11

Permutations, Combinations and the Binomial Theorem

General Outcome

Develop algebraic and numeric reasoning that involves combinatorics.

Specific Outcomes

PCBT1 Apply the fundamental counting principle to solve problems.

PCBT2 Determine the number of permutations of n elements taken r at a time to solve problems.

PCBT3 Determine the number of combinations of n different elements taken r at a time to solve problems.

PCBT4 Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers).

By the end of this chapter, students will be able to:

Section	Understanding Concepts, Skills, and Processes
11.1	✓ solve counting problems using the fundamental counting principle
	✓ determine, using a variety of strategies, the number of permutations of n elements taken r at a time
	✓ solve counting problems when two or more elements are identical
	✓ solve an equation that involves ${}_nP_r$ notation
11.2	✓ explain the differences between a permutation and a combination
	✓ determine the number of ways to select r elements from n different elements
	✓ solve problems using the number of combinations of n different elements taken r at a time
	✓ solve an equation that involves ${}_nC_r$ notation
11.3	✓ relate the coefficients in the expansion of $(x + y)^n$, $n \in \mathrm{N}$, to Pascal's triangle and to combinations
	✓ expand $(x + y)^n$, $n \in \mathrm{N}$, in a variety of ways, including the binomial theorem
	✓ determine a specific term in the expansion of $(x + y)^n$

Assessment	Supporting Learning
Assessment *for* Learning	
Method 1: Use the introduction on page 514 in *Pre-Calculus 12* to activate students' prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal entry to explain what they personally know about permutations, combinations, and the binomial theorem.	• Have students update their list of what they need to work on and keep track of the skills and processes that need attention. • Students who require activation of prerequisite skills may wish to complete **BLM 11–1 Chapter 11 Prerequisite Skills**. This material is on the Teacher CD of this Teacher's Resource and mounted on the www.mcgrawhill.ca/school/learningcentres book site.
Assessment *as* Learning	
As students work on each section in Chapter 11, have them keep track of any problems they are having.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled. • Encourage students to write definitions for the Key Terms in their own words, including reminder tips that may be helpful for review throughout the chapter. • Encourage students to write examples of their own in their notebook or math portfolio. Students should have an example for each method that is covered in the chapter.
Assessment *for* Learning	
BLM 11–1 Chapter 11 Prerequisite Skills This master provides a review of prerequisite skills needed for the chapter.	• Use the Prerequisite Skills blackline master to provide additional opportunities for students to demonstrate their readiness for the chapter material.

Chapter 11 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning	Web Link www.mcgrawhill.ca/school/learningcentres
Chapter Opener • 30–45 min (TR page 279)			BLM 11–1 Chapter 11 Prerequisite Skills BLM U4–1 Unit 4 Project Checklist					• information on careers and educational programs in actuarial science • information on famous combinatorial problems
11.1 Permutations • 120–180 min (TR page 280)	Students should be familiar with • multiplying and combining like terms • factoring polynomials • creating tree diagrams • entering expressions, setting windows, and graphing using technology	• graphing technology • index cards (optional)	BLM 11–2 Section 11.1 Extra Practice TM 11–1 How to Do Page 519 Example 2a) Using TI 83/84	**Essential:** #1–5, 7–9, 12, 14, 16, 17 **Typical:** #1, 2, 4–7, 9–16, one of 18–22, 24, 25, one of 26–29, C1–C4 **Extension/Enrichment:** #20, 21, 24, 25, 28–32, C1, C3–C5	TR pages 282, 286	TR pages 284–286		
11.2 Combinations • 90–120 min (TR page 287)	Students should be familiar with • factoring polynomials • analyzing rational functions	• standard deck of cards (optional) • coloured markers, beads, or marbles (optional)	BLM 11–3 Section 11.2 Extra Practice	**Essential:** #1–5, 7, 9–12, one of 17–19 **Typical:** #1–3, 6–14, one of 17–19, 21, C1–C3 **Extension/Enrichment:** #8, 14–16, 20–24, C2–C4	TR pages 288, 292	TR pages 290, 292		• information on Canadian First Nations native Cree artist George Fagnan
11.3 The Binomial Theorem • 90–120 min (TR page 293)	Students should be familiar with • expanding a power of a binomial expression • analyzing rational functions	• counters • copies of Pascal's triangle • grid paper (optional)	Master 3 Centimetre Grid Paper BLM 11–4 Section 11.3 Extra Practice TM 11–2 How to Do Page 536 #C4 Using *The Geometer's Sketchpad®* TM 11–3 How to Do Page 536 #C4 Using *GeoGebra*	**Essential:** #1, 3–6, 7b), d), 8, 11, 12, 16, 17a), b) **Typical:** #2–6, 7a), c), e), 9–13, one of 14–16, 17c), d), 18, C1–C4 **Extension/Enrichment:** #7c), e), 13, 17c), 19–24, C1–C4	TR pages 294, 297	TR pages 296, 297		• information on patterns in Pascal's triangle
Chapter 11 Review and Practice Test • 75–105 min (TR page 298)			BLM 11–2 Section 11.1 Extra Practice BLM 11–3 Section 11.2 Extra Practice BLM 11–4 Section 11.3 Extra Practice BLM 11–5 Chapter 11 Study Guide BLM 11–6 Chapter 11 Test	Have students do at least one question related to any concept, skill, or process that has been giving them trouble. **Chapter 11 Review minimum:** #1–5, 7–9, 11, 12, 14, and 16–18 Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Chapter 11 Practice Test minimum:** #1–13		TR page 298	TR page 298 BLM 11–6 Chapter 11 Test	
Unit 4 Project Wrap-Up • 60–90 min (TR page 299)			Master 1 Holistic Project Rubric Master 2 Ana-Holistic Project Rubric BLM U4–1 Unit 4 Project Checklist				TR page 300 Master 1 Holistic Project Rubric Master 2 Ana-Holistic Project Rubric	• sample Unit 4 Project Holistic Rubric • sample Unit 4 Project Ana-Holistic Rubric
Unit 4 Cumulative Review and Test • 60–90 min (TR page 301)			BLM U4–2 Unit 4 Test BLM 11–7 Chapter 11 BLM Answers	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.		TR page 301	TR page 301	