McGraw-Hill Ryerson

Pre-Calculus 12

TEACHER'S RESOURCE

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Contents

Blackline Masters

The blackline masters for *Pre-Calculus 12* are available on the *Pre-Calculus 12* Teacher's Resource CD-ROM.

These include generic masters, chapter-specific masters, and a series of Technology Masters. The Technology Masters provide how-to information on how to do certain Examples and questions in *Pre-Calculus 12* using appropriate software.

TIME LINES FOR *PRE-CALCULUS 12*

The chart below shows estimated times, in minutes, for covering the material in *Pre-Calculus 12*. Please note that times will vary depending on your particular class and its individual students.

Also note that there are alternative ways to cover and assess many outcomes. For example, student achievement of unit outcomes can be checked by having students do the unit review and unit test, or, more holistically, by having students complete the unit project, or by doing a combination of these things.

Pre-Calculus 12 Time Frames

	1	2	3	4	5	6	7	8	9	10	11
Unit Opener	30–45			30–45		30–45			45–60		
Chapter Opener	45–60	45–60	45–60	30–45	30–45	45–60	30–45	45–60	30–40	30–45	30–45
Section 1	60–90	90–120	90–120	90–120	135–180	60–90	90–120	60–90	90–120	90–120	120–180
Section 2	90–120	90–120	90–120	90–120	90–135	90–120	90–120	60–90	60–90	90–120	90–120
Section 3	90–120	60–90	60–90	90–120	60–90	60–90	90–120	60–90	90–120	120–180	90–120
Section 4	60–90		60–90	90–120	90–135	60–90		90–120			
Chapter Review and Practice Test	60–90	60–90	90–135	60–135	90–135	60–90	90–120	60–90	90–120	60–90	75–105
Unit Project			60			90–120		60–90			75–105
Unit Review Unit Test			90–135			60–90		60–90			60–90

AN INTRODUCTION TO *PRE-CALCULUS 12* TEACHER'S RESOURCE

Unit Material

Each unit begins with **unit opener** notes that list the General and Specific Outcomes for the unit and provide suggestions for teaching the unit opener material, which includes an introduction to the **unit project**. **Unit reviews** reinforce the chapters in the unit.

Chapter Material

Each chapter starts with a four-page **foldout** that provides

- an overview of the chapter outcomes and concepts, skills, and processes
- assessment suggestions for the use of the prerequisite skills blackline master
- suggested timing for the numbered sections, chapter review, and practice test
- a list of prerequisite skills for each section
- suggested assignments for most students
- a list of related blackline masters available on the CD-ROM
- a list of materials and technology tools needed for each lesson
- the location of Assessment *as* Learning, Assessment *for* Learning, and Assessment *of* Learning opportunities in the chapter

The **Chapter Opener** includes

- a description of the math that will be covered in the chapter
- suggestions for introducing students to the chapter's topics

The opening page of each numbered section expands on the information provided in the foldout.

The **teaching notes** include the following:

- Answers for the **Investigate** and **Your Turn** questions
- **Planning Notes**
- **Assessment** boxes that provide a variety of short assessment strategies and related supported learning
- A question planning chart that suggests the questions to assign. You may want to assign either the Essential or Typical set depending on the level of your students. After students have completed these questions, you may want to assign Extension/Enrichment questions to students who would benefit from greater challenge.
  - Essential: the minimum, usually knowledge and skill questions, that all students should be able to complete to address the outcomes
  - Typical: questions that most students should be fairly successful with; includes some Extend and Create Connections questions
  - Extension/Enrichment: questions that extend the concepts horizontally and provide additional challenge; includes more challenging Extend and Create Connections questions
- Each chapter ends with a **chapter review** and a **practice test**.

CHARACTERISTICS OF MCGRAW-HILL RYERSON'S *PRE-CALCULUS 12* PROGRAM

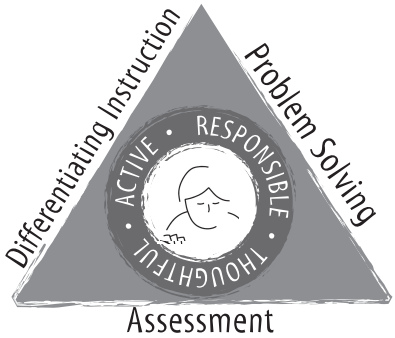
The *Pre-Calculus 12* program was designed for students and educators using the outcomes and achievement indicators published in *The WNCP Common Curriculum Framework for Grades 10–12 Mathematics, January 2008*. This resource package will support educators and all grade 12 students enrolled in Pre-Calculus Grade 12 in Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory.

The *Pre-Calculus 12* design is based on current educational philosophy and pedagogy. The instructional design adheres to the principles set out in the WNCP Common Curriculum Framework, which include beliefs about students and mathematics learning, the affective domain, mathematical processes, the nature of mathematics learning, and instructional focus. Other considerations include chapter sequence, the role of technology, and layout.

Because concern for teachers and students is paramount, the program was developed around two central questions:

1. How would the instructional design benefit/support students?

2. What would teachers require to support their implementation of the new curriculum?



Pedagogical Approach

The program is based on the philosophy that the focus of student learning is to develop a deeper understanding of mathematics and its connection to student lives, careers, and interests. For that reason, the instructional design is based on the premise that students can, and will, take responsibility for their learning, and that they are active and thoughtful learners. With these beliefs in mind, the resource supports a wide range of student interests and learning styles.

Mathematics: Making Links

Throughout the *Pre-Calculus 12* student resource, students are given the opportunity to see the links between real life and mathematics.

- Every unit includes a **unit project** that models mathematics in the real world, engages students' interest, and gives students a meaningful purpose for learning the mathematics presented in the unit. The project is designed to engage students by making links between the mathematics in the unit and students' personal experiences and interests, as well as between mathematics and the real world.
- Most concepts or procedures are introduced in a real-life context.
- The **Career Link** allows students to see how the math they are learning applies to a career in the real world. The visuals in many chapter openers show people performing work related to the math skills in the chapter. These jobs and careers require varying amounts of education, thus connecting more students to how mathematics may be used in their future lives.

Procedural Fluency and Conceptual Understanding

The three-part lesson structure in McGraw-Hill Ryerson's *Pre-Calculus 12* program is designed to engage students in learning that develops their conceptual understanding and procedural fluency. The three parts are described below.

1. *Investigate*

- Each investigation is designed to help students build their own understanding of the new concept by working individually and in groups to explore a mathematical concept or procedure.
- The investigations emphasize personal strategies and alternative methods for solving problems.
- **Reflect and Respond** questions at the end of the Investigate help students to generalize learning about the key concept or methods being investigated.

2. *Link the Ideas*

- Explanations at the beginning of this section help students connect what they did in the Investigate to the **examples** that follow.
- Examples and **solutions** demonstrate how to use the concept explored in the Investigate. Some examples demonstrate how to use commonly available concrete materials and manipulatives. Many include multiple approaches to a solution.
- **Your Turn** questions after each example allow students to check their understanding of concepts.
- **Key Ideas** summarize the key concepts of the lesson.

3. *Check Your Understanding*

- **Practise** allows for the practice of new skills through approximately eight questions.
- **Apply** introduces 10 to 15 problems in a range of real-world contexts.
- **Extend** includes challenging questions that require higher-order thinking. Some may require additional research or connect to one or more other topics within the curriculum.
- **Create Connections** questions require explanation of students' strategies or reasoning. Each question involves the use of communication and connects to other subjects, careers, or the mathematics of another section.
- Most chapters have a **Mini Lab**. This provides a hands-on activity that encourages students to further explore the concepts they are learning.

Problem Solving

Problem solving is central to the McGraw-Hill Ryerson *Pre-Calculus 12* program. Significant emphasis has been placed on incorporating problems that accomplish the following:

- have a range of contexts
- can be solved using different problem solving strategies
- may have multiple solutions

A variety of problem solving experiences are provided throughout the units:

- Each unit is based around a **unit project**. Unit project questions throughout the unit ask students to solve a number of problems as they connect the mathematics they are learning to a real-life context. The unit project is wrapped up at the end of the unit as a performance task.

- Students are frequently asked to discuss their methods for solving problems. Doing so reinforces thinking and helps students realize that there may be multiple methods for solving a problem.
- A problem provides the focus for learning in the **Investigate** at the beginning of each section, often making use of concrete materials.
- Students are challenged to higher levels of thinking and to extend their thinking in the **Extend** and **Create Connections** sections of the exercises and the **Extended Response** section in the practice test.

Differentiating Instruction

Differentiating instruction provides educators with the tools needed to create a learning environment in which students are actively involved and working together. Hands-on activities engage students and help to meet their diverse needs. Significant emphasis has been placed on

- variety—provides opportunities for students to be thoughtful about what and how they learn
- choice—encourages students to develop responsibility by making good personal decisions
- balance—is essential in having students actively involved in their learning. Students' needs are best met when they experience a variety of ways to develop and understand concepts.

Care has been taken in the McGraw-Hill Ryerson *Pre-Calculus 12* program to ensure that all students—including special needs students (with learning disabilities or gifted), students at risk, English language learners, or students from different cultures—can access the mathematics and experience success.

- Visuals that illustrate how to carry out investigations accompany the instructions, where appropriate.
- Visuals and graphics are paired with questions and content in other strategic locations in the student resource.
- New terms are listed in the chapter opener. When these **Key Terms** are first used, they are highlighted in green and defined in the margin
- Key Terms, as well as other useful words, are defined in the **Glossary**.
- The Teacher's Resource provides strategies and blackline master support for accommodating different learning styles, special needs, English language learners, First Nations, Métis, Inuit, Francophone, and at-risk students.
- The Assessment *for* Learning suggestions on the back page of each chapter foldout serve to activate student knowledge and concepts related to the topics in the upcoming chapter.
- The open-ended nature of many of the problems and tasks accommodate the needs of all students by allowing for multiple entry points.
- **Did You Know?** boxes present interesting information related to the math or context of the lesson. Some provide literacy information or connections to other subject areas.

ASSESSMENT

Teachers are encouraged to assess students on an ongoing basis, using Assessment *as* Learning, Assessment *for* Learning, and Assessment *of* Learning. Through the use of self-assessment masters, unit project checklists, and reflection, students are encouraged to assess their own progress, to identify their own strengths and weaknesses, and then to consider what they need to do in order to progress. Teachers are encouraged to coach students through this process.

Many opportunities for Assessment *as* Learning and Assessment *for* Learning assist teachers in identifying ways they can facilitate student progress to a higher level of conceptual and procedural understanding and skill development. Assessment *of* Learning further contributes to growth as teachers and students begin to use this summative assessment as a time for communication and reflections about future goals and strategies for improving.

Assessment *as* Learning (Diagnostic)

These assessment tools include student reflection. They are provided throughout the *Pre-Calculus 12* student resource and Teacher's Resource to assist the teacher in programming by identifying student weaknesses and gaps.

- **Reflect and Respond** questions at the end of each **Investigate** provide early opportunities for students to construct knowledge about the section content.
- The **Create Connections** questions allow students to explore their understandings of a concept.
- The **prerequisite skills**, and **study guide** blackline masters in the Teacher's Resource provide additional support in identifying and facilitating student learning.
- The suggested assignments, questions, and activities in the **Meeting Student Needs** boxes in the Teacher's Resource address a variety of learner needs, including those of English language learners and gifted and enrichment students.
- Diagnostic support in the form of introductory questions designed to open discussion in the classroom and in the form of exploration activities are provided in the Teacher's Resource, where appropriate.

Assessment *for* Learning (Formative)

Formative assessment tools are provided throughout the *Pre-Calculus 12* student resource and the Teacher's Resource.

- The **unit opener** information and related discussion, **chapter opener** and related discussion, and the **prerequisite skills** blackline master in the Teacher's Resource activate learning necessary for students' success in the upcoming unit and chapter.
- The Assessment *for* Learning box on the back of the chapter foldout is designed to provide teachers with an opportunity to activate student knowledge and assess the understanding that students should have to begin the chapter.
- The **Reflect and Respond** questions provide an opportunity to determine students' understanding of concepts through conversations and/or written work.
- The **Your Turn** questions target key skills of a section.
- Students can use the **Practise** assignments in each section to check their understanding.
- The **chapter reviews** and **unit reviews** provide opportunities to assess knowledge/understanding, applications, communications, mental math, and problem solving.

Assessment *of* Learning (Summative)

Summative assessment is provided in the following ways:

- **Practice tests** and **unit tests** are provided at the end of the chapters in the student resource, and **chapter tests** are provided as blackline masters in the Teacher's Resource.
- The **unit project wrap-up** at the end of each unit provides teachers with an opportunity to check whether students have synthesized the concepts and procedures. A **unit project checklist** blackline master helps students assess whether or not they have completed the unit project. A holistic and an ana-holistic rubric for the unit projects are included in the Teacher's Resource.

Teachers are encouraged to use alternative assessments beyond formal testing. For example, student work on the unit project displays how well a student understands mathematical concepts and processes.

Portfolio Assessment

Student-selected portfolios provide a powerful platform for assessing students' mathematical thinking. Portfolios provide the following benefits:

- help teachers assess students' growth and mathematical understanding
- give insight into students' self-awareness about their own progress
- help parents/guardians understand their child's growth

Pre-Calculus 12 has many components that provide ideal portfolio items. Including any or all of the following chapter items is a non-threatening, formative way to gain insight into students' progress:

- student responses to the chapter and unit openers
- answers to the **Reflect and Respond** questions, which give students early opportunities to construct knowledge about the section content
- answers to the **Create Connections** questions, which allow students to explore their understanding of concepts
- journal responses, which show student understanding of the chapter skills and processes
- student responses to the **unit project** questions

Project Rubrics

Master 1 Holistic Project Rubric and **Master 2 Ana-Holistic Project Rubric** are generic rubrics provided as templates for assessing student work. Teachers are encouraged to work with the class to create a specific rubric for each project using either of these rubrics. These project rubrics highlight the level of development of conceptual and procedural understanding within a particular topic, and provide consistent assessment strategies for multiple approaches and/or for multiple solutions to problems and problem solving.

Teachers are encouraged to develop the rubric with students early in each project. This will help students become active participants in their own assessment and program planning.

Specific holistic rubrics and ana-holistic rubrics for each project are provided on the *McGraw-Hill Ryerson Pre-Calculus 12* web site at www.mcgrawhill.ca/school/learningcentres in the Teacher Learning Centre.

TECHNOLOGY IN THE CLASSROOM

Where appropriate, lessons are designed to provide students with the opportunity to develop their skills in the use of various technologies, but not to rely on this technology to think mathematically. Students are also asked to use the Internet to research information related to problems they are required to solve.

The student resource provides technology learning that matches technology requirements for curriculum expectations and that deepens students' conceptual understanding.

Blackline masters of technology activities are included in the Teacher's Resource when grade-specific outcomes suggest these are needed. The masters include directions for using different softwares and graphing calculators common in many classrooms. These worksheets can easily be used in a computer laboratory.

CAPITALIZING ON DIVERSITY AND REAL LIFE

Throughout the student resource, students are given opportunities to see how mathematics connects to real life by engaging in meaningful problem solving situations. Chapters are introduced with problems that model real life. Visual images used to introduce lessons, as well as those in the exercise sets, depict the cultural diversity within classrooms. Examples of mathematics from various cultures are evident throughout the text. Names used in the lessons and exercises reflect the diversity of Canadian society.

ALTERNATIVE LEARNING ENVIRONMENTS AND HOME CONNECTIONS

The design of the McGraw-Hill Ryerson *Pre-Calculus 12* program recognizes that students' learning in mathematics may take place in a variety of learning environments outside of the traditional classroom. For example, students learn mathematics as they complete their homework, work with parents/guardians, and employ their mathematical skills in everyday life. The following features support learning outside of the traditional classroom:

- **Key Ideas** provide summaries to serve as references for students and parents/guardians when doing homework.
- Visuals and **Key Ideas** allow investigations to be easily followed independently.
- Opportunities for bringing mathematics activities home are provided through **Practise**, **Apply**, **Extend**, and **Create Connections** questions.
- Suggestions for alternative and independent learning are provided as support notes in the Teacher's Resource.

COOPERATIVE LEARNING

There are multiple opportunities throughout the program for teachers to use different types of classroom learning environments and groupings. The investigations at the beginning of each section lend themselves to being completed in groups, but teachers are free to choose class groupings that meet the needs of their students, whether or not they are in a traditional classroom setting. Additional suggestions are also provided in this Teacher's Resource.

Students learn effectively when they are actively engaged in the process of learning. Most sections of *Pre-Calculus 12* begin with a hands-on activity that fosters this approach. These activities are best done through cooperative learning during which students work together—either with a partner or in a small group of three or four—to complete the activity and develop generalizations about the topic or process. For students who are distance or distributed learners, these investigations can be explored independently, allowing for valuable student-based learning.

Group learning such as this is an important aspect of a constructivist educational approach. It encourages interactions and increases chances for students to communicate and learn from each other.[1]

Teachers' Role—In classrooms or alternative learning environments where students are adept at cooperative learning, the teacher becomes the facilitator, guide, and progress monitor. Until students have reached that level of group cooperation, however, the teacher will need to coach them in how to learn cooperatively. This may include

- making sure that the materials are at hand and directions perfectly clear, so that students know what they are doing before starting group work
- carefully structuring activities so that students can work together
- coaching how to provide peer feedback in a way that allows the listener to hear and attend
- constantly monitoring student progress and providing assistance to groups having problems with either group cooperation or the math at hand
- using a discussion board or other medium to facilitate partner or group work for distance or distributed learners

Group Composition—The size of the group may vary from activity to activity. Small-group settings allow students to take risks that they might not take in a whole class.[2] Research suggests that small groups are fertile environments for developing mathematical reasoning.[3]

Results of international studies suggest that groups of mixed ability work well in mathematics classrooms.[4] If your class is new to cooperative learning, you may wish to assign students to groups according to the specific skills of each individual. For example, pair a student who is talkative but weaker in number sense and numeration with a quiet student who is stronger in those areas. Pair a student who is weaker in many parts of mathematics but has excellent spatial sense with a stronger mathematics student who has poor spatial sense. In this way, student strengths and weaknesses complement each other, and peers have a better chance of recognizing the value of working together.

Cooperative Learning Skills—When coaching students about cooperative learning, consider task skills and working relationship skills.

Task Skills	Working Relationship Skills
• following directions • communicating information and ideas • seeking clarification • ensuring that others understand • actively listening to others • staying on task	• encouraging others to contribute • acknowledging and responding to the contributions of others • checking for agreement • disagreeing in an agreeable way • mediating disagreements within the group • sharing • showing appreciation for the efforts of others

[1] Sternberg, R.J., and W.M. Williams, *Educational Psychology* (Boston, MA: Allyn & Bacon, 2002).

[2] Van De Walle, J., *Elementary and Middle School Mathematics: Teaching Developmentally*, 4th ed. (Boston, MA: Addison Wesley Longman, 2000).

[3] Artzt, A.F., and S. Yaloz-Femia, "Mathematical Reasoning During Small-Group Problem Solving," in L. Stiff and F. Curcio (eds.), *Developing Mathematical Reasoning in Grades K–12* (Reston, VA: National Council of Teachers of Mathematics, 1999), 115–26.

[4] Kilpatrick, J., J. Swafford, and B. Findell, *Adding It Up: Helping Children Learn Mathematics* (Washington, DC: National Academy Press, 2001).

Discuss common group roles and how group members can use them. Make sure students understand that the same person can play more than one role.

Role	Job	Sample Comment
Leader	• makes sure the group is on task and everyone is participating • pushes group to come to a decision	Let's do this. Can we decide ...? This is what I think we should do ...
Recorder	• manages materials • writes down data collected or measurements made	This is what I wrote down. Is that what you mean?
Presenter	• presents the group's results and conclusions	This is what the group thinks ...
Organizer	• watches time • keeps on topic • encourages getting the job done	Let's get started. Where should we start? So far we've done the following ... Are we on topic? What else do we need to do?
Clarifier	• checks that members understand and agree	Does everyone understand? So, what I hear you saying is ... Do you mean that ... ?

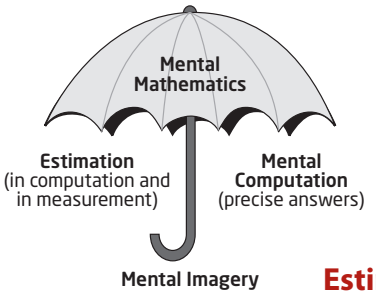
Types of Groups

Three group types are commonly used in the mathematics classroom.

Think/Pair/Share—This consists of having students individually think about a concept and then pick a partner to share their ideas. For example, students might work on the **Create Connections** questions and then choose a partner to discuss the concepts with. Working together, the partners could expand on what they understood individually. In this way, they learn from each other, learn to respect each other's ideas, and learn to listen.

Cooperative Task Group—Task groups of two to four students can work on activities in the investigations at the beginning of each section. As a group, students can share their understanding of what is happening during the activity and how that relates to the mathematics topic, at the same time as they develop group cooperation skills.

Jigsaw—Another common cooperative learning group is called a jigsaw. In this technique, individual group members are responsible for researching and understanding a specific area of information for a project. Individual students then share what they have learned so that the entire group gets information about all areas being studied.



MENTAL MATHEMATICS

A major goal of mathematics instruction for the twenty-first century is for students to make sense of the mathematics in their lives. The development of all areas of mental mathematics is a major contributor to this comfort and understanding. Mental mathematics is the mental manipulation of knowledge dealing with numbers, shapes, and patterns to solve problems.

Estimation

Estimation refers to the approximate answers for calculations, a very practical skill in today's world. The development of estimation skills helps refine mental computation skills, enhances number sense, and fosters confidence in math abilities—all of which are key in problem solving. Over 80% of out-of-school problem solving situations involve mental computation and estimation.[5]

Estimation does not mean guessing at answers. Rather, it involves a host of computational strategies that are selected to suit the numbers involved. The goal is to refine these strategies over time with regular practice, so that estimates become more precise. The ultimate goal is for students to estimate automatically and quickly when faced with a calculation. These estimations allow for recognition of errors on calculator displays, provide learners with a strategy for checking the reasonableness of their calculations, and give students a strategy for finding an answer when only an approximation is necessary.

Mental Imagery

Mental imagery in mathematics refers to the images in the mind when one is doing mathematics. It is this mental representation, or conceptual knowledge, that needs to be developed in all areas of mathematics. Capable math students "see" the math and are able to perform mental manoeuvres in order to make connections and solve problems. These images are formed when students manipulate objects, explore numbers and their meanings, and talk about their learning. Students must be encouraged to look into their mind's eye and "think about their thinking."

Mental Computation

Mental computation refers to an operation used to obtain the precise answer for a calculation. Unlike traditional algorithms, which involve one method of calculation for each operation, mental computations include a number of strategies—often in combination with each other—for finding the exact answer. As with estimation, strategies for mental computation develop in quantity and quality over time. A thorough understanding of, and facility with, mental computation allows students to solve complicated multi-step problems without spending needless time figuring out calculations and is a valuable prerequisite for proficiency with algebra. Students need regular practice in these strategies.

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[5] Reys, B. J., and R.E. Reys, "One Point of View: Mental Computation and Computational Estimation—Their Time Has Come," *Arithmetic Teacher* (Vol. 33, No. 7, 1986), 4–5.

Some Points Regarding Mental Mathematics

- The various estimation and mental calculation strategies must be taught and are best developed in context; opportunities must be provided for regular practice of these strategies. Having students share their various strategies is vital, as it provides possible options for classmates to add to their repertoire.
- Key to the development of skills in mental math is the understanding of place value (number sense) and the number operations. This understanding is enhanced when students make mental math a focus as they calculate.
- Mental math strategies are flexible; the student needs to select one that is appropriate for the numbers in the computation. Practice should be in the form of practising the strategy itself, selecting appropriate strategies for a variety of computation examples, and using the strategies in problem solving situations.
- Sometimes mental math strategies are used in conjunction with paper-and-pencil tasks. The questions are rewritten to make the calculation easier.
- The ultimate goal of mental mathematics is for students to estimate for reasonableness and to look for opportunities to calculate mentally.
- Students need to identify why particular procedures work; they should not be taught computation "tricks" without understanding.

Keep in Mind

Practice in classrooms has traditionally been in the form of asking students to write the answers to questions presented orally. This is particularly challenging for students who are primarily visual learners. Although we are sometimes faced with computations of numbers we cannot see, most often the numbers are written down. This makes it easier to select a strategy. In daily life, we see the numbers when solving written problems (e.g., when checking calculations on a bill or invoice, when determining what to leave for tips, when calculating discounted prices from a price tag). Provide students with mental math practice that is sometimes oral and sometimes visual.

Curriculum Correlation

Unit 1 Transformations and Functions

Chapter 1 Function Transformations

Strand/Outcome	Chapter/Section	Pages
Topic: Relations and Functions		
General Outcome *Develop algebraic and graphical reasoning through the study of relations.*		
Specific Outcomes		
2. Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations. [C, CN, R, V]	1.1 Unit 1 Project	pp. 6–15, 56, 58–59, 158, 160–161 pp. 3, 157
3. Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations. [C, CN, R, V]	1.2 Unit 1 Project	pp. 16–31, 56, 58–59, 158, 160–161 pp. 3, 157
4. Apply translations and stretches to the graphs and equations of functions. [C, CN, R, V]	1.3 Unit 1 Project	pp. 32–43, 57–59, 158, 160–161 pp. 3, 43. 157
5. Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the: • x-axis • y-axis • line $y = x$. [C, CN, R, V]	1.2, 1.4 Unit 1 Project	pp. 16–31, 44–59, 158, 160–161 pp. 3, 157
6. Demonstrate an understanding of inverses of relations. [C, CN, R, V]	1.4 Unit 1 Project	pp. 44–55, 57–59, 158, 160–161 pp. 3, 157

Chapter 2 Radical Functions

Strand/Outcome	Chapter/Section	Pages
Topic: Relations and Functions		
General Outcome *Develop algebraic and graphical reasoning through the study of relations.*		
Specific Outcomes		
2. Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations. [C, CN, R, V]	2.1 Unit 1 Project	pp. 62–77, 99, 102–103, 158–161 pp. 3, 157
3. Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations. [C, CN, R, V]	2.1 Unit 1 Project	pp. 62–77, 99, 102–103, 158–161 pp. 3, 157
4. Apply translations and stretches to the graphs and equations of functions. [C, CN, R, V]	2.1 Unit 1 Project	pp. 62–77, 99, 102–103, 158–161 pp. 3, 157
13. Graph and analyze radical functions (limited to functions involving one radical). [CN, R, T, V]	2.1–2.3 Unit 1 Project	pp. 62–103, 158–161 pp. 3, 89, 157

Chapter 3 Polynomial Functions

Strand/Outcome	Chapter/Section	Pages
Topic: Relations and Functions		
General Outcome *Develop algebraic and graphical reasoning through the study of relations.*		
Specific Outcomes		
11. Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree ≤ 5 with integral coefficients). [C, CN, ME]	3.2–3.3 Unit 1 Project	pp. 118–135, 153–156, 159–161 pp. 3, 157
12. Graph and analyze polynomial functions (limited to polynomial functions of degree ≤ 5). [C, CN, T, V]	3.1, 3.4 Unit 1 Project	pp. 106–117, 136–156, 159–161 pp. 3, 117, 157

Unit 2 Trigonometry

Chapter 4 Trigonometry and the Unit Circle

Strand/Outcome	Chapter/Section	Pages
Topic: Trigonometry		
General Outcome *Develop trigonometric reasoning.*		
Specific Outcomes		
1. Demonstrate an understanding of angles in standard position, expressed in degrees and radians. [CN, ME, R, V]	4.1 Unit 2 Project	pp. 166–179, 215, 218–219, 326, 328–329 pp. 163, 325
2. Develop and apply the equation of the unit circle. [CN, R, V]	4.2 Unit 2 Project	pp. 180–190, 215–216, 218–219, 326, 328–329 pp. 163, 325
3. Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees. [ME, PS, R, T, V]	4.3 Unit 2 Project	pp. 191–205, 216–217, 219, 326, 328–329 pp. 163, 205, 325
5. Solve, algebraically, first and second degree trigonometric equations with the domain expressed in degrees and radians. [CN, PS, R, T, V]	4.4 Unit 2 Project	pp. 206–214, 217–219, 326, 328–329 pp. 163, 325

Chapter 5 Trigonometric Functions and Graphs

Strand/Outcome	Chapter/Section	Pages
Topic: Trigonometry		
General Outcome *Develop trigonometric reasoning.*		
Specific Outcomes		
4. Graph and analyze the trigonometric functions sine and cosine and tangent to solve problems. [CN, PS, T, V]	5.1–5.4 Unit 2 Project	pp. 222–287, 326–329 pp. 163, 281, 325
5. Solve, algebraically and graphically, first and second degree trigonometric equation with domain expressed in degrees and radians. [CN, PS, T, V]	5.4 Unit 2 Project	pp. 266–281, 284–287, 326–329 pp. 163, 281, 325

Chapter 6 Trigonometric Identities

Strand/Outcome	Chapter/Section	Pages
Topic: Trigonometry		
General Outcome *Develop trigonometric reasoning.*		
Specific Outcomes		
6. Prove trigonometric identities, using: • reciprocal identities • quotient identities • Pythagorean identities • sum or difference identities (restricted to sine, cosine and tangent) • double-angle identities (restricted to sine, cosine and tangent). [R, T, V]	6.1–6.3 Unit 2 Project	pp. 290–315, 322–324, 327–329 pp. 163, 308, 325
5. Solve, algebraically, first and second degree trigonometric equation with domain expressed in degrees and radians [CN, PS, T, V]	6.4 Unit 2 Project	pp. 316–324, 327–329 pp. 163, 325

Unit 3 Exponential and Logarithm Functions

Chapter 7 Exponential Functions

Strand/Outcome	Chapter/Section	Pages
Topic: Relations and Functions		
General Outcome *Develop algebraic and graphical reasoning through the study of relations.*		
Specific Outcomes		
2. Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations. [C, CN, R, V]	7.1 Unit 3 Project	pp. 334–345, 366, 368–369, 422, 424–425 pp. 331, 403, 421
3. Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations. [C, CN, R, V]	7.1 Unit 3 Project	pp. 334–345, 366, 368–369, 422, 424–425 pp. 331, 403, 421
4. Apply translations and stretches to the graphs and equations of functions. [C, CN, R, V]	7.1 Unit 3 Project	pp. 334–345, 366, 368–369, 422, 424–425 pp. 331, 403, 421
9. Graph and analyze exponential and logarithmic functions. [C, CN, T, V]	7.1–7.2 Unit 3 Project	pp. 334–357, 366–369, 422, 424–425 pp. 331, 357, 403, 421
10. Solve problems that involve exponential equations. [C, CN, PS, R]	7.3 Unit 3 Project	pp. 358–365, 367–369, 422, 424–425 pp. 331, 403, 421

Chapter 8 Logarithmic Functions

Strand/Outcome	Chapter/Section	Pages
Topic: Relations and Functions		
General Outcome *Develop algebraic and graphical reasoning through the study of relations.*		
Specific Outcomes		
7. Demonstrate an understanding of logarithms. [CN, ME, R]	8.1, 8.2, 8.3, 8.4 Unit 3 Project	pp. 372–420, 422–425 pp. 331, 403, 421
8. Demonstrate an understanding of the product, quotient and power laws of logarithms. [C, CN, R, T]	8.3, 8.4 Unit 3 Project	pp. 392–420, 422–425 pp. 331, 403, 421
9. Graph and analyze exponential and logarithmic functions. [C, CN, T, V]	8.1, 8.2, 8.4 Unit 3 Project	pp. 372–391, 404–420, 422–425 pp. 331, 421
10. Solve problems that involve exponential and logarithmic equations. [C, CN, PS, R]	8.1, 8.2, 8.3, 8.4 Unit 3 Project	pp. 372–420, 422–425 pp. 331, 403, 421

Unit 4 Equations and Functions

Chapter 9 Rational Functions

Strand/Outcome	Chapter/Section	Pages
Topic: Relations and Functions		
General Outcome *Develop algebraic and graphical reasoning through the study of relations.*		
Specific Outcomes		
14. Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials or trinomials). [CN, R, T, V]	9.1–9.3 Unit 4 Project	pp. 430–471, 550, 552–553 pp. 427, 456, 498, 545, 549

Chapter 10 Function Operations

Strand/Outcome	Chapter/Section	Pages
Topic: Relations and Functions		
General Outcome *Develop algebraic and graphical reasoning through the study of relations.*		
Specific Outcomes		
1. Demonstrate an understanding of operations on, and compositions of, functions. [CN, R, T, V]	10.1–10.3 Unit 4 Project	pp. 474–513, 550, 552–553 pp. 427, 456, 498, 545, 549

Chapter 11 Permutations, Combinations, and the Binomial Theorem

Strand/Outcome	Chapter/Section	Pages
Topic: Permutations, Combinations, and Binomial Theorem		
General Outcome *Develop algebraic and reasoning that involves combinatorics.*		
Specific Outcomes		
1. Apply the fundamental counting principle to solve problems. [C, PS, R, V]	11.1 Unit 4 Project	pp. 516–527, 546, 548, 551–553 pp. 427, 456, 498, 545, 549
2. Determine the number of permutations of n elements taken r at a time to solve problems. [C, PS, R, V]	11.1 Unit 4 Project	pp. 518–527, 546, 548, 551–553 pp. 427, 456, 498, 545, 549
3. Determine the number of combinations of n different elements taken r at a time to solve problems. [C, PS, R, V]	11.2 Unit 4 Project	pp. 528–536, 546–548, 551–553 pp. 427, 456, 498, 545, 549
4. Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers). [CN, R, V]	11.3 Unit 4 Project	pp. 537–545, 547–548, 551–553 pp. 427, 456, 498, 545, 549