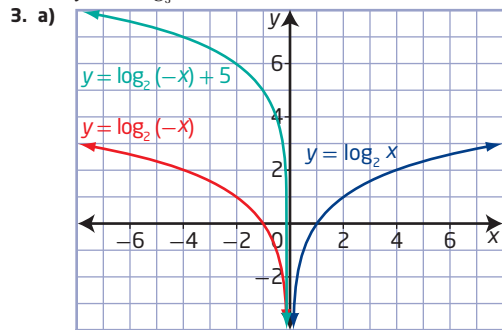
b) i) $\theta = \frac{\ln r}{0.14}$ ii) $\theta = 17.75$

8.2 Transformations of Logarithmic Functions, pages 389 to 391

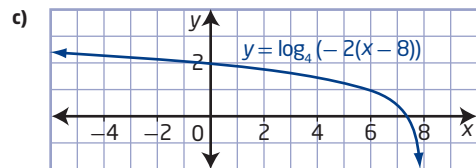
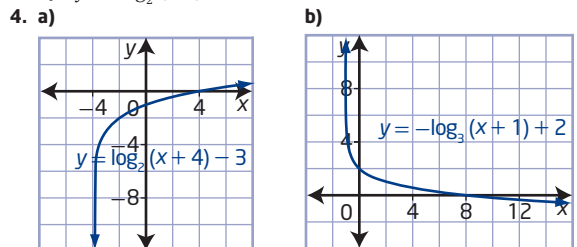
- Translate 1 unit right and 6 units up.
 - Reflect in the x -axis, stretch vertically about the x -axis by a factor of 4, and stretch horizontally about the y -axis by a factor of $\frac{1}{3}$.
 - Reflect in the y -axis, stretch vertically about the x -axis by a factor of $\frac{1}{2}$, and translate 7 units up.



b) $y = 2 \log_3(x + 3)$

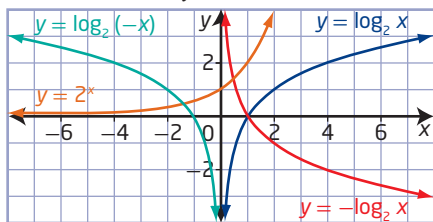


b) $y = \log_2(-x) + 5$



- vertical asymptote $x = -3$
 - domain $\{x \mid x > -3, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$
 - y-intercept -5 iv) x-intercept -2
- vertical asymptote $x = -9$
 - domain $\{x \mid x > -9, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$
 - y-intercept 2 iv) x-intercept -8.75
- vertical asymptote $x = -3$
 - domain $\{x \mid x > -3, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$
 - y-intercept -1.3 iv) x-intercept 22
- vertical asymptote $x = -1$
 - domain $\{x \mid x > -1, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$
 - y-intercept -6 iv) x-intercept $-\frac{3}{4}$

6. a) $y = 5 \log x$ b) $y = \log_8 2x$
 c) $y = \frac{1}{3} \log_2 x$ d) $y = \log_4 \left(\frac{x}{2}\right)$
7. a) stretch horizontally about the y -axis by a factor of $\frac{1}{4}$; translate 5 units left and 6 units up
 b) stretch horizontally about the y -axis by a factor of 3; stretch vertically about the x -axis by a factor of 2; reflect in the y -axis; translate 1 unit right and 4 units down
8. a) $a = -1, b = 1, h = -6, k = 3; y = -\log_3(x + 6) + 3$
 b) $a = 5, b = 3, h = 0, k = 0; y = 5 \log_3 3x$
 c) $a = 0.75, b = -0.25, h = 2, k = -5;$
 $y = \frac{3}{4} \log_3 \left(-\frac{1}{4}(x - 2)\right) - 5$
9. a) Reflect in the y -axis, stretch vertically about the x -axis by a factor of 5, stretch horizontally about the y -axis by a factor of $\frac{1}{4}$, and translate 3 units right and 2 units down.
 b) Reflect in the x -axis, reflect in the y -axis, stretch vertically about the x -axis by a factor of $\frac{1}{4}$, translate 6 units right and 1 unit up.
10. a) $y = \log_3 x - 6$ b) $y = \log_2 \left(\frac{x}{4}\right)$
11. Stretch vertically about the x -axis by a factor of 3 and translate 4 units right and 2 units down.
12. a) Stretch vertically about the x -axis by a factor of 0.67, stretch horizontally about the y -axis by a factor of $\frac{25}{9}$ or approximately 2.78, and translate 1.46 units up.
 b) 515 649 043 kWh
13. a) 0.8 μ L b) 78 mmHg
14. a) 172 cm b) 40 kg
15. $a = \frac{1}{3}$
16. a) $y = -2 \log_5 x + 13$ b) $y = \log 2x$
17. $a = \frac{1}{2}, k = -8$
- C1 $a = \frac{1}{4}, b = \frac{1}{3}, h = 4, k = -1;$
 $g(x) = 0.25 \log_5 \left(\frac{1}{3}\right)(x - 4) - 1$
- C2 a) $y = -\log_2 x, y = \log_2(-x), y = 2^x$
 b) Reflect in the x -axis, reflect in the y -axis, and reflect in the line $y = x$.



- C3 a) $y = \frac{1}{2} \log_7 \left(\frac{x-5}{3}\right) + \frac{1}{2}$ b) $y = 3^{\frac{x-8}{2}} + 1$
- C4 Answers will vary.

8.3 Laws of Logarithms, pages 400 to 403

1. a) $\log_7 x + 3 \log_7 y + \frac{1}{2} \log_7 z$
 b) $8(\log_5 x + \log_5 y + \log_5 z)$
 c) $2 \log x - \log y - \frac{1}{3} \log z$
 d) $y = \log_3 x + \left(\frac{1}{2}\right)(\log_3 y - \log_3 z)$
2. a) 2 b) 3 c) 3.5 d) 3

3. a) $\log_9 \left(\frac{xz^4}{y}\right)$ b) $y = \log_3 \frac{\sqrt{x}}{y^2}$
 c) $\log_6 \left(\frac{x}{\sqrt[5]{xy^2}}\right)$ d) $\log \sqrt[3]{xy}$
4. a) 1.728 b) 1.44 c) 1.2
5. a) 27 b) 49
6. a) Stretch horizontally about the y -axis by a factor of $\frac{1}{8}$.
 b) Translate 3 units up.
7. a) False; the division must take place inside the logarithm.
 b) False; it must be a multiplication inside the logarithm.
 c) True
 d) False; the power must be inside the logarithm.
 e) True
8. a) $P - Q$ b) $P + Q$ c) $P + \frac{Q}{2}$ d) $2Q - 2P$
9. a) $6K$ b) $1 + K$ c) $2K + 2$ d) $\frac{K}{5} - 3$
10. a) $\frac{1}{2} \log_5 x, x > 0$ b) $\frac{2}{3} \log_{11} x, x > 0$
11. a) $\log_2 \left(\frac{x+5}{3}\right), x < -5$ or $x > 5$
 b) $\log_7 \left(\frac{x+4}{x+2}\right), x < -4$ or $x > 4$
 c) $\log_8 \left(\frac{x+3}{x-2}\right), x > 2$
12. a) Left Side = $\log_c 48 - (\log_c 3 + \log_c 2)$
 $= \log_c 48 - \log_c 6$
 $= \log_c 8$
 = Right Side
 b) Left Side = $7 \log_c 4$
 $= 7 \log_c 2^2$
 $= 2(7) \log_c 2$
 $= 14 \log_c 2$
 = Right Side
 c) Left Side = $\frac{1}{2}(\log_c 2 + \log_c 6)$
 $= \frac{1}{2}(\log_c 2 + \log_c 3 + \log_c 2)$
 $= \frac{1}{2}(2 \log_c 2) + \frac{1}{2} \log_c 3$
 $= \log_c 2 + \log_c \sqrt{3}$
 = Right Side
 d) Left Side = $\log_c (5c)^2$
 $= 2 \log_c 5c$
 $= 2(\log_c 5 + \log_c c)$
 $= 2(\log_c 5 + 1)$
 = Right Side
13. a) 70 dB b) approximately 1995 times as loud
 c) approximately 98 dB
14. Decibels must be changed to intensity to gauge loudness. The function that maps the change is not linear.
15. 3.2 V
16. a) 10^{-7} mol/L b) 12.6 times as acidic c) 3.4
17. 0.18 km/s
18. a) The graphs are the same for $x > 0$. However, the graph of $y = \log x^2$ has a second branch for $x < 0$, which is the reflection in the y -axis of the branch for $x > 0$.
 b) The domains are different. The function $y = \log x^2$ is defined for all values of x except 0, while the function $y = 2 \log x$ is defined only for $x > 0$.
 c) $x > 0$

19. a) $y = \log_c x$
 $c^y = x$
 $\log_d c^y = \log_d x$
 $y \log_d c = \log_d x$
 $y = \frac{\log_d x}{\log_d c}$

b) 3.2479

c) $\varphi = -\frac{\log D}{\log 2}$

d) 207.9 times larger

20. a) Left Side
 $= \log_{q^3} p^3$
 $= \frac{\log_q p^3}{\log_q q^3}$
 $= \frac{3 \log_q p}{3 \log_q q}$
 $= \frac{\log_q p}{\log_q q}$
 $= \frac{1}{1}$
 $= \text{Right Side}$

b) Left Side
 $= \frac{1}{\log_p 2} - \frac{1}{\log_q 2}$
 $= \frac{1}{\log_2 2} - \frac{1}{\log_2 2}$
 $= \frac{\log_2 p}{\log_2 2} - \frac{\log_2 q}{\log_2 2}$
 $= \frac{\log_2 p - \log_2 q}{\log_2 2}$
 $= \log_2 \frac{p}{q}$
 $= \text{Right Side}$

c) Left Side
 $= \frac{1}{\log_q p} + \frac{1}{\log_q p}$
 $= \frac{1}{\log q} + \frac{1}{\log p}$
 $= \frac{\log q}{\log p} + \frac{\log q}{\log p}$
 $= \frac{2 \log q}{\log p}$

Right Side
 $= \frac{1}{\log_q p}$
 $= \frac{1}{\log p}$
 $= \frac{\log q^2}{\log p}$
 $= \frac{2 \log q}{\log p}$

Left Side = Right Side

d) Left Side = $\log_{\frac{1}{q}} p$
 $= \frac{\log_q p}{\log_q q^{-1}}$
 $= -\log_q p$
 $= \log_q \frac{1}{p}$
 $= \text{Right Side}$

- C1 a) Stretch vertically about the x -axis by a factor of 3.
b) Stretch vertically about the x -axis by a factor of 5 and translate 2 units left.
c) Reflect in the x -axis.
d) Reflect in the x -axis, stretch vertically about the x -axis by a factor of $\frac{1}{2}$, and translate 6 units right.

C2 -1

C3 a) $\log 2$ b) $15 \log 2$

C4 Answers will vary.

8.4 Logarithmic and Exponential Equations, pages 412 to 415

1. a) 1000 b) 14 c) 3 d) 108
2. a) 1.61 b) 10.38 c) 4.13 d) 0.94
3. No, since $\log_3(x-8)$ and $\log_3(x-6)$ are not defined when $x = 5$.
4. a) $x = 0$ is extraneous.
b) Both roots are extraneous.
c) $x = -6$ is extraneous.
d) $x = 1$ is extraneous.
5. a) $x = 8$ b) $x = 25$ c) $x = 96$ d) $x = 9$

6. a) Rubina subtracted the contents of the log when she should have divided them. The solution should be
 $\log_6 \left(\frac{2x+1}{x-1} \right) = \log_6 5$
 $2x+1 = 5(x-1)$
 $1+5 = 5x-2x$
 $6 = 3x$
 $x = 2$

- b) Ahmed incorrectly concluded that there was no solution. The solution is $x = 0$.
c) Jennifer incorrectly eliminated the log in the third line. The solution, from the third line on, should be

$$x(x+2) = 2^3$$

$$x^2 + 2x - 8 = 0$$

$$(x-2)(x+4) = 0$$

So, $x = 2$ or $x = -4$.

Since $x > 0$, the solution is $x = 2$.

7. a) 0.65 b) -0.43 c) 81.37 d) 4.85
8. a) no solution ($x = -3$ not possible)
b) $x = 10$ c) $x = 4$ d) $x = 2$ e) $x = -8, 4$
9. a) about 2.64 pc b) about 8.61 light years
10. 64 kg
11. a) 10 000 b) 3.5%
c) approximately 20.1 years
12. a) 248 Earth years b) 228 million kilometres
13. a) 2 years b) 44 days c) 20.5 years
14. 30 years
15. approximately 9550 years
16. 8 days
17. 34.0 m
18. $x = 4.5, y = 0.5$
19. a) The first line is not true.
b) To go from line 4 to line 5, you are dividing by a negative quantity, so the inequality sign must change direction.

20. a) $x = 100$ b) $x = \frac{1}{100}, 100$ c) $x = 1, 100$

21. a) $x = 16$ b) $x = 9$

22. $x = -5, 2, 4$

C1 a) $\log 8 + \log 2^x = \log 512$
 $x \log 2 = \log 512 - \log 8$
 $x \log 2 = \log 64$
 $x = 6$

b) She could have divided by 8 as the first step.

c) Answers will vary.

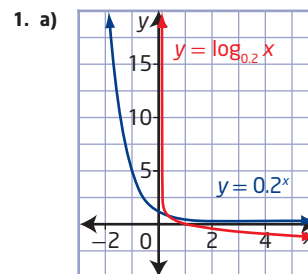
C2 12

C3 14

C4 a) $x = \frac{\pi}{4}, \frac{7\pi}{4}$ b) $x = \frac{\pi}{2}$

C5 Answers will vary.

Chapter 8 Review, pages 416 to 418



- b) i) domain
 $\{x \mid x > 0, x \in \mathbb{R}\}$,
range
 $\{y \mid y \in \mathbb{R}\}$
ii) x -intercept 1
iii) no y -intercept
iv) vertical asymptote
 $x = 0$
c) $y = \log_{0.2} x$

2. $c = 4$

3. $2^4 = 16$ and $2^5 = 32$, so the answer must be between 4 and 5.

4. a) 25 b) -2 c) 3.5 d) 16 e) 0.01

5. 40 times as great

6. a) b) $a = -1, b = 2, c = 4, h = 0, k = -5$

7. $y = \log_2 4x$

8. a) Reflect in the x -axis, stretch horizontally about the y -axis by a factor of $\frac{1}{3}$, and translate 12 units right and 2 units up.

- b) Reflect in the y -axis, stretch vertically about the x -axis by a factor of $\frac{1}{4}$, and translate 6 units right and 7 units down.

9. a) $x = -8$

- b) domain $\{x \mid x > -8, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$

- c) y -intercept 15 d) x -intercept -7.75

10. a) Transform by stretching the graph horizontally about the y -axis by a factor of 440 and stretching vertically about the x -axis by a factor of 12.

- b) 5 notes above c) 698.46 Hz

11. a) $5 \log_5 x - \log_5 y - \frac{1}{3} \log_5 z$

- b) $\frac{1}{2}(\log x + 2 \log y - \log z)$

12. a) $\log \frac{xz^{\frac{2}{3}}}{y^3}$ b) $\log_7 \frac{x}{y^{\frac{1}{3}}z^{\frac{2}{3}}}$

13. a) $\log \sqrt{x}, x > 0$ b) $\log \frac{x-5}{x+5}, x < -5$ or $x > 5$

14. a) 2 b) 0.5

15. 6.3 times as acidic

16. 398 107 times as bright

17. 93 dB

18. a) 1.46 b) 4.03

19. a) 5 b) 10 c) $\frac{5}{3}$ d) -4, 25

20. 6.5 years

21. 35 kg

22. 2.5 h

23. a) 14 years b) 25.75 years

Chapter 8 Practice Test, pages 419 to 420

1. D 2. A 3. B 4. A 5. C 6. B

7. a) $\frac{1}{81}$ b) 25 c) 5 d) 3 e) $\frac{13}{3}$

8. $m = 2.5, n = 0.5$

9. Example: Stretch vertically about the x -axis by a factor of 5, stretch horizontally about the y -axis by a factor of $\frac{1}{8}$, reflect in the x -axis, and translate 1 unit right.

10. a) $x = -5$

- b) domain $\{x \mid x > -5, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$

- c) y -intercept 8 d) $-4\frac{124}{125}$

11. a) no solution b) $x = 6$ c) $x = -2, 4$

12. a) 1.46 b) 21.09

13. 33 years

14. 875 times as great

15. She should not be worried: adding another refrigerator will only increase the decibels to 48 dB.

16. 4.8 h

17. 2029

Cumulative Review, Chapters 7-8, pages 422 to 423

1. a) b) The two functions have the same domain, $x \in \mathbb{R}$; the same range, $y > 0$; the same y -intercept, 1; and the same horizontal asymptote, $y = 0$.

- c) $y = 4^x$ is an increasing function: as x increases, the corresponding values of y also increase.
 $y = \frac{1}{4}^x$ is a decreasing function: as x increases, the corresponding values of y decrease.

2. a) B b) D c) A d) C

3. a) 1000 b) 3 h c) 256 000 d) 21 h

4. a) a vertical stretch by a factor of 2 about the x -axis, a horizontal translation of 4 units left, and a vertical translation of 1 unit up

- b) c) The domain remains the same: $x \in \mathbb{R}$; the range changes from $y > 0$ to $y > 1$ due to the vertical translation; the equation of the horizontal asymptote changes from $y = 0$ to $y = 1$ due to the vertical translation; the y -intercept changes from 1 to 163 due to the vertical stretch and the vertical translation.

5. a) 2^{3x+6} and 2^{3x-15} or 8^{x+2} and 8^{x-5}

- b) 3^{12-3x} and 3^{-4x} or $\left(\frac{1}{3}\right)^{3x-12}$ and $\left(\frac{1}{3}\right)^{4x}$

6. a) -1 b) $\frac{1}{8}$

7. a) -0.72 b) 0.63

8. a) 39% b) 3.7 s

9. a) $\log_3 y = x$ b) $\log_2 m = a + 1$

10. a) $x^4 = 3$ b) $a^b = x + 5$

11. a) -4 b) 4.5 c) -1 d) 49

12. a) 2 b) 32 c) $\frac{1}{125}$ d) $\frac{243}{32}$

13. a vertical stretch by a factor of $\frac{1}{3}$ about the x -axis, a horizontal stretch by a factor of $\frac{1}{2}$ about the y -axis, a horizontal translation of 4 units right and a vertical translation of 5 units up

14. a) $y = 3 \log(x + 5)$ b) $y = -\log 2x - 2$

15. a) 1.6×10^{-8} mol/L to 6.3×10^{-7} mol/L

- b) yes

16. a) $\log \frac{m^2}{\sqrt{np}^3}, m > 0, n > 0, p > 0$

- b) $\log_3 3x^6, x > 0$ c) $\log(x + 1), x > 1$

- d) $\log_2 3^{2x}, x \in \mathbb{R}$

17. In the last step, Zack incorrectly factored the quadratic equation; $x = -5$ and 13.

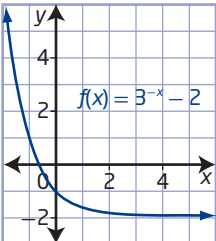
18. a) 0.53 b) 9 c) 3 d) 2

19. a) $E = 10^{10}$ J and $E = 10^{11.4}$ J

- b) approximately 25.1 times

20. 54.25 years

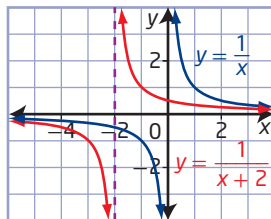
Unit 3 Test, pages 424 to 425

- D 2. B 3. A 4. C 5. A 6. A 7. D
- $y = -2\left(\frac{1}{4}\right)^{x-3}$
- 3^{-1}
- (2, -2)
- 0.001
- 2
- 
 - domain $\{x \mid x \in \mathbb{R}\}$, range $\{y \mid y > -2, y \in \mathbb{R}\}$
 - $x = -0.6$
- 7
 - 2
- domain $\{x \mid x > 2, x \in \mathbb{R}\}$, range $\{y \mid y \in \mathbb{R}\}$, asymptote $x = 2$
 - $y = 10^{1-x} + 2$
 - 12
- $\frac{1}{3}, 4$
 - 7
- Giovanni multiplied the base by 2, which is not correct. The second line should be $3^x = 4$. Giovanni also incorrectly applied the quotient law of logarithms in the sixth line. This line should be deleted. This leads to the solution $x = 1.26$.
- 5.0
- $P(t) = 6(1.013^t)$, where t is the number of years since 2000
 - year 2040
- 12 deposits

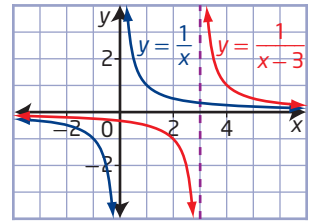
Chapter 9 Rational Functions

9.1 Exploring Rational Functions Using Transformations, pages 442 to 445

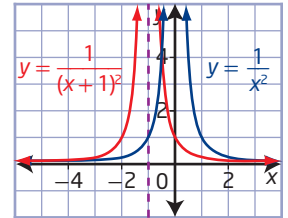
- Since the graph has a vertical asymptote at $x = -1$, it has been translated 1 unit left;
 $B(x) = \frac{2}{x+1}$.
 - Since the graph has a horizontal asymptote at $y = -1$, it has been translated 1 unit down;
 $A(x) = \frac{2}{x} - 1$.
 - Since the graph has a horizontal asymptote at $y = 1$, it has been translated 1 unit up;
 $D(x) = \frac{2}{x} + 1$.
 - Since the graph has a vertical asymptote at $x = 1$, it has been translated 1 unit right;
 $C(x) = \frac{2}{x-1}$.
- Base function $y = \frac{1}{x}$;
vertical asymptote $x = -2$,
horizontal asymptote $y = 0$



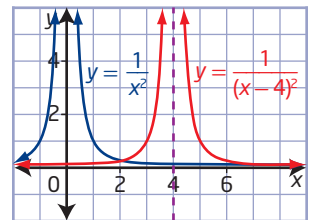
- Base function $y = \frac{1}{x}$; vertical asymptote $x = 3$, horizontal asymptote $y = 0$



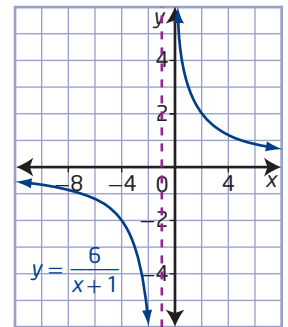
- Base function $y = \frac{1}{x^2}$; vertical asymptote $x = -1$, horizontal asymptote $y = 0$



- Base function $y = \frac{1}{x^2}$; vertical asymptote $x = 4$, horizontal asymptote $y = 0$



- Apply a vertical stretch by a factor of 6, and then a translation of 1 unit left to the graph of $y = \frac{1}{x}$.
domain $\{x \mid x \neq -1, x \in \mathbb{R}\}$,
range $\{y \mid y \neq 0, y \in \mathbb{R}\}$,
no x -intercept,
 y -intercept 6,
horizontal asymptote $y = 0$, vertical asymptote $x = -1$



- Apply a vertical stretch by a factor of 4, and then a translation of 1 unit up to the graph of $y = \frac{1}{x}$.
domain $\{x \mid x \neq 0, x \in \mathbb{R}\}$,
range $\{y \mid y \neq 1, y \in \mathbb{R}\}$,
 x -intercept -4, no y -intercept, horizontal asymptote $y = 1$, vertical asymptote $x = 0$

