Name: _____ Date: _____

Unit 3 Test

Multiple Choice

For # 1 to 6, select the best answer.

1. The graph of the function $f(x) = 5^x$ is transformed by a vertical stretch by a factor of 3 about the *x*-axis, a reflection in the *y*-axis, and a horizontal translation 1 unit right. The range of the new function is
   A $\{y \mid y > 0, y \in R\}$
   B $\{y \mid y > 1, y \in R\}$
   C $\{y \mid y > 2, y \in R\}$
   D $\{y \mid y > 3, y \in R\}$

2. The logarithmic form of $y = 2^{x+1} - 5$ is
   A $\log_2 (y + 5) - 1 = x$
   B $\log_2 (x + 5) - 1 = y$
   C $\log_2 (y + 1) - 5 = x$
   D $\log_2 (x + 1) - 5 = y$

3. The equation of the asymptote of the inverse of $f(x) = 3 \log_7 (x - 2) + 1$ is
   A $x = 1$
   B $y = 1$
   C $x = 2$
   D $y = 2$

4. If $5^{2x-3} = 3^x$, then x is
   A $\dfrac{3 \log 5}{2 \log 5 + \log 3}$
   B $\dfrac{3 \log 5}{2 \log 5 - \log 3}$
   C $\dfrac{2 \log 5 - \log 3}{3 \log 5}$
   D $\dfrac{2 \log 5 + \log 3}{3 \log 5}$

5. If the graph of a logarithmic function has a vertical asymptote of $x = 1$ and an *x*-intercept equal to 5, a possible equation is
   A $y = 5 \log_4 (x - 1) - 1$
   B $y = \log_4 (5(x - 1))$
   C $y = \log_4 (x - 1) - 1$
   D $y = \log_4 (x - 1) + 4$

6. The half-life of a certain radioactive substance is 8 days. There are 5 g present initially. The best approximation when there will be 1 g remaining is
   A 10 days
   B 15 days
   C 16 days
   D 19 days

Numerical Response

7. The graph of the exponential function $f(x) = k(a^x)$ passes through the points $(1, 4.5)$ and $(-1, 0.5)$. Determine the values of a and k.

8. Determine the roots of the equation $5 \times 2^{x-1} = 10^{-x+2} - 5$.

9. Evaluate $b^{\log_b 9 + \log_b 3}$.

10. If $\log_2 x = 1$ and $\log_3 y = x$, what is the value of y?

11. If $\log_c x = 2$ and $\log_c y = 3$, what is the value of $\left(\log_c \dfrac{x}{y} \right)^2$?

Written Response

12. The exponential function $f(x) = 2^x$ is transformed to $y = -3f(x + 2)$.
    a) State the equation of the transformed function in exponential form.
    b) Sketch the graph of $y = -3f(x + 2)$ and state the domain, range, and any intercepts.

13. Determine the coordinates of the points of intersection of the graphs of the functions $y = 3^{x-1}$ and $y = \log_3 x + 1$. Explain the significance of the points of intersection.

14. Solve for *x*. Verify your solution.
    a) $5^{x+6} = 625^{x-3}$
    b) $\log_3 (3x + 6) - \log_3 (x - 4) = 2$
    c) $2 \log (x - 1) - \log (x + 1) = \log (x - 2)$

15. Express y as a function of x. State the domain.

   a) $\log (y + 1) = 2 \log x$

   b) $\log_4 (2y) = \log_4 (x + 1) - \log_4 (x - 1)$

   c) $\log (y) + 1 = \log (x - 3)$

16. The population of Calgary was 906 000 on January 1, 1996 and 1 188 000 on January 1, 2006.

   a) If the growth rate of the city can be modelled as an exponential function, determine the equation of the function, $P(t) = ab^t$, where $P(t)$ is population, in thousands, and t is time, in years, since 1996. Express the value of a as a whole number and b to 3 decimal places.

   b) State the average annual growth rate of the city, expressed to the nearest tenth of a percent.

   c) Predict the year the population will first exceed 1.5 million people.

17. The pH of a solution is given by pH $= -\log$ [H+], where [H+] is the concentration of hydrogen ions in moles per litre.

   a) Determine the concentration of hydrogen ions in a solution with pH $= 6.2$.

   b) If the concentration of the hydrogen ions in a solution is 3.2×10^{-6} moles per litre, what is the pH of the solution?

18. An investment of $2500 in a guaranteed investment certificate is paying interest at a rate of 3.25% per year, compounded monthly.

   a) Determine the equation of the exponential function.

   b) Graph the function, stating the domain, range, and any intercepts.

   c) How long will it take for the investment to double in value? Express your answer to the nearest year.