Radical Functions

Opener

***Pre-Calculus 12*, pages 60–61**

Suggested Timing
45–60 min

Blackline Masters
BLM 2–1 Chapter 2 Prerequisite Skills
BLM U1–1 Unit 1 Project Checklist

Planning Notes

Introduce the chapter by having a discussion about functions. Have students recall what they already know about functions, including the vertical line test to identify a function, set notation and interval notation, determining domain and range, and transformations.

Have a discussion as well about radical expressions and equations. Encourage students to write as much as they can remember about radicals. Can they simplify radicals? What are radical expressions? Equations? How do you solve radical equations?

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. Students may have used different types of graphic organizers. Ask students which one(s) might be useful in this chapter. Encourage students to use a summary method of their choice.

Unit Project

In this chapter, students revisit the Unit 1 project introduced in Chapter 1. Some students may have already begun gathering ideas for their piece of artwork, photo, or image of a function. Suggest that students consider the shape of a radical function and where they may have seen one that they would like to use for their project. The Project Corner in this chapter provides examples of the type of image students may choose.

Meeting Student Needs

- Provide students with a checklist containing the learning outcomes for this unit. Discuss specific terms. Develop a sense of understanding of what students need to learn by the end of the unit.
- Consider having students complete the questions on **BLM 2–1 Chapter 2 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- Hand out to students **BLM U1–1 Unit 1 Project Checklist**, which provides a list of all the requirements for the Unit 1 Project.

Career Link

Discuss with students what they know about remote sensing. Ask:

- What do you know about remote sensing? satellite images?
- How do you think maps are produced?
- How is mathematics involved in remote sensing?

Suggest that students go online to research careers in remote sensing. Are there any aspects of the career that surprise them?

2.1 Radical Functions and Transformations

***Pre-Calculus 12*, pages 62–77**

Suggested Timing

90–120 min

Materials

- grid paper
- graphing technology

Blackline Masters

Master 3 Centimetre Grid Paper
BLM 2–2 Section 2.1 Extra Practice
TM 2–1 How to Do Page 71 Example 4d) Using TI-Nspire™ With Touchpad
TM 2–2 How to Do Page 71 Example 4d) Using TI-83/84

Mathematical Processes for Specific Outcomes

RF2 Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations.

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Reasoning (R)
- ✓ Visualization (V)

RF3 Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations.

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Reasoning (R)
- ✓ Visualization (V)

RF4 Apply translations and stretches to the graphs and equations of functions.

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Reasoning (R)
- ✓ Visualization (V)

RF13 Graph and analyze radical functions (limited to functions involving one radical).

- ✓ Connections (CN)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–9, 10a), b), 18
Typical	#4, 5, 7, 9, 10c), d), one of 11–14, 15–17, C1–C4
Extension/Enrichment	#16, 17, 19, C1–C4

Planning Notes

Discuss the outcomes and achievement indicators for this section. Encourage students to develop a list of the terms and definitions found within this section.

Consider setting up a short time for students to perform an experiment similar to that completed by the Apollo 15 astronaut. Use a baseball and a foam or hollow plastic ball about the same size. Drop them from the top of a ladder and record their fall with a video camera. Did they fall at the same rate?

Section 2.1 introduces students to the radical function, its graph, and its domain and range. Students are expected to build their understanding by graphing radical functions using a table of values and graph paper. Hands-on techniques, such as building a table of values and plotting points, promote a concrete and deeper understanding of the characteristics of radical functions, including the flow and personality of their graphs.

Students are expected to use their knowledge of transformations to sketch graphs of any radical function. Given a radical function, they need to determine all reflections, stretches, and translations indicated by the function as compared to $y = \sqrt{x}$.

Students also develop skills to determine the domain and range of any radical function, with and without the use of a graph. They should be able to extend this understanding to predict the effect of the domain and range on any application of radical functions.

Investigate a Radical Function

The investigation introduces students to the graphs of radical functions and helps them understand the relationships between quadratic functions and radical functions. It requires students to sketch graphs to determine the domain and range of the quadratic and radical functions. Provide **Master 3 Centimetre Grid Paper** for students to draw the quadratic and radical functions to scale. Organize the class into groups of two or three students. Have each group complete the investigation. Then, discuss the results with the entire class. The investigation could be used as a math lab and included in each student's portfolio.

In step 3, when students are relating one equation to another, have them focus on comparing the restrictions on the variable and domains, and determining what is used to describe each function. For example, in the distance–time function, distance is described in terms of time.

Meeting Student Needs

- Study the given equation. Which variable is independent? Dependent? Once students have established that time (t), the independent variable, is on the horizontal axis they can then discuss domain and range.
- Encourage students taking physics to use formulas relating velocity and gravitational acceleration to determine the exact equation used in the Investigate.

Common Errors

- When determining the t-values of the new function, students may not know how to plot an exact value.

R_x Have students determine an approximate value for t to the nearest hundredth, and plot those values.

- Students may not know how to determine the domain of the original function, $d = 5t^2$.

R_x Discuss with student the context of the function. Since it describes distance over time, can time have a negative value? What is the smallest value that can represent t? How does the help to determine the domain of the function?

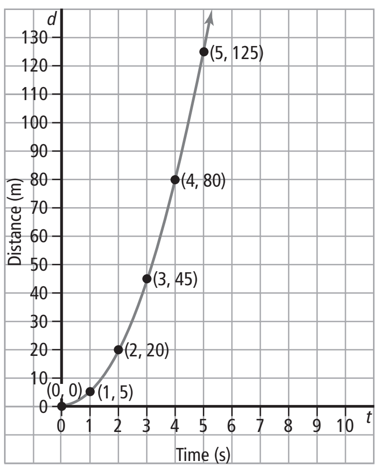
Answers

Investigate a Radical Function

1. a) domain: $\{t \mid t \geq 0, t \in \mathrm{R}\}$. Since the time begins when the object is dropped, each value for time must be a positive real number. range: $\{d \mid d \geq 0, d \in \mathrm{R}\}$.

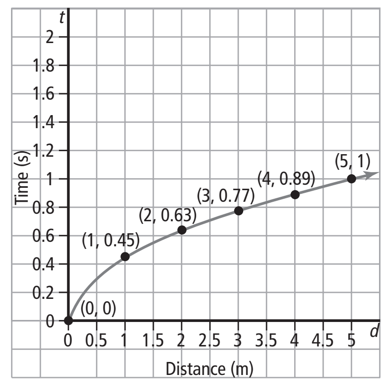
b)

t	d
0	0
1	5
2	20
3	45
4	80
5	125



2. $t = \sqrt{\frac{d}{5}}$

d	t
0	0
1	0.45
2	0.63
3	0.77
4	0.89
5	1



3. Example: The original function is quadratic, whereas the new function is a radical function involving a square root of the variable. For each value of x, the y-value in the original function is equal to 125 (y_{new}^4). The domain and range of both functions are the same.

4. a) Example: time–distance function. I would use the original function to find the distance. I would use the new function when I know the distance and want to find the time.

b) half parabola; half parabola on its side

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• You may wish to have students work with a classmate for this activity. • If students are having difficulty, solve the problem on the board algebraically to show how the equations vary and clarify any misunderstandings. • Ensure students can distinguish between the different shapes of the graph.

When discussing the examples with your class, consider having students attempt the example first, either by themselves or in pairs. Give them a couple of minutes to discuss and explore how they might solve the given question before they read the solutions. Share with students the importance of fully communicating what they are doing. They should appreciate that good communication will help them remember how they solved a problem. Communication also helps them to find and correct errors much more easily.

Example 1

This example helps students develop an understanding of the translations of radical functions. By building a table of values, plotting the points, and graphing, students should recognize the aspect of a radical function that causes horizontal and vertical translations of the graph $y = \sqrt{x}$. Some students may have difficulty choosing appropriate x-values for their table of values. Guide them to remember the relationship between perfect square values and their square roots. Consider asking leading questions. What type of number is produced when you take the square root of a perfect square value? Is it easy to plot these numbers?

Example 2

Review function transformations with students as an introduction to this example.

Students can use horizontal and vertical translations with and without reflections to determine the domain and range of any radical function. They should rewrite the given radical function in the form $y - k = a\sqrt{b(x - h)}$ to help.

Ask

- Does a horizontal stretch affect the domain of a function?
- Does a reflection on the y-axis affect the domain of a function?
- What transformations of a function affect the range?
- Do horizontal and vertical translations affect the domain and range of a function? If so, how?

Once students understand the importance of the translations and reflections in determining the domain and range of radical functions, have them develop a process to determine domain and range without graphing.

The student resource suggests two methods of sketching graphs. Students should try both methods to decide which one they feel most comfortable using.

Example 3

It is important that students recognize that the given example involves only a vertical or horizontal stretch with no reflections or translations. It is much more difficult to determine functions involving both vertical and horizontal stretches. Including reflections and translations may be best approached later through the practice problems. Two methods are given in the student resource to determine a radical function from a graph. If you have a preferred method, share it with the class and have students chose one of the given methods.

Example 4

Example 4 illustrates how knowledge of radical functions can be used to solve problems. In application problems, context becomes important. What do the domain and range represent in the problem involving the speed of sound and the effect of air temperature? How will this affect the graph and solution of the given question? Is it important to determine the domain and range before solving the problem? What possible solutions can you expect or not expect given the domain and range? These questions will add depth to students' understanding and help instill confidence in solving radical function application questions.

To help students use technology in part d), provide **TM 2–1 How to Do Page 71 Example 4d) Using TI-Nspire™ With Touchpad** or **TM 2–2 How to Do Page 71 Example 4d) Using TI-83/84**.

Meeting Student Needs

- Have students record their own summary of the Key Ideas, including examples. Have them store the summary for each section in the same location for review purposes.
- Students can be encouraged to use graphing technology to view the graphs prior to making the table of values.
- Provide students with three to five factoring questions where the coefficient on the x^2 term is a fraction. They have not had much exposure to removing a common rational factor.
- Students should be encouraged to use visual representations of functions to assist them with the understanding of domain and range.
- For Example 2, students need to understand that a vertical stretch means to **multiply** each y-value by 3. Discuss how to reflect the points on the y-axis (change the sign of the x-value). Method 2 for both parts a) and b) provides great detail to assist students.
- For Example 4, part b), have the students simplify, showing all necessary steps. Some students will not understand the process found in the textbook.

Common Errors

- Students may confuse domain and range.

R_x Ordered alphabetically, *domain* precedes *range* and x precedes y. Keeping this in mind will help students remember that x-values are the domain of a function and y-values are the range.

- Students may not factor out the value of b in any given radicand.

Rx Use a specific example to discuss the following, and then generalize with any students who need help. Given the function $y = a\sqrt{b(x - h)} + k$, what is the radicand? What is the difference between $b(x - h)$ and $bx - bh$? Which form is better to help determine the horizontal stretch and reflection of the function?

- Students do not recognize a vertical or horizontal reflection.

Rx Have students use graphing technology to graph $y = \sqrt{x}$, $y = -\sqrt{x}$, and $y = \sqrt{-x}$. What is the value of a in each case? What is the value of b in each case? How does a negative value of a and b change the shape of the graph? In which axis does $-$a cause a reflection? In which axis does $-b$ cause a reflection? What generalization can you make about reflections and the given radical functions?

- Students may have difficulty determining the domain and range of a function without a graph or without using technology.

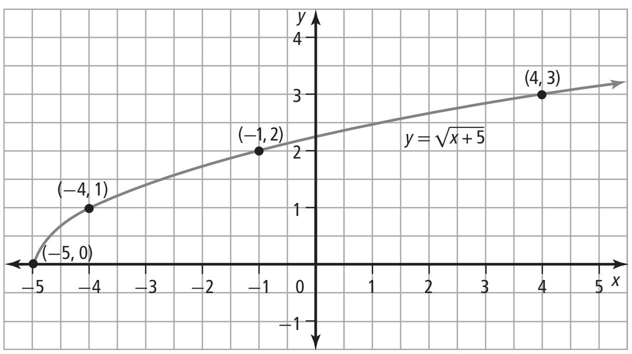
Rx Have students discuss with you the effect of each of the transformation parameters on the domain and range. What happens when a or b is negative? How does this affect the graph of a radical function? How does this affect the domain and range of a given radical function? Does the radicand affect the domain or range? How? How does the vertical translation affect the range of the function? Can you rearrange the function to connect the y-variable and the vertical translation? How can you use this connection to determine the range of the function? How does this change if the graph is reflected in the x-axis?

Answers

Example 1: Your Turn

Example: table of values

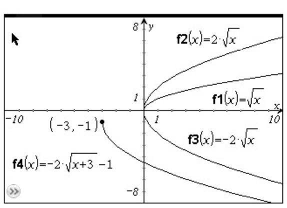
x	y
−5	0
−4	1
−1	2
4	3



domain: $\{x \mid x \geq -5, x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, x \in \mathrm{R}\}$

Example 2: Your Turn

a)



b) domain of $y = \sqrt{x}$: $\{x \mid x \geq 0, x \in \mathrm{R}\}$; after a translation left 3 units, domain becomes $\{x \mid x \geq -3, x \in \mathrm{R}\}$;

range of $y = \sqrt{x}$: $\{y \mid y \geq 0, x \in \mathrm{R}\}$; after a translation down 1 unit and a reflection in the x-axis, range becomes $\{y \mid y \leq -1, y \in \mathrm{R}\}$.

Example 3: Your Turn

a) $y = \frac{1}{2}\sqrt{x}$, $y = \sqrt{\frac{1}{4}x}$

b) $y = \sqrt{\frac{1}{4}x}$

$= \sqrt{\frac{1}{4}}\sqrt{x}$

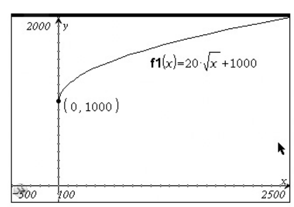
$= \frac{1}{2}\sqrt{x}$

Therefore, $y = \frac{1}{2}\sqrt{x}$ is equivalent to $y = \sqrt{\frac{1}{4}x}$.

Example 4: Your Turn

a) vertical stretch by a factor of 20; translation up 1000 units

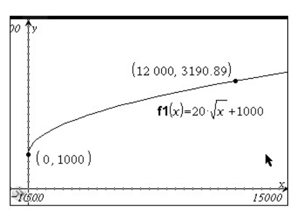
b)



The graph implies that the minimum cost of production is \$1000.

c) domain: $\{n \mid n \geq 0, n \in \mathrm{N}\}$; domain implies that the number of items is a positive whole number; range: $\{C(n) \mid C(n) \geq 1000, C(n) \in \mathrm{R}\}$; range implies that the minimum cost of production is \$1000

d)



$C(12\,000)$
$= \$3190.89$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work in pairs. • Have students verbalize how $x + 5$ will transform the graph. Encourage them to think about what they know about how transformations in quadratics are handled. • Ask students to identify the domain and have them verbalize what it means.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work in pairs. • Have students identify which method they find easier to use. • Ask students to describe the order in which to deal with the parameters (a, b, h, and k).
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work in pairs. • Ask students in what order the parameters must be dealt with (a, b, h, and k). • Have them verbalize what each parameter should do to the graph. • Have students complete first the transformation that is most obvious to them. This will likely be a vertical stretch. • Ask students to identify what they must consider to make the transformation go in the other direction (horizontal).
Example 4 Have students do the Your Turn related to Example 4.	• You may wish to have students work in pairs. • Have students verbalize what each parameter should do to the graph. • Some students may find it easier to use the table of values generated on their calculator.

Check Your Understanding

Provide class time for students to complete the Check Your Understanding questions. It is best for students to have an opportunity to get help when practising skills and solving problems.

For #4, have students explain which parameter of the expression $y = a\sqrt{b(x - h)} + k$ describes vertical stretches and reflections on the x-axis, horizontal stretches and reflections on the y-axis, horizontal translations, and vertical translations. How can they use these parameters to develop a radical function?

For #5, students should choose one of the two methods described in Example 2 to sketch a graph of each function. Work with students to sketch the graphs of functions, such as the one in part d), which include all types of transformations. Ask coaching questions to have students identify any transformations. Then, go through the four-step graphing process to sketch the graph. Have students summarize the process in their own words before completing the remaining parts of this question.

Question #6 helps students develop the understanding that all vertical and horizontal stretches can be expressed in the form $y = a\sqrt{x}$ or $y = \sqrt{bx}$.

Have students use exact and approximate values for r in their table of values in #7b). This will make sketching the graph easier and add to students' understanding of the questions and appropriate uses of exact and approximate values.

The important aspect of #8b) is to have students explain why they prefer a particular form of a transformational stretch.

For #10, students should be able to determine the horizontal and vertical translations by moving the starting point of the graph to the origin. Using the distance and direction that the point needs to be moved will allow them to determine the values of the parameters h and k. It may be helpful for students to draw a new set of axes at the starting point of each graph. Use the starting point as a relative origin and the axes to determine the vertical or horizontal stretches of the function. Once they have determined the values of a and b, students should use the reflections to determine the signs of those parameters.

It is assumed in #11 that no stretches are involved. However, students need to consider reflections, so the form of the answers should be $y = \pm\sqrt{\pm(x - h)} + k$ or $y - k = \pm\sqrt{\pm(x - h)}$.

For #14 and 15, discuss with students if they think that the given graphs involve any translations. Do they involve reflections? If so, the reflections are on which axis? What type of stretch do they prefer to determine? In addition to $y = a\sqrt{x}$ or $y = \sqrt{bx}$, what do they require to determine the value of a or b? Have them describe a point on the graph that they will use and how they will use it to solve for a or b.

Question #16 involves translations, reflections, and stretches. Have students use their sketch to determine the stretches, reflections, and translations as they did in #10.

Some students may have difficulties in #17 determining the percents in parts b) and d). For part b), consider using the expression $\frac{P_N}{P_W} \times 100$, where P_N is the population of a nation and P_W is the world population. Use the expression $\frac{V(x)}{765} \times 100$ in part d).

The Mini Lab in #18 is a good hands-on exploration that can be done either in class or as a homework activity. In step 3, for very short lengths, it may be easier to video record the swinging pendulum and use the video to determine the time taken for ten complete swings.

Question #20 requires students to express all stretches as a single vertical or horizontal stretch before they graph or describe transformations.

Meeting Student Needs

- Students should complete the checklist of understanding provided in the opener of this section. This will guide them when deciding which questions they need to practice to gain a deeper understanding of the necessary outcomes for this section.
- Encourage students to update the bookmark created at the first of the section. Use the information there to work through the practice questions.
- Students should also use graphing technology to graph each function. The visual representation will greatly assist students in their understanding of the concepts in this section.
- For #5, encourage students to try two methods of graphing—transform the graph directly and map individual points.
- Allow students to work in pairs for the Apply questions. They may choose to complete different questions and then share their answers. When students have completed all assigned questions, encourage various students to present the solutions to the class using the whiteboard, poster paper, or another method of choice.
- Provide **Master 3 Centimetre Grid paper** to students to help them in sketching graphs.
- Provide **BLM 2–2 Section 2.1 Extra Practice** to students who would benefit from more practice.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–9, 10a), b), and 18. Students who have no problems with these questions can go on to the remaining questions.	• Ensure students know how to locate the table of values in their calculator. This may assist them in answering #1–3 and 5, depending on their preferred approach. • Questions #2–5 rely on students being able to transfer their earlier knowledge of transformations to how the same parameters affect a radical equation. You may wish to have them explain how the parameters of *a*, *b*, *h*, and *k* affect a graph. • Question #4 gives students the opportunity to demonstrate their understanding by identifying a required parameter and writing the equation that would result in a scribed move. • Questions #6–8 require students to describe/visualize transformations by working from graphs. It may be helpful for them to graph $y = \sqrt{x}$ and compare it to the new graph for changes in position. • You may wish to review how the domain and range help in determining the parameters of an equation. You may wish to show students how to use the values to determine a vertex point and how to sketch the radical graph from this point. • Consider doing #18 as a demonstration if sufficient materials are not available.
Assessment *as* Learning	
Create Connections Have all students complete C1–C4.	• Have students refer to #5f) and how they answered this question before attempting C1. • Allow students to try several different possibilities for C2 before they answer the questions. Do they see any parallels? • If students are unsure about C3, have them sketch a graph. • Suggest that students complete C4 with a classmate.

2.2 Square Root of a Function

***Pre-Calculus 12*, pages 78–89**

Suggested Timing

90–120 min

Materials

- grid paper and ruler
- graphing technology (optional)

Blackline Masters

Master 3 Centimetre Grid Paper
BLM 2–3 Section 2.2 Check Your Understanding Graphs
BLM 2–4 Section 2.2 Extra Practice

Mathematical Processes for Specific Outcomes

RF13 Graph and analyze radical functions (limited to functions involving one radical).

- ✓ Connections (CN)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 5a), b), 6, 8, 9
Typical	#2, 3, 5c), d), 8–11, one of 12 or 13, 15, 16, C1–C4
Extension/Enrichment	#11, 14–19, C1–C4

Planning Notes

Section 2.2 helps students develop an understanding of the relationships between a function and the square root of the function. Students are expected to be able to compare graphs, domains, and ranges for any linear or quadratic function $y = f(x)$ to those for the square root of the same function, $y = \sqrt{f(x)}$. Students should be able to generalize how the domain will change, including indicating any intervals where the domain is undefined. They should also be able to generalize how the range of the square root of a function changes from the original range. The use of technology is suggested to help students develop an understanding of the relationships between the original function and the square root of the function, their graphs, and changes in domain and range. Once they have developed a process for graphing the square root of a function, students should be able to communicate their strategy for graphing $y = \sqrt{f(x)}$ given the graph of $y = f(x)$.

Discuss the outcome and achievement indicators for this section. Encourage students to develop a list of the terms and definitions found within this section.

Present students with two right triangles and ask them to solve for the missing side (one side and then the hypotenuse). Discuss using the exact answer versus an approximate answer. Encourage the students to look for right-triangle formations in the structures within the school. Can they find evidence of the use of right-triangles in construction?

Investigate Related Functions: $y = f(x)$ and $y = \sqrt{f(x)}$

Organize the class into groups of two or three students. Allow students to use graphing technology to produce their graphs.

Have students complete steps 1–4. Discuss the step 4 Reflect and Respond as a class before having groups complete steps 5–8. Have each group design a summary sheet that they can use in their notes or add to a Foldable. Discuss the investigation as a class, asking students to share how the two sets of equations are similar and how they are different. What type of function is $y = 2x + 4$? What type of function is $A = 25 - h^2$? When the square root of each function is graphed, do they produce the same changes to the graph of the original function? Do they produce the same changes to their domain and range? Do they have similar restrictions on the range of $y = \sqrt{2x + 4}$ and $v = \sqrt{25 - h^2}$? What restrictions occur for the range of the square root of a function?

Have students summarize their discoveries in their notes or on their Foldable. Students having difficulty may need some coaching leading up to steps 7 and 8. Ask leading questions:

Step 2:

- Is there an easy way to determine an appropriate domain for the graph of $v = \sqrt{25 - h^2}$?
- Are there any restrictions on the radicand when taking the square root of a number?
- Do these restrictions exist when taking the square root of an algebraic expression?
- Can you write an inequality to express the restrictions on the expression $25 - h^2$?

- What is the least value h can have so that $25 - h^2 \geq 0$ and so that the measures of each side have a positive value?
- What is the greatest value h can have so that $25 - h^2 \geq 0$?
- Do these values help to determine the domain of the function $v = \sqrt{25 - h^2}$?

Step 3:
- What is the formula for the area of a square?
- How can you use this formula to write the area, A, in terms of h instead of v?

Step 4:
- Are the domains of the functions exactly the same?
- Can the length of a side or the area of a square have a negative value?
- What are the minimum and maximum lengths of v?
- What are the minimum and maximum areas?
- What is the relationship between the minimum values of the areas and the length of v?
- What is the relationship between the maximum values of the areas and the length of v?
- How does this information help to determine the range of the function?

Step 5:
- Do the graphs have the same domain?
- How do they differ?
- Does the function $y = \sqrt{2x + 4}$ exist where $2x + 4 < 0$? Why not?
- What relationship exists between the values of y where $x = 0$?
- Does the same relationship exist for y-values sharing other values of x?

For step 6, have students determine their own linear or quadratic function before comparing the original function to the square roots of the functions.

Meeting Student Needs

- Teachers might assign one group to create the right triangle/area diagram on the floor using a scale of 1 m represents 1 cm. Another group may draw the diagram on poster paper using a scale of 2 cm represents 1 cm. The rest of the students draw the diagram in their notebooks. All triangles will likely be different as the only criterion is that the hypotenuse is 5 cm (or to scale). The larger representation may assist some students.
- For part B, have students work with a classmate. Each student creates either $y = 2x + 4$ OR $y = \sqrt{2x + 4}$. Each student can verbally share the graphs and individual thoughts about the connections between the two graphs. Together, they can write a summary of their findings.

Common Errors

- Students may graph functions for values that do not make sense given the context of the question (for example, negative measures of length).

R_x As students work on the investigation, begin a discussion with any group showing graphs that include negative values for v, A, or h. Ask them if it is possible to measure the length of a side or the area of a square using negative values.

- Students may not be able to compare the graphs of a function and its square root.

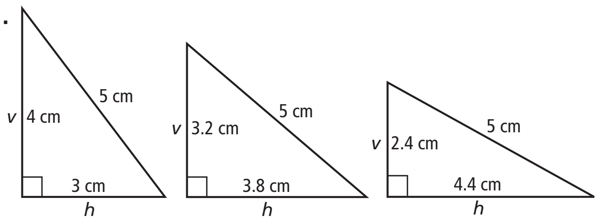
R_x Ask coaching questions such as Are there any points on both graphs that are exactly the same? What is the y-value at these invariant points? What do you notice about the relative position of the graphs between these two invariant points? What do you notice about the relative positions of the graphs when y is greater than 1? What do you notice about the graph of the square root function when y is less than 0?

- Some students may have difficulty creating their own functions.

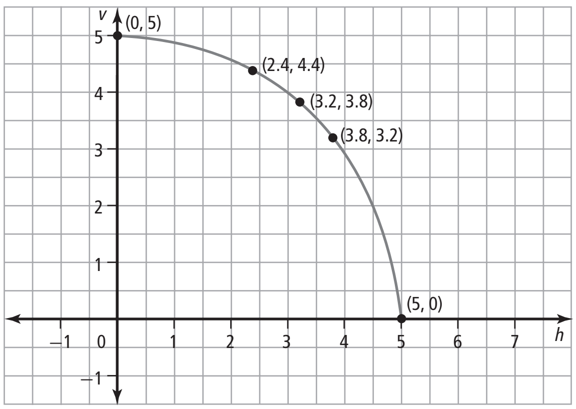
R_x Help students by limiting their choices to linear and quadratic functions. Have them choose one of each and then take the square roots of these functions.

Investigate Related Functions: $y = f(x)$ and $y = \sqrt{f(x)}$

1.

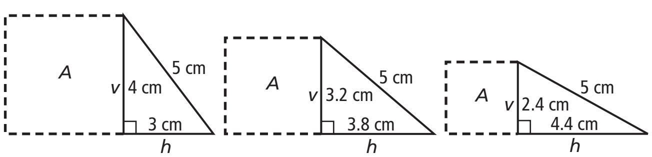


2. a) $v = \sqrt{25 - h^2}$; domain: $\{h \mid 0 \le h \le 5, h \in \mathrm{R}\}$

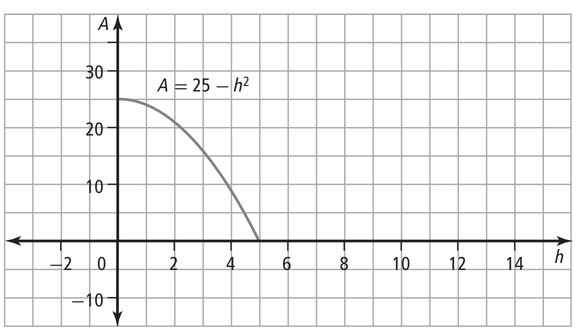


b) Measured values are equal to the calculated values.

3. a)



b)



domain: $\{h \mid 0 \le h \le 5, h \in \mathrm{R}\}$

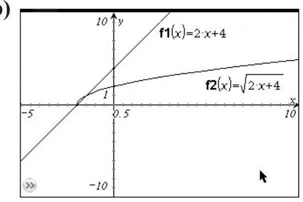
4. a) The equation of one function is the square root of the equation of the other function.

b) The domains are the same.

c) The range of the function $v = \sqrt{25 - h^2}$ is the square root of the range of the function of $A = 25 - h^2$.

5. a) The equation of the second function is the square root of the equation of the first function.

b)



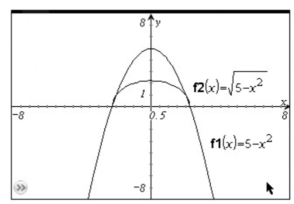
Both graphs have the same x-intercept and rise above the x-axis, for values of x greater than the x-intercept.

c) The y-values for $y = \sqrt{2x + 4}$ are square roots of the y-values for $y = 2x + 4$ for similar x coordinates.

6. a) Example: Consider a comparison of $y = 5 - x^2$ and $y = \sqrt{5 - x^2}$.

x	$y = 5 - x^2$
−4	−11
−2	1
0	5
2	1
4	−11

x	$y = \sqrt{5 - x^2}$
−4	undefined
−2	1
0	$\sqrt{5}$
2	1
4	undefined



For the function $y = 5 - x^2$, the domain is $\{x \mid x \in \mathrm{R}\}$.

The domain of $y = \sqrt{5 - x^2}$ is $\{x \mid -\sqrt{5} \le x \le \sqrt{5}, x \in \mathrm{R}\}$.

The maximum value of $y = \sqrt{5 - x^2}$ is the square root of the maximum value of $y = 5 - x^2$.

7. a) The domain of the square root of the original function exists only where the original function has values of $y \ge 0$ and does not exist where $y < 0$.

b) The range of the square root of the function is the square root of the range of the original function.

c) The range of the square root function does not exist where the range of the original function is less than 0.

8. Example: Invariant points exist for both graphs where $y = 0$ and $y = 1$.The graph for the square root of the original function does not exist where the graph of the original function is below the x-axis. The square root graph is above the graph of the original function over the interval $0 \le y \le 1$ and below it where $y > 1$. The maximum values of the square root graph are the square roots of the maximum values of the original graph. As well, any y-value of the square root graph is the square root of any y-value of the original graph that shares the same x-coordinate.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• If students are having difficulty finding similarities and differences between the equations, have them graph them and work from the visual. That may make it easier to identify the domain and range. • Give students hints to look at the intercepts and zeros. • Encourage students to locate the x- and y-intercepts and make suggestions as to how the two graphs are related and how they are different. • Ensure students understand that the domain is not open for every function. Students should be able to explain why.

When discussing the examples with the class, consider having students try to solve the question in the example first, either by themselves or in pairs. Give them a couple of minutes to discuss and explore how they might solve the question before you discuss the solutions.

Example 1

Once the graphs have been sketched, students compare them to look for any shared (invariant) points and intervals where $f(x) < \sqrt{f(x)}$, $\sqrt{f(x)} < f(x)$, and $\sqrt{f(x)}$ is undefined ($f(x) < 0$).

Some students may need guidance to determine which aspects of the graphs they need to examine. Ask

- Are there any points where the graphs intersect?
- What are the y-coordinates at the points of intersection?
- Why do the graphs intersect where $y = 0$? Can you show algebraically that the functions are equal if $y = 0$?
- Why do the graphs intersect where $y = 1$?
- For what values of the function $f(x)$ is the graph of $f(x) = 3 - 2x$ higher than the graph of $y = \sqrt{3 - 2x}$?
- For what values of the function $f(x)$ is the graph of $f(x) = 3 - 2x$ lower than the graph of $y = \sqrt{3 - 2x}$?
- What happens to the graph of $y = \sqrt{3 - 2x}$ where $f(x) < 0$? Why?

When comparing graphs of $f(x)$ and $\sqrt{f(x)}$, students should get in the habit of listing any invariant points, as well as regions where $f(x) < \sqrt{f(x)}$, $\sqrt{f(x)} < f(x)$, and $\sqrt{f(x)}$ is undefined.

Example 2

When determining the domains and ranges of the functions, students should first try to identify any restrictions on the given functions. Ask

- Are there any values of x where the function is undefined? What algebraic operations are not defined for the real numbers?
- Are the square roots of negative values real numbers?
- How can you use these restrictions to determine the domain of the function?
- How does the domain of a function affect its range?
- Within a determined domain, are there any minimum and maximum values of the function?
- How does having minimum and maximum values affect the range of a function?

When comparing the domains, students should be able to make generalizations about where the domains are the same and where they differ. Is the domain of one function included in the domain of the other? Which function has a larger domain? What restrictions on the domain of $\sqrt{f(x)}$ cause it to differ from the domain of $f(x)$?

Example 3

Students should develop a process of sketching graphs similar to the graph in the example by building a framework of points determined by using invariant points; maximum or minimum points within the domain of $\sqrt{f(x)}$; and any key points where $f(x)$ is a perfect square, thus producing an integer value for $\sqrt{f(x)}$. Students should also be aware that the graph of $\sqrt{f(x)}$ produces endpoints at the x-intercepts of the original graph. Students complete the sketch of the graph of $y = \sqrt{f(x)}$ by joining all points with a smooth curve.

Meeting Student Needs

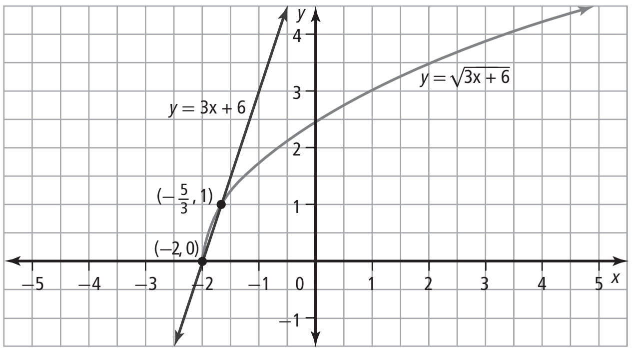
- Have students record their own summary of the Key Ideas, including examples. Have them store the summary for each section in the same location for review purposes.
- Together, students should review the section "Relative Locations of $y = f(x)$ and $y = \sqrt{f(x)}$". Look at the examples and discuss their features until an understanding is developed.
- After Example 2, students should be able to provide a written explanation about the domain and range of $y = f(x)$ and $y = \sqrt{f(x)}$.

- Work through Example 3 as a whole group. Coloured markers or highlighters will be beneficial when working through step-by-step instructions. If possible, use a different colour for each step of the sketch.
- Add the Key Ideas to the bookmark created in Section 2.1.

Answers

Example 1: Your Turn

a)

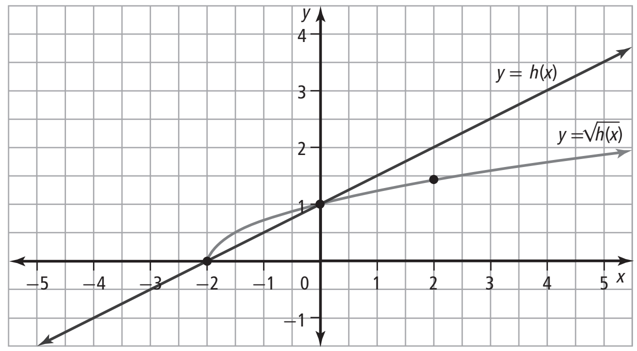


b) For $y = g(x)$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{3x + 6}$, the domain is $\{x \mid x \geq -2, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
Invariant points occur at $(-2, 0)$ and $\left(-\frac{5}{3}, 1\right)$.

Example 2: Your Turn

The domain of function $y = x^2 - 1$ is $\{x \mid x \in \mathrm{R}\}$. The range of the function is $\{y \mid y \geq -1, y \in \mathrm{R}\}$. For the function $y = \sqrt{x^2 - 1}$, the domain is $\{x \mid x \leq -1 \text{ and } x \geq 1, x \in \mathrm{R}\}$ because the original function is greater than or equal to zero where $x \leq -1$ and where $x \geq 1$. The range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$ because the square root is not defined for negative values.

Example 3: Your Turn



Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• A review of invariant points may be necessary. A graph would assist in modelling this concept. This should assist students in finding commonalities. • Question students to ensure they understand why the negative *y*-values are not part of the root of *f*(*x*). • Encourage students to use a table of values to find similarities and how the radical differs.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work in pairs. • Have students identify which method is easier for them to use in solving. • Encourage students to determine the zeros of the function and ask them to explain how these numbers can help determine where the square root of the function is not defined. If they have difficulty doing this, suggest they try numbers on either side of the zeros and determine whether they can take the root of the answer.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work in pairs. • Students may struggle knowing where to begin. Suggest they always look for places where $y = 1$ or $y = 0$, since these are invariant points. Sketch these pieces in first. • Ask if students can identify parts of the graph that will not be included (values below the *x*-axis). • Remind students that although they are making a sketch, the values should be reasonable estimated roots of the points on the curve that they can quickly calculate.

Check Your Understanding

Provide **BLM 2–3 Section 2.2 Check Your Understanding Graphs** to assist students to complete #8, 11, and 17.

For #9, have students graph the original function and the square root of the function on the same screen. This will help them to compare the graphs, domains, and ranges.

For #12, what is the distance from the centre of Earth to the surface? What is the total distance from the centre of Earth to the point on the horizon? How can you express this distance using an algebraic expression? What relationship exists between the sides of the triangle drawn on the diagram? Can you write an equation to express this relationship for d in terms of the variable h?

In #16, students may have difficulty recognizing that some of the transformations apply only to the function $f(x)$ and therefore transform the point $(-24, 12)$. Once the value of the radicand is determined, students may be expected to transform the new y-value indicated as transformations of the square root. Suggest to students that they first list and apply transformations to the ordered pair, indicated as transformations to the radicand. Then, using the y-value of the new ordered pair, conduct transformations on that value as listed in the form of transformations of the square root.

Meeting Student Needs

- Encourage students to try the Project Corner on page 89 in the student resource. You may wish to have students work with a classmate and share their methods. This will assist students when working on the Unit 1 Project.
- Students should complete the checklist of understanding provided in the opener of this section. This will guide them when deciding which questions they need to practice to gain a deeper understanding of the necessary outcomes for this section.
- Students should use graphing technology to provide visual assistance for questions. Some students would benefit from the graphs being projected onto an overhead screen as the larger images allow for more details to be illustrated.
- Provide **BLM 2–4 Section 2.2 Extra Practice** to students who would benefit from more practice.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–4, 5a), b), 6, 8, and 9. Students who have no problems with these questions can go on to the remaining questions.	• A good prompt for #2 is #1. Coach students through #2f) if they have difficulty generalizing the information. • Assign #4 before doing #3 if students are having difficulty interpreting graphs. Coach them through how the square root graph changes the graph in #4 to assist them in completing #3. Suggest they look for similar graphs in #3 as the one they graphed in #4. • Assign #5 and 6 only if students have demonstrated an understanding of the graphs and how they link to their respective equations. Use the graphs they have completed in #3 to review domain and range before beginning #5 and 6. • You may wish to consider assigning #9 before #8, depending on the students comfort level. Encourage them to solve these questions using more than one approach where possible.
Assessment *as* Learning	
Create Connections Have all students complete #C1, C2, and C4.	• Students completing these questions may move on to complete the rest. • Student responses for C1 could be shared with a classmate and edited for correctness. This information would be valuable for their graphic organizer or notebook. • Encourage students to include annotated notes beside their graphs for C2 and C4. These will serve as valuable study notes. Remind students to identify invariant points where they exist and explain what these points mean and how to locate them.

2.3 Solving Radical Equations Graphically

Pre-Calculus 12, pages 90–98

Suggested Timing
90–120 min

Materials
- graphing technology

Blackline Masters
BLM 2–5 Section 2.3 Extra Practice

Mathematical Processes for Specific Outcomes
RF13 Graph and analyze radical functions (limited to functions involving one radical).

- ✓ Connections (CN)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–6, 8–10
Typical	#2–5, 7–9, 11 or 12, 13, 14, C1–C3
Extension/Enrichment	#14–17, C1–C3

Planning Notes

Section 2.3 guides students to develop techniques to solve radical equations. Students solve equations using graphical and algebraic methods. Some students may benefit from a discussion about extraneous roots of a radical equation and why they occur. Students should solve a radical equation containing one radical and then one radical equation containing two radicals. Ensure that the students check all possible solutions.

Investigate Solving Radical Equations Graphically

Organize the class into groups of two or three students. Have each group complete the investigation. Then, gather the class together to discuss their results.

Students using technology may experience difficulty finding the x-intercepts or endpoints of radical functions. Older graphing calculators require the students to use lower and upper bounds to determine x-intercepts or zeros. Newer graphing calculators do not have this problem.

In step 2, some students may require help to solve radical equations algebraically. Give them a short tutorial on solving radical equations involving one radical. Solve one equation with the group, and through discussion develop a method they can use to solve radical equations.

Students having difficulty with step 3 may need some help to explore when graphical and algebraic solutions differ. In the whole class discussion, have groups share their answers with the class. Some students may remember from previous courses that extraneous roots may exist when solving radical equations algebraically. How does a graphing calculator show extraneous roots? Using an example such as $\sqrt{x} = -6$, have students solve the equation graphically first, and then algebraically. Discuss their results before and after they check their algebraic solutions by substitution.

Have students discuss step 4 in class, making sure that each student has an opportunity to share and explain their preference to at least one classmate.

Meeting Student Needs

- Students can work in small groups. Research may be necessary using computers. Allow students to access any technology requested.
- Within the group, students should individually solve the equation algebraically then compare the method(s) used.
- Ensure that students take time to write a response for #3 and #4.

Common Errors

- Students may have a graphing calculator that will not determine x-intercepts of a radical function.

R_x Use the table feature of the graphing calculator to determine the value of x where $y = 0$.

- Students may not recognize extraneous roots of radical equations.

R_x Have students verify their algebraic solutions by substitution.

Answers

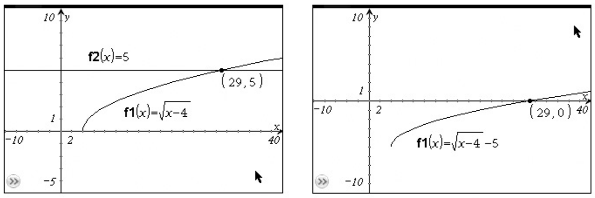
Investigate Solving Radical Equations Graphically

1. Example:

a) There is more than one method to solve the equations graphically.

b) Graph $y_1 = \sqrt{x - 4}$ and $y_2 = 5$ on the same coordinate plane. Identify points of intersection. Check your solution by testing the x-coordinate value in the original equation.
OR
Determine the x-intercepts. Check your x-intercept solution in the original equation.

c)



2. Example:

a) • Square both sides of the equation.
• Simplify both sides of the equation.
• Collect all like terms on one side of the equation.
• Simplify if possible.

b)
$$\begin{aligned}\sqrt{x-4} &= 5\\ \left(\sqrt{x-4}\right)^2 &= 5^2\\ x - 4 &= 25\\ x - 4 + 4 &= 25 + 4\\ x &= 29\end{aligned}$$

c) Verify by substitution.

3. a) The answers are the same.

b) Not always. Some radical equations have extraneous roots when solved algebraically. These roots will not exist on the graphical solution.

4. Example:
Case 1: I prefer algebraic solutions. They take longer to determine but are more complete because they include all solutions including any extraneous roots.
Case 2: I prefer graphical solutions. I can use graphing technology and the solutions do not confuse me by including any extraneous roots.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• You may wish to have students work in pairs for this activity. • This investigation also could be done as a class. The different ways to solve the problem would serve for good class discussion, since some students are more comfortable using calculator graphing whereas others prefer an algebraic approach. • It is not necessary for the class to respond to #3b) correctly immediately. It might serve students better to develop this answer through the examples.

Example 1

After students solve the equation algebraically, have them verify their solutions by substitution. To help students develop a better understanding of the relationship between the solution and the restrictions on the variable, ask

- Does your solution satisfy the restrictions that you determined on the variable?
- Do you think it is possible to have a solution that does not satisfy the restrictions on the variable?
- What do you call a solution that does not satisfy the restrictions on a variable?

Have students graph the radical function. To help them discover the relationship between x-intercepts and algebraic solutions, ask

- What aspect of the graph provides a solution for the radical equation?
- What is the value of y along the x-axis?
- Why do the x-intercepts represent the solution to the equation?
- Are the solutions the same when solved algebraically and graphically?
- From reading your graph, what is the smallest value of the domain?
- How does the domain determined by graphing compare with the restrictions on the variable?

Example 2

It is important that students encounter this situation so that they can learn the importance of verifying their algebraic solutions. Have the class solve the given equation algebraically and then graphically. Discuss any differences that they notice in their solutions. Ask

- What is the difference between the two methods concerning the number of solutions?
- Why do you think there is a difference in the number of solutions?
- Verify both algebraic solutions by substitution. What do you notice about the solution $x = -4$?
- Why do you think -4 is an extraneous root?
- What is the value of $\sqrt{1}$?

- What is the value of x where $x^2 = 1$?
- Why are these answers different?

Students often forget the difference between determining principle square roots, where they must determine the square root of a value resulting in only positive values, and taking the square root of an equation where $x^2 = 1$:

$$\begin{aligned} x^2 &= 1 \\ \sqrt{x^2} &= \sqrt{1} \\ |x| &= 1 \\ x = 1 &\text{ or } x = -1 \end{aligned}$$

Example 3

Have students attempt to solve the equation graphically in two different ways. Ask

- What methods can you use to solve systems of equations graphically?
- Can you solve this radical equation using a system of equations?
- How do you determine the solution to the equation?
- Can you think of a way to solve the radical equation without using a system of equations?
- Is it possible to manipulate the equation so that it is expressed as a single function?
- Can you graph this function? How?
- Which aspect of the graph provides a solution for the radical equation?

Example 4

Have students attempt the example. Discuss with the class their solutions and remind students that they should develop a consistent approach to solving word problems:

- Read the question once to gain a general understanding of the question and its context.
- Read the question again to identify the important information.
- Substitute all known values into any given formula or equation.
- Determine any restrictions on the unknown variables.
- Solve the radical equation either algebraically or graphically. Does the answer seem reasonable? Does it satisfy the restrictions on the variable?
- Check solutions by substitution into the original radical equation.

Meeting Student Needs

- Have students record their own summary of the Key Ideas, including examples. Have them store the summary for each section in the same location for review purposes.
- For Example 3, break students into pairs. One student can graph $f(x) = x + 4 - \sqrt{3x^2 - 5}$ and the other can graph $f(x) = \sqrt{3x^2 - 5} - x - 4$. They can discuss why the graphs look different. Why does this not matter to the question?

Common Errors

- Students may not eliminate extraneous roots as solutions.

R$_x$ Have students verify the solution using substitution into the original radical equation. Discuss why their solution is not correct if it does not satisfy their verification.

- Students may not be able to solve quadratic equations by factoring.

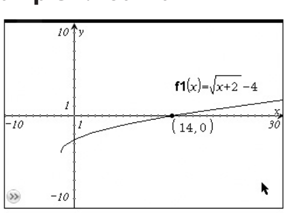
R$_x$ Remind students that they can solve quadratic equations using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Answers

Example 1: Your Turn

a)



b)
$$\begin{aligned} \sqrt{x+2} - 4 &= 0 \\ \sqrt{x+2} &= 4 \\ x + 2 &= 16 \\ x &= 14 \end{aligned}$$

c) The solution to the radical equation ($x = 14$) is equal to the x-intercept of the graph of the radical function.

Left Side	Right Side
$\sqrt{x+2} - 4$	0
$\sqrt{(14)+2} - 4$	
$\sqrt{16} - 4$	
$4 - 4$	
0	

Answers

Example 2: Your Turn

Algebraic solution:

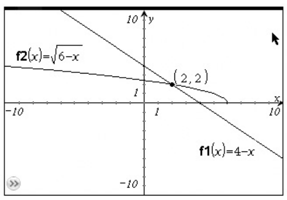
$$\begin{aligned} 4 - x &= \sqrt{6 - x} \\ (4 - x)^2 &= \left(\sqrt{6 - x}\right)^2 \\ x^2 - 8x + 16 &= 6 - x \\ x^2 - 7x + 10 &= 0 \\ (x - 5)(x - 2) &= 0 \end{aligned}$$

Therefore, $x = 5$ and $x = 2$.

Check:

$x = 5$		$x = 2$	
Left Side	**Right Side**	**Left Side**	**Right Side**
$4 - x$	$\sqrt{6 - x}$	$4 - x$	$\sqrt{6 - x}$
$4 - (5)$	$\sqrt{6 - (5)}$	$4 - (2)$	$\sqrt{6 - (2)}$
-1	$\sqrt{1}$	2	$\sqrt{4}$
	1		2

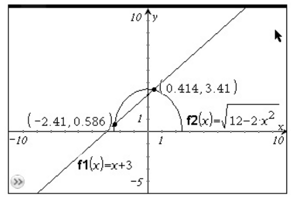
Graphic solution:



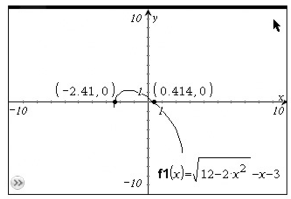
Example 3: Your Turn

The solution is $x \approx -2.41$ and $x \approx 0.41$.

Method 1:



Method 2:



Example 4: Your Turn

approximately 3.1 m/s

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Students should find the link between the roots and the graph fairly easy to connect. You may also wish to review the table function where they can see the zeros as ordered pairs. • It may be necessary to review the Trace function with students, and how to read the values off their graphing calculator or a similar technology program.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work in pairs. • Remind students that squaring often results in the elimination of a lead negative and therefore can give a false or extraneous solution. • Ensure that students understand that they can arrive at extraneous roots algebraically, but this should not be the case graphically.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work in pairs. • Students should be able to solve graphically and algebraically. Coach them on their weaker method. • Remind students how they determine the difference between an exact answer and an approximate answer (no rounding vs. rounding). • Some students may need a review of multiplying binomials and simplifying radicals placed in the quadratic formula. Discuss what a reasonable answer is when working with decimals. • You may need to coach some students through the meaning and the effect of $\|x\|$ as it applies to a restriction on the domain.
Example 4 Have students do the Your Turn related to Example 4.	• You may wish to have students work in pairs. • Ensure students are able to change the range on their calculator for graphing. • Suggest that students identify their known and unknown values when solving a given formula.

Check Your Understanding

For #3, ask students which method they suggest using when the radical equation involves algebraic expressions on both sides of the equal sign. Which method do they suggest when the radical equation involves an algebraic expression on only one side of the equation?

For #5, how do you determine restrictions on the variables?

For #10, how can you determine the number of people affected by the medical condition after 6 years? after 1.5 years? Are 7.4 million people affected after 1.5 years? after 6 years? Does either time value affect 7.4 million or more people? Expressed as an inequality, how much time will it take for the condition to affect 7.4 million or more people?

Students should recognize in #11 that the period of a pendulum is the time required for the pendulum to swing from one side to the other and back.

When solving the equation $\sqrt{x^2} = 9$ in #13, what is the first operation to use to isolate x? Given that $x^2 = 81$, what values do you get when taking the square root of 81? When solving the equation $(\sqrt{x})^2 = 9$, what is the first operation to use to isolate x? What is the value of x when $\sqrt{x} = \sqrt{9}$? How are these two solutions similar? How are they different?

If the real number in #14 is exactly one greater than its square root, ask students what equation can be written to express this relationship.

For #16, given the function $y = \sqrt{-3(x + c)} + c$ and the point $(-1, 1)$ on the graph of the function, what substitutions will you make to solve for c?

For #17, have students consider the expressions to use to express the length of an unknown side and another side that is twice the length.

Meeting Student Needs

- For #1, can the students write each given equation in a second form?
- Ensure that students understand the importance of restrictions to questions #5 to 7.
- For question #10, students may choose to graph $N(t) = 1.3\sqrt{t} + 4.2$ and $f(x) = 7.4$. Can they explain why?
- Question #14 is not as easy as it appears. Students could start by using a guess and check method. In a short time they will understand that the real number is not a whole number. They will then need to create an equation and solve.
- Provide **BLM 2–5 Section 2.3 Extra Practice** to students who would benefit from more practice.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–6 and 8–10. Students who have no problems with these questions can go on to the remaining questions.	• In #1, students demonstrate whether they can manipulate the values on each side of the equation to develop an equivalent one for graphing. This process should be competently demonstrated before you assign any questions (such as application questions) in which students may need to isolate a variable. • For #2–6, emphasize the importance of solving both graphically and algebraically; however, allow students to start each question with the method they feel most comfortable with. Have them use the answers from this method to verify their work in the alternate approach. • You may need to review manipulating variables for #8. Ensure students can verbalize why it is considered an approximate answer.
Assessment *as* Learning	
Create Connections Have all students complete C1–C3.	• Discuss the response to C1 as a class to clarify any misunderstandings. Have students summarize the response in their notebook or graphic organizer. • You may wish to have students work with a classmate to complete C2. Have them compare their equation with that of another pair before proceeding with parts b)–d). • You may wish to have students work in small groups to brainstorm the answer for C3. Have groups share their answers with the class. Clarify any misunderstandings. Ask students to use the examples given to generate one of their own for their graphic organizer.

Chapter 2 Review and Practice Test

2

***Pre-Calculus 12*, pages 99–103**

Suggested Timing
60–90 min

Materials
- grid paper
- graphing technology (optional)

Blackline Masters
Master 3 Centimetre Grid Paper
BLM 2–2 Section 2.1 Extra Practice
BLM 2–4 Section 2.2 Extra Practice
BLM 2–5 Section 2.3 Extra Practice
BLM 2–6 Chapter 2 Study Guide
BLM 2–7 Chapter 2 Test

Planning Notes

Have students who are not confident discuss strategies with you or a classmate. Encourage them to refer to their summary notes, worked examples, and previously completed questions in the related sections of the student resource. You may wish to provide students with **BLM 2–6 Chapter 2 Study Guide**.

The practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1–5, 6a), b), 7b), c), 8–10, 12.

Though a tipi is used to illustrate #18, no tipi will have a base that is a perfect circle. Plains people, like the Blackfoot, use a foundational pod of four poles for greater strength in windy conditions. This results in a base that is ovoid, not circular. Cree and Lakota traditionally use a tripod as a foundation, giving the tipi a more circular base. To avoid insult, specific protocols must be followed regarding the teachings surrounding the building of tipis, the number of poles, who may set them up, and the direction they face.

Meeting Student Needs

- For #16 on the Practice Test, suggest that students locate the origin at the base of the centre of the roof and consider the function formed on the right edge of the roof.
- Students who require more practice on a particular topic may refer to **BLM 2–2 Section 2.1 Extra Practice**, **BLM 2–4 Section 2.2 Extra Practice**, and **BLM 2–5 Section 2.3 Extra Practice**.
- You may wish to provide students with **Master 3 Centimetre Grid Paper**.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 2 Review The Chapter 2 Review provides an opportunity for students to assess themselves by checking their answers against answers in the student resource. **Minimum:** #1–7, 8a), 10a), 12, 13, 14b), 15, 16a), c), d), 17	• Encourage students to use **BLM 2–6 Chapter 2 Study Guide** to identify areas they may need some extra work in before starting the practice test. • Encourage students to revisit questions in sections where they are still experiencing difficulty.
Chapter 2 Study Guide This master will help students identify and locate reinforcement for skills that are developed in this chapter.	• Encourage students to use the practice test as a guide for any areas in which they require further assistance. Students should be able to answer confidently the suggested minimum questions. Encourage students to try additional questions beyond the minimum. • Consider allowing students to use any summative charts, concept maps, or graphic organizers in completing the practice test.
Assessment *of* Learning	
Chapter 2 Test After students complete the practice test, you may wish to use **BLM 2–7 Chapter 2 Test** as a summative assessment.	• Before the test, coach students in areas in which they are having difficulty. • You may wish to have students refer to **BLM 2–6 Chapter 2 Study Guide** and identify areas they need reinforcement in before beginning the chapter test.