# Chapter 9 Rational Functions

## 9.1 Exploring Rational Functions Using Transformations, pages 442 to 445

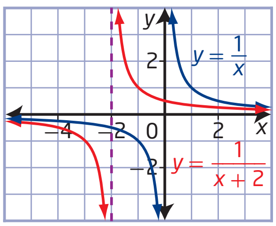
**1. a)** Since the graph has a vertical asymptote at $x = -1$, it has been translated 1 unit left; $B(x) = \frac{2}{x + 1}$.

**b)** Since the graph has a horizontal asymptote at $y = -1$, it has been translated 1 unit down; $A(x) = \frac{2}{x} - 1$.

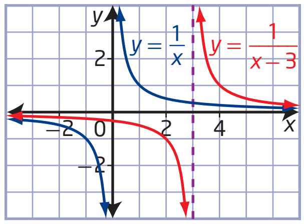
**c)** Since the graph has a horizontal asymptote at $y = 1$, it has been translated 1 unit up; $D(x) = \frac{2}{x} + 1$.

**d)** Since the graph has a vertical asymptote at $x = 1$, it has been translated 1 unit right; $C(x) = \frac{2}{x - 1}$.

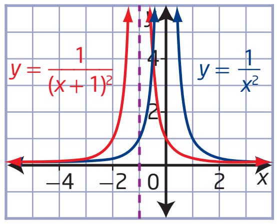
**2. a)** Base function $y = \frac{1}{x}$; vertical asymptote $x = -2$, horizontal asymptote $y = 0$



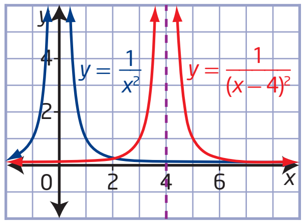

**b)** Base function $y = \frac{1}{x}$; vertical asymptote $x = 3$, horizontal asymptote $y = 0$



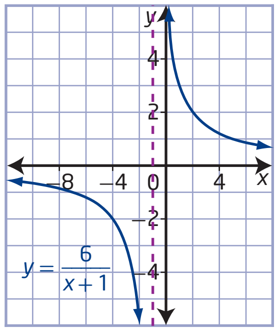

**c)** Base function $y = \frac{1}{x^2}$; vertical asymptote $x = -1$, horizontal asymptote $y = 0$



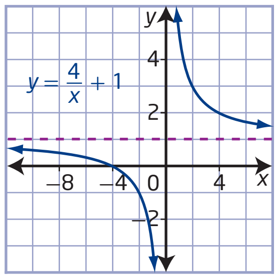

**d)** Base function $y = \frac{1}{x^2}$; vertical asymptote $x = 4$, horizontal asymptote $y = 0$



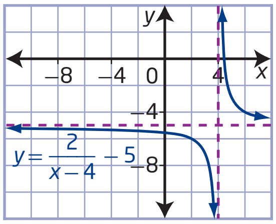

**3. a)** Apply a vertical stretch by a factor of 6, and then a translation of 1 unit left to the graph of $y = \frac{1}{x}$.
domain $\{x \mid x \neq -1, x \in \mathrm{R}\}$, range $\{y \mid y \neq 0, y \in \mathrm{R}\}$, no $x$-intercept, $y$-intercept 6, horizontal asymptote $y = 0$, vertical asymptote $x = -1$




**b)** Apply a vertical stretch by a factor of 4, and then a translation of 1 unit up to the graph of $y = \frac{1}{x}$.
domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y \neq 1, y \in \mathrm{R}\}$, $x$-intercept $-4$, no $y$-intercept, horizontal asymptote $y = 1$, vertical asymptote $x = 0$

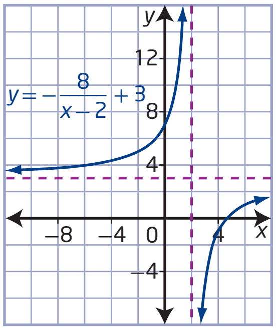


**c)** Apply a vertical stretch by a factor of 2, and then a translation of 4 units right and 5 units down to the graph of $y = \frac{1}{x}$.



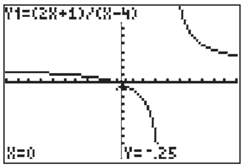

domain $\{x \mid x \neq 4, x \in \mathrm{R}\}$, range $\{y \mid y \neq -5, y \in \mathrm{R}\}$, $x$-intercept 4.4, $y$-intercept −5.5, horizontal asymptote $y = -5$, vertical asymptote $x = 4$

**d)** Apply a vertical stretch by a factor of 8 and a reflection in the $x$-axis, and then a translation of 2 units right and 3 units up to the graph of $y = \frac{1}{x}$.



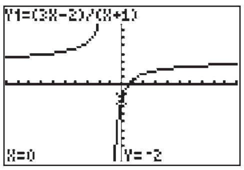

domain $\{x \mid x \neq 2, x \in \mathrm{R}\}$, range $\{y \mid y \neq 3, y \in \mathrm{R}\}$, $x$-intercept $\frac{14}{3}$, $y$-intercept 7, horizontal asymptote $y = 3$, vertical asymptote $x = 2$

**4. a)**



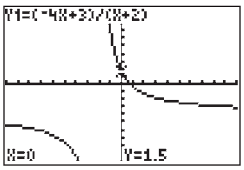

horizontal asymptote $y = 2$, vertical asymptote $x = 4$, $x$-intercept −0.5, $y$-intercept −0.25

**b)**



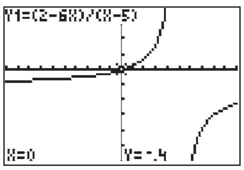

horizontal asymptote $y = 3$, vertical asymptote $x = -1$, $x$-intercept 0.67, $y$-intercept −2

**c)**




horizontal asymptote $y = -4$, vertical asymptote $x = -2$, $x$-intercept 0.75, $y$-intercept 1.5

**d)**



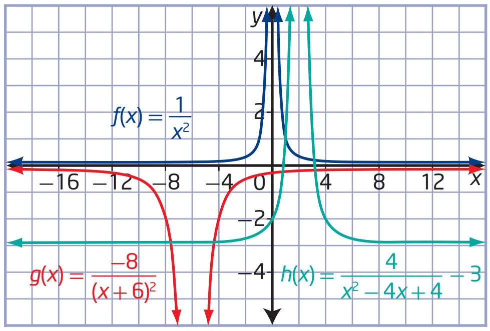

horizontal asymptote $y = -6$, vertical asymptote $x = 5$, $x$-intercept 0.33, $y$-intercept −0.4

**5. a)** $y = \frac{12}{x} + 11$; horizontal asymptote $y = 11$, vertical asymptote $x = 0$, $x$-intercept −1.09, no $y$-intercept

**b)** $y = -\frac{8}{x + 8} + 1$; horizontal asymptote $y = 1$, vertical asymptote $x = -8$, $x$-intercept $x = 0$, $y$-intercept $y = 0$

**c)** $y = \frac{4}{x + 6} - 1$; horizontal asymptote $y = -1$, vertical asymptote $x = -6$, $x$-intercept −2, $y$-intercept −0.33

**6.**




For $f(x) = \frac{1}{x^2}$:

- Non-permissible value: $x = 0$
- Behaviour near non-permissible value: As $x$ approaches 0, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, $y$ approaches 0.
- Domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$
- Asymptotes: $x = 0$, $y = 0$

For $g(x) = \frac{-8}{(x + 6)^2}$:

- Non-permissible value: $x = -6$
- Behaviour near non-permissible value: As $x$ approaches −6, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, $y$ approaches 0.
- Domain $\{x \mid x \neq -6, x \in \mathrm{R}\}$, range $\{y \mid y < 0, y \in \mathrm{R}\}$
- Asymptotes: $x = -6$, $y = 0$

For $h(x) = \frac{4}{x^2 - 4x + 4} - 3$:

- Non-permissible value: $x = 2$
- Behaviour near non-permissible value: As $x$ approaches 2, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, $y$ approaches −3.
- Domain $\{x \mid x \neq 2, x \in \mathrm{R}\}$, range $\{y \mid y > -3, y \in \mathrm{R}\}$
- Asymptotes: $x = 2$, $y = -3$

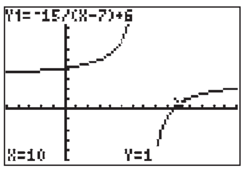
Each function has a single non-permissible value, a vertical asymptote, and a horizontal asymptote. The domain of each function consists of all real numbers except for a single value. The range of each function consists of a restricted set of the real numbers. $|y|$ becomes very large for each function when the values of $x$ approach the non-permissible value for the function.

**7. a)** $y = -\frac{4}{x}$ **b)** $y = \frac{1}{x + 3}$

**c)** $y = \frac{8}{x - 2} + 4$ **d)** $y = \frac{-4}{x - 1} - 6$

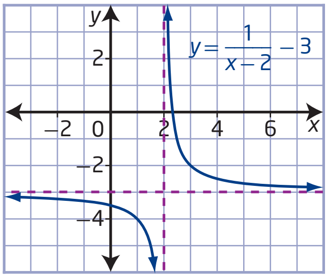
**8. a)** $a = -15$, $k = 6$

**b)**



**9. a)** $y = \frac{1}{x - 2} - 3$

**b)**




domain $\{x \mid x \neq 2, x \in R\}$, range $\{y \mid y \neq -3, y \in R\}$

**c)** No, there are many functions with different values of $a$ for which the asymptotes are the same.

**10. a)** When factoring the 3 out of the numerator, Mira forgot to change the sign of the 21.

$y = \frac{-3x + 21 - 21 + 2}{x - 7}$

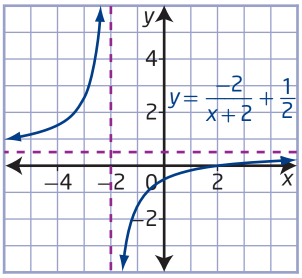
$y = \frac{-3(x - 7) - 19}{x - 7}$

$y = \frac{-19}{x - 7} - 3$

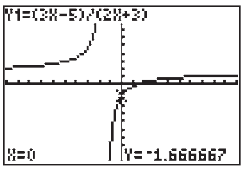
**b)** She could try sample points without technology. With technology, she could check if the asymptotes are the same.

**11. a)** $y = \frac{-2}{x + 2} + \frac{1}{2}$

**b)**



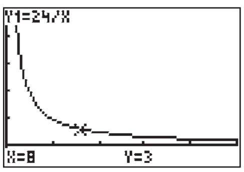

**12.**




$x$-intercept $\frac{5}{3}$, $y$-intercept $-\frac{5}{3}$, horizontal asymptote $y = 1.5$, vertical asymptote $x = -1.5$

**13.** As $p$ increases, $N$ decreases, and vice versa. This shows that as the average price of a home increases, the number of buyers looking for a house decreases.

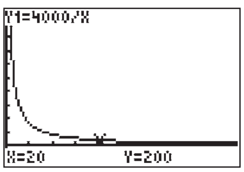
**14. a)** $l = \frac{24}{w}$




**b)** As the width increases, the length decreases to maintain the same area.

**15. a)** $y = \frac{4000}{x}$

**b)**

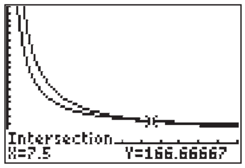



**c)** If 4000 students contribute, they will only need to donate $1 each to reach their goal.

**d)** $y = \frac{4000}{x} + 1000$; This amounts to a vertical translation of 1000 units up.

**16. a)** $y = \frac{100x + 500}{x}$, $y = \frac{60x + 800}{x}$

**b)**



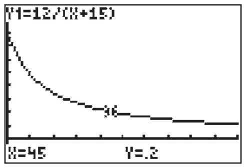

**c)** The graph shows that the more years you run the machine, the less the average cost per year is. One of the machines is cheaper to run for a short amount of time, while the other is cheaper if you run it for a longer period of time.

**d)** If Hanna wants to run the machine for more than 7.5 years, she should choose the second model. Otherwise, she is better off with the first one.

**17. a)** $I = \frac{12}{x + 15}$

**b)** Domain $\{x \mid 0 \leq x \leq 100, x \in R\}$; the graph does not have a vertical asymptote for this domain.

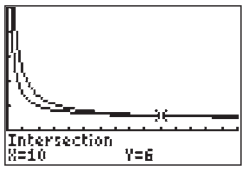
**c)**




A setting of 45 Ω is needed for 0.2 A.

**d)** In this case, there would be an asymptote at $x = 0$.

**18. a)** $y = \frac{4x + 20}{x}$, $y = \frac{5x + 10}{x}$

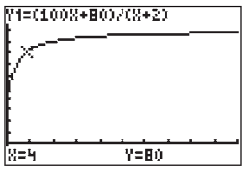



**b)** The graph shows that for a longer rental the average price goes down.

**c)** No. For rentals of less than 10 h, the second store is cheaper. For any rental over 10 h, the first store is cheaper.

**19. a)** $v = \frac{100t + 80}{t + 2}$

**b)**



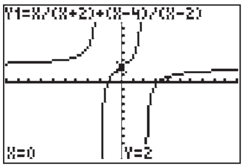

**c)** Horizontal asymptote $y = 100$; the horizontal asymptote demonstrates that the average speed gets closer and closer to 100 km/h but never reaches it. Vertical asymptote $t = -2$; the vertical asymptote does not mean anything in this context, since time cannot be negative.

**d)** 4 h after the construction zone

**e)** Example: Showing the average speed is a good indication of your fuel economy.

**20.** $y = \frac{-4x - 4}{x - 6}$

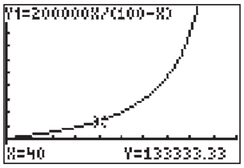
**21. a)** $y = \frac{-x - 3}{x - 1}$ **b)** $y = \frac{5(x - 4)}{x - 6}$

**22.** 


This rational function has two vertical asymptotes ($x = -2$ and $x = 2$) and appears to have a horizontal asymptote ($y = 2$) for values of $x$ less than $-2$ and greater than 2.

**C1** Answers may vary.

**C2 a)** Domain $\{p \mid 0 \leq p < 100, p \in R\}$; you can nearly eliminate 100% of emissions.

**b)** 


The shape of the graph indicates that as the percent of emissions eliminated increases, so does the cost.

**c)** It costs almost 6 times as much. This is not a linear function, so doubling the value of $p$ does not correspond to a doubling of the value of $C$.

**d)** No it is not possible. There is a vertical asymptote at $p = 100$.

**C3** Example: Both functions are vertically stretched by a factor of 2, and then translated 3 units right and 4 units up. In the case of the rational function, the values of the parameters $h$ and $k$ represent the locations of asymptotes. For the square root function, the point $(h, k)$ gives the location of the endpoint of the graph.

## 9.2 Analysing Rational Functions, pages 451 to 456

**1. a)**

| Characteristic | $y = \frac{x - 4}{x^2 - 6x + 8}$ |
|---|---|
| Non-permissible value(s) | $x = 2$, $x = 4$ |
| Feature exhibited at each non-permissible value | vertical asymptote, point of discontinuity |
| Behaviour near each non-permissible value | As $x$ approaches 2, $\lvert y \rvert$ becomes very large. As $x$ approaches 4, $y$ approaches 0.5. |
| Domain | $\{x \mid x \neq 2, 4, x \in R\}$ |
| Range | $\{y \mid y \neq 0, 0.5, y \in R\}$ |

**b)** There is an asymptote at $x = 2$ because 2 is a zero of the denominator only. There is a point of discontinuity at (4, 0.5) because $x - 4$ is a factor of both the numerator and the denominator.

**2. a)**

| x | y |
|---|---|
| −1.5 | −4.5 |
| −1.0 | −4.0 |
| −0.5 | −3.5 |
| 0.5 | −2.5 |
| 1.0 | −2.0 |
| 1.5 | −1.5 |

Since the function does not increase or decrease drastically as $x$ approaches the non-permissible value, it must be a point of discontinuity.

**b)**

| x | y |
|---|---|
| 1.7 | 40.7 |
| 1.8 | 60.8 |
| 1.9 | 120.9 |
| 2.1 | −118.9 |
| 2.2 | −58.8 |
| 2.3 | −38.7 |

Since the function changes sign at the non-permissible value and $\lvert y \rvert$ increases, it must be a vertical asymptote.

**c)**

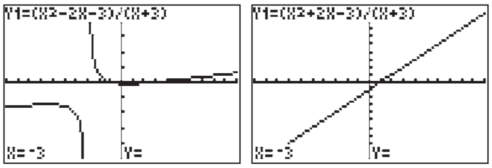
| x | y |
|---|---|
| −3.7 | 74.23 |
| −3.8 | 120.6 |
| −3.9 | 260.3 |
| −4.1 | −300.3 |
| −4.2 | −160.6 |
| −4.3 | −114.23 |

Since the function changes sign at the non-permissible value and $\lvert y \rvert$ increases, it must be a vertical asymptote.

**d)**

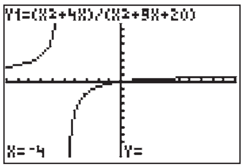
| x | y |
|---|---|
| 0.17 | 1.17 |
| 0.18 | 1.18 |
| 0.19 | 1.19 |
| 0.21 | 1.21 |
| 0.22 | 1.22 |
| 0.23 | 1.23 |

Since the function does not increase or decrease drastically as $x$ approaches the non-permissible value, it must be a point of discontinuity.

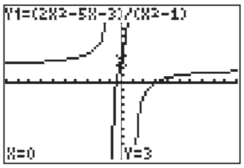
**3. a)** 


Both of the functions have a non-permissible value of −3. However, the graph of $f(x)$ has a vertical asymptote, while the graph of $g(x)$ has a point of discontinuity.

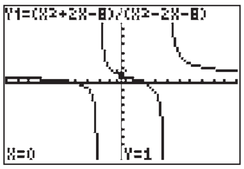
**b)** The graph of $f(x)$ has a vertical asymptote at $x = -3$ because $x + 3$ is a factor of the denominator only. The graph of $g(x)$ has a point of discontinuity at $(-3, -4)$ because $x + 3$ is a factor of both the numerator and the denominator.

**4. a)** 

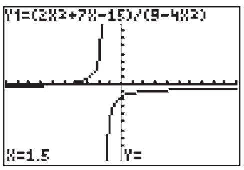

Vertical asymptote $x = -5$; point of discontinuity $(-4, -4)$; $x$-intercept 0; $y$-intercept 0

**b)** 


Vertical asymptotes $x = \pm 1$; no points of discontinuity; $x$-intercepts −0.5, 3; $y$-intercept 3

**c)** 


Vertical asymptotes $x = -2, 4$; no points of discontinuity; $x$-intercepts −4, 2; $y$-intercept 1

**d)** 


Vertical asymptote $x = -1.5$; point of discontinuity $(1.5, -1.083)$; $x$-intercept −5; $y$-intercept −1.67

**5. a)** The graph of $A(x) = \frac{x(x + 2)}{x^2 + 4}$ has no vertical asymptotes or points of discontinuity and $x$-intercepts of 0 and −2; C.

**b)** The graph of $B(x) = \dfrac{x - 2}{x(x - 2)}$ has a vertical asymptote at $x = 0$, a point of discontinuity at (2, 0.5), and no $x$-intercept; A.

**c)** The graph of $C(x) = \dfrac{x + 2}{(x - 2)(x + 2)}$ has a vertical asymptote at $x = 2$, a point of discontinuity at (−2, −0.25), and no $x$-intercept; D.

**d)** The graph of $D(x) = \dfrac{2x}{x(x + 2)}$ has a vertical asymptote at $x = -2$, a point of discontinuity at (0, 1), and no $x$-intercept; B.

**6. a)** Since the graph has vertical asymptotes at $x = 1$ and $x = 4$, the equation of the function has factors $x - 1$ and $x - 4$ in the denominator only; the $x$-intercepts of 2 and 3 mean that the factors $x - 2$ and $x - 3$ are in the numerator; C.

**b)** Since the graph has vertical asymptotes at $x = -1$ and $x = 2$, the equation of the function has factors $x + 1$ and $x - 2$ in the denominator only; the $x$-intercepts of 1 and 4 mean that the factors $x - 1$ and $x - 4$ are in the numerator; B.

**c)** Since the graph has vertical asymptotes at $x = -2$ and $x = 5$, the equation of the function has factors $x + 2$ and $x - 5$ in the denominator only; the $x$-intercepts of −4 and 3 mean that the factors $x + 4$ and $x - 3$ are in the numerator; D.

**d)** Since the graph has vertical asymptotes at $x = -5$ and $x = 4$, the equation of the function has factors $x + 5$ and $x - 4$ in the denominator only; the $x$-intercepts of −2 and 1 mean that the factors $x + 2$ and $x - 1$ are in the numerator; A.

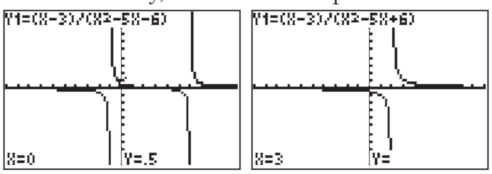
**7. a)** $y = \dfrac{x^2 + 6x}{x^2 + 2x}$ **b)** $y = \dfrac{x^2 - 4x - 21}{x^2 + 2x - 3}$

**8. a)** $y = \dfrac{(x + 10)(x - 4)}{(x + 5)(x - 5)}$ **b)** $y = \dfrac{(2x + 11)(x - 8)}{(x + 4)(2x + 11)}$

**c)** $y = \dfrac{(x + 2)(x + 1)}{(x - 3)(x + 2)}$ **d)** $y = \dfrac{x(4x + 1)}{(x - 3)(7x - 6)}$

**9. a)** Example: The graphs will be different. Factoring the denominators shows that the graph of $f(x)$ will have two vertical asymptotes, no points of discontinuity, and an $x$-intercept, while the graph of $g(x)$ will have one vertical asymptote, one point of discontinuity, and no $x$-intercept.

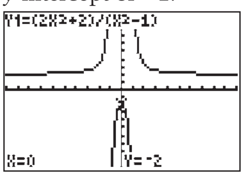
**b)**




**10.** $y = -\dfrac{3(x - 2)(x + 3)}{(x - 2)(x + 3)}$

**11. a)** The function will have two vertical asymptotes at $x = -1$ and $x = 1$, no $x$-intercept, and a $y$-intercept of −2.

**b)**

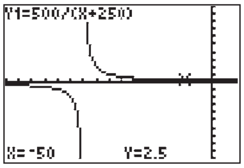



**c)** **i)** The graph will be a line at $y = 2$, but with points of discontinuity at (−1, 2) and (1, 2).

**ii)** The graph will be a line at $y = 2$.

**12. a)** $t = \dfrac{500}{w + 250}$, $w \neq -250$

**b)**

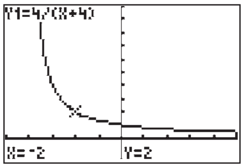



**c)** When the headwind reaches the speed of the aircraft, theoretically it will come to a standstill, so it will take an infinite amount of time for the aircraft to reach its destination.

**d)** Example: The realistic part of the graph would be in the range of normal wind speeds for whichever area the aircraft is in.

**13. a)** $t = \dfrac{4}{w + 4}$; $\{w \mid -4 < w \leq 4, w \in R\}$

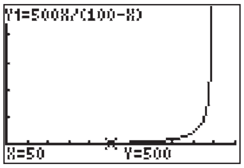
**b)**




**c)** As the current increases against the kayakers, in other words as the current reaches −4 km/h, the time it takes them to paddle 4 km approaches infinity.

**14. a)** The non-permissible value will result in a vertical asymptote. It corresponds to a factor of the denominator only.

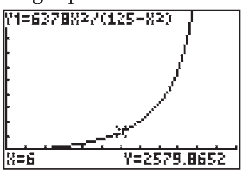
**b)**




It is not possible to vaccinate 100% of the population.

**c)** Yes, the vaccination process will get harder after you have already reached the major urban centres. It will be much more costly to find every single person.

**15. a)**



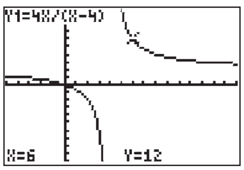

The only parts of the graph that are applicable are when $0 \leq x < \sqrt{125}$.

**b)** As the initial velocity increases, the maximum height also increases but at a greater rate.

**c)** The non-permissible value represents the vertical asymptote of the graph; this models the escape velocity since when the initial velocity reaches the escape velocity the object will leave Earth and never return.

**16.** $y = \dfrac{-(x + 6)(x - 2)}{2(x + 2)(x - 3)}$

**17. a)**



b) The image distance decreases while the object distance is still less than the focal length. The image distance starts to increase once the object distance is more than the focal length.

c) The non-permissible value results in a vertical asymptote. As the object distance approaches the focal length, it gets harder to resolve the image.

**18. a)** Example: Functions $f(x)$ and $h(x)$ will have similar graphs since they are the same except for a point of discontinuity in the graph of $h(x)$.

b) All three graphs have a vertical asymptote at $x = -b$, since $x + b$ is a factor of only the denominators. All three graphs will also have an $x$-intercept of $-a$, since $x + a$ is a factor of only the numerators.

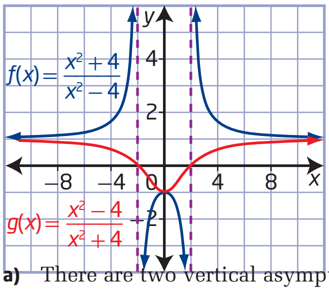
**19.** The $x$-intercept is 3 and the vertical asymptote is at $x = \frac{3}{4}$.

**20.** $y = \dfrac{x^2 - 4x + 3}{2x^2 - 18x - 20}$

**21. a)** $y = \dfrac{(x + 4)(x - 2)(3x + 4)}{4(x + 4)(x - 2)}$

b) $y = \dfrac{(x - 1)(x + 2)^2(x - 2)}{(x - 1)(x + 2)}$

**22.**




They are reciprocals since when one of them approaches infinity the other approaches 0.

**23. a)** There are two vertical asymptotes at $x = \pm 2$.

b) There is a point of discontinuity at $\left(5, \frac{65}{9}\right)$ and a vertical asymptote at $x = -4$.

**C1** Examples:

a) No. Some rational functions have no points of discontinuity or asymptotes.

b) A rational function is a function that has a polynomial in the numerator and/or in the denominator.

**C2** Example: True. It is possible to express a polynomial function as a rational function with a denominator of 1.

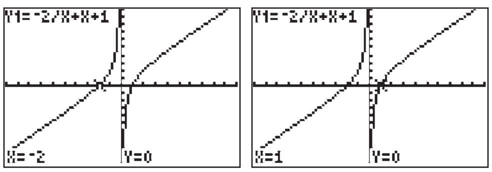
**C3** Answers may vary.

## 9.3 Connecting Graphs and Rational Equations, pages 465 to 467

**1. a)** B **b)** D **c)** A **d)** C

**2. a)** $x = -2, x = 1$

b) $x = -2, x = 1$




c) The value of the function is 0 when the value of $x$ is $-2$ or 1. The $x$-intercepts of the graph of the corresponding function are the same as the roots of the equation.

**3. a)** $x = -\frac{7}{4}$ **b)** $x = 4$ **c)** $x = \frac{3}{2}$ **d)** $x = -\frac{6}{5}$

**4. a)** $x = -8, x = 1$ **b)** $x = 0, x = 3$

c) $x = 4$ **d)** $x = 1, x = \frac{5}{3}$

**5. a)** $x \approx -0.14, x \approx 3.64$ **b)** $x \approx -2.30, x \approx 0.80$

c) $x \approx -2.41, x \approx 0.41$ **d)** $x \approx -5.74, x \approx -0.26$

**6. a)** $x = -\frac{2}{5}$ **b)** $x = 1$

c) $x = -5$ **d)** $x = -\frac{1}{3}$

**7.** Example: Her approach is correct but there is a point of discontinuity at (1, 4). Multiplying by $(x - 1)$ assumes that $x \neq 1$.

**8.** $x = -1, x = -\frac{2}{7}$

**9.** No solutions

**10.** 2.82 m

**11.** 20.6 h

**12.** 15 min

**13. a)** $y = \dfrac{0.5x + 2}{x + 28}$

b) After she takes 32 shots, she will have a 30% shooting percentage.

**14. a)** 200.4 K **b)** 209.3 K

**15. a)** $C(x) = \dfrac{0.01x + 10}{x + 200}$ **b)** 415 mL

**16.** $x \approx 1.48$

**17. a)** $x \leq -\frac{13}{4}$ or $x > 1$ **b)** $-8 \leq x < -6, 2 < x \leq 4$

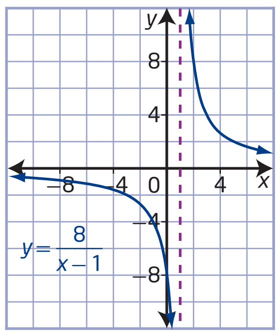
**C1** Example: No, this is incorrect. For example, $\frac{1}{x} = 0$ has no solution.

**C2** Example: The extraneous root in the radical equation occurs because there is a restriction that the radicand be positive. This same principle of restricted domain is the reason why the rational equation has an extraneous root.

**C3** Answers may vary.

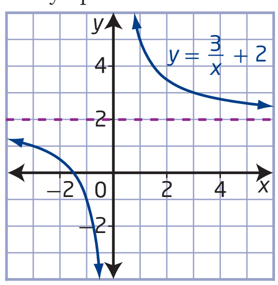
## Chapter 9 Review, pages 468 to 469

**1. a)** Apply a vertical stretch by a factor of 8, and then a translation of 1 unit right to the graph of $y = \frac{1}{x}$.



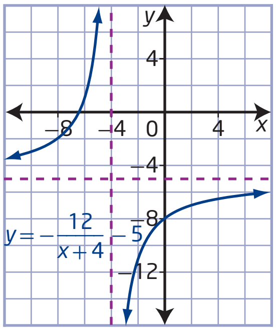

domain $\{x \mid x \neq 1, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$, no $x$-intercept, $y$-intercept $-8$, horizontal asymptote $y = 0$, vertical asymptote $x = 1$

b) Apply a vertical stretch by a factor of 3 and then a translation of 2 units up to the graph of $y = \frac{1}{x}$.



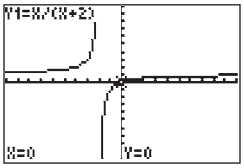

domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 2, y \in R\}$, $x$-intercept $-1.5$, no $y$-intercept, horizontal asymptote $y = 2$, vertical asymptote $x = 0$

**c)** Apply a vertical stretch by a factor of 12 and a reflection in the $x$-axis, and then a translation of 4 units left and 5 units down to the graph of $y = \frac{1}{x}$.



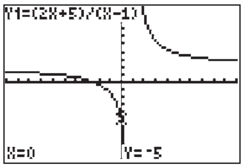

domain $\{x \mid x \neq -4, x \in \mathrm{R}\}$, range $\{y \mid y \neq -5, y \in \mathrm{R}\}$, $x$-intercept $-6.4$, $y$-intercept $-8$, horizontal asymptote $y = -5$, vertical asymptote $x = -4$

**2. a)**



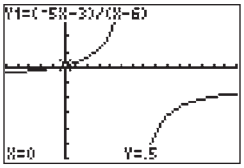

Horizontal asymptote $y = 1$, vertical asymptote $x = -2$, $x$-intercept 0, $y$-intercept 0

**b)**



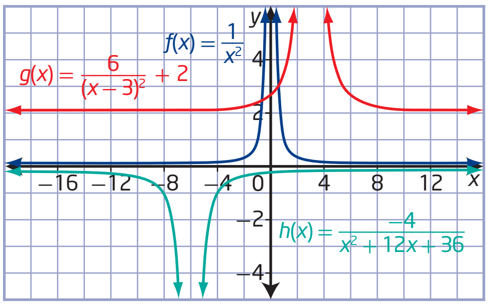

Horizontal asymptote $y = 2$, vertical asymptote $x = 1$, $x$-intercept $-2.5$, $y$-intercept $-5$

**c)**




Horizontal asymptote $y = -5$, vertical asymptote $x = 6$, $x$-intercept $-0.6$, $y$-intercept 0.5

**3.**




For $f(x) = \frac{1}{x^2}$:

- Non-permissible value: $x = 0$
- Behaviour near non-permissible value: As $x$ approaches 0, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, $y$ approaches 0.
- Domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$
- Asymptotes: $x = 0$, $y = 0$

For $g(x) = \frac{6}{(x-3)^2} + 2$:

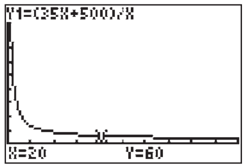
- Non-permissible value: $x = 3$
- Behaviour near non-permissible value: As $x$ approaches 3, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, $y$ approaches 2.
- Domain $\{x \mid x \neq 3, x \in \mathrm{R}\}$, range $\{y \mid y > 2, y \in \mathrm{R}\}$
- Asymptotes: $x = 3$, $y = 2$

For $h(x) = \frac{-4}{x^2 + 12x + 36}$:

- Non-permissible value: $x = -6$
- Behaviour near non-permissible value: As $x$ approaches $-6$, $|y|$ becomes very large.
- End behaviour: As $|x|$ becomes very large, $y$ approaches 0.
- Domain $\{x \mid x \neq -6, x \in \mathrm{R}\}$, range $\{y \mid y < 0, y \in \mathrm{R}\}$
- Asymptotes: $x = -6$, $y = 0$

Each function has a single non-permissible value, a vertical asymptote, and a horizontal asymptote. The domain of each function consist of all real numbers except for a single value. The range of each function is a restricted set of real numbers. $|y|$ becomes very large for each function when the values of $x$ approach the non-permissible value for the function.

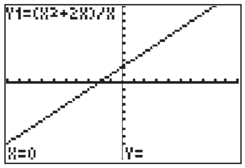
**4. a)** $y = \frac{35x + 500}{x}$




**b)** The more uniforms that are bought, the less expensive their average cost.

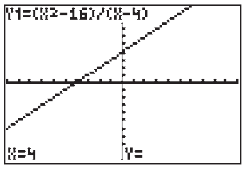
**c)** They will need to buy 100 uniforms.

**5. a)**



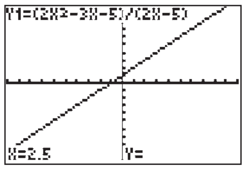

linear with a point of discontinuity at (0, 2)

**b)**




linear with a point of discontinuity at (4, 8)

**c)**



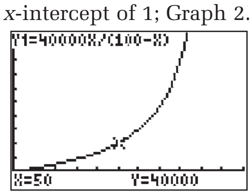

linear with a point of discontinuity at (2.5, 3.5)

**6.** The graph of $A(x) = \frac{x - 4}{(x - 4)(x - 1)}$ has a vertical asymptote at $x = 1$, a point of discontinuity at $\left(4, \frac{1}{3}\right)$, and no $x$-intercept; Graph 3.

The graph of $B(x) = \frac{(x + 4)(x + 1)}{x^2 + 1}$ has no vertical asymptotes or points of discontinuity and $x$-intercepts of $-4$ and $-1$; Graph 1.

The graph of $C(x) = \frac{x - 1}{(x - 2)(x + 2)}$ has vertical asymptotes at $x = \pm 2$, no points of discontinuity, and an $x$-intercept of 1; Graph 2.

**7. a)**




**b)** As the percent of the spill cleaned up approaches 100, the cost approaches infinity.

**c)** No, since there is a vertical asymptote at $p = 100$.

# Chapter 10 Function Operations

## 10.1 Sums and Differences of Functions, pages 483 to 487

**1. a)** $h(x) = |x - 3| + 4$ **b)** $h(x) = 2x - 3$
**c)** $h(x) = 2x^2 + 3x + 2$ **d)** $h(x) = x^2 + 5x + 4$

**2. a)** $h(x) = 5x + 2$ **b)** $h(x) = -3x^2 - 4x + 9$
**c)** $h(x) = -x^2 - 3x + 12$ **d)** $h(x) = \cos x - 4$

**3. a)** $h(x) = x^2 - 6x + 1$; $h(2) = -7$
**b)** $m(x) = -x^2 - 6x + 1$; $m(1) = -6$
**c)** $p(x) = x^2 + 6x - 1$; $p(1) = 6$

**4. a)** $y = 3x^2 + 2 + \sqrt{x + 4}$; domain $\{x \mid x \geq -4, x \in \mathrm{R}\}$
**b)** $y = 4x - 2 - \sqrt{x + 4}$; domain $\{x \mid x \geq -4, x \in \mathrm{R}\}$
**c)** $y = \sqrt{x + 4} - 4x + 2$; domain $\{x \mid x \geq -4, x \in \mathrm{R}\}$
**d)** $y = 3x^2 + 4x$; domain $\{x \mid x \in \mathrm{R}\}$