Answers

Chapter 1

1.1 Horizontal and Vertical Translations, pages 1–8

1. **a)** $h = 10, k = 0$ **b)** $h = -2, k = 3$
c) $h = 17, k = 13$ **d)** $h = -1, k = -7$
e) $h = 0, k = 4$

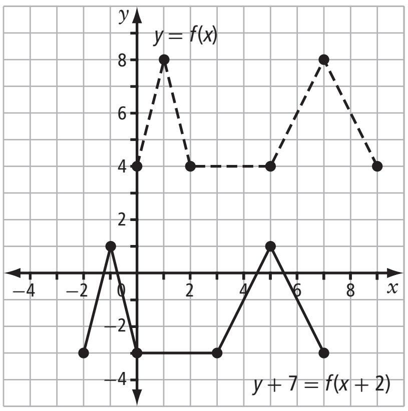
2. **a)** $y + 5 = (x - 2)^2$ **b)** $y + 5 = |x - 2|$
c) $y + 5 = \frac{1}{x - 2}, x \neq 2$

3. **a)** $(x, y) \rightarrow (x + 25, y)$; horizontal translation 25 units to the right

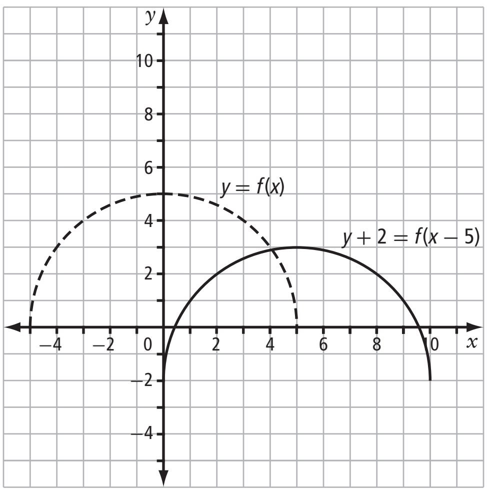
b) $(x, y) \rightarrow (x, y - 50)$; vertical translation 50 units down

c) $(x, y) \rightarrow (x - 20, y + 10)$; horizontal translation 20 units to the left and vertical translation 10 units up

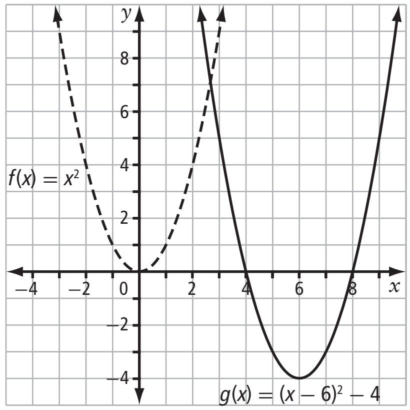
4. **a)** $(x, y) \rightarrow (x - 2, y - 7)$



b) $(x, y) \rightarrow (x + 5, y - 2)$



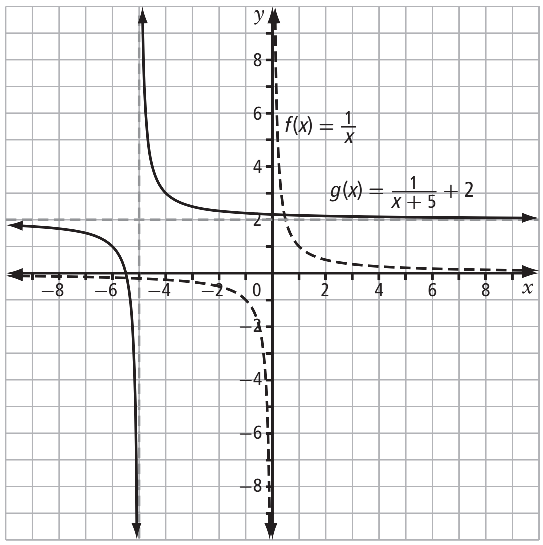
5. **a)** $h = 6, k = -4$ **b)** $(x, y) \rightarrow (x + 6, y - 4)$
c) $y = (x - 6)^2 - 4$
d)



e) (0, 0), (6, −4); vertex has coordinates (h, k)

f) domain of each function: $\{x \mid x \in R\}$; range of $f(x)$: $\{y \mid y \geq 0, y \in R\}$, range of $g(x)$: $\{y \mid y \geq -4, y \in R\}$; in general, the range is $\{y \mid y \geq k, y \in R\}$

6. **a)** $h = -5, k = 2$ **b)** $(x, y) \rightarrow (x - 5, y + 2)$
c) $y = \frac{1}{x + 5} + 2$
d)



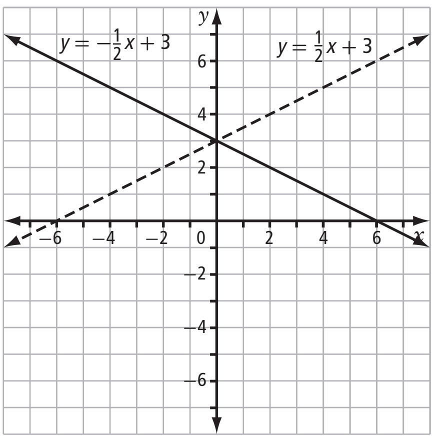
e) For $f(x)$: domain $\{x \mid x \neq 0, x \in R\}$, range $\{y \mid y \neq 0, y \in R\}$, asymptotes $y = 0$, $x = 0$; For $g(x)$: domain $\{x \mid x \neq -5, x \in R\}$, range $\{y \mid y \neq 2, y \in R\}$, asymptotes $y = 2$, $x = -5$; restriction on the domain of $g(x)$ is $x \neq h$, restriction on the range of $g(x)$ is $y \neq k$, asymptotes are at $x = h$ and $y = k$

7.

Function	Horizontal Translation		Vertical Translation	
	to the right 1 unit	to the left 3 units	up 2 units	down 4 units
Quadratic $y = x^2$	$y = (x - 1)^2$ $(x, y) \rightarrow (x + 1, y)$ vertex at (1, 0)	$y = (x + 3)^2$ $(x, y) \rightarrow (x - 3, y)$ vertex at (−3, 0)	$y - 2 = x^2$ $(x, y) \rightarrow (x, y + 2)$ vertex at (0, 2)	$y + 4 = x^2$ $(x, y) \rightarrow (x, y - 4)$ vertex at (0, −4)
Absolute value $y = \|x\|$	$y = \|x - 1\|$ $(x, y) \rightarrow (x + 1, y)$ vertex at (1, 0)	$y = \|x + 3\|$ $(x, y) \rightarrow (x - 3, y)$ vertex at (−3, 0)	$y - 2 = \|x\|$ $(x, y) \rightarrow (x, y + 2)$ vertex at (0, 2)	$y + 4 = \|x\|$ $(x, y) \rightarrow (x, y - 4)$ vertex at (0, −4)
Reciprocal $y = \frac{1}{x}$	$y = \frac{1}{x - 1}$ $(x, y) \rightarrow (x + 1, y)$ vertical asymptote; $x = 1$; horizontal asymptote: $y = 0$	$y = \frac{1}{x + 3}$ $(x, y) \rightarrow (x - 3, y)$ vertical asymptote; $x = -3$; horizontal asymptote: $y = 0$	$y - 2 = \frac{1}{x}$ $(x, y) \rightarrow (x, y + 2)$ vertical asymptote; $x = 0$; horizontal asymptote: $y = 2$	$y + 4 = \frac{1}{x}$ $(x, y) \rightarrow (x, y - 4)$ vertical asymptote; $x = 0$; horizontal asymptote: $y = -4$
Any function $y = f(x)$	$y = f(x - 1)$ $(x, y) \rightarrow (x + 1, y)$	$y = f(x + 3)$ $(x, y) \rightarrow (x - 3, y)$	$y - 2 = f(x)$ $(x, y) \rightarrow (x, y + 2)$	$y + 4 = f(x)$ $(x, y) \rightarrow (x, y - 4)$

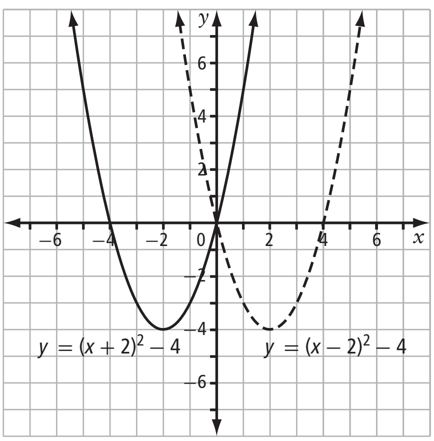
1.2 Reflections and Stretches, pages 9–17

1. a)



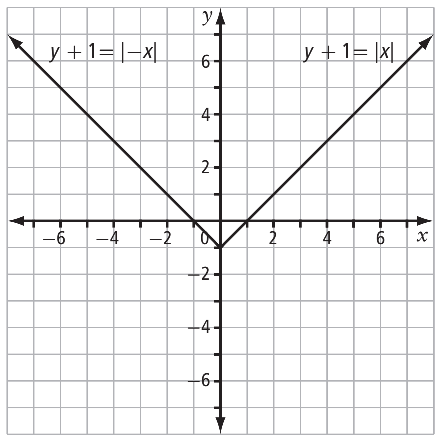
$y = -\frac{1}{2}x + 3$; same y-intercept, different x-intercepts, opposite slopes, same domain and range

b)



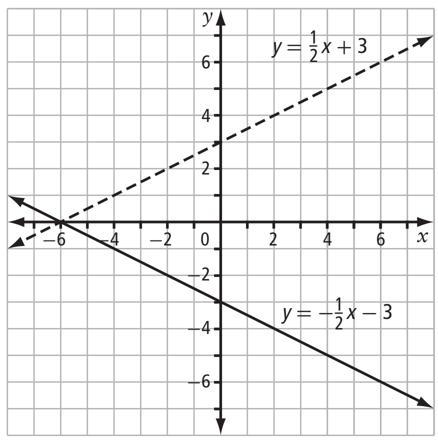
$y + 4 = (x + 2)^2$; same y-intercept, different x-intercepts, same domain and range, same shape, same orientation, vertex has opposite x-coordinate (h) but same y-coordinate (k)

c)

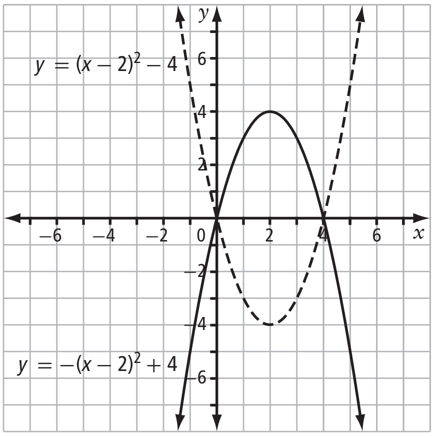


$y + 1 = |-x|$; reflection maps to the original graph

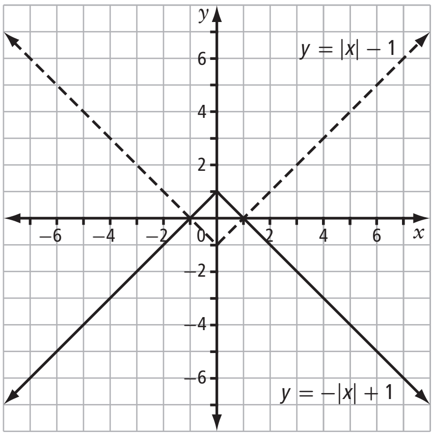
2. a)



$y = -\frac{1}{2}x - 3$; same x-intercept, different y-intercepts, opposite slopes, same domain and range

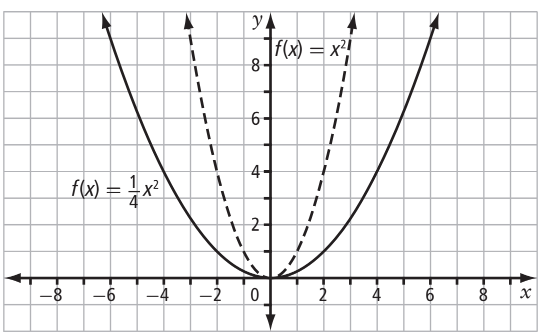
b) 

$y - 4 = -(x - 2)^2$; same y-intercept, same x-intercepts (zeros), different orientation, one has a maximum value and one has a minimum value, same shape, vertex has same x-coordinate (h) and opposite y-coordinate (k)

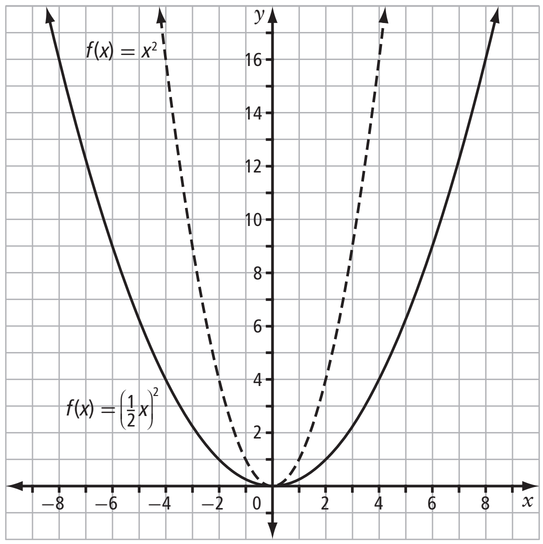
c) 

$y - 1 = -|x|$; same x-intercepts (zeros), different y-intercepts, different orientation, one has a maximum value and one has a minimum value, same shape, vertex has same x-coordinate (h) and opposite y-coordinate (k)

3. a) $(x, y) \rightarrow \left(x, \frac{1}{4}y\right)$; $f(x) = \frac{1}{4}x^2$



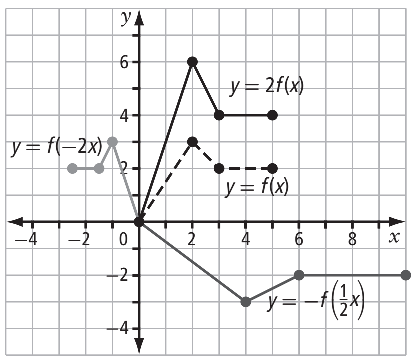
b) $(x, y) \rightarrow (2x, y)$; $f(x) = \left(\frac{1}{2}x\right)^2$



4. a) $\left(\frac{1}{2}x\right)^2 = \left(\frac{1}{2}\right)^2 (x)^2 = \frac{1}{4}x^2$

b) Example: Given $f(x) = x^2$, any horizontal stretch by a factor of p is equivalent to a vertical stretch by a factor of $\frac{1}{p^2}$.

5. a) $y = 2f(x)$ b) $y = -f\left(\frac{1}{2}x\right)$ c) $y = f(-2x)$



6. Answers may vary.

1.3 Combining Transformations, pages 18–25

1. Steps i) and ii) may be reversed and the answer will still be correct.

a) i) reflection in the y-axis, ii) vertical stretch by a factor of 4, iii) translation 5 units down

b) i) horizontal stretch by a factor of $\frac{1}{2}$, ii) reflection in the x-axis, iii) translation 7 units to the left

c) i) horizontal stretch by a factor of 4, ii) vertical stretch by a factor of 1.75, iii) translation 1.5 units to the right

d) i) horizontal stretch by a factor of $\frac{1}{3}$ and reflection in the y-axis, ii) vertical stretch by a factor of $\frac{1}{2}$ and reflection in the x-axis, iii) translation 3 units up and 1 unit to the left

2. **a)** $y + 7 = -f\left(\frac{1}{6}x\right)$

b) $y = \frac{1}{2}|-(x - 3)|$

c) $y + 4 = -\frac{1}{9}(x - 10)^2$ or $y + 4 = -\left[\frac{1}{3}(x - 10)\right]^2$

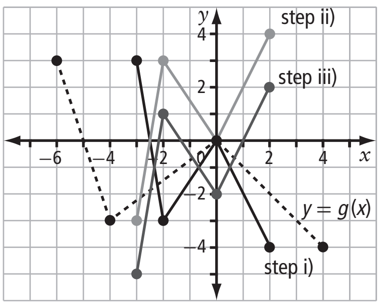
3. **a)** (6, 6)

b) (−11, −10)

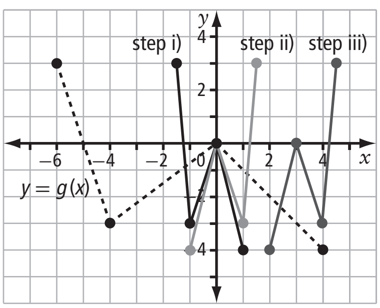
c) (18, 30)

4. (3, −12), (−14, 8), and (24, −24)

5. **a)** i) horizontal stretch by a factor of $\frac{1}{2}$, ii) reflection in the x-axis, iii) translation 2 units down

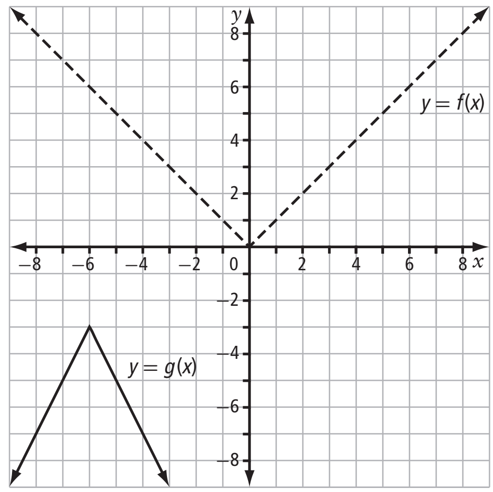


b) i) horizontal stretch by a factor of $\frac{1}{4}$,
ii) reflection in the y-axis,
iii) translation 3 units to the right



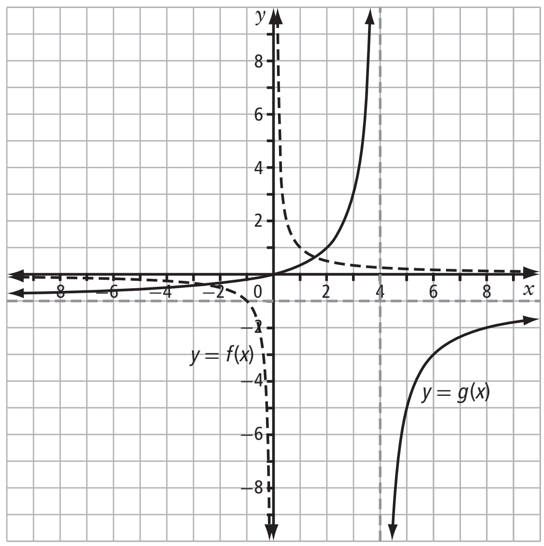
6. **a)** $y = -2|x + 6| - 3$

b)



7. **a)** $y = -\dfrac{1}{\frac{1}{4}(x - 4)} - 1$ or $y = -\dfrac{4}{x - 4} - 1$

b)



8. $y - 7 = -2f(x + 5)$

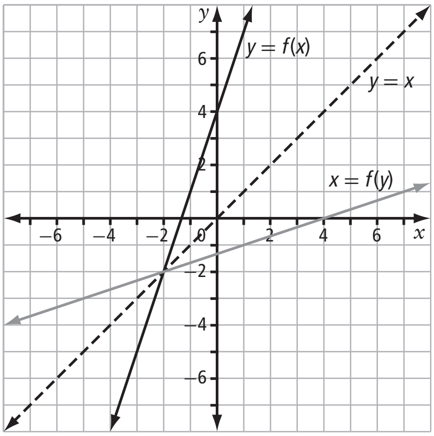
9. $y = 2f\left(-\frac{1}{2}x\right)$

10. $y = f(-2x) + 3$

11. Answers may vary.

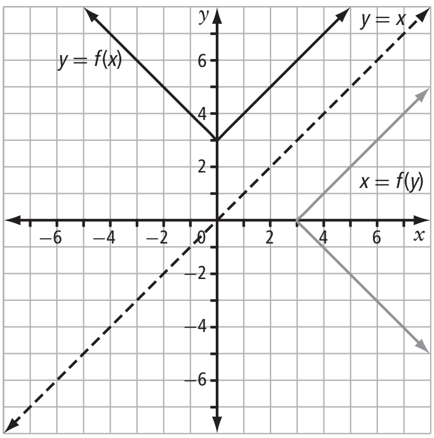
1.4 Inverse of a Relation, pages 26–34

1. a)



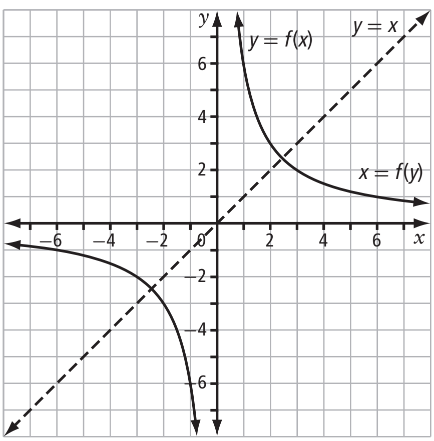
The inverse of $f(x)$ is a function; invariant point at (−2, −2).

b)



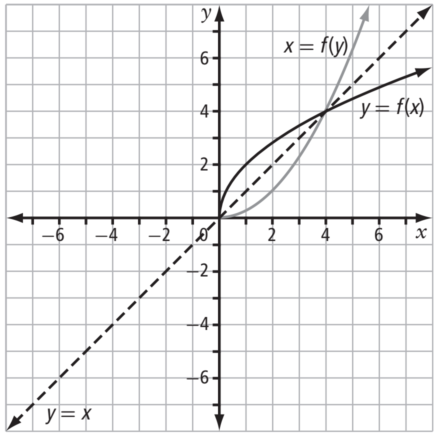
The inverse of $f(x)$ is not a function.

c)



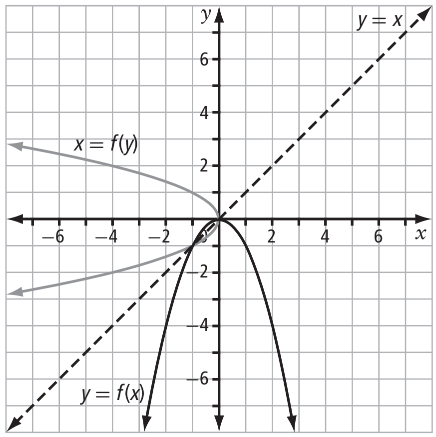
The inverse of $f(x)$ is a function; invariant points at approximately (2.5, 2.5) and (−2.5, −2.5).

d)



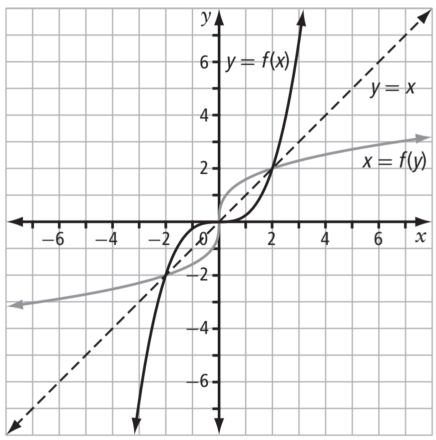
The inverse of $f(x)$ is a function; invariant points at (0, 0) and (4, 4).

e)



The inverse of $f(x)$ is not a function; invariant points at (−1, −1) and (0, 0).

f)



The inverse of $f(x)$ is a function; invariant points at (−2, −2), (0, 0), and (2, 2).

2. **a)** $f^{-1}(x) = x + 4$ **b)** $f^{-1}(x) = -\frac{1}{6}x - \frac{1}{3}$
c) $f^{-1}(x) = \frac{5}{3}x + 5$ **d)** $f^{-1}(x) = 2x - 6$

3. Examples: **a)** $\{x \mid x \geq 2, x \in R\}$ or $\{x \mid x \leq 2, x \in R\}$
b) $\{x \mid x \geq -4, x \in R\}$ or $\{x \mid x \leq -4, x \in R\}$

4. **a)** For $f(x) = -x^2 + 6$, $x \geq 0$, the inverse is $f^{-1}(x) = \sqrt{-(x - 6)}$. For $f(x) = -x^2 + 6$, $x \leq 0$, the inverse is $f^{-1}(x) = -\sqrt{-(x - 6)}$.
b) For $f(x) = \frac{1}{2}x^2 + 4$, $x \geq 0$, the inverse is $f^{-1}(x) = \sqrt{2(x - 4)}$. For $f(x) = \frac{1}{2}x^2 + 4$, $x \leq 0$, the inverse is $f^{-1}(x) = -\sqrt{2(x - 4)}$.

5. $y = \pm\sqrt{x + 2} - 3$

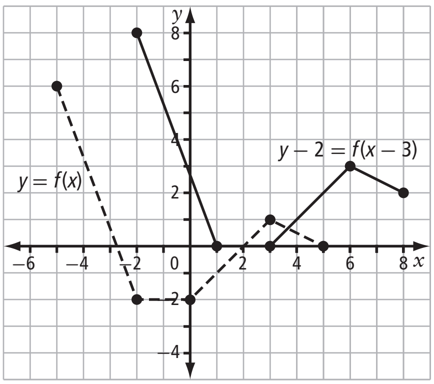
6. **a)** $42 < x < 105$
b) $f^{-1}(x) = \sqrt{\frac{x}{0.01634}} + 26.643$, where x = CRL, in millimetres
c) 14.3 weeks

7. Answers may vary.

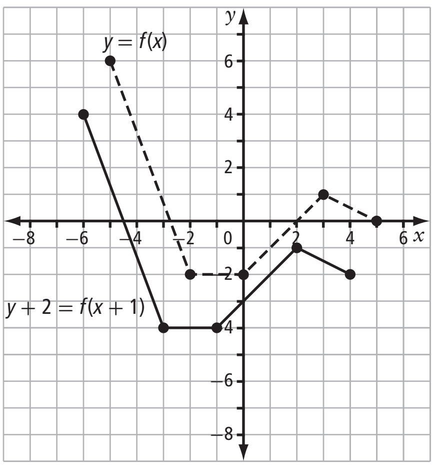
Chapter 1 Review, pages 35–37

1. **a)** $y + 3 = |x - 5|$ **b)** $y - 1 = |x + 4|$

2. **a)**

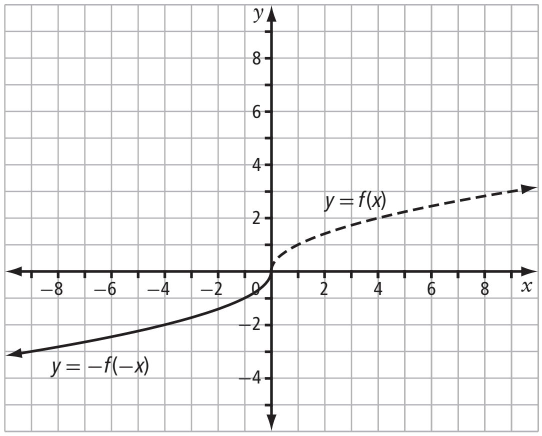


b)

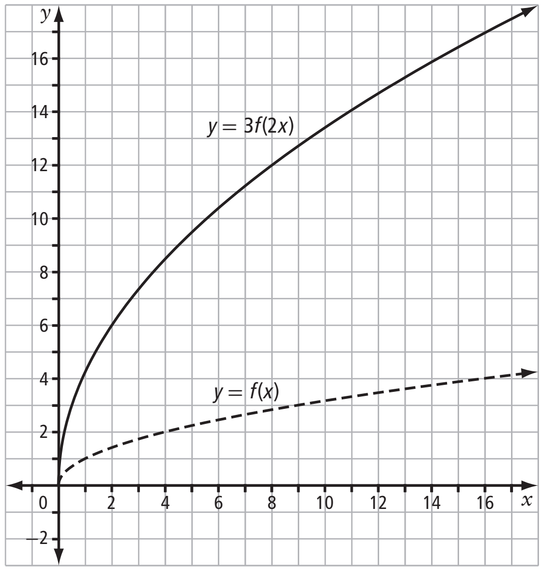


3. **a)** (12, 5) **b)** (−3, −5) **c)** (36, −10)

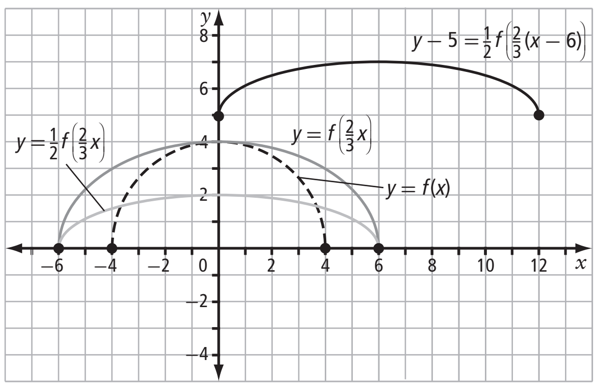
4. **a)** reflection in the y-axis and reflection in the x-axis



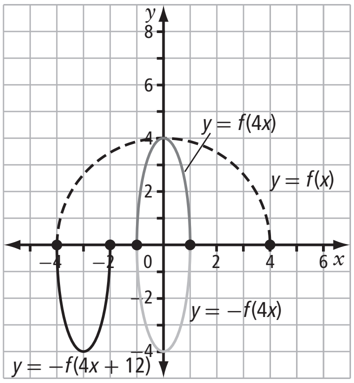
b) horizontal stretch by a factor of $\frac{1}{2}$, vertical stretch by a factor of 3



5. **a)**



b)



6. **a)** $f^{-1}(x) = -2x + 10$

b) Example: restricted domain of $f(x)$: $\{x \mid x \geq 1, x \in R\}, f^{-1}(x) = \sqrt{\frac{1}{2}x} + 1$

Chapter 2

2.1 Radical Functions and Transformations, pages 39–46

1. **a)** vertical stretch by a factor of 3, reflection in the y-axis, translation 4 units left and 2 units down; domain: $\{x \mid x \leq -4, x \in R\}$; range: $\{y \mid y \geq -2, y \in R\}$

b) vertical stretch by a factor of 2, reflection in the x-axis, horizontal stretch by a factor of $\frac{1}{4}$, translation of 3 units right and 5 units up; domain: $\{x \mid x \geq 3, x \in R\}$; range: $\{y \mid y \leq 5, y \in R\}$

c) vertical stretch by a factor of 4, horizontal stretch by a factor of $\frac{1}{5}$, translation of 1 unit left and 4 units down; domain: $\{x \mid x \geq -1, x \in R\}$; range: $\{y \mid y \geq -4, y \in R\}$

d) horizontal stretch by a factor of $\frac{1}{3}$, reflection in the x-axis and y-axis, translation 2 units left; domain: $\{x \mid x \leq -2, x \in R\}$; range: $\{y \mid y \leq 0, y \in R\}$

2. **a)** $y = -3\sqrt{x - 4} - 2$

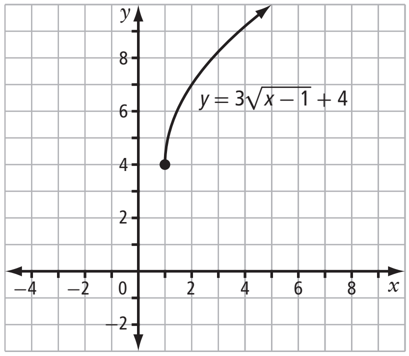
b) $y = \sqrt{-4(x + 5)} + 3$

c) $y = 2\sqrt{\frac{1}{3}(x + 4)} + 1$

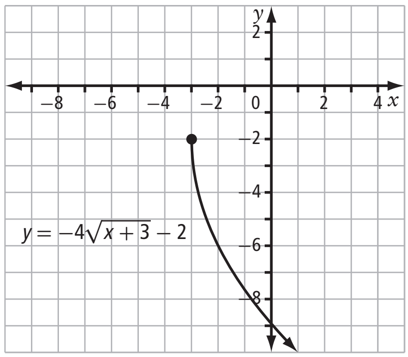
d) $y = -3\sqrt{-2(x + 6)}$

3. **a)** B **b)** C **c)** D **d)** A

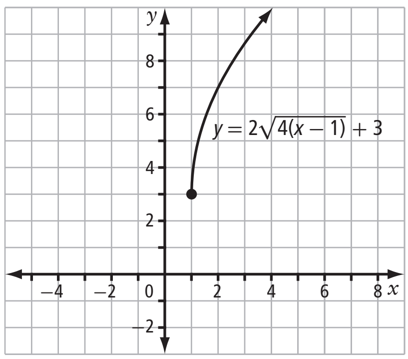
4. **a)**



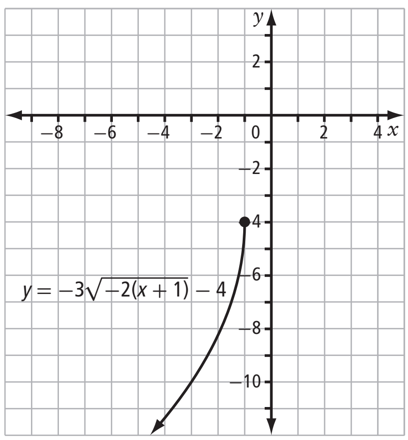
b)



c)



d)



5. a) $y = 3\sqrt{(x - 4)} + 1$ or $y = \sqrt{9(x - 4)} + 1$
b) $y = 2\sqrt{(x + 3)} - 2$ or $y = \sqrt{4(x + 3)} - 2$
c) $y = -4\sqrt{x - 2} + 3$ or $y = -\sqrt{16(x - 2)} + 3$
d) $y = -2\sqrt{x + 3} - 4$ or $y = -\sqrt{4(x + 3)} - 4$

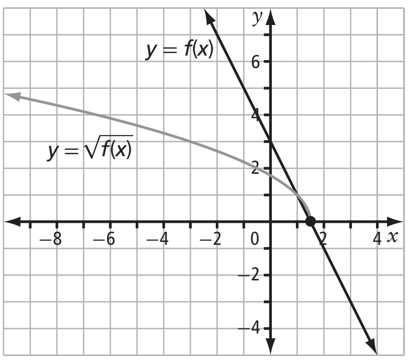
6. a) vertical stretch by a factor of $\frac{1}{2}$ and horizontal stretch by a factor of $\frac{1}{6}$
b) $y = \frac{\sqrt{6}}{2}\sqrt{x}$; vertical stretch by a factor of $\frac{\sqrt{6}}{2}$
c) $y = \sqrt{\frac{3}{2}x}$; horizontal stretch by a factor of $\frac{2}{3}$

7. Yes. You only need to find the translations, h and k, and either the vertical or the horizontal stretch. Example: $y = 3\sqrt{(x - 2)} - 3$ and $y = \sqrt{9(x - 2)} - 3$ are the same function, one with a vertical stretch and the other with a horizontal stretch.

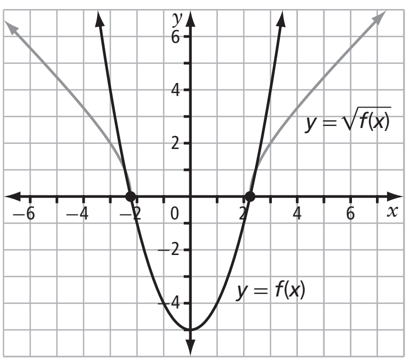
2.2 Square Root of a Function, pages 47–54

1. a) (3, 0) b) (−5, 5) c) (9, 3.9)
d) This is not possible because you cannot take the square root of a negative number.

2. a)



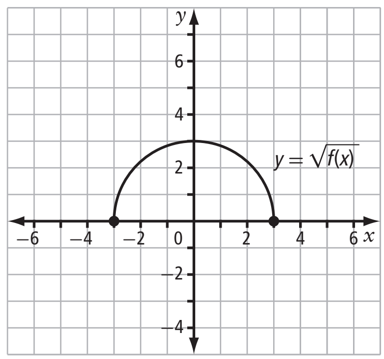
b)



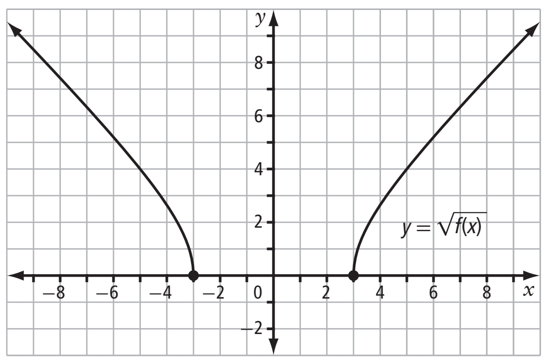
3. a) C b) A c) D d) B

4. a) $f(x)$: domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in R\}$
$\sqrt{f(x)}$: domain: $\{x \mid x \geq 2, x \in R\}$;
range: $\{y \mid y \geq 0, y \in R\}$
b) $f(x)$: domain: $\{x \mid x \in R\}$;
range: $\{y \mid y \geq 2, y \in R\}$
$\sqrt{f(x)}$: domain: $\{x \mid x \in R\}$;
range: $\{y \mid y \geq \sqrt{2}, y \in R\}$
c) $f(x)$: domain: $\{x \mid x \in R\}$;
range: $\{y \mid y \geq -4, y \in R\}$
$\sqrt{f(x)}$: domain: $\{x \mid x \leq -2 \text{ and } x \geq 2, x \in R\}$;
range: $\{y \mid y \geq 0, y \in R\}$
d) $f(x)$: domain: $\{x \mid x \in R\}$;
range: $\{y \mid y \leq 3, y \in R\}$
$\sqrt{f(x)}$: domain: $\{x \mid -\sqrt{3} \leq x \leq \sqrt{3}, x \in R\}$;
range: $\{y \mid 0 \leq y \leq \sqrt{3}, y \in R\}$

5. a)



b)

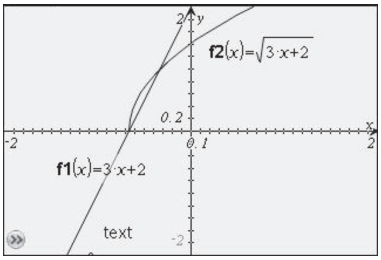


6. The values of 0 and 1 are unchanged when the square root is taken. That is, $1 = \sqrt{1}$ and $0 = \sqrt{0}$.

7. a) When you graph the square root of a function, the graph of $y = \sqrt{f(x)}$ is always above the graph of $y = f(x)$ between the invariant points when $f(x) = 0$ and $f(x) = 1$. This means that the value of $y = \sqrt{f(x)}$ is greater than $y = f(x)$ for the corresponding x-values.

b) Example: He could change the window settings so that the focus is more on the x-values between the invariant points. He could also use the table function on his calculator to create a table of values.

c)

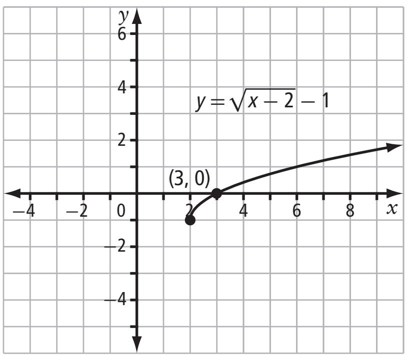


2.3 Solving Radical Equations Graphically, pages 55–62

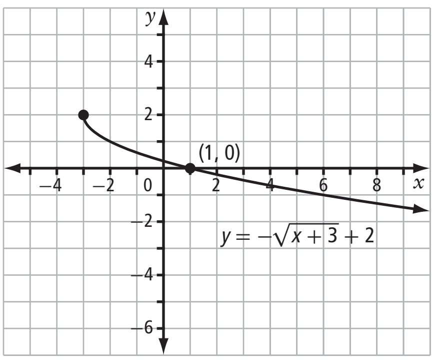
1. **a)** $x = 22$ **b)** $x = 43$

c) $x = 20$ **d)** $x = 3$

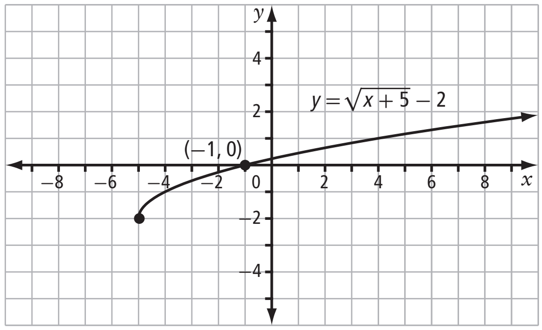
2. **a)** $x = 3$



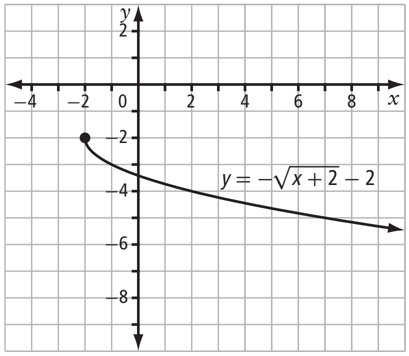
b) $x = 1$



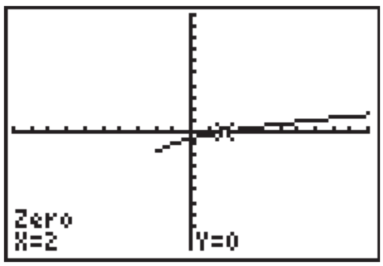
c) $x = -1$



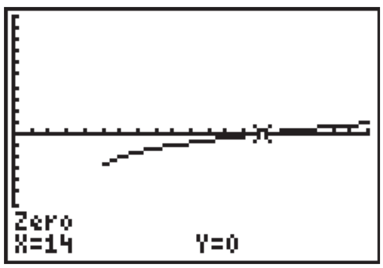
d) no solution



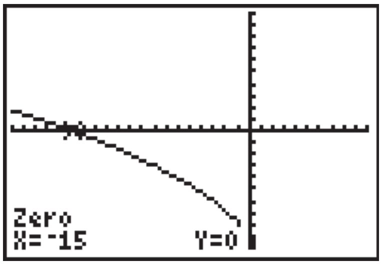
3. **a)** $x \geq 2$; $x = 2$



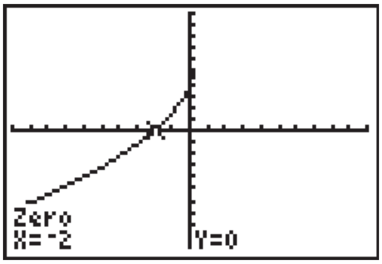
b) $x \geq 5$; $x = 14$



c) $x \le 1$; $x = -15$



d) $x \le 0.25$; $x = -2$



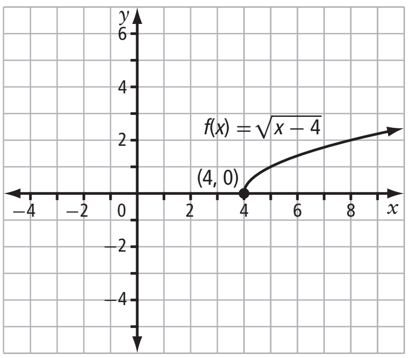
4. **a)** $x \ge -10$; $x = 6$ **b)** $x \le -2$; $x = -2$

c) $x \le 4$; $x = 4$ **d)** $x \le 5.2$; $x = 5$

5. **a)** In solving the equation algebraically you obtain $x = 7$, but when you substitute $x = 7$ into the original equation it does not satisfy the equation.

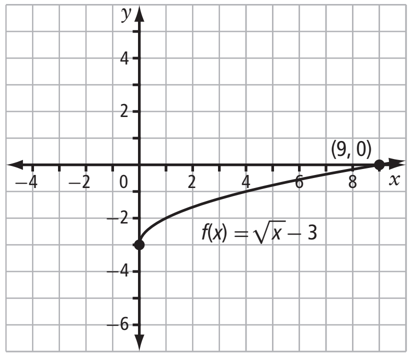
b) If you graph a single function, $y = \sqrt{2x - 5} + 3$, there is no x-intercept. If you graph two functions, $y = \sqrt{2x - 5} + 4$ and $y = 1$, there is no point of intersection.

6. **a)** The graph of $y = \sqrt{x}$ is translated 4 units right.



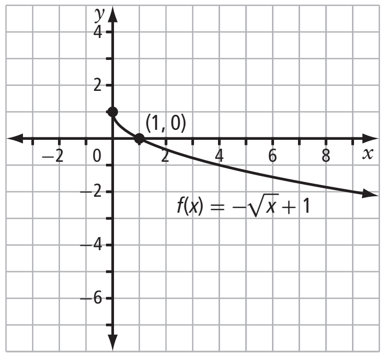
$x = 4$

b) The graph of $y = \sqrt{x}$ is translated 3 units down.



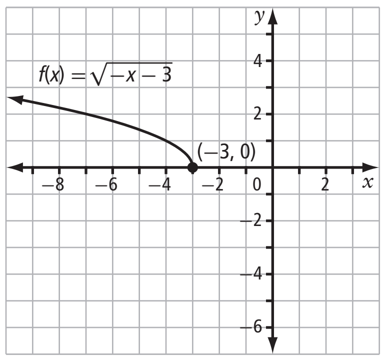
$x = 9$

c) The graph of $y = \sqrt{x}$ is reflected in the x-axis, and translated 1 unit up.



$x = 1$

d) The graph of $y = \sqrt{x}$ is reflected in the y-axis, and then translated 3 units left.



$x = -3$

7. $f(x) = \sqrt{2x} - 4$

8. Her error is that she squared each term. The correct solution is

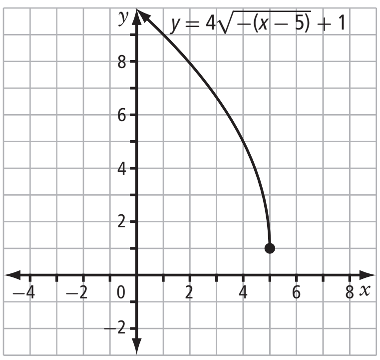
$$\sqrt{3x-1} - 4 = 1$$
$$\sqrt{3x-1} = 5$$
$$(\sqrt{3x-1})^2 = 5^2$$
$$3x - 1 = 25$$
$$3x = 26$$
$$x = 8\frac{2}{3}$$

9. a) It has no solution because $\sqrt{2x+7} \neq -2$.

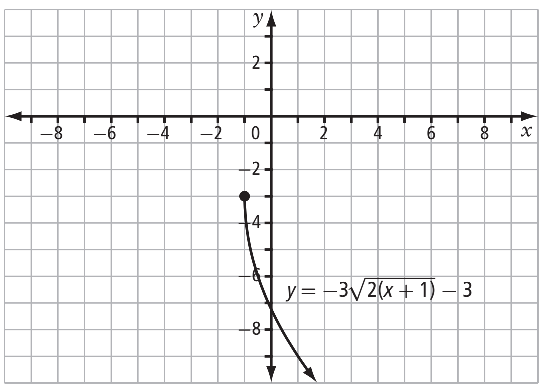
b) Example: $\sqrt{4x+10} + 6 = 2$.

Chapter 2 Review, pages 63–64

1. a) vertical stretch by a factor of 4, reflection in the y-axis, and a translation of 5 units right and 1 unit up



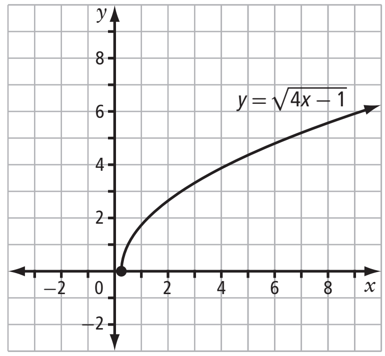
b) vertical stretch by a factor of 3, reflection in the x-axis, horizontal stretch by a factor of 0.5, and a translation of 1 unit left and 3 units down



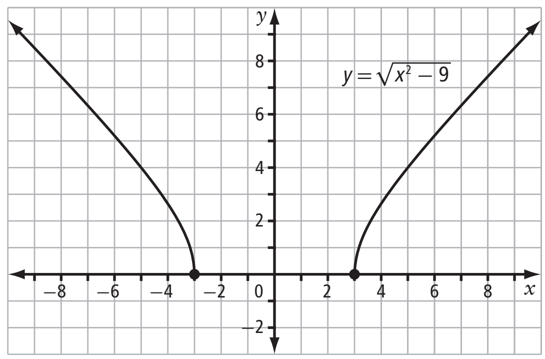
2. a) $y = -4\sqrt{x+5} + 3$; domain: $\{x \mid x \geq -5, x \in \mathrm{R}\}$; range: $\{y \mid y \leq 3, y \in \mathrm{R}\}$

b) $y = 3\sqrt{x-2} - 5$; domain: $\{x \mid x \geq 2, x \in \mathrm{R}\}$; range: $\{y \mid y \geq -5, y \in \mathrm{R}\}$

3. a) domain: $\{x \mid x \geq 0.25, x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$

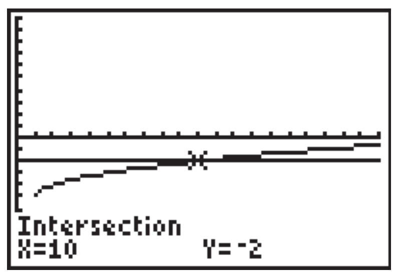


b) domain: $\{x \mid x \leq -3 \text{ and } x \geq 3, x \in \mathrm{R}\}$; range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$

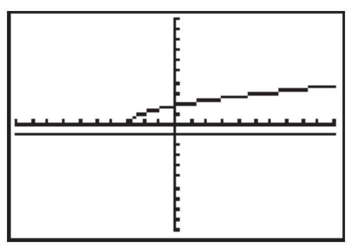


4. a) $x = 11$ b) $x = 6$

5. a) $x \geq 1$; $x = 10$



b) $x \geq -3$; no solution



Chapter 3

3.1 Characteristics of Polynomial Functions, pages 66–77

1. a) Yes; polynomial function of degree 4
   b) No; exponential function
   c) Yes; polynomial function of degree 0
   d) No; function has a variable with a negative exponent

2.

Polynomial Function	Degree	Type	Leading Coefficient	Constant Term
a) $f(x) = 6x^3 - 5x^2 + 2x - 8$	3	Cubic	6	−8
b) $y = -2x^5 + 5x^3 + x^2 + 1$	5	Quintic	−2	1
c) $g(x) = x^3 - 7x^4$	4	Quartic	−7	0
d) $p(x) = 10x - 9$	1	Linear	10	−9
e) $y = -0.5x^2 + 4x + 3$	2	Quadratic	−0.5	3
f) $h(x) = 3x^4 - 8x^3 + x^2 + 2$	4	Quartic	3	2
g) $y = -5$	0	Constant	0	−5

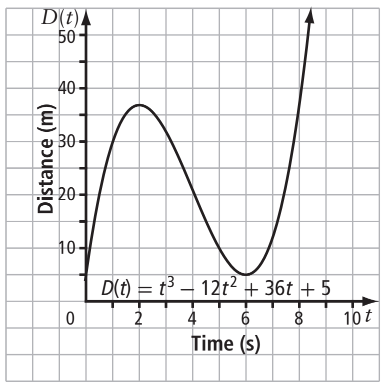
3. a) odd degree; negative leading coefficient; 2 x-intercepts; domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in R\}$
   b) even degree; positive leading coefficient; 2 x-intercepts; domain: $\{x \mid x \in R\}$; range: $\{y \mid y \geq -5, y \in R\}$
   c) odd degree; positive leading coefficient; 5 x-intercepts; domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in R\}$

4. a) degree 5; positive leading coefficient; extends from quadrant III to I; maximum of 5 x-intercepts; y-intercept of 2
   b) degree 5; negative leading coefficient; extends from quadrant II to IV; maximum of 5 x-intercepts; y-intercept of 0
   c) degree 3; negative leading coefficient; extends from quadrant II to IV; maximum of 3 x-intercepts; y-intercept of −6
   d) degree 5; positive leading coefficient; extends from quadrant III to I; maximum of 5 x-intercepts; y-intercept of 3
   e) degree 4; positive leading coefficient; opens upward; maximum of 4 x-intercepts; y-intercept of −1

5. No. Example: $y = 5x^3 + x + 1$ extends from quadrant III to I

6. a) degree 4
   b) leading coefficient: 1.25; constant: −3200; The constant represents the initial cost.
   c) degree 4; positive leading coefficient; opens upward
   d) domain: $\{x \mid x \geq 0, x \in R\}$; it is impossible to have negative skateboard sales
   e) The positive x-intercept represents the break-even point.
   f) $P(12) = 34\ 720$. The profit from the sale of 1200 skateboards is $34 720.

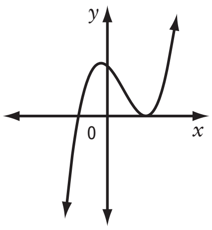
7. a) 3
   b) leading coefficient: 1; constant: 5; The constant represents the initial distance from the tree.
   c) degree 3; positive leading coefficient; extends from quadrant III to quadrant I
   d) domain: $\{t \mid t \geq 0, t \in R\}$; time cannot be negative
   e) $D(7) = 12$; 12 m
   f)



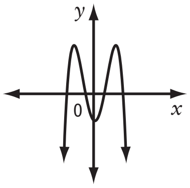
8. a) degree 4; positive leading coefficient; opens upward; extends from quadrant II to I; domain: $\{x \mid x \in R\}$; range: $\{y \mid y \geq 11\ 738, y \in R\}$; The range for the time period $\{x \mid 0 \leq x \leq 15, x \in R\}$ that the population model can be used is $\{y \mid 12\ 000 \leq y \leq 298\ 500, y \in R\}$; no x-intercepts, y-intercept: 12 000
   b) 12 000
   c) 68 000
   d) approximately 13 years from now

9. Examples:

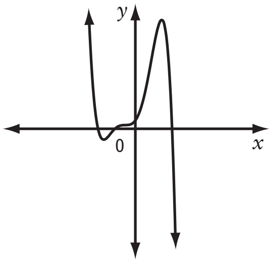
a)



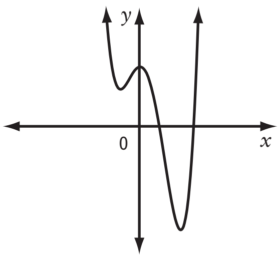
b)



c)



d)



3.2 The Remainder Theorem, pages 78–83

1. **a)** $\frac{x^3 + 3x^2 - 2x + 5}{x + 1} = x^2 + 2x - 4 + \frac{9}{x + 1}$

b) $x \neq -1$

c) $(x + 1)(x^2 + 2x - 4) + 9 = x^3 + 3x^2 - 2x + 5$

2. **a)** $Q(x) = 2x - 7$, $R = 26$

b) $Q(x) = x^2 - 4x + 15$, $R = -70$

c) $Q(x) = 3x^3 + 2x^2 - 6$, $R = 1$

d) $Q(x) = -4x^3 - 12x^2 - 36x - 97$, $R = -298$

3. **a)** $\frac{2x^2 - x + 5}{x + 3} = 2x - 7 + \frac{26}{x + 3}, x \neq -3$

b) $\frac{x^3 - x - 10}{x + 4} = x^2 - 4x + 15 - \frac{70}{x + 4}, x \neq -4$

c) $\frac{3x^4 + 2x^3 - 6x + 1}{x} = 3x^3 + 2x - 6 + \frac{1}{x}, x \neq 0$

d) $\frac{-4x^4 + 11x - 7}{x - 3} = -4x^3 - 12x^2 - 36x - 97 - \frac{298}{x - 3},$
$x \neq 3$

4. **a)** −36 **b)** 8

5. **a)** 22 **b)** −9 **c)** −2 **d)** −4

6. **a)** $k = 1$ **b)** $k = -1$ **c)** $k = -2$ **d)** $k = 2$

7. $m = 3$

8. $x - 2$, $3x + 1$

9. **a)** $(x - a)$ is a factor of $bx^3 + cx^2 + dx + e$.

b) $e + ad + a^2c + a^3b$

3.3 The Factor Theorem, pages 84–90

1. **a)** $x - 2$ **b)** $x + 4$ **c)** $x - b$ **d)** $x + d$

2. **a)** Yes **b)** Yes **c)** No **d)** Yes

3. **a)** No **b)** Yes **c)** Yes **d)** Yes

4. **a)** ±1, ±2, ±4, ±8, ±16

b) ±1, ±2, ±3, ±4, ±6, ±12

c) ±1, ±2, ±4, ±8, ±16, ±32

d) ±1, ±2, ±3, ±6, ±9, ±18

5. **a)** $(x - 2)(x + 2)(x - 1)$

b) $(x - 2)^2(x + 2)$

c) $(x + 1)^3$

d) $(x - 1)(x + 2)(x^2 + x + 1)$

6. **a)** $(x + 2)(x - 3)(x + 3)$

b) $(2x + 1)(x - 1)(2x - 3)$

c) $(x - 2)(2x + 5)(3x - 1)$

d) $(x + 1)^2(x + 3)(x - 4)$

7. **a)** $k = 9$ **b)** $k = 3$

8. $x - 1$, $x + 3$, and $x + 5$

9. $V(x) = (x - 1)(x + 2)(3x - 1)$

10. Example: Start by using the integral zero theorem to check for a first possible integer value. Apply the factor theorem using the value found from the integral zero theorem. Use division to determine the remaining factor. Repeat the process until all factors are found.

3.4 Equations and Graphs of Polynomials Functions, pages 91–102

1. **a)** $x = 0, -2, \frac{1}{2}$

b) $x = -1, 3, 5$

c) $x = 2$

2. **a)** $f(x) = -2(x - 1)(x + 1)(x - 3)$; −1, 1, 3

b) $f(x) = 0.5(x - 2)^2(x + 1)(x + 3)$; −1, −3, 2

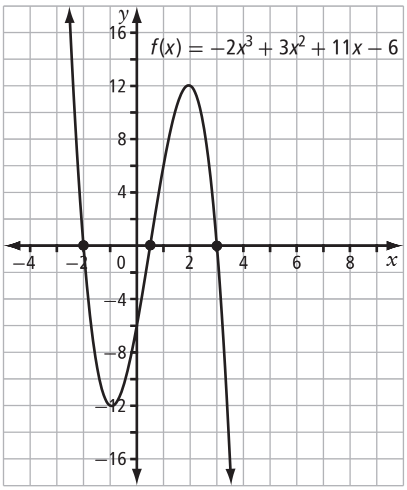
c) $f(x) = -0.2(x - 2)^3(x + 4)^2$; −4, 2

3. **a)** −4 and 5; positive for $-4 < x < 5$; negative for $x < -4$ and $x > 5$; −4 (multiplicity 1) and 5 (multiplicity 3); the function changes sign at both, but is flatter at $x = 5$

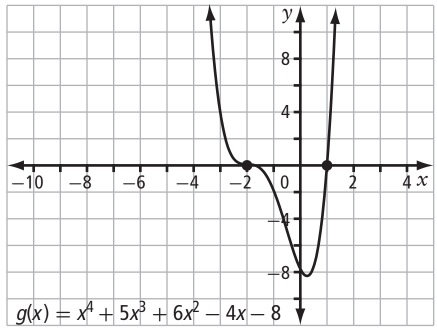
b) −6 and 3; positive for $-6 < x < 3$ and $x > 3$; negative for $x < -6$; −6 (multiplicity 3) and 3 (multiplicity 2); the function changes sign at $x = -6$, but not at $x = 3$

c) −4, −1, and 3; positive for $x < -4$, $-4 < x < -1$, and $x > 3$; negative for $-1 < x < 3$; −4 (multiplicity 2), −1 (multiplicity 1), and 3 (multiplicity 1); the function changes sign at $x = -1$ and at $x = 3$, but not at $x = -4$

4. **a)** x-intercepts: −2, 0.5, 3 (all of multiplicity 1); y-intercept: −6



b) x-intercepts: −2 (multiplicity 3) and 1 (multiplicity 1); y-intercept: −8



5. **a)** $f(x) = (x + 4)(x - 1)(x + 2)$

b) $f(x) = -(2x + 1)(x - 3)(x + 2)$

c) $f(x) = -0.25\,(x + 2)^2(x - 3)^3$

6. **a)** $a = \frac{1}{2}$; vertical stretch by a factor of $\frac{1}{2}$

$b = 3$; horizontal stretch by a factor of $\frac{1}{3}$

$h = -4$; translation of 4 units left

$k = -5$; translation of 5 units down

b) domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in R\}$

7. **a)** $y = -\frac{3}{4}(x - 2)^3(x + 5)$

b) $y = (x + 1)^2(x - 3)(x + 2)^2$

8. 26 ft by 46 ft

9. h and k; these parameters represent the horizontal translation and the vertical translation, respectively, of the graph and do not change its shape or orientation.

Chapter 3 Review, pages 103–107

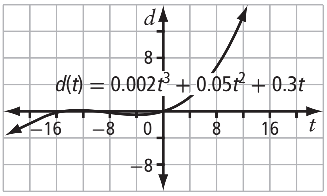
1.

Polynomial Function	Degree	Type	Leading Coefficient	Constant Term
a) $f(x) = -2x^4 - x^3 + 3x - 7$	4	Quartic	−2	−7
b) $y = 3x^5 + 2x^4 - x^3 + 3$	5	Quintic	3	3
c) $g(x) = 0.5x^3 - 8x^2$	3	Cubic	0.5	0
d) $p(x) = 10$	0	Constant	0	10

2. **a)** even degree; negative leading coefficient; 2 x-intercepts; domain: $\{x \mid x \in R\}$; range: $\{y \mid y \leq 19, y \in R\}$

b) odd degree; positive leading coefficient; 3 x-intercepts; domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in R\}$

3. a) degree 3
   b) leading coefficient: 0.002; constant: 0; The constant represents the distance that the boat is from the shore at time 0 s (the initial position of the boat).
   c) degree: 3; positive leading coefficient; extends from quadrant III to I
   d) domain: $\{t \mid t \geq 0, t \in R\}$; it is impossible to have negative time
   e) When $t = 15$, $d(15) = 22.5$. After 15 s, the boat is 22.5 m from the shore.
   f)

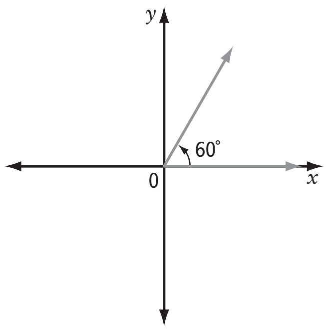


4. a) $\frac{5x^3 - 7x^2 - x + 6}{x - 1} = 5x^2 - 2x - 3 + \frac{3}{x - 1}$
   b) $x \neq 1$
   c) $(x - 1)(5x^2 - 2x - 3) + 3 = 5x^3 - 7x^2 - x + 6$
5. a) $R = 9$ b) $R = 15$
   c) $R = 41$ d) $R = 595$
6. a) $m = 4$ b) 28
7. $P(x) = x^3 + 2x^2 - 15x + 10$
8. a) $x - 7$ b) $x + 6$ c) $x - c$
9. a) Yes b) No
10. a) ±1, ±3, ±9, ±27
    b) ±1, ±2, ±3, ±4, ±6, ±9, ±12, ±18, ±36
11. a) $(x - 3)(x - 2)(x + 1)$
    b) $(x - 4)(x + 2)(3x + 1)$
    c) $(x - 3)(x - 1)(x + 6)(5x + 2)$
    d) $x(x - 2)(x + 2)(2x + 5)$
12. $x - 1$, $x + 2$, and $5x + 2$
13. a) degree 5; negative leading coefficient; −3 (multiplicity 2) and 1 (multiplicity 3); the function changes sign at $x = 1$, but not at $x = -3$; positive for $x < -3$ and $-3 < x < 1$; negative for $x > 1$; $f(x) = -0.25(x + 3)^2(x - 1)^3$
    b) degree 4; positive leading coefficient; −2 (multiplicity 1), −0.5 (multiplicity 1), and 2 (multiplicity 2); the function changes sign at $x = -2$ and at $x = -0.5$, but not at $x = 2$; positive for $x < -2$, $-0.5 < x < 2$, and $x > 2$; negative for $-2 < x < -0.5$; $f(x) = 0.5(x + 2)(2x + 1)(x - 2)^2$
14. a) $a = -2$; vertical stretch by a factor of 2 and reflection in the x-axis
    $b = \frac{1}{3}$; horizontal stretch by a factor of 3
    $h = 1$; translation of 1 unit to the right
    $k = 4$; translation of 4 units up
    b) domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in R\}$
15. $y = -3(x + 2)^2(x - 3)$

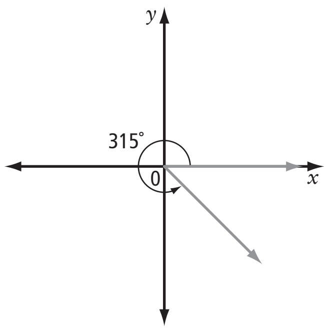
Chapter 4

4.1 Angles and Angle Measure, pages 109–119

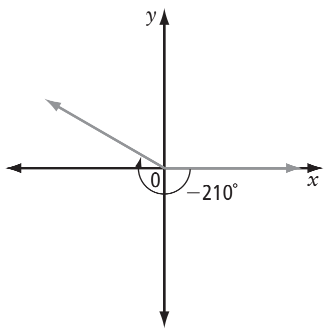
1. a) $\frac{\pi}{3}$



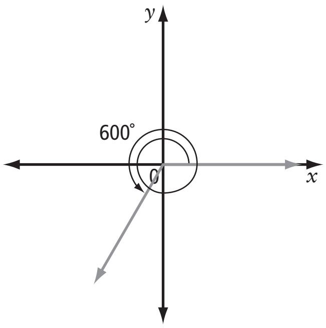
   b) $\frac{7\pi}{4}$



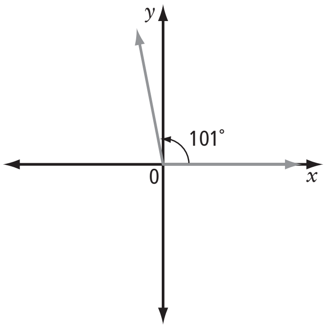
   c) $-\frac{7\pi}{6}$



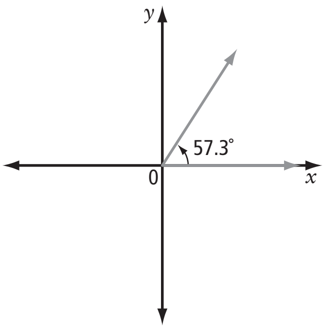
d) $\frac{10\pi}{3}$



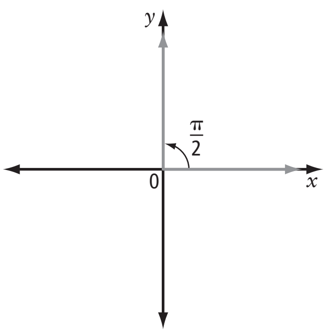
2. **a)** 1.76



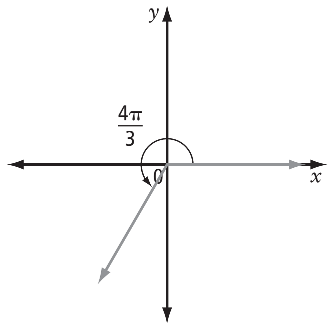
b) 1.00



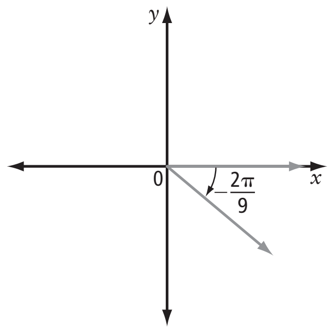
3. **a)** 90°



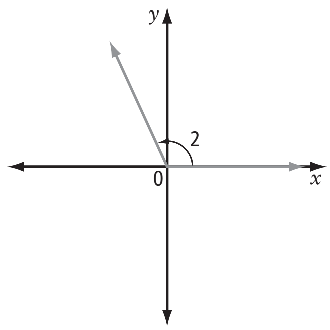
b) 240°



c) −40°



d) 114.59°



4. Examples: **a)** −11°, 709° **b)** −127°, 233°

c) $\frac{8\pi}{3}, -\frac{4\pi}{3}$ **d)** $\frac{\pi}{4}, -\frac{7\pi}{4}$

5. **a)** −465°, −105°, 615°; $255° \pm 360°n$, $n \in N$

b) $-3\pi, -\pi, 3\pi$; $\pi \pm 2\pi n$, $n \in N$

c) $-\frac{7\pi}{6}, \frac{17\pi}{6}, \frac{29\pi}{6}; \frac{5\pi}{6} \pm 2\pi n$, $n \in N$

6. **a)** 41.89 cm **b)** 51.05 mm

7. **a)** Example: Arc length, $a = \theta r$, is dependent on the radius of the circle. For a given central angle, θ, as the radius increases or decreases, the arc length also increases or decreases. Angular velocity, $\omega = \frac{\theta}{t}$, is independent of the radius of the circle. The angular velocity does not change as the radius of the circle increases or decreases.
   b) $\frac{\pi}{10}$ or 0.31 radians/min; $\frac{33\pi}{200}$ or 0.52 m/s
   c) 39π or 122.5 m
   d) approximately 3706 km/h

8. Angles in first rotation:

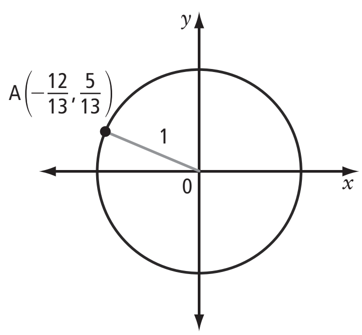
   0°, 0 radians; 30°, $\frac{\pi}{6}$ radians; 45°, $\frac{\pi}{4}$ radians; 60°, $\frac{\pi}{3}$ radians; 90°, $\frac{\pi}{2}$ radians; 120°, $\frac{2\pi}{3}$ radians; 135°, $\frac{3\pi}{4}$ radians; 150°, $\frac{5\pi}{6}$ radians; 180°, π radians; 210°, $\frac{7\pi}{6}$ radians; 225°, $\frac{5\pi}{4}$ radians; 240°, $\frac{4\pi}{3}$ radians; 270°, $\frac{3\pi}{2}$ radians; 300°, $\frac{5\pi}{3}$ radians; 315°, $\frac{7\pi}{4}$ radians; 330°, $\frac{11\pi}{6}$ radians

   Angles in second rotation:

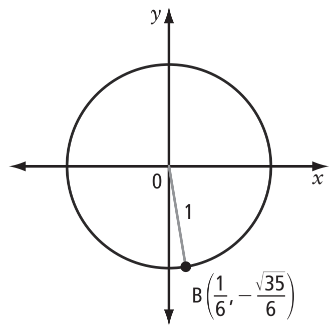
   360°, 2π radians; 390°, $\frac{13\pi}{6}$ radians; 405°, $\frac{9\pi}{4}$ radians; 420°, $\frac{7\pi}{3}$ radians; 450°, $\frac{5\pi}{2}$ radians; 480°, $\frac{8\pi}{3}$ radians; 495°, $\frac{11\pi}{4}$ radians; 510°, $\frac{17\pi}{6}$ radians; 540°, 3π radians; 570°, $\frac{19\pi}{6}$ radians; 585°, $\frac{13\pi}{4}$ radians; 600°, $\frac{10\pi}{3}$ radians; 630°, $\frac{7\pi}{2}$ radians; 660°, $\frac{11\pi}{3}$ radians; 675°, $\frac{15\pi}{4}$ radians; 690°, $\frac{23\pi}{6}$ radians

4.2 The Unit Circle, pages 120–128

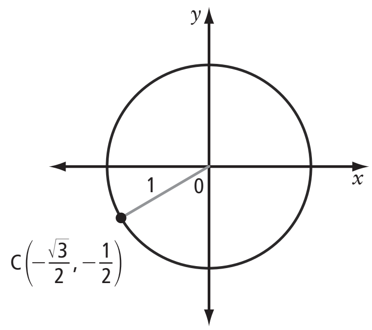
1. **a)** $x^2 + y^2 = 625$ **b)** $x^2 + y^2 = 1.21$
2. **a)** Yes **b)** Yes **c)** No
3. **a)** $-\frac{12}{13}$



   b) $-\frac{\sqrt{35}}{6}$



   c) $-\frac{\sqrt{3}}{2}$



4. **a)** (0, 1) **b)** (1, 0)
   c) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **d)** $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$
   e) $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ **f)** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
5. **a)** π **b)** $\frac{\pi}{6}$
   c) $\frac{3\pi}{4}$ **d)** $\frac{4\pi}{3}$
6. **a)** $\frac{\pi}{2}$ **b)** $\frac{5\pi}{6}$
7. **a)** $x^2 + y^2 = 2.25 \times 10^{16}$
   b) $x^2 + y^2 = 1$
   c) Mars has a larger circle with radius 1.38.
   d) 2.76π or 8.67 radians; 503.7 days

8. $P(0) = (1, 0)$; $P\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$; $P\left(\frac{\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$;
$P\left(\frac{\pi}{3}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; $P\left(\frac{\pi}{2}\right) = (0, 1)$;
$P\left(\frac{2\pi}{3}\right) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; $P\left(\frac{3\pi}{4}\right) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$;
$P\left(\frac{5\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$; $P(\pi) = (-1, 0)$;
$P\left(\frac{7\pi}{6}\right) = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$; $P\left(\frac{5\pi}{4}\right) = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$;
$P\left(\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$; $P\left(\frac{3\pi}{2}\right) = (0, -1)$;
$P\left(\frac{5\pi}{3}\right) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$; $P\left(\frac{7\pi}{4}\right) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$;
$P\left(\frac{11\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$; $P(2\pi) = (1, 0)$

4.3 Trigonometric Ratios, pages 129–137

1. $\sin\theta = -\frac{24}{25}$, $\cos\theta = \frac{7}{25}$, $\tan\theta = -\frac{24}{7}$,
$\csc\theta = -\frac{25}{24}$, $\sec\theta = \frac{25}{7}$, $\cot\theta = -\frac{7}{24}$

2. **a)** − **b)** +
c) + **d)** −

3. **a)** I, III **b)** III
c) I, II **d)** II

4. **a)** $\frac{1}{2}$ **b)** $\frac{1}{\sqrt{2}}$ **c)** 0
d) $\frac{1}{\sqrt{3}}$ **e)** $-\frac{2}{\sqrt{3}}$ **f)** $-\sqrt{2}$

5. **a)** 3.628 **b)** −0.249
c) 2.985 **d)** −1.701

6. **a)** 108°, 288°, 468°, 648°
b) −197°, −163°, 163°, 197°

7. **a)** $\frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$
b) $-\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

8. $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$,
$\sec\theta = -\frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

4.4 Introduction to Trigonometric Equations, pages 138–144

1. **a)** 30°, 150° **b)** $\frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$

2. **a)** 42.8°, 317.2° **b)** 224.9°, 315.1°

3. **a)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** 11.54°, 168.46°, 210°, 330°

4. **a)** $\frac{\pi}{6} + 2\pi n, n \in I$ **b)** $\frac{11\pi}{6} + 2\pi n, n \in I$

5. **a)** $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$ **b)** $\frac{\pi}{4} + \pi n, n \in I$

6. It is not permissible to divide by tan θ since tan θ can equal 0. To solve, it is necessary to factor the expression to obtain (tan θ)(tan θ − 3) = 0. Setting the factor (tan θ) equal to 0 gives two of the solutions. Setting the factor (tan θ − 3) equal to 0 gives the other two solutions.

Correct solution: 0, 1.25, 3.14, 4.39

7. **a)** $0, \frac{\pi}{2}, \frac{3\pi}{2}$
b) $2\pi n, n \in I$; $\frac{\pi}{2} + 2\pi n, n \in I$;
$\frac{3\pi}{2} + 2\pi n, n \in I$; The three expressions cannot be combined in a single expression, because the intervals between solutions in one revolution are not consistent.

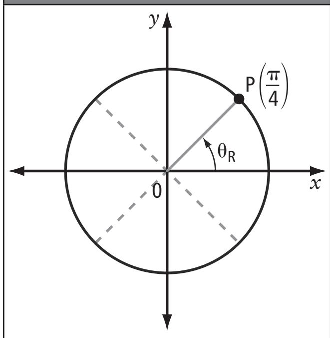
8. No. Example: $n = 1$ does not work.

9. If sin θ < 0, the solutions will be in quadrants III and IV. If csc θ < 0, the solutions will be in quadrants III and IV. If sin θ > 0, the solutions will be in quadrants I and II. If csc θ > 0, the solutions will be in quadrants I and II.

If cos θ < 0, the solutions will be in quadrants II and III. If sec θ < 0, the solutions will be in quadrants II and III. If cos θ > 0, the solutions will be in quadrants I and IV. If sec θ > 0, the solutions will be in quadrants I and IV.

If tan θ < 0, the solutions will be in quadrants II and IV. If cot θ < 0, the solutions will be in quadrants II and IV. If tan θ > 0, the solutions will be in quadrants I and III. If cot θ > 0, the solutions will be in quadrants I and III.

9. Example:

$\theta_R = \frac{\pi}{4}$	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
	$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\csc\frac{\pi}{4} = \sqrt{2}$	$\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$ $\csc\frac{3\pi}{4} = \sqrt{2}$	$\sin\frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$ $\csc\frac{5\pi}{4} = -\sqrt{2}$	$\sin\frac{7\pi}{4} = -\frac{1}{\sqrt{2}}$ $\csc\frac{7\pi}{4} = -\sqrt{2}$
	$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\sec\frac{\pi}{4} = \sqrt{2}$	$\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$ $\sec\frac{3\pi}{4} = -\sqrt{2}$	$\cos\frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$ $\sec\frac{5\pi}{4} = -\sqrt{2}$	$\cos\frac{7\pi}{4} = \frac{1}{\sqrt{2}}$ $\sec\frac{7\pi}{4} = \sqrt{2}$
	$\tan\frac{\pi}{4} = 1$ $\cot\frac{\pi}{4} = 1$	$\tan\frac{3\pi}{4} = -1$ $\cot\frac{3\pi}{4} = -1$	$\tan\frac{5\pi}{4} = 1$ $\cot\frac{5\pi}{4} = 1$	$\tan\frac{7\pi}{4} = -1$ $\cot\frac{7\pi}{4} = -1$

Chapter 4 Review, pages 145–147

1. **a)** $\frac{3\pi}{2}$ **b)** 300°

c) $\frac{5\pi}{3}$ **d)** $\frac{-720°}{\pi}$

e) $\frac{11\pi}{4}$ **f)** 585°

2. Examples:

a) $\frac{23\pi}{6}, -\frac{\pi}{6}$,
general form: $\frac{11\pi}{6} \pm 2\pi n, n \in N$

b) 345°, −735°,
general form: $-375° \pm (360°)n, n \in N$

3. **a)** 6.3 **b)** 28.6°

4. $-\frac{\sqrt{5}}{3}$

5. **a)** $\frac{\pi}{3}$ **b)** $\frac{7\pi}{4}$

6. 56°, 304°, 416°, 664°

7. **a)** $-\sqrt{3}$ **b)** $-\frac{2}{\sqrt{3}}$

8. $\theta_1 \approx 128.7° + 360°n, n \in I$;
$\theta_2 \approx 231.3° + 360°n, n \in I$

9. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

Chapter 5

5.1 Graphing Sine and Cosine Functions, pages 149–157

1. **a)** 2 **b)** $\frac{1}{4}$

c) 5 **d)** 3

2. **a)** 360°, 2π **b)** 180°, π

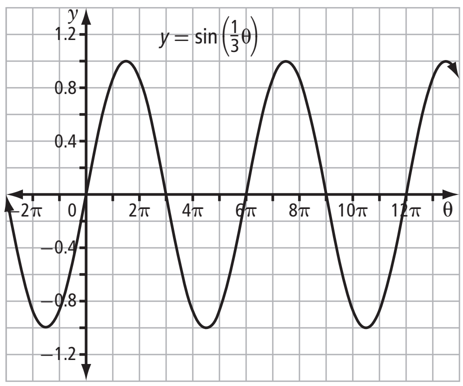
c) 1440°, 8π **d)** 240°, $\frac{4\pi}{3}$

3. **a)** $2\pi; \frac{1}{2}$ **b)** $\frac{2\pi}{3}$; 1

c) $\frac{\pi}{2}$; 2 **d)** 6π; 1.5

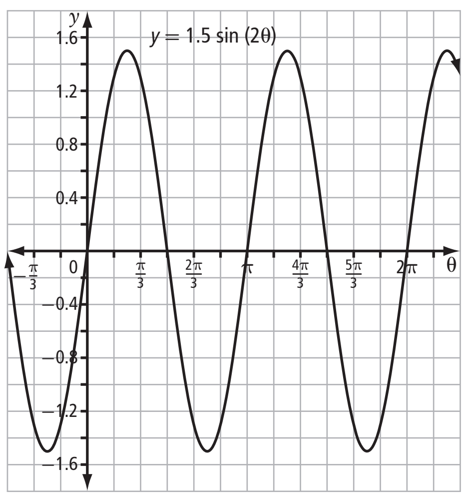
4. **a)** For $y = \sin \theta$:
amplitude: 1; maximum value: 1; minimum value: −1; period: 2π; θ-intercepts: $\pi n, n \in I$; y-intercept: 0

For $y = \sin\left(\frac{1}{3}\theta\right)$:
amplitude: 1; maximum value: 1; minimum value: −1; period: 6π; θ-intercepts: $3\pi n$, $n \in I$; y-intercept: 0



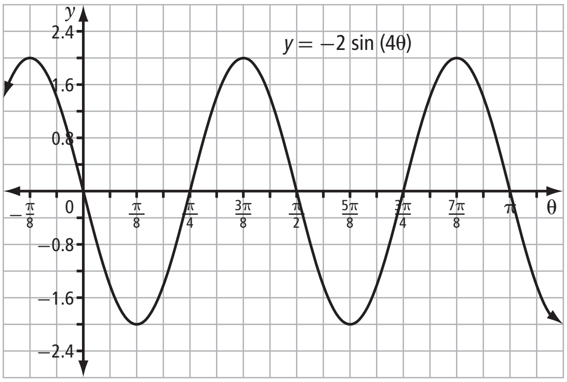
b) For $y = \sin \theta$:
amplitude: 1; maximum value: 1; minimum value: −1; period: 2π; θ-intercepts: $\pi n, n \in I$; y-intercept: 0

For $y = 1.5 \sin (2\theta)$:
amplitude: 1.5; maximum value: 1.5; minimum value: −1.5; period: π; θ-intercepts: $\frac{\pi}{2}n, n \in I$; y-intercept: 0

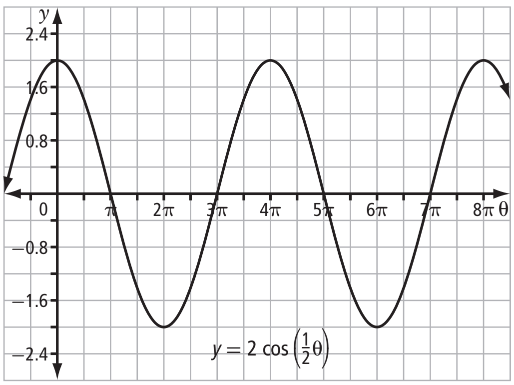


c) For $y = \sin \theta$:
amplitude: 1; maximum value: 1; minimum value: −1; period: 2π; θ-intercepts: πn, $n \in I$; y-intercept: 0

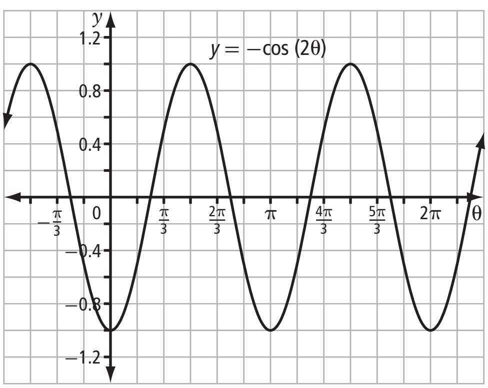
For $y = y = -2 \sin (4\theta)$:
amplitude: 2; reflected in x-axis; maximum value: 2; minimum value: −2; period: $\frac{\pi}{2}$; θ-intercepts: $\frac{\pi}{4}n$, $n \in I$; y-intercept: 0



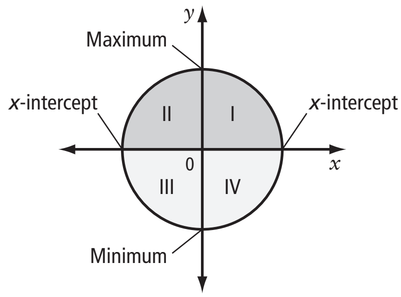
5. a) amplitude: 2; maximum value: 2; minimum value: −2; period: 4π; θ-intercepts: $\pi + 2\pi n$, $n \in I$; y-intercept: 2



b) amplitude: 1; reflected in the x-axis; maximum value: 1; minimum value: −1; period: π; θ-intercepts: $\frac{\pi}{4} + \frac{\pi}{2}n$, $n \in I$; y-intercept: −1



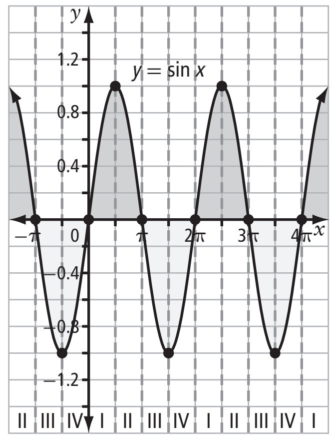
6. $y = \sin x$:



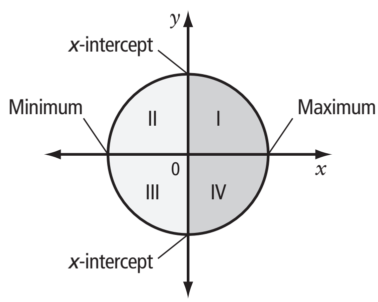
maximum values: $\frac{\pi}{2}, \frac{5\pi}{2}$

minimum values: $-\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}$

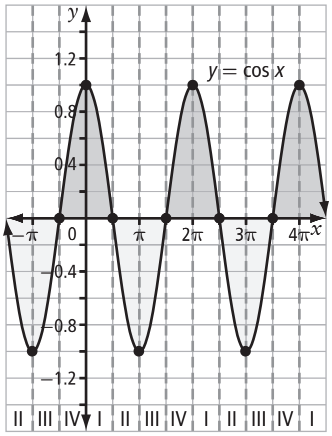
x-intercepts: $-\pi, 0, \pi, 2\pi, 3\pi, 4\pi$



$y = \cos x$:

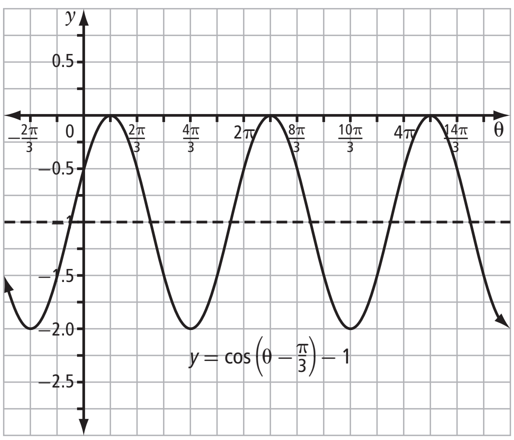


maximum values: 0, 2π, 4π
minimum values: $-\pi$, π, 3π
x-intercepts: $-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

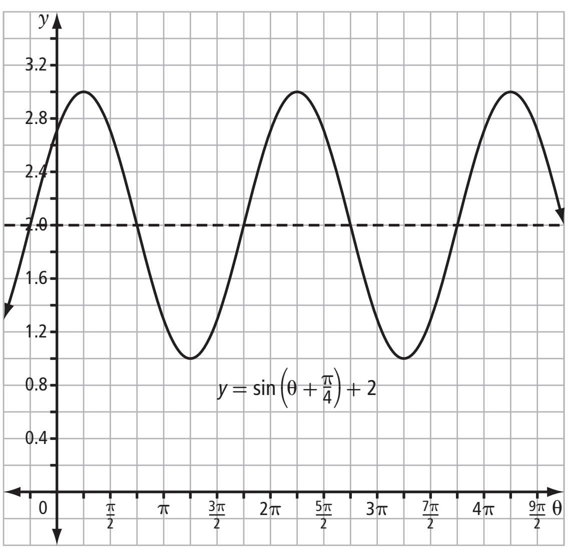


5.2 Transformations of Sinusoidal Functions, pages 158–166

1. **a)** $\frac{\pi}{3}$ units to the right; 1 unit down



b) $\frac{\pi}{4}$ units to the left; 2 units up



2. **a)** amplitude: 5; period: 720°; phase shift: 90° to the right; vertical displacement: 15 units up; domain: $\{x \mid x \in R\}$; range: $\{y \mid 10 \le y \le 20, y \in R\}$

b) amplitude: 0.1; period: 180°; phase shift: 45° to the left; vertical displacement: 1 unit down; domain: $\{x \mid x \in R\}$; range: $\{y \mid -1.1 \le y \le -0.9, y \in R\}$

c) amplitude: 1; period: π; phase shift: $\frac{\pi}{12}$ units to the right; vertical displacement: 0.5 units up; domain: $\{x \mid x \in R\}$; range: $\{y \mid -0.5 \le y \le 1.5, y \in R\}$

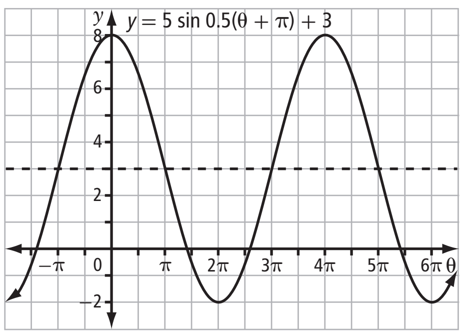
d) amplitude: 1.5; period: 4π; phase shift: $\frac{\pi}{2}$ units to the left; vertical displacement: 1 unit down; domain: $\{x \mid x \in R\}$; range: $\{y \mid -2.5 \le y \le 0.5, y \in R\}$

3. **a)** $y = 2 \sin 2\left(x + \frac{\pi}{3}\right) - 1$

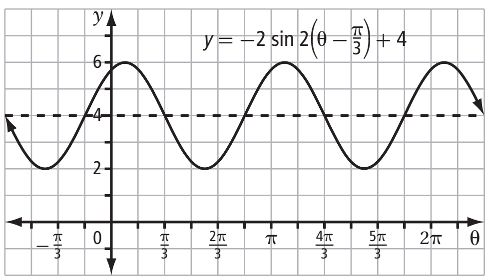
b) $y = \frac{1}{4} \sin \frac{1}{3}(x + \pi) + 2$

c) $y = 4 \sin \frac{2}{3}(x - 60°)$

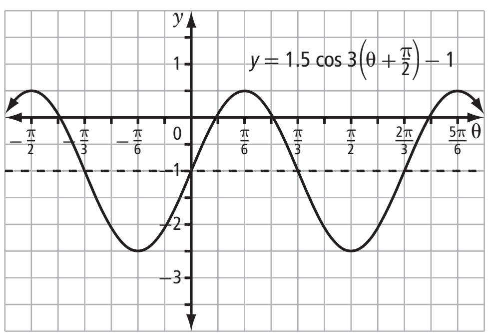
4. **a)**



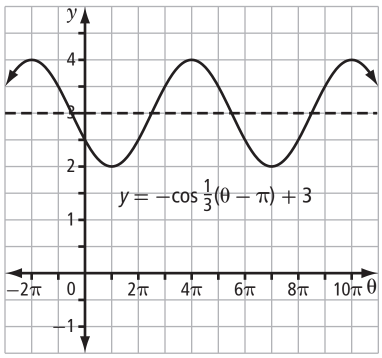
b)



c)



d)



5. Example: $y = -4 \sin (0.5\theta) + 1$;
$y = 4 \sin (0.5\theta - 2\pi) + 1$

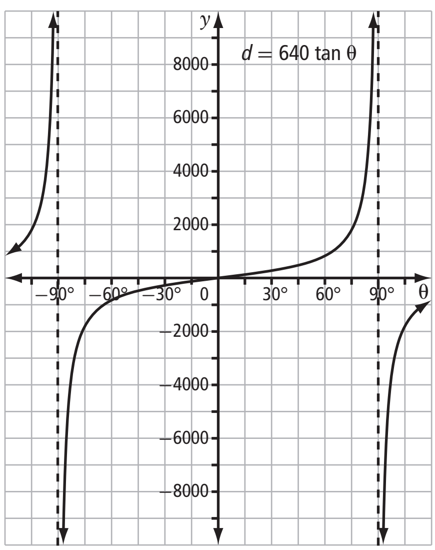
6. Example: $y = 2.2 \cos \left(2\left(\theta - \frac{\pi}{6}\right)\right) - 1.8$;
$y = 2.2 \cos \left(2\left(\theta + \frac{7\pi}{6}\right)\right) - 1.8$

7. Example: $y = 3 \sin \left(3\left(\theta - \frac{\pi}{6}\right)\right) + 2$;
$y = -3 \cos (3\theta) + 2$

8. Examples:
   a) $a = 3, b = 2, c = \frac{\pi}{2}, d = 2$
   b) $a = -3, b = 2, c = 0, d = 2$
   c) $a = 3, b = 2, c = -\frac{\pi}{4}, d = 2$

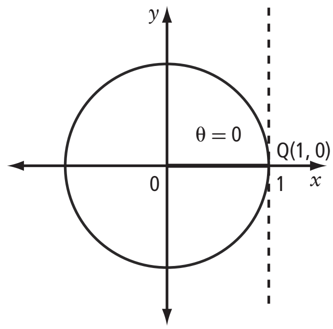
5.3 The Tangent Function, pages 167–174

1. **a)** 0 **b)** undefined **c)** 1 **d)** −1
2. **a)** 0 **b)** 0 **c)** 0
3. **a)** 1 **b)** 1 **c)** 1
4. **a)** $n\pi, n \in I$ **b)** $\frac{\pi}{4} + n\pi,\ n \in 1$
5. **a)** 0.70 **b)** 0.70 **c)** −0.70
6. **a)** $d = 640 \tan \theta$
   b) Example:
   domain: $\{\theta \mid -90° < \theta < 90°, \theta \in R\}$;
   range: $\{d \mid -8000 \le d \le 8000, d \in R\}$



7. **a)** $s = 10 \tan \theta, -75° \le \theta \le 75°$
   b) Example: the sun passes directly overhead with no tilt; it is a sunny day
   c) 10 cm **d)** 17.3 cm

8. **a)** **A**



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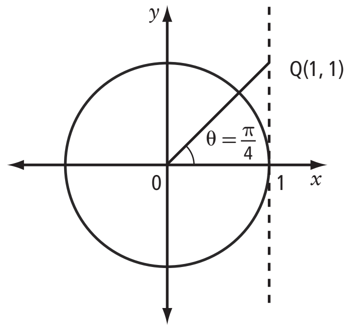

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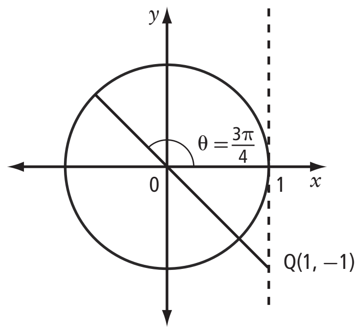

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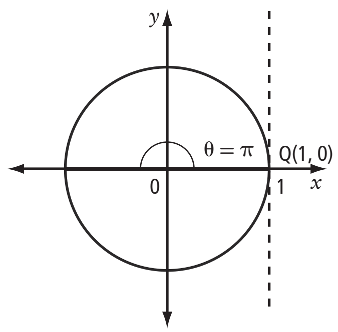

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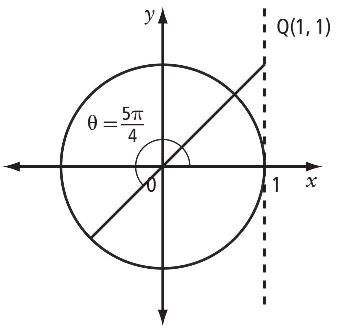

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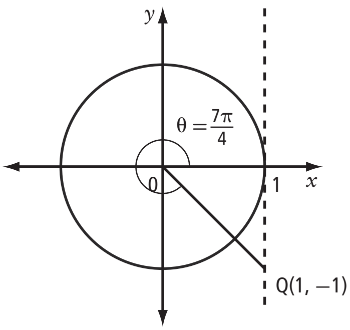

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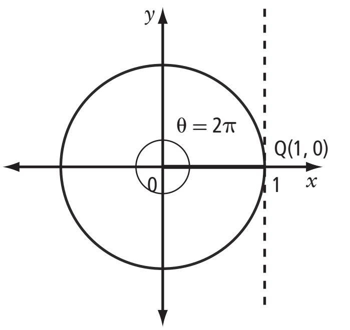
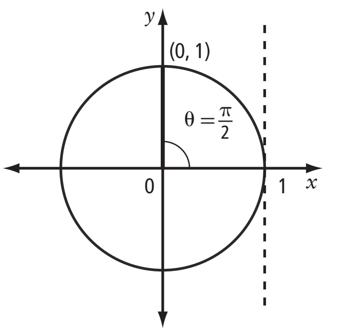
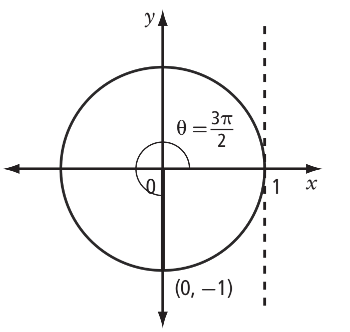


b) i)



ii)



5.4 Equations and Graphs of Trigonometric Functions, pages 175–182

1. **a)** $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$ **b)** $x = \frac{\pi}{4} + \frac{\pi}{2}n, n \in \mathrm{I}$

2. Examples:

a) $t \approx 0.0008, 0.0075, 0.0175, 0.0242$

b) $t \approx 0.0104, 0.0146, 0.0269$

c) $t = \frac{n}{120}, n \in \mathrm{I}$

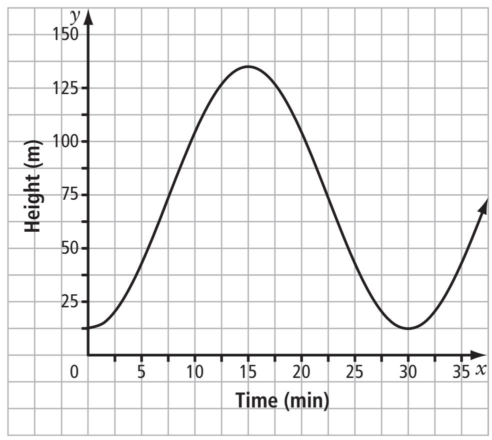
3. **a)** $\frac{1}{440}$ s **b)** $y = \sin(880\pi x)$

c) $y = \sin(523.26\pi x)$ or $y \approx \sin(1643.87x)$

4. **a)** $y = 168 \sin(120\pi x)$

b) $y = 308 \sin(100\pi x)$

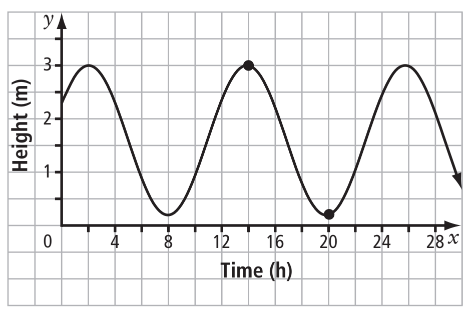
5. **a)**



Example: I assume that the ride does not stop to let on passengers, and that the wheel is vertical (perpendicular to the ground).

b) $y = -61 \cos\left(\frac{\pi}{15}x\right) + 74$;
domain: $\{x \mid 0 \le x \le 30, x \in \mathrm{R}\}$ unless the passenger goes around more than once

6. **a)**



Example: assume that the amplitude of the tide is equal each occurrence, and that the tide comes in every 12 h exactly.

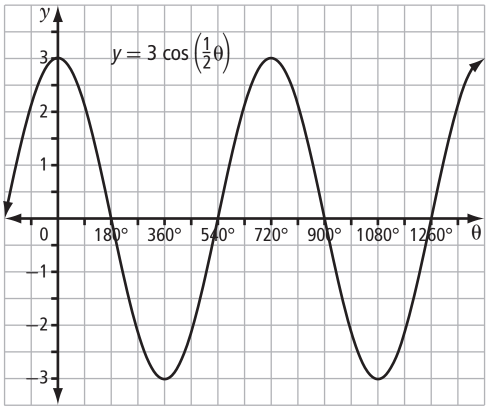
b) $y = 1.4 \sin\left(\frac{\pi}{6}(x + 1)\right) + 1.6$; The domain should be restricted to some reasonable amount of time such that the assumptions made in part a) are roughly correct.

7. $y = -18.6 \cos\left(\frac{\pi}{6}(x - 1)\right) + 0.3$

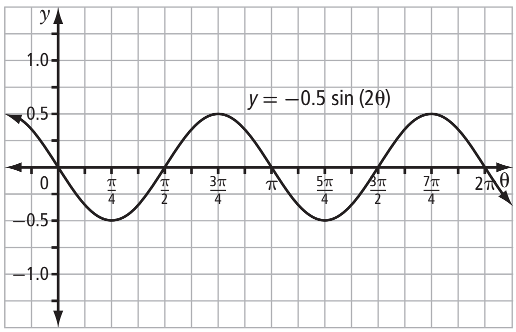
8. Examples: precipitation, ocean tides, temperatures, hours of daylight

Chapter 5 Review, pages 183–186

1. amplitude: 3; period: 720°



2. amplitude: 0.5; period: π



3. **a)** amplitude: 2; period: 120° or $\frac{2\pi}{3}$

b) amplitude: $\frac{1}{3}$; period: 360° or 2π

c) amplitude: $\frac{3}{4}$; period: 180° or π

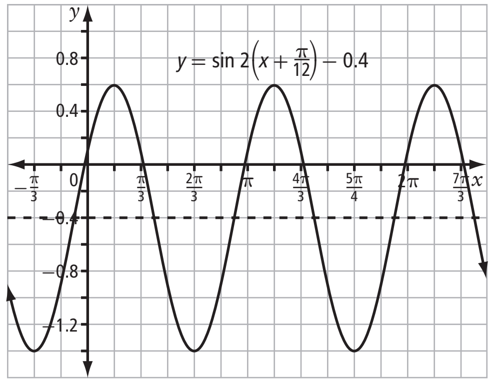
d) amplitude: 4; period: 540° or 3π

4. **a)** amplitude: 5; period: 8π; phase shift: $\frac{\pi}{3}$ units to the left; vertical displacement: 1 unit down

b) amplitude: $\frac{1}{2}$; period: π; phase shift: π units to the right; vertical displacement: 3 units down

c) amplitude: 3; period: 90°; phase shift: 50° to the left; vertical displacement: 6 units up

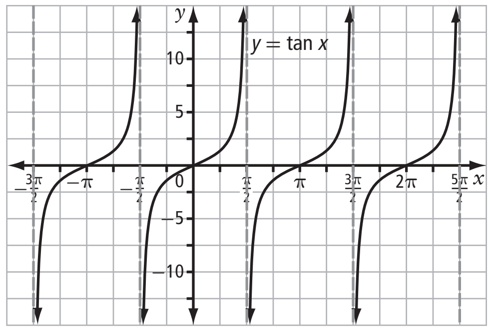
5.



6. Examples: $y = 10 \cos 4\left(\theta - \frac{\pi}{4}\right) - 3$,

$y = 10 \cos 4\left(\theta + \frac{\pi}{4}\right) - 3$

7.



8. **a)** $\tan \theta = 0.58$; $\theta \approx 30.1°$

b) $\tan \theta = -0.8$; $\theta \approx 141.3°$

9. $y = -7.7 \cos \frac{\pi}{6}(x - 1) + 9.6$

10. **a)** $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ **b)** $\frac{\pi}{2}$

c) $15° + 180°n$ and $135° + 180°n$, $n \in I$

Chapter 6

6.1 Reciprocal, Quotient, and Pythagorean Identities, pages 188–196

1. **a)** $x \neq \frac{\pi}{2} + \pi n$, where $n \in I$

b) $x \neq \frac{\pi}{2}n$, where $n \in I$

c) $x \neq \pi n$ and $x \neq \frac{3\pi}{2} + 2\pi n$, where $n \in I$

d) $x \neq \frac{\pi}{2} + \pi n$ and $x \neq 2\pi n$, where $n \in I$

2. a) $\cos x$ b) $\tan x$ c) $\sin x$

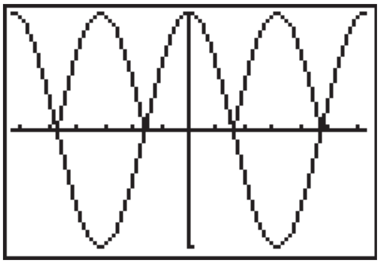
3. a) $\cot x$ b) $\sec x$ c) $\csc x$

4. a) When substituted, both values satisfy the equation.
   b) $x \neq 0°, 90°, 180°$, and $270°$

5. a) $x \neq \frac{\pi}{2}n$, where $n \in I$
   b) The graph of both functions, $f(x) = \tan x + \frac{1}{\tan x}$ and $g(x) = \frac{1}{\cos x \sin x}$, look the same, so this may be an identity.
   c) The equation is verified for $x = \frac{\pi}{4}$.

6. a) $\sin^2 \theta$ b) $\frac{3}{4}$ c) 25%

7. a) $\cos x = \sqrt{1 - \sin^2 x}$ is true only for $x = \frac{\pi}{6}$.
   b)



   c) Example: Both the graph and the value substitution methods suggest that $\cos x = \sqrt{1 - \sin^2 x}$ is not an identity. In fact, the equation is *not* an identity because $y = \sqrt{1 - \sin^2 x}$ takes the square and then the square root, which removes the negative sign. Cos x is negative from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$.

8. $\csc x$, $x \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

9. a) graph appears to be equivalent to $\sin x$; $\sin x = \csc x - \frac{\cot x}{\sec x}$
   b) $x \neq \frac{\pi}{2}n$, where $n \in I$
   c)

$$\begin{aligned}\csc x - \frac{\cot x}{\sec x} &= \frac{1}{\sin x} - \frac{1}{\sec x}\left(\frac{\cos x}{\sin x}\right)\\ &= \frac{1}{\sin x} - \cos x\left(\frac{\cos x}{\sin x}\right)\\ &= \frac{1 - \cos^2 x}{\sin x}\\ &= \frac{\sin^2 x}{\sin x}\\ &= \sin x\end{aligned}$$

10. 0

11. a)

Trigonometric Ratio	Non-Permissible Values (degrees)	Non-Permissible Values (radians)
$\tan x$	$x \neq 90° + 180°n$, where $n \in I$	$x \neq \frac{\pi}{2} + \pi n$, where $n \in I$
$\csc x$	$x \neq 180°n$, where $n \in I$	$x \neq \pi n$, where $n \in I$
$\sec x$	$x \neq 90° + 180°n$, where $n \in I$	$x \neq \frac{\pi}{2} + \pi n$, where $n \in I$
$\cot x$	$x \neq 180°n$, where $n \in I$	$x \neq \pi n$, where $n \in I$

   b) Example: Both $\sin x$ and $\cos x$ are defined for all values of x and there are no non-permissible values.
   c) Example: While both $\sin x$ and $\cos x$ are defined for all values of x, there are values for x where these trigonometric functions are equal to zero. So, if one of these functions is the sole term in the denominator, the fraction will be undefined for these values of x.

12. a)

Trigonometric Ratio	Zero Values (degrees)	Zero Values (radians)
$\sin x$	$x = 180°n$, where $n \in I$	$x = \pi n$, where $n \in I$
$\cos x$	$x = 90° + 180°n$, where $n \in I$	$x = \frac{\pi}{2} + \pi n$, where $n \in I$
$\tan x$	$x = 180°n$, where $n \in I$	$x = \pi n$, where $n \in I$
$\cot x$	$x = 90° + 180°n$, where $n \in I$	$x = \frac{\pi}{2} + \pi n$, where $n \in I$

   b) Example: The zeros are non-permissible if they occur in the denominator, in which case the expression is undefined.
   c) Example: A zero only exists if there are values of x for which the numerator of the ratio is 0. For both $\csc x$ and $\sec x$, the numerator is 1.

13. No. Non-permissible values occur for values where a trigonometric ratio is undefined and/or values where the denominator of a trigonometric identity has values of zero.

6.2 Sum, Difference, and Double-Angle Identities, pages 197–204

1. a) $\cos 65°$ b) $\sin 26°$ c) $\tan 61°$
   d) $3 \sin \frac{\pi}{5}$ e) $\cos \frac{\pi}{4}$ f) $\tan \frac{2\pi}{3}$

2. a) $\sin \frac{\pi}{2} = 1$ b) $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$
   c) $\tan 120° = -\sqrt{3}$ d) $\cos \pi = -1$

3. a) $\sin 40°$ b) $\cos \frac{3\pi}{4}$ c) $\tan \frac{7\pi}{12}$

4. a) $-\tan x$ b) $\cos \frac{\theta}{2}$ c) $\frac{1}{4} \sin \theta$ d) $-4 \cos 4\theta$

5. a) $x \neq \pi n, n \in I$
   b) $\frac{1 - \cos 2x}{\sin x}$
   $= \frac{1 - (1 - 2\sin^2 x)}{\sin x}$
   $= \frac{2\sin^2 x}{\sin x}$
   $= 2 \sin x$

6. a) $\frac{\sqrt{6} + \sqrt{2}}{4}$ b) $\frac{\sqrt{6} - \sqrt{2}}{4}$ c) $\frac{1 - \sqrt{3}}{1 + \sqrt{3}}$ d) $\frac{4}{\sqrt{6} + \sqrt{2}}$

7. $2 \sin x \cos y$

8. a) $\frac{336}{625}$ b) $\frac{527}{625}$ c) $\frac{336}{527}$

9. a) $-\frac{16}{25}$ b) $-\frac{33}{65}$ c) $\frac{56}{33}$

10. a) $\frac{\sin 2x}{1 - \cos 2x} = \frac{2 \sin x \cos x}{1 - (1 - 2\sin^2 x)}$
    $= \frac{2 \sin x \cos x}{2 \sin^2 x}$
    $= \frac{\cos x}{\sin x}$
    $= \cot x$
    b) $\sin (x + y) \sin (x - y)$
    $= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y)$
    $= \sin^2 x \cos^2 y - \sin x \cos y \cos x \sin y + \cos x \sin y \sin x \cos y - \cos^2 x \sin^2 y$
    $= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y$
    $= \sin^2 x(1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y$
    $= \sin^2 x - \sin^2 x \sin^2 y - \sin^2 y + \sin^2 x \sin^2 y$
    $= \sin^2 x - \sin^2 y$

11. a) $-\sqrt{2} \sin x$ b) $2 \sin x$

12. a) The double-angle identities are developed by using equal angles in the sum identities, rather than two different angles.
    b) There are three forms of the double-angle identity for cosine because once the sum identity is used to obtain the first cosine identity, the Pythagorean identity can be used to convert to the other two forms.

6.3 Proving Identities, pages 205–214

1. a) $\cos x$ b) $\frac{\cos x - 5}{4}$ c) $\frac{3 \sec x}{2 \sin x - 1}$

2. a) $\frac{\sin x + \sin^2 x}{\cos x + \sin x \cos x} = \frac{\sin x(1 + \sin x)}{(1 + \sin x)\cos x}$
   $= \frac{\sin x}{\cos x}$
   $= \tan x$
   b) $\frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin x \cos x} = \frac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x(\cos x + \sin x)}$
   $= \frac{\cos x - \sin x}{\cos x}$
   $= 1 - \frac{\sin x}{\cos x}$
   $= 1 - \tan x$
   c) $\frac{3\cos^2 x + 5 \cos x - 2}{9 \cos^2 x - 1} = \frac{(3 \cos x - 1)(\cos x + 2)}{(3 \cos x - 1)(3 \cos x + 1)}$
   $= \frac{\cos x + 2}{3 \cos x + 1}$

3. a) $\frac{\cos^2 x + \sin x}{\sin x \cos x}$ b) $2 \cot x \csc x$ c) $\sec x$

4. a) Left Side $= \frac{1 - \sin^2 x}{\cos x}$
   $= \frac{\cos^2 x}{\cos x}$
   $= \cos x$
   Right Side $= \frac{\sin 2x}{2 \sin x}$
   $= \frac{2 \sin x \cos x}{2 \sin x}$
   $= \cos x$
   Left Side = Right Side
   b) Left Side $= \frac{\csc^2 x - 1}{\csc^2 x}$
   $= \frac{\csc^2 x}{\csc^2 x} - \frac{1}{\csc^2 x}$
   $= 1 - \sin^2 x$
   $= \cos^2 x$
   = Right Side
   c) Left Side $= (\cos x - \sin x)^2$
   $= \cos^2 x - 2 \cos x \sin x + \sin^2 x$
   $= \cos^2 x + \sin^2 x - 2 \cos x \sin x$
   $= 1 - 2 \cos x \sin x$
   $= 1 - \sin 2x$
   = Right Side

5. a) C; $\cot x = \frac{\cos x}{\sin x}$, so $\sin x\left(\frac{\cos x}{\sin x}\right) = \cos x$

b) D; Both are forms of the double-angle identity for cos.

c) B; The quadratic expands to $\sin^2 x + \cos^2 x + 2\sin x\cos x$. Applying the Pythagorean identity, $1 + 2\sin x\cos x$.

d) A;

$$\begin{aligned}\sin^2 x + \cos^2 x + \tan^2 x &= 1 + \tan^2 x\\ &= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}\\ &= \frac{1}{\cos^2 x}\\ &= \sec^2 x\end{aligned}$$

6. a) The graphs look the same, so this might be an identity.

b)
$$\begin{aligned}\text{Right Side} &= \frac{\sin x + \sin 2x}{\cos 2x + 1 + \cos x}\\ &= \frac{\sin x + 2\sin x\cos x}{2\cos^2 x - 1 + 1 + \cos x}\\ &= \frac{\sin x(1 + 2\cos x)}{2\cos^2 x + \cos x}\\ &= \frac{\sin x(1 + 2\cos x)}{\cos x(2\cos x + 1)}\\ &= \frac{\sin x}{\cos x}\\ &= \tan x\\ &= \text{Left Side}\end{aligned}$$

c) $x \neq \frac{\pi}{2} + \pi n, \frac{2\pi}{3} + 2\pi n$, and $\frac{4\pi}{3} + 2\pi n, n \in \text{I}$

7. a)
$$\begin{aligned}\text{Right Side} &= \frac{2(\cos x\sin 2x - \sin x\cos 2x)}{\sin 2x}\\ &= \frac{2(\cos x(2\sin x\cos x) - \sin x(2\cos^2 x - 1))}{2\sin x\cos x}\\ &= \frac{2\cos^2 x\sin x - 2\sin x\cos^2 x + \sin x}{\sin x\cos x}\\ &= \frac{\sin x}{\sin x\cos x}\\ &= \frac{1}{\cos x}\\ &= \sec x\\ &= \text{Left Side}\end{aligned}$$

b)
$$\begin{aligned}\text{Right Side} &= \frac{2\csc 2x\tan x}{\sec x}\\ &= 2\csc 2x\tan x\left(\frac{1}{\sec x}\right)\\ &= 2\left(\frac{1}{\sin 2x}\right)\left(\frac{\sin x}{\cos x}\right)\cos x\\ &= 2\left(\frac{1}{2\sin x\cos x}\right)\sin x\\ &= \frac{1}{\cos x}\\ &= \sec x\\ &= \text{Left Side}\end{aligned}$$

c)
$$\begin{aligned}\text{Left Side} &= \tan 2x - \sin 2x\\ &= \frac{\sin 2x}{\cos 2x} - \sin 2x\\ &= \frac{\sin 2x - \cos 2x\sin 2x}{\cos 2x}\\ &= \frac{\sin 2x(1 - \cos 2x)}{\cos 2x}\\ &= \frac{\sin 2x}{\cos 2x}(1 - \cos 2x)\\ &= \tan 2x(2\sin^2 x)\\ &= 2\tan 2x\sin^2 x\\ &= \text{Right Side}\end{aligned}$$

d)
$$\begin{aligned}\text{Left Side} &= \frac{1 + \tan x}{1 + \cot x}\\ &= \frac{1 + \frac{\sin x}{\cos x}}{1 + \frac{\cos x}{\sin x}}\\ &= \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\sin x + \cos x}{\sin x}}\\ &= \frac{\cos x + \sin x}{\cos x}\left(\frac{\sin x}{\cos x + \sin x}\right)\\ &= \frac{\sin x}{\cos x}\\ &= \tan x\end{aligned}$$

$$\begin{aligned}\text{Right Side} &= \frac{1 - \tan x}{\cot x - 1}\\ &= \frac{1 - \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} - 1}\\ &= \frac{\frac{\cos x - \sin x}{\cos x}}{\frac{\cos x - \sin x}{\sin x}}\\ &= \frac{\cos x - \sin x}{\cos x}\left(\frac{\sin x}{\sin x - \cos x}\right)\\ &= \frac{\sin x}{\cos x}\\ &= \tan x\end{aligned}$$

Left Side = Right Side

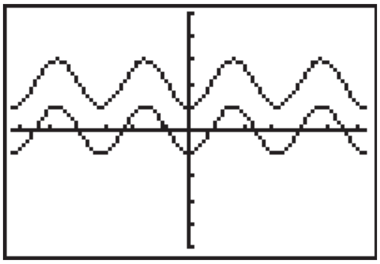
8. a)
$$\begin{aligned}\text{Left Side} &= \sin(45° + x) + \sin(45° - x)\\ &= (\sin 45°\cos x + \cos 45°\sin x)\\ &\quad + (\sin 45°\cos x - \cos 45°\sin x)\\ &= 2\sin 45°\cos x\\ &= 2\left(\frac{\sqrt{2}}{2}\right)\cos x\\ &= \sqrt{2}\cos x\\ &= \text{Right Side}\end{aligned}$$

b) Left Side = $\sin(x + \pi)$
$= \sin x \cos \pi + \cos x \sin \pi$
$= -\sin x$

Right Side $= -\cos\left(x + \frac{3\pi}{2}\right)$
$= -\left(\cos x \cos \frac{3\pi}{2} - \sin x \sin \frac{3\pi}{2}\right)$
$= -(-\sin x(-1))$
$= -\sin x$

Left Side = Right Side

9. a)



The two graphs are different, so this is not an identity.

b) Evaluating the equation for $x = 0$ is a counterexample.

10. Left Side $= 2(\sin x \cos y + \cos x \sin y) \times (\cos x \cos y - \sin x \sin y)$
$= 2(\sin x \cos x \cos^2 y - \sin^2 x \cos y \sin y + \cos^2 x \sin y \cos y - \cos x \sin x \sin^2 y)$
$= 2(\sin x \cos x(\cos^2 y - \sin^2 y) + \cos y \sin y(\cos^2 x - \sin^2 x))$
$= 2 \sin x \cos x(\cos^2 y - \sin^2 y) + 2 \cos y \sin y(\cos^2 x - \sin^2 x)$
$= \sin 2x \cos 2y + \sin 2y \cos 2x$
= Right Side

11. a) The graphs appear to be the same, so this could be an identity.

b) Left Side $= \sin^4 x + \cos^2 x$
$= (\sin^2 x)^2 + \cos^2 x$
$= (1 - \cos^2 x)^2 + \cos^2 x$
$= (1 - 2\cos^2 x + \cos^4 x) + \cos^2 x$
$= 1 - 2\cos^2 x + \cos^4 x + \cos^2 x$
$= 1 - \cos^2 x + \cos^4 x$
$= \sin^2 x + \cos^4 x$
= Right Side

c) Example: In part a), the two graphs look the same, but this is not enough to say with certainty that the equation is an identity. The proof in part b) indicates that the two sides are equal for all permissible values.

6.4 Solving Trigonometric Equations Using Identities, pages 215–223

1. a) 60°, 120° **b)** 60°, 300°
c) 45°, 225° **d)** 135°, 315°

2. a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **b)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
c) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **d)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

3. a) $0, \frac{\pi}{2}, \pi$ **b)** $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$
c) $0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}$ **d)** $\frac{\pi}{2}, \frac{3\pi}{2}$

4. a) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}$ **b)** $0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi$
c) $0, \pi$ **d)** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
e) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

5. a) $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **b)** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
c) $2.30, 3.98, \frac{\pi}{3}, \frac{5\pi}{3}$ **d)** $0.41, 2.73, \frac{7\pi}{6}, \frac{11\pi}{6}$

6. a) $0.25, 2.89, \frac{\pi}{6}, \frac{5\pi}{6}$ **b)** $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$
c) $0, \frac{\pi}{3}, \frac{5\pi}{3}$ **d)** $\frac{\pi}{2}$
e) $0.46, 3.61, \frac{\pi}{4}, \frac{5\pi}{4}$ **f)** $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

7. a) i) $\frac{2\pi}{3}, \frac{4\pi}{3}$ **ii)** $-\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$ **iii)** $-\frac{2\pi}{3}, \frac{2\pi}{3}$
b) The greater the domain, the greater the number of solutions.
c) The general solution is $x = \frac{2\pi}{3} + 2\pi n$ and $x = \frac{4\pi}{3} + 2\pi n$, where $n \in I$.

8. a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ **b)** π
c) $\frac{\pi}{6}, \frac{5\pi}{6}$ **d)** $1.91, 4.37, \frac{2\pi}{3}, \frac{4\pi}{3}$

9. B = 5, C = −2

10. $x = \frac{\pi}{2}n$, where $n \in I$

11. The general solution is $x = \frac{\pi}{2} + \pi n$, $x = \frac{5\pi}{4} + 2\pi n$, and $x = \frac{7\pi}{4} + 2\pi n$, where $n \in I$.

12. The given equation is not factorable since there are no two integers whose product is 3 and whose sum is −5.

13. The step that is wrong in the solution is the line $\cos x(\cos x + 1) = 0$. Brooke mistakenly thought that $\cos 2x$ meant $\cos^2 x$. The correct solution is:

$$\cos 2x + \cos x = 0$$
$$2\cos^2 x - 1 + \cos x = 0$$
$$2\cos^2 x + \cos x - 1 = 0$$
$$(2\cos x - 1)(\cos x + 1) = 0$$
$$\cos x = \frac{1}{2} \text{ or } \cos x = -1$$

For $\cos x = \frac{1}{2}$: $x = 60° + 360°n$ or $x = 300° + 360°n$, where $n \in I$

For $\cos x = -1$: $x = 180° + 360°n$, where $n \in I$

Chapter 6 Review, pages 224–227

1. a) $x \neq \frac{\pi}{2}n$, where $n \in I$

b) $x \neq \pi n$, where $n \in I$

c) $x \neq \frac{\pi}{2} + \pi n$, where $n \in I$

2. a) $\cos x$ **b)** $\sin x$

3. a) $\csc x$ **b)** $\sec x$

4. a) When substituted, both values satisfy the equation.

b) $x \neq 90°, 270°$

5. a) $\cos 30° = \frac{\sqrt{3}}{2}$

b) $\sin 135° = \frac{\sqrt{2}}{2}$

c) $\cos 150° = -\frac{\sqrt{3}}{2}$

6. a) $\frac{\sqrt{2} + \sqrt{6}}{4}$ **b)** $\frac{\sqrt{6} + \sqrt{2}}{4}$

c) $\frac{\sqrt{2} - \sqrt{6}}{4}$ **d)** $\frac{-\sqrt{6} + \sqrt{2}}{4}$

7. a) $-\frac{336}{625}$ **b)** $\frac{527}{625}$

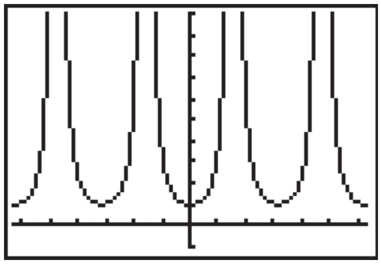
c) $-\frac{336}{527}$

8. Left Side $= \sin(\pi - x) - \tan(\pi + x)$
$$= \sin\pi\cos x - \sin x\cos\pi - \frac{\tan\pi + \tan x}{1 - \tan\pi\tan x}$$
$$= \sin x - \tan x$$

Right Side $= \frac{\sin x(\cos x - 1)}{\cos x}$
$$= \frac{\sin x\cos x - \sin x}{\cos x}$$
$$= \frac{\sin x\cos x}{\cos x} - \frac{\sin x}{\cos x}$$
$$= \sin x - \tan x$$

Left Side = Right Side

9. a)



Yes. The graphs appear to be equal.

b) Left Side $= \sin^2 x + \tan^2 x + \cos^2 x$
$$= 1 + \tan^2 x$$
$$= \sec^2 x$$
= Right Side

10. a) Left Side $= \cos x\tan^2 x$
$$= \cos x\frac{\sin^2 x}{\cos^2 x}$$
$$= \frac{\sin^2 x}{\cos x}$$
$$= \sin x\frac{\sin x}{\cos x}$$
$$= \sin x\tan x$$
= Right Side

b) Right Side $= \tan x + \tan x\cos 2x$
$$= \tan x(1 + \cos 2x)$$
$$= \tan x(1 + (2\cos^2 x - 1))$$
$$= \tan x(2\cos^2 x)$$
$$= 2\frac{\sin x}{\cos x}\cos^2 x$$
$$= 2\sin x\cos x$$
$$= \sin 2x$$
= Left Side

11. a) $\frac{\pi}{2}$ **b)** 0

c) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

12. a) $\frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ **b)** $\frac{3\pi}{2}$

c) no solution **d)** π

13. The general solution is $x = \frac{7\pi}{6} + 2\pi n$, $x = \frac{11\pi}{6} + 2\pi n$, $x = \frac{\pi}{2} + 2\pi n$, where $n \in I$. These can be combined as $x = \frac{\pi}{2} + \frac{2\pi}{3}n$, where $n \in I$.

Chapter 7

7.1 Characteristics of Exponential Functions, pages 229–237

1. **a)** No, the variable is not the exponent.

b) Yes, the base is greater than 0 and the variable is the exponent.

c) Yes, the base is greater than 0 and the variable is the exponent.

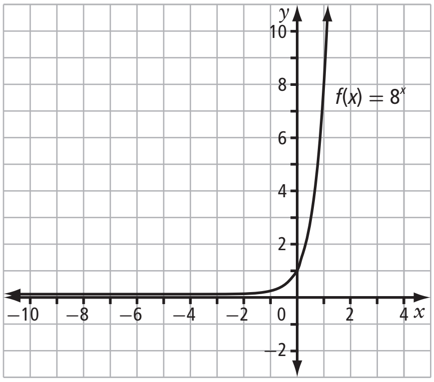
d) No, the variable is not the exponent.

e) No, the variable is not the exponent.

2. **a)** C **b)** A **c)** D **d)** B

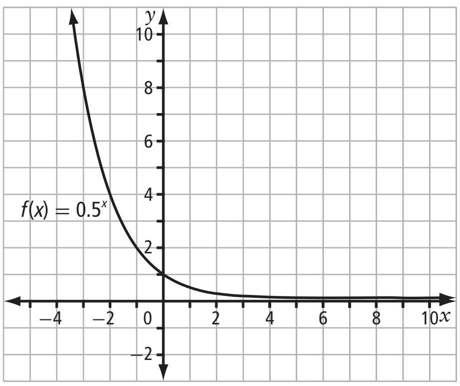
3. **a)** $y = 10^x$ **b)** $y = 5^x$ **c)** $y = \left(\frac{1}{4}\right)^x$

4. **a)**



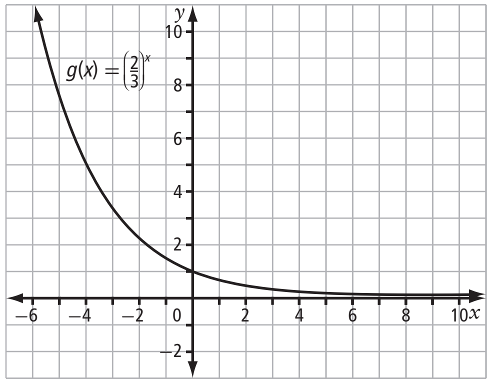
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; function increasing; horizontal asymptote $y = 0$

b)



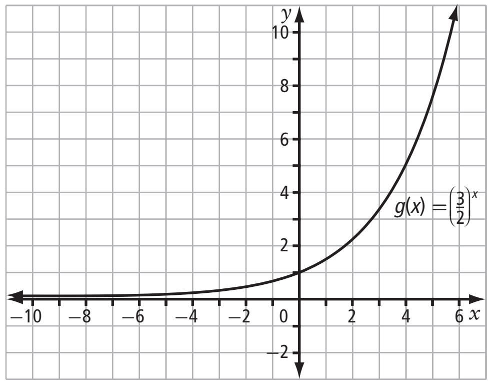
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; function decreasing; horizontal asymptote $y = 0$

c)



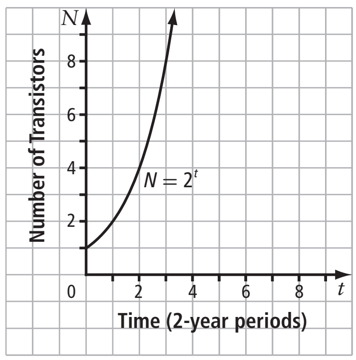
domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; function decreasing; horizontal asymptote $y = 0$

d)



domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y > 0, y \in \mathrm{R}\}$; y-intercept 1; function increasing; horizontal asymptote $y = 0$

5. **a)**

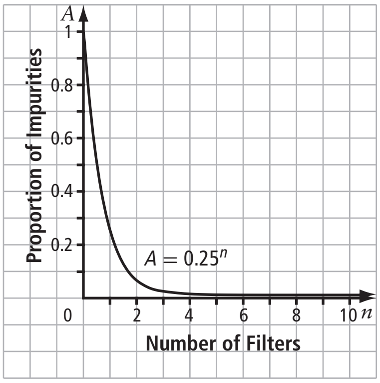


function increasing

b) domain: $\{t \mid t \geq 0, t \in \mathrm{R}\}$; range: $\{N \mid N \geq 1, N \in \mathrm{N}\}$

c) 2 transistors; 32 transistors; 1024 transistors

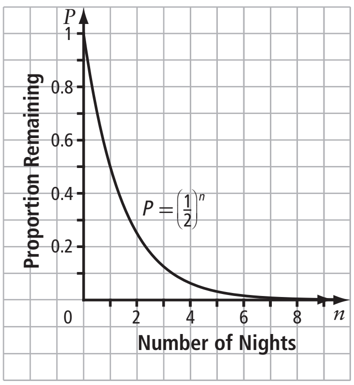
6. a) Example: Since 75% of impurities are removed, 25% remain, or as a decimal, 0.25.

b) 

c) domain: $\{n \mid n \geq 0, n \in W\}$; range: $\{A \mid 0 < A \leq 1, A \in R\}$

d) 0.0039 of original impurities remain

7. a) $P = \left(\frac{1}{2}\right)^n$

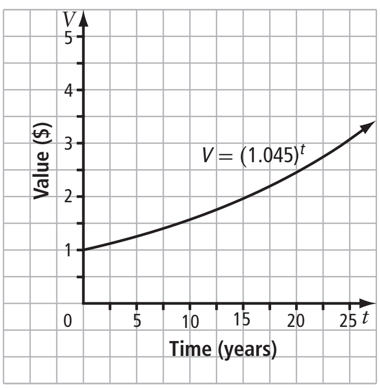
b) 

c) 3.13% after 5 nights;

0.39% after 8 nights

d) No; $y = 0$ is a horizontal asymptote.

8. a) $V = (1.045)^t$

b) 

c) $1.94

d) 25 years

9. a) Example: All exponential functions are defined for all real numbers, and all powers of positive bases are positive.

b) Example: All exponential functions have a y-intercept of 1 because $c^0 = 1$ for all $c > 0$.

c) Example: whether the function is increasing or decreasing

7.2 Transformations of Exponential Functions, pages 238–248

1. a) No b) Yes
c) No d) No

2. a) No b) No
c) No d) Yes

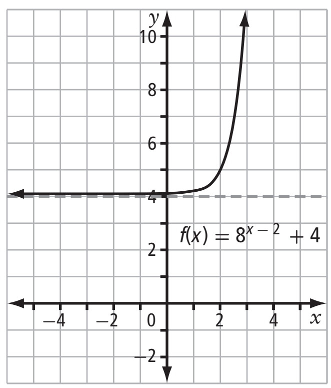
3. a) No b) No
c) Yes d) No

4. a) horizontally stretched by a factor of $\frac{1}{2}$, translated 5 units right and 6 units down

b) vertically stretched by a factor of $\frac{2}{3}$, reflected in the y-axis, translated 9 units up

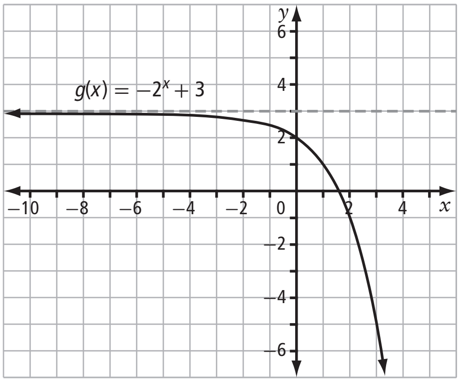
c) vertically stretched by a factor of 2, reflected in the x-axis, horizontally stretched by a factor of 4

d) vertically stretched by a factor of 500, horizontally stretched by a factor of $\frac{1}{2}$, translated 3 units left and 8 units down

5. a) 

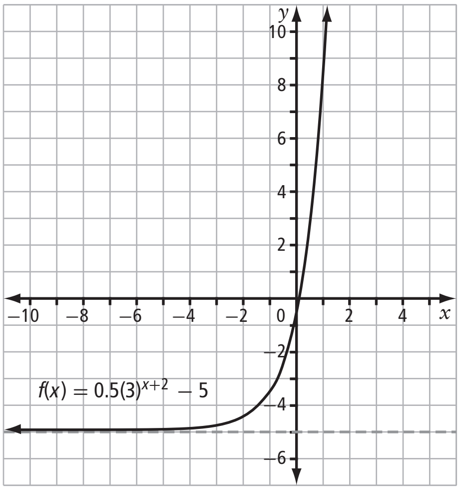
domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 4, y \in R\}$; y-intercept $\frac{257}{64}$ or ≈ 4.02; function increasing; horizontal asymptote $y = 4$

b)



domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y < 3, y \in \mathrm{R}\}$; y-intercept 2; function decreasing; horizontal asymptote $y = 3$

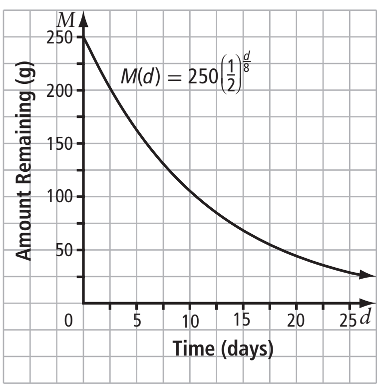
c)



domain: $\{x \mid x \in \mathrm{R}\}$; range $\{y \mid y > -5, y \in \mathrm{R}\}$; y-intercept $-\frac{1}{2}$; function increasing; horizontal asymptote $y = -5$

6. **a)** $M(d) = 250\left(\frac{1}{2}\right)^{\frac{d}{8}}$; vertical stretch by a factor of 250; horizontal stretch by a factor of 8

b)

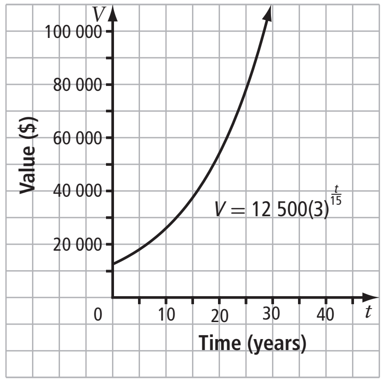


domain: $\{d \mid d \geq 0, d \in \mathrm{R}\}$;
range: $\{M \mid 0 < M \leq 250, M \in \mathrm{R}\}$; $M = 0$;
M-intercept 250

c) approximately 192.8 g; Example: substitute $x = 3$ and check against the graph

7. **a)** $V = 12\,500(3)^{\frac{t}{15}}$; vertically stretched by a factor of 12 500; horizontally stretched by a factor of 15

b)

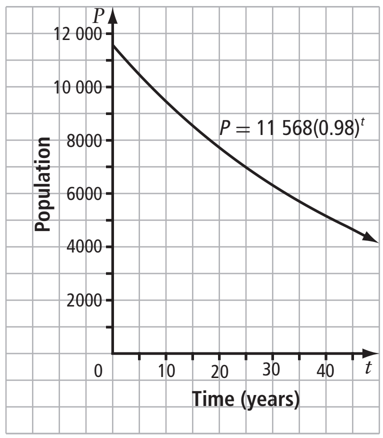


domain: $\{t \mid t \geq 0, t \in \mathrm{R}\}$;
range: $\{V \mid V \geq 12\,500, V \in \mathrm{R}\}$

c) $234 009.43; assumes that the growth rate does not change

d) approximately 19 years

8. **a)** $P = 11\,568(0.98)^t$



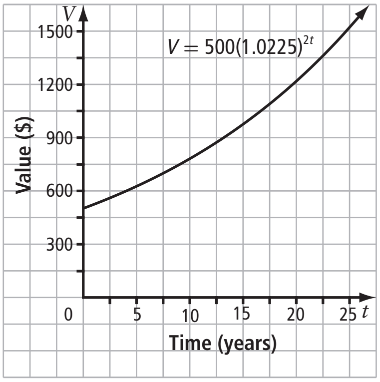
b) vertically stretched by a factor of 11 568

c) approximately 10 456

d) approximately 7722

e) Example: According to the equation, no, but since fractions of people are not possible, the population may reach zero in the future.

9. **a)** vertically stretched by a factor of 500, horizontally stretched by a factor of $\frac{1}{2}$



b) \$500; This is the initial investment.

c) \$974.70

d) approximately 16 years

10. **a)** Example: They are both correct. David is correct because the y-intercepts of the graphs are 1 and 2, which suggests a vertical stretch of 1. But Jodi is also correct because the points on the transformed graph are 1 unit left of the same points on the base function, suggesting a horizontal translation by 1 unit left.

b) Example: The laws of exponents mean that $2(2)^x = 2^1 \times 2^x = 2^{x+1}$.

c) Example: $y = 3^{-x}$ and $y = \left(\frac{1}{3}\right)^x$

7.3 Solving Exponential Equations, pages 249–255

1. **a)** 3^4 **b)** 3^{15}

c) $3^{\frac{3}{2}}$ **d)** $3^{\frac{5}{3}}$

e) $3^{\frac{14}{3}}$ **f)** 3^{-6}

g) $3^{\frac{21}{2}}$

2. **a)** 2^3, 2^6 **b)** 3^2, 3^6

c) 5^{x+6}, 5^3 **d)** 2^{3x}, 2^{6x+12}

e) 3^{15x+12}, 3^{-2x-6} **f)** 2^{-2x-14}, 2^{-9x}

3. **a)** $x \approx 3.2$ **b)** $x \approx 3.4$

4. **a)** 3 **b)** 4

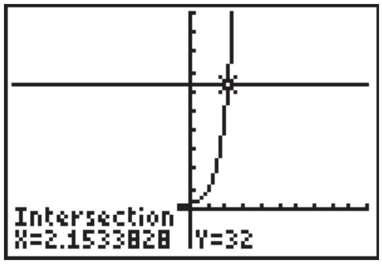
c) −6 **d)** −2

5. **a)** 1 **b)** $\frac{13}{4}$

c) −2 **d)** $-\frac{12}{5}$

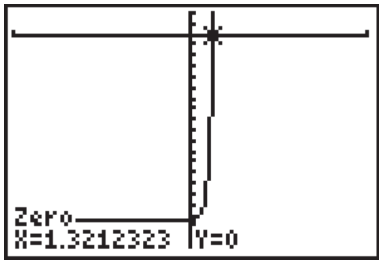
e) 3

6. **a)** Example: Graph $y = 5^x$ and $y = 32$, and calculate the point of intersection.



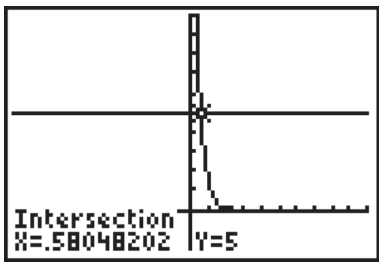
$x \approx 2.15$

b) Example: Graph $y = 10^{2x} - 439$ and calculate the x-intercept.



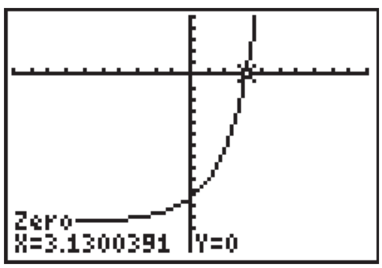
$x \approx 1.32$

c) Example: Graph $y = 25\left(\frac{1}{2}\right)^{4x}$ and $y = 5$ and calculate the point of intersection.



$x \approx 0.58$

d) Example: Graph $y = 200(1.05)^{12x} - 1250$ and calculate the x-intercept.



$x \approx 3.13$

7. a) $N = 4(2)^t$, where N is the number of bacteria, and t is the time, in hours.

b) 10 h

8. a) $V = 1000(2)^{\frac{t}{8}}$, where V is the value of the painting and t is the time, in years.

b) approximately 13.5 years

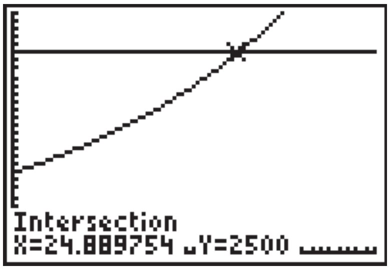
9. a) $M = 350(1.03)^t$, where M is the number of members and t is the time, in years.

b) 16 years

10. a) $V = 1000(1.0375)^t$, where V is the value of the investment and t is the time, in years.

b) systematic trial or use technology to graph either one or two functions

c)



approximately 25 years

d) Example: There is no common base, so once I reach the point in the equation $1.0375^t = 2.5$, I do not have an algebraic method to calculate t other than systematic trial.

Chapter 7 Review, pages 256–258

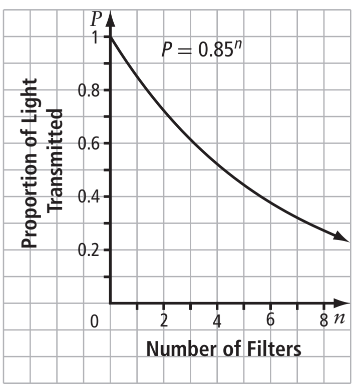
1. a) domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$; y-intercept 1; horizontal asymptote $y = 0$; function increasing

b) domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$; y-intercept 1; horizontal asymptote $y = 0$; function decreasing

2. a) $y = 5^x$ **b)** $y = \left(\frac{1}{3}\right)^x$

3. a) $P = 0.85^n$

b)



approximately 4 filters

4. a) vertically stretched by a factor of 2, translated 3 units down

b) translated 3 units left

c) horizontally stretched by a factor of $\frac{1}{2}$, translated 4 units right and 1 unit up

d) vertically stretched by a factor of 5, horizontally stretched by a factor of $\frac{1}{6}$, translated 2 units left

5. a) $y = \frac{1}{2}(4)^{x+2} - 6$; domain: $\{x \mid x \in R\}$; range: $\{y \mid y > -6, y \in R\}$

b) $y = 5(4)^{3x}$; domain: $\{x \mid x \in R\}$; range: $\{y \mid y > 0, y \in R\}$

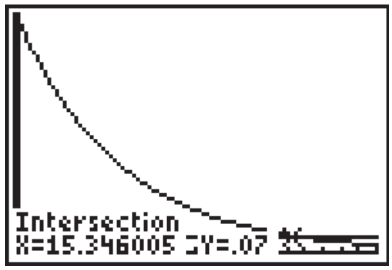
c) $y = (4)^{\frac{1}{2}(x-3)} - 1$; domain: $\{x \mid x \in R\}$; range: $\{y \mid y > -1, y \in R\}$

6. a) 3 **b)** -2

c) $\frac{15}{8}$ **d)** $-\frac{20}{7}$

7. a) $P = \left(\frac{1}{2}\right)^{\frac{t}{4}}$ **b)** 8 days

c)



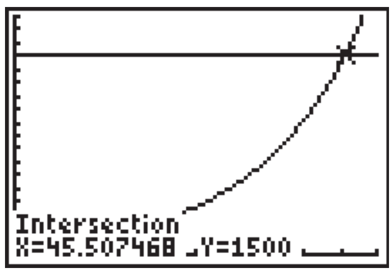
15.3 days

8. a) $N = 64(2)^{\frac{t}{10}}$

b) 337 colonies

c) 40 h

d)

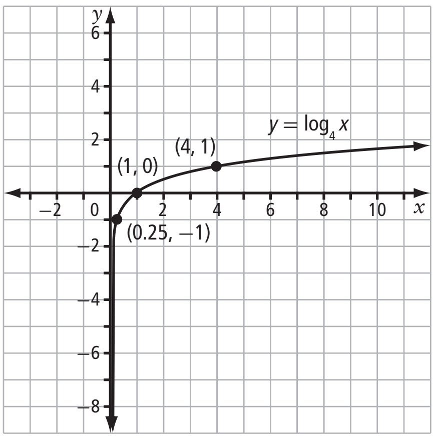


46 h

Chapter 8

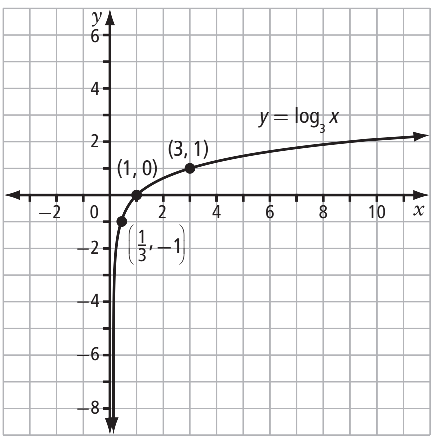
8.1 Understanding Logarithms, pages 260–266

1. a)



domain: $\{x \mid x > 0, x \in R\}$; range: $\{y \mid y \in R\}$; x-intercept 1; vertical asymptote $x = 0$; $y = \log_4 x$

b)



domain: $\{x \mid x > 0, x \in R\}$; range: $\{y \mid y \in R\}$; x-intercept 1; vertical asymptote $x = 0$; $y = \log_3 x$

2. a) $\log_3 243 = 5$ **b)** $\log 10\,000 = 4$ **c)** $\log_{16} 4 = \frac{1}{2}$ **d)** $\log_8 \frac{1}{64} = -2$ **e)** $\log 0.01 = -2$ **f)** $\log_{27} 9 = \frac{2}{3}$ **g)** $\log_{12} 2y = x$ **h)** $\log_2 (y - 1) = 2x - 5$

3. a) $2^5 = 32$ **b)** $8^3 = 512$ **c)** $5^4 = 625$ **d)** $10^3 = 1000$ **e)** $10^{-4} = 0.0001$ **f)** $\left(\frac{1}{2}\right)^{-3} = 8$ **g)** $3^y = x + 1$ **h)** $4^{(y+1)} = 2x$

4. a) 2 **b)** 6 **c)** 2 **d)** −2 **e)** −3 **f)** 1 **g)** 4 **h)** $\frac{11}{2}$ **i)** 0 **j)** $\frac{1}{2}$

5. $\log_{10} 300$, $\log_6 400$, $\log_2 100$

6. a) 3 **b)** 3 **c)** 16 **d)** $\frac{1}{25}$ **e)** 4 **f)** $\frac{1}{2}$ **g)** 3 **h)** 36 **i)** 5 **j)** 4

7. a) 216 **b)** 4 **c)** 12 **d)** 7 **e)** −8 **f)** 10

8. a) 13 dB **b)** 1000 times

9. Examples:

a) The graph will increase more slowly.

b) The graph of $y = 5^x$ increases more quickly.

c) Because the graph of $y = 5^x$ increases more quickly, the graph of $y = \log_5 x$ will increase more slowly.

d) A larger base leads to a logarithmic graph that increases more slowly.

10. Examples:

a) If $c < 0$, the function is discontinuous and is not defined for many real numbers.

b) Since $1^x = 1$ for all values of x, the only domain of $y = \log_1 x$ is 1.

8.2 Transformations of Logarithmic Functions, pages 267–274

1. a) translated 1 unit left and 8 units down

b) vertically stretched by a factor of 2 and horizontally stretched by a factor of $\frac{1}{4}$

c) reflected in the x-axis and horizontally stretched by a factor of $\frac{1}{3}$

d) vertically stretched by a factor of 5, horizontally stretched by a factor of $\frac{1}{2}$, reflected in the y-axis, translated 4 units left

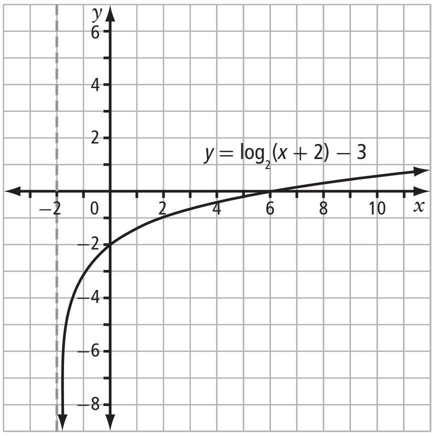
2. a) $y = 3 \log_5 (x - 2)$

b) $y = -\log_5 (x + 4) - 1$

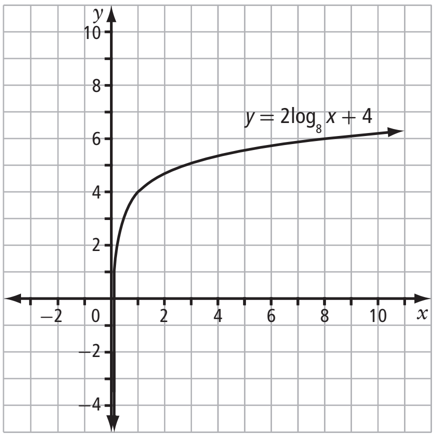
c) $y = \frac{1}{2}\log_5 (2x)$

d) $y = 4 \log_5 (-x) - 2.5$

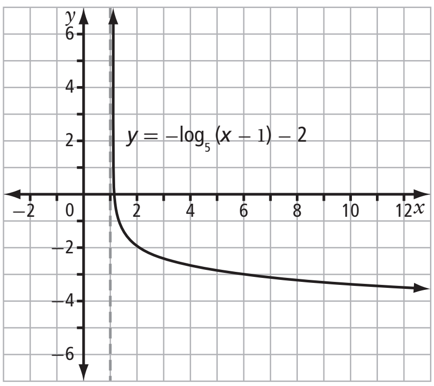
3. a)



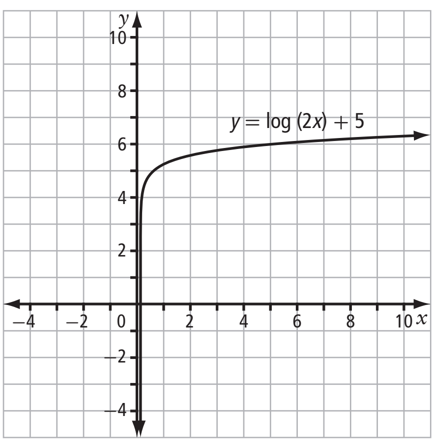
b)



c)



d)



4. a) horizontally stretched by a factor of $\frac{1}{2}$, translated 3 units to the left

b) horizontally stretched by a factor of $\frac{1}{3}$, translated 4 units to the right

c) horizontally stretched by a factor of 2, translated 6 units to the right

d) horizontally stretched by a factor of 3, translated 18 units to the left

5. a) vertically stretched by a factor of B and translated 1 unit left

b) 50 000 bits per second

6. a) domain: $\{x \mid x > 8, x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$; x-intercept: 244 140 633; no y-intercept; vertical asymptote $x = 8$

b) domain: $\{x \mid x > 1, x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$; x-intercept: 2.1; no y-intercept; vertical asymptote $x = 1$

c) domain: $\{x \mid x > 0, x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$; x-intercept: 71 663 616; no y-intercept; vertical asymptote $x = 0$

d) domain: $\{x \mid x > -3, x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$; x-intercept 125; y-intercept −5.4; vertical asymptote $x = -3$

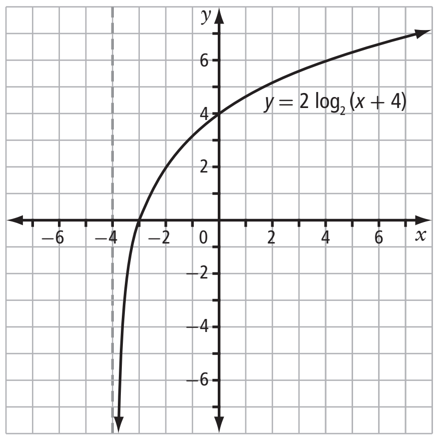
7. a) translated 2 units right and 5 units down

b) translated 1 unit left and 2 units down

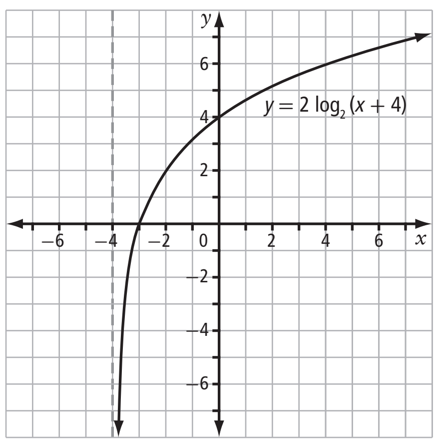
8. a) vertically stretched by a factor of 3

b) vertically stretched by a factor of $\frac{1}{2}$

9. a)



b)



c) The graphs are identical. The order of the transformations does not matter when performing a vertical stretch and a horizontal translation.

d) Example: a horizontal stretch by a factor of 2 and a translation 3 units up; a vertical stretch by a factor of 3 and a translation 1 unit right

e) Example: when a stretch and a translation are in the same direction

8.3 Laws of Logarithms, pages 275–281

1. **a)** 2 **b)** 3 **c)** 5 **d)** 2 **e)** 2 **f)** 3 **g)** 5 **h)** 11 **i)** 16 **j)** 1000

2. **a)** $4 \log_7 x + \frac{3}{2}\log_7 y$

b) $3 \log_{12} x + 6 \log_{12} y + 15 \log_{12} z$

c) $3 \log_8 x - \frac{1}{2}\log_8 y - \frac{5}{2}\log_8 z$

d) $\frac{1}{2}\log x - \frac{3}{2}\log y$

3. **a)** $2 + \frac{5}{3}\log_7 x$ **b)** $2 - 2\log x - 2\log y$

c) $\frac{7}{3}\log_5 y - 3 - \log_5 x$ **d)** $6\log_2 x - 5 - 2\log_2 y$

4. **a)** $\log_6 54x^4$ **b)** $\log_2 \frac{y^8}{2x}$ **c)** $\log_4 x^{16}y^{20}$

d) $\log_3 (xy)^{\frac{11}{3}}$ **e)** $\log \frac{2}{25x^{\frac{3}{2}}y^{\frac{1}{4}}}$ **f)** $\log_7 \frac{x^{\frac{9}{2}}}{\sqrt[6]{5}}$

g) $\log \frac{2x^{\frac{5}{3}}}{3}$ **h)** $\log_9 x^6y^9$

5. **a)** $L = \log I^{10} - \log I_0^{10}$ **b)** $I = 10^{0.1L + \log I_0}$

6. **a)** $[H_3O]^+ = 10^{-pH}$

b) $pH = \log([H_3O]^+)^{-1} = \log \frac{1}{[H_3O^+]}$

7. **a)** False; it must be a multiplication inside the logarithm.

b) False; the division must take place inside the logarithm.

c) True **d)** True

e) False; the exponent must apply to the entire argument of the logarithm.

8. **a)** $\frac{1}{6}$ **b)** 3 **c)** 8

9. **a)** $7P$ **b)** $P + 1$ **c)** $2P$

d) $P - 1$ **e)** $-P$ **f)** $\frac{P}{2}$

10. **a)** The function $y = \log_2 x^2$ can be written as $y = 2\log_2 x$, which is a vertical stretch by a factor of 2 of $y = \log_2 x$.

b) The function $y = \log_2 3x$ is of the form $y = \log_2 bx$. This is a horizontal stretch by a factor of $\frac{1}{3}$ of the function $y = \log_2 x$.

c) The function $y = \log_2 3x$ can be written as $y = \log_2 x + \log_2 3$, which is a translation of $\log_2 3$ units up.

d) No. Example: $y = \log_2 \frac{1}{x}$ can be written as $y = -\log_2 x$, which is a reflection in the x-axis, not the reciprocal transformation.

8.4 Logarithmic and Exponential Equations, pages 282–291

1. **a)** 1024 **b)** 25 **c)** 32 **d)** 213 **e)** 5 **f)** 1005

2. **a)** 0.93 **b)** 1.13 **c)** −3.64 **d)** 8.00

3. **a)** $\frac{\log 205}{\log 5}$ **b)** $\frac{\log 311}{\log 4} + 3$

c) $\frac{\log 7539 - 1}{2}$ **d)** $\frac{\log 40}{\log 4} - 2$

e) $\frac{2\log 85}{\log 6}$

4. **a)** 6 **b)** 10 or −10

c) 16 **d)** 9

5. **a)** $x > 0$ **b)** $x > 2$

c) undefined for all x

6. **a)** $\frac{3\log 5}{\log 5 - 1} \approx -6.97$

b) $\frac{-3\log 8}{2\log \frac{2}{3}} \approx 7.69$

c) $\frac{2\log 6 + 5\log 2}{2\log 2 - \log 6} \approx -17.39$

d) $\frac{3\log 3 + 2\log 6 + \log 2}{2\log 3 - \log 6} \approx 18.68$

7. **a)** $\frac{5}{2}$ **b)** $\frac{17}{4}$ **c)** 2 **d)** no soluion

8. **a)** 2 **b)** 6 **c)** 6 **d)** 3

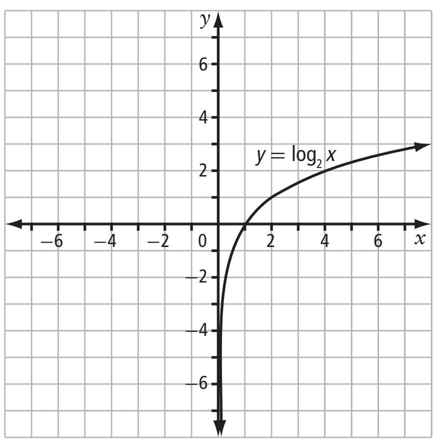
9. **a)** $m = 65\left(\frac{1}{2}\right)^{\frac{t}{88}}$ **b)** 43.84 g **c)** 149.6 years

10. **a)** $p = 974(1.015)^t$ **b)** 1049 **c)** 14 years

11. **a)** $\frac{3}{2}$ or 1 **b)** −3 or $\frac{3}{2}$

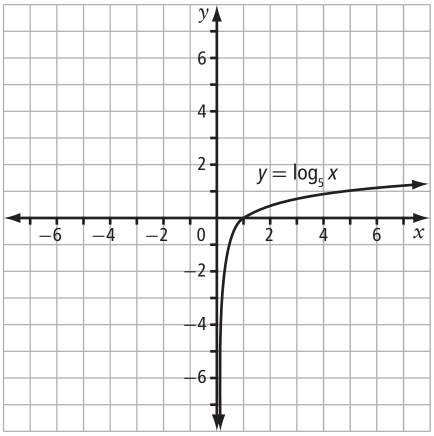
Chapter 8 Review, pages 292–295

1. **a)**



domain: $\{x \mid x > 0, x \in R\}$; range: $\{y \mid y \in R\}$; x-intercept 1; vertical asymptote $x = 0$

b)



domain: $\{x \mid x > 0, x \in R\}$;
range: $\{y \mid y \in R\}$; x-intercept 1;
vertical asymptote $x = 0$

2. **a)** $\log_6 216 = 3$ **b)** $\log_2 1024 = 10$
c) $\log 0.001 = -3$ **d)** $\log_5 125 = x$

3. **a)** $3^4 = 81$ **b)** $25^{\frac{1}{2}} = 5$
c) $10^0 = 1$ **d)** $2^9 = 3x - 4$

4. **a)** vertically stretched by a factor of 2, translated 1 unit left; domain: $\{x \mid x > -1, x \in R\}$; range: $\{y \mid y \in R\}$; x-intercept: 0; y-intercept: 0
b) translated 3 units right and 5 units up; domain: $\{x \mid x > 3, x \in R\}$; range: $\{y \mid y \in R\}$; x-intercept: 3.0; no y-intercept

5. **a)** $y = \log(x - 5) - 4$; domain: $\{x \mid x > 5, x \in R\}$; range: $\{y \mid y \in R\}$
b) $y = 3\log(x + 2) - 6$; domain: $\{x \mid x > -2, x \in R\}$; range: $\{y \mid y \in R\}$
c) $y = \log(3x) + 1$; domain: $\{x \mid x > 0, x \in R\}$; range: $\{y \mid y \in R\}$

6. **a)** 2 **b)** 3 **c)** 2 **d)** 0

7. **a)** $2 + 4\log_5 x + \frac{3}{4}\log_5 y$
b) $5\log y - 2 - \frac{1}{2}\log x$

8. **a)** $3\log_4 xy$ **b)** $\log \frac{x^5}{\sqrt{y^5}}$

9. **a)** $\frac{2}{\log 3} \approx 4.19$ **b)** $\frac{\log 517}{\log 7} + 3 \approx 6.21$
c) $\frac{\log 5500 - 1}{2} \approx 1.37$ **d)** $\frac{4\log 2}{\log 2 - \log 5} \approx -3.03$

10. **a)** 128 **b)** $\frac{117}{2}$ **c)** no solution **d)** 5

11. Example: $10^{9.3 - 9.0} \approx 2$

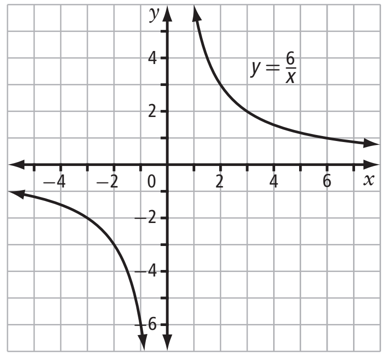
12. **a)** $N = 40(2)^{\frac{t}{4}}$ **b)** 18.58 h
c) 6.34 h; does not depend on the number of bacteria present at the beginning

13. **a)** $P = 100(0.6)^n$ **b)** 21.6%
c) 9 filters

Chapter 9

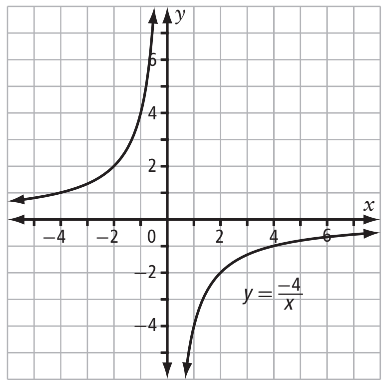
9.1 Exploring Rational Functions Using Transformations, pages 297–304

1. **a)**



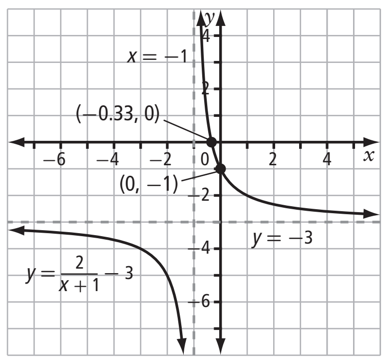
vertical asymptote: $x = 0$
horizontal asymptote: $y = 0$

b)

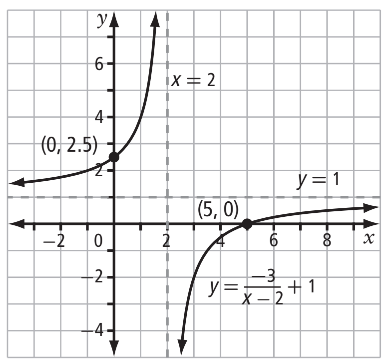


vertical asymptote: $x = 0$
horizontal asymptote: $y = 0$

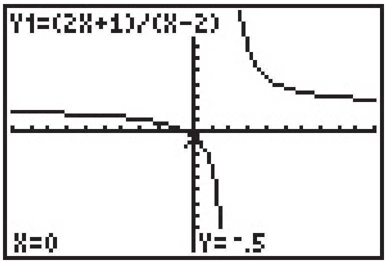
2. **a)**



b)

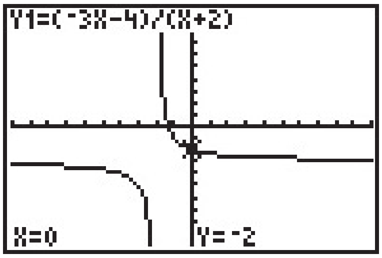


3. a)



vertical asymptote: $x = 2$; horizontal asymptote $y = 2$; x-intercept -0.5; y-intercept -0.5

b)



vertical asymptote: $x = -2$; horizontal asymptote $y = -3$; x-intercept $-\frac{4}{3}$; y-intercept -2

4. a) C **b)** A

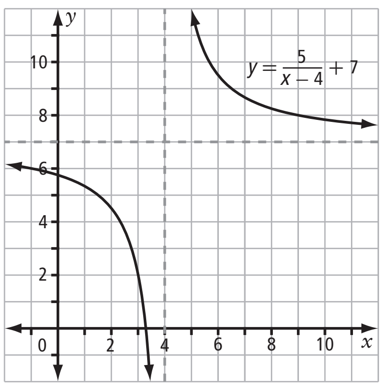
c) D **d)** B

5. a) $y = \frac{1}{x - 2} + 3$

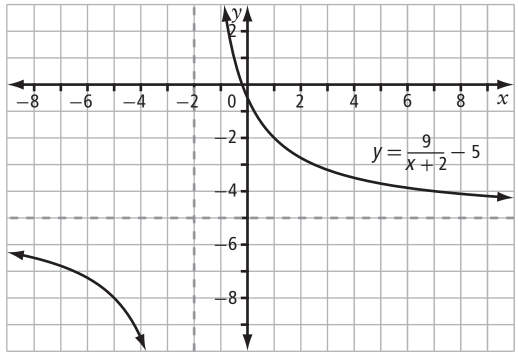
b) $y = \frac{2}{x - 1} + 3$

c) $y = \frac{3}{x + 2} - 4$

6. a) $y = \frac{5}{x - 4} + 7$

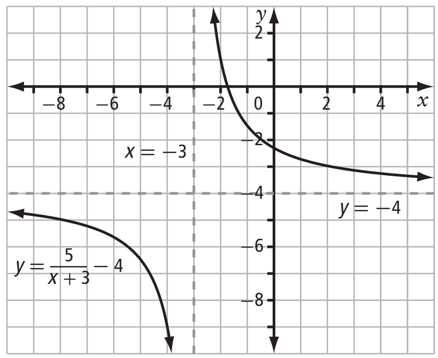


b) $y = \frac{9}{x + 2} - 5$



7. a) Example: $y = \frac{5}{x + 3} - 4$; The non-permissible value is the vertical asymptote and the horizontal asymptote is equal to parameter k, or -4.

b) Example:



c) domain: $\{x \mid x \neq -3, x \in \mathrm{R}\}$, range: $\{y \mid y \neq -4, y \in \mathrm{R}\}$

d) Yes, there are many functions with different a values for which the asymptotes are the same.

8. Example: Similarity: The graphs are all transformed by a vertical stretch by a factor of 2 and a translation of 4 units right and 3 units down. Difference: Their base functions are all different. The base functions are $y = \frac{1}{x}$, $y = x^2$, and $y = \sqrt{x}$.

9.2 Analysing Rational Functions, pages 305–313

1. a) vertical asymptote; the numerator and denominator have no common factors

b) point of discontinuity; the numerator and denominator have a common factor and simplify to become a linear function

c) both; the numerator and denominator have a common factor and simplify to become a rational function

d) vertical asymptote; the numerator and denominator have no common factors

2. **a)** vertical asymptote: $x = 4$; x-intercept: -1; y-intercept: -0.25

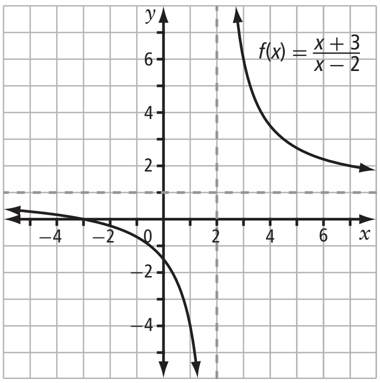
b) point of discontinuity: $(-3, 1)$; no vertical asymptotes; x-intercept: -4; y-intercept: 4

c) point of discontinuity: $(5, 0.5)$; vertical asymptote: $x = -1$; x-intercept: 2; y-intercept: -2

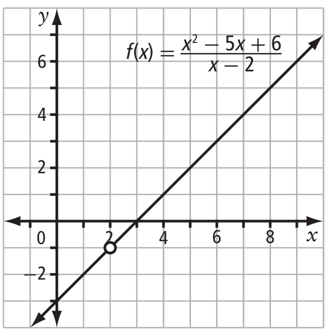
d) no points of discontinuity; vertical asymptotes: $x = 6$, $x = -1$; x-intercepts: -2, -4; y-intercept: $-\frac{4}{3}$

3. **a)** C **b)** B **c)** D **d)** A

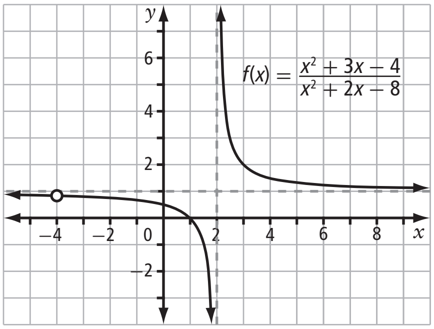
4. **a)** vertical asymptote: $x = 2$; horizontal asymptote: $y = 1$; x-intercept: -3; y-intercept: $-\frac{3}{2}$



b) point of discontinuity: $(2, -1)$; x-intercept: 3; y-intercept: -3



c) vertical asymptote: $x = 2$; horizontal asymptote: $y = 1$; x-intercept: 1; y-intercept: $\frac{1}{2}$; point of discontinuity: $\left(-4, \frac{5}{6}\right)$



5. **a)** $f(x) = \frac{x + 2}{x - 4}$ or $f(x) = \frac{6}{x - 4} + 1$

b) $f(x) = \frac{x^2 - 5x - 6}{x + 1}$ or $f(x) = \frac{(x - 6)(x + 1)}{x + 1}$

c) $f(x) = \frac{x^2 - 2x - 3}{x^2 - 4x + 3}$ or $f(x) = \frac{(x + 1)(x - 3)}{(x - 1)(x - 3)}$

6. **a)** Example: $f(x) = \frac{(x - 2)(x + 4)}{(x + 4)}$

b) Example: $f(x) = \frac{(x + 1)(x - 3)}{(x - 1)(x - 3)}$

c) Example: $f(x) = \frac{x}{(x - 4)(x + 3)}$

d) Example: $f(x) = \frac{(x + 1)(x + 3)}{x - 7}$

7. **a)** $\left(\frac{3}{2}, \frac{1}{2}\right)$

b) There is a "hole" or gap in the graph at this point. This means that the value at this point is indeterminate because value of the function at this point is $\frac{0}{0}$.

8. **a)** Example: Factor the numerator and denominator. The factor common to both the numerator and the denominator corresponds to a point of discontinuity.

b) Example: Factor the numerator and denominator. The factor of only the denominator corresponds to a vertical asymptote.

c) Example: Factor the numerator and denominator. The factor common to both the numerator and the denominator corresponds to a point of discontinuity, while the factor of only the denominator corresponds to a vertical asymptote.

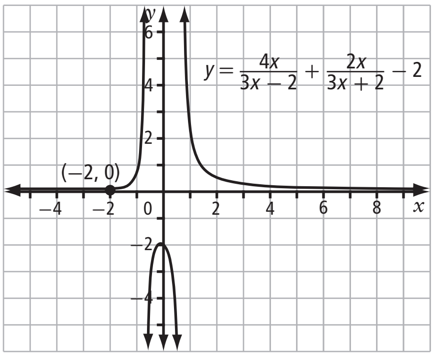
9. Example: Yes. If the polynomial in the denominator is a constant, there will be neither a point of discontinuity nor a vertical asymptote.

9.3 Connecting Graphs and Rational Equations, pages 314–320

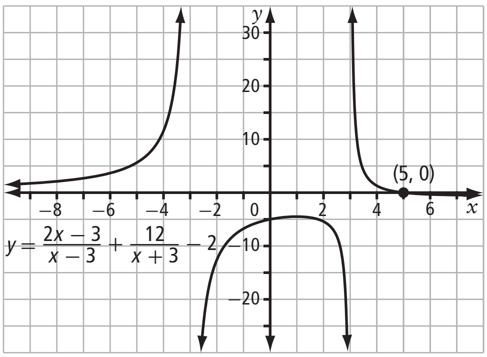
1. **a)** $x = -2, x = 7$ **b)** $x = -3, x = -8$

c) $x = -2$ **d)** $x = \frac{5}{2}, x = -1$

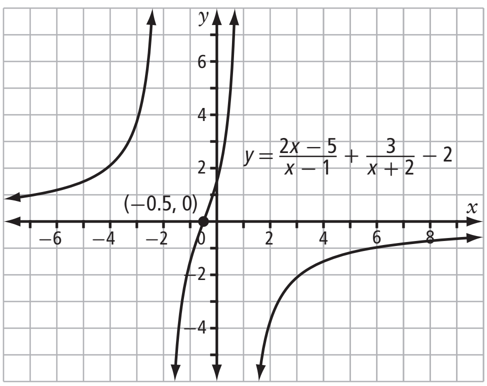
2. **a)** $x = -2$



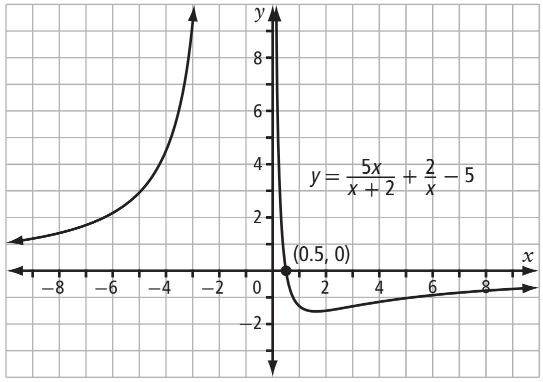
b) $x = 5$



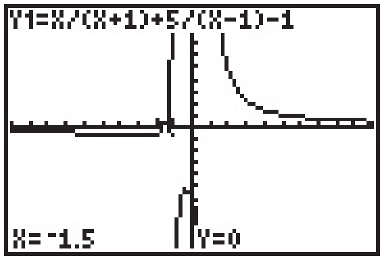
c) $x = -0.5$



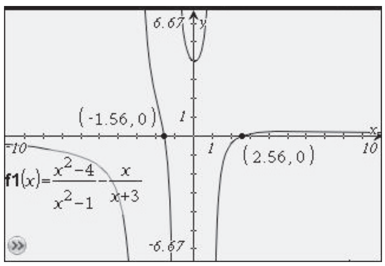
d) $x = 0.5$



3. **a)** $x = -1.5$



b) $x = \frac{1 \pm \sqrt{17}}{2}$ or $x \approx -1.56, x \approx 2.56$



4. **a)** $x = -5$; no extraneous root

b) $x = -3$; extraneous root: $x = 1$

c) $x = -7$; extraneous root: $x = 2$

d) $x = 2$; extraneous root: $x = -2$

5. **a)** Amber: $\frac{400}{x}$, Matteo: $\frac{400}{(x-1)}$

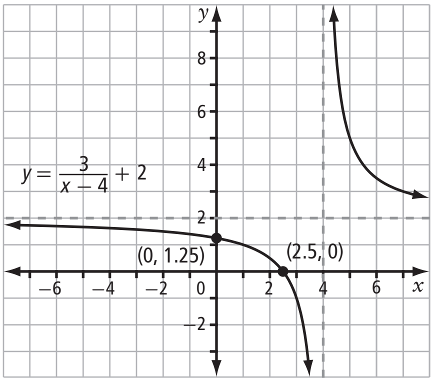
b) $\frac{400}{(x-1)} - \frac{400}{x} = 20$; Amber: 80 km/h, Matteo: 100 km/h

6. Example: Solving an equation algebraically gives an exact solution because you can answer as a fraction or a radical, if necessary. Solving graphically gives a whole number value or a decimal approximation.

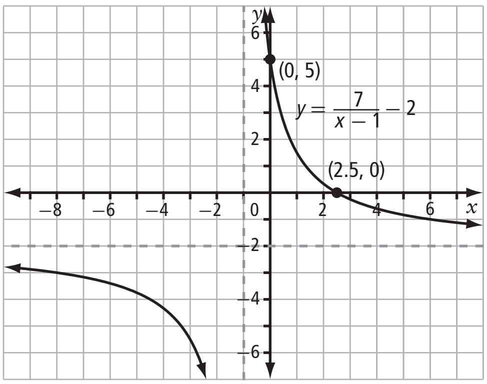
7. Example: Solving algebraically, extraneous solutions are determined by comparing the answer to the restrictions. Solving graphically only solves equations in their simplest form.

Chapter 9 Review, pages 321–323

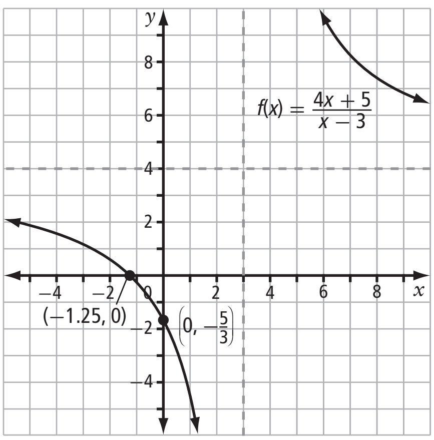
1. a)



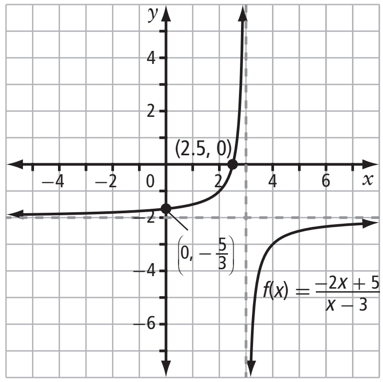
b)



2. a)



b)



3. a) vertical asymptotes at $x = -3$ and $x = 2$, no points of discontinuity, x-intercept of 0; D

b) vertical asymptote at $x = 6$, point of discontinuity (−4, 0.7), x-intercept of 3; A

c) point of discontinuity at (3, 1), no vertical asymptotes, x-intercept of 2; C

d) vertical asymptotes at $x = 1$ and $x = 2$, no points of discontinuity, x-intercepts of −3 and 4; B

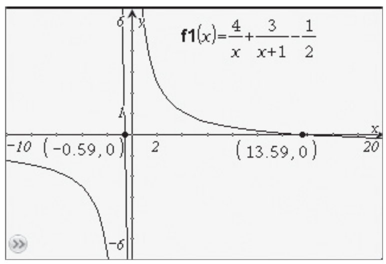
4. a) no points of discontinuity; vertical asymptote: $x = -5$; x-intercept: $-\frac{1}{2}$; y-intercept: $\frac{1}{5}$

b) point of discontinuity: (2, −4); no vertical asymptotes; x-intercept: 6; y-intercept: −6

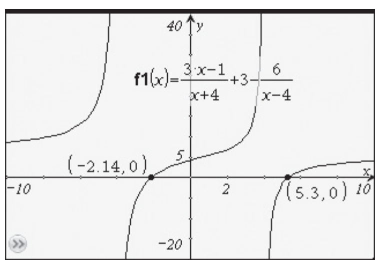
5. a) $x = -4, x = -6$ **b)** $x = 2, x = 6$

c) $x = -7$ **d)** $x = -2, x = 5$

6. a) $x \approx -0.6, x \approx 13.6$



b) $x \approx -2.1, x \approx 5.3$



Chapter 10

10.1 Sums and Differences of Functions, pages 325–334

1.

x	$f(x)$	$g(x)$	$(f + g)(x)$	$(f - g)(x)$
−6	2	4	6	−2
−4	4	2	6	2
−2	6	0	6	6
0	8	−2	6	10
2	10	−4	6	14
4	12	−6	6	18
6	14	−8	6	22

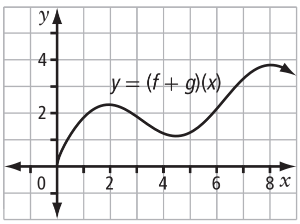
2. **a)** $(f + g)(x) = \{(-6, 7), (-4, 8), (-2, 9), (0, 10)\}$
b) $(f - g)(x) = \{(-6, 1), (-4, -4), (-2, -9), (0, -14)\}$

3. **a)** $(f + g)(x) = 5x - 1$
b) $(f - g)(x) = x - 3$
c) $(g - f)(x) = -x + 3$

4. **a)** 36 **b)** 6 **c)** 4

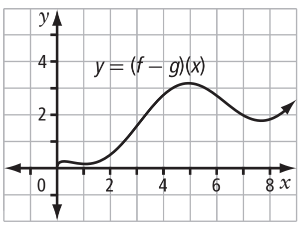
5. **a)** $y = 5x^2 + \sqrt{x^2 - 4}$; $\{x \mid x \le -2 \text{ and } x \ge 2, x \in R\}$
b) $y = 5x^2 - \sqrt{x^2 - 4}$; $\{x \mid x \le -2 \text{ and } x \ge 2, x \in R\}$
c) $y = \sqrt{x^2 - 4} - 5x^2$; $\{x \mid x \le -2 \text{ and } x \ge 2, x \in R\}$

6. **a)**



domain: $\{x \mid x \ge 0, x \in R\}$;
range: $\{y \mid y \ge 0, y \in R\}$

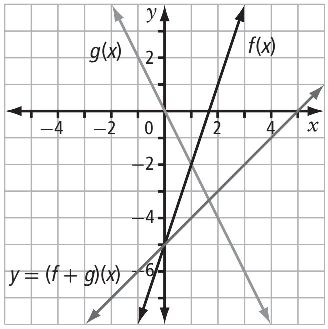
b)



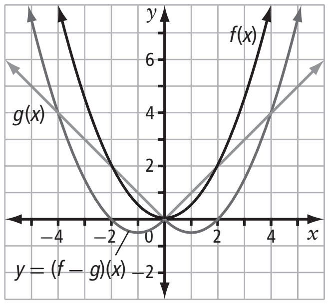
domain: $\{x \mid x \ge 0, x \in R\}$;
range: $\{y \mid y \ge 0, y \in R\}$

7. **a)** 9 **b)** −1 **c)** 3 **d)** 3
e) The domain of $f(x)$ does not include the value 5.

8. **a)**



b)

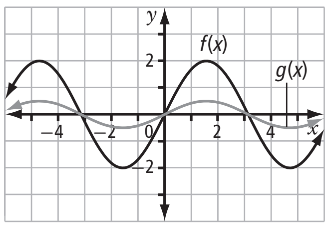


9. **a)** $y = -x^2 + x + 8$ **b)** $y = x^2 + 3x - 2$
c) $y = x^2 + x - 2$

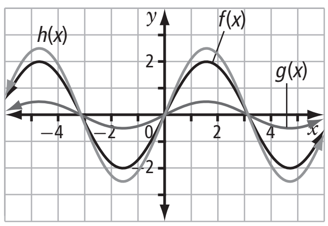
10. **a)** $g(x) = x^2 - 6x + 7$ **b)** $g(x) = -5x + 11$
c) $g(x) = -2x^2 - 3x + 8$

11. **a)** $C(n) = 25n + 45\ 000$ **b)** $R(n) = 100n$
c) $P(n) = 75n - 45\ 000$ **d)** (600, 0)

12. **a)**



b)



c) 2.5 units

13. Example: $f(x) = \sqrt{x - 1}$ and $g(x) = \sqrt{-x}$
The domain of $f(x)$ is $\{x \mid x \ge 1, x \in R\}$ and the domain of $g(x)$ is $\{x \mid x \le 0, x \in R\}$. Since there are no values of x common to the two domains, there are no possible values for the range.

14. Example: No, it is not possible. Subtraction is not commutative.

10.2 Products and Quotients of Functions, pages 335–344

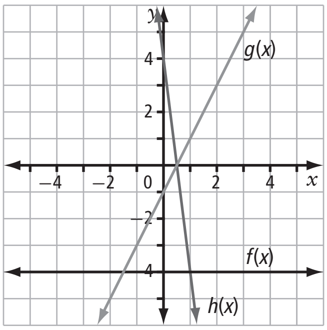
1. **a)** $h(x) = 4x^2 - 8x + 3$; $k(x) = \dfrac{2x-1}{2x-3}$, $x \neq \dfrac{3}{2}$

b) $h(x) = -3x^3 - 12x$; $k(x) = \dfrac{3x}{-x^2-4}$

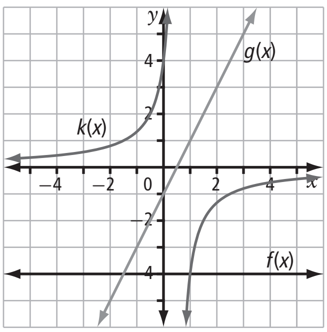
c) $h(x) = (5x-1)(\sqrt{4-x})$; $k(x) = \dfrac{\sqrt{4-x}}{5x-1}$, $x \neq \dfrac{1}{5}$

2. **a)** -20 **b)** $-\dfrac{2}{3}$ **c)** 0 **d)** -24

3. **a)**



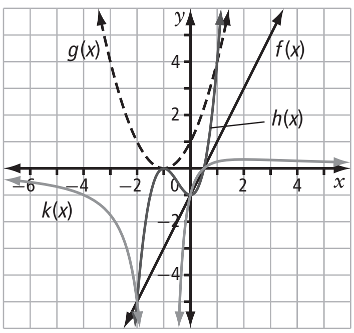
b)



4. **a)** $h(x) = 2x^3 + 3x^2 - 1$

b) $k(x) = \dfrac{2x-1}{x^2+2x+1}$, $x \neq -1$

c)



d) $h(x)$: domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \in \mathrm{R}\}$; $k(x)$: domain: $\{x \mid x \neq -1, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0.33, y \in \mathrm{R}\}$

5. **a)** -260 **b)** $\dfrac{5}{4}$ **c)** $-\dfrac{4}{5}$

6. **a)** $y = (4x^3 - 5x^2)(\sqrt{x-1})$, $x \geq 1$

b) $y = \dfrac{(4x-5)(\sqrt{x-1})}{x^2}$, $x \geq 1$

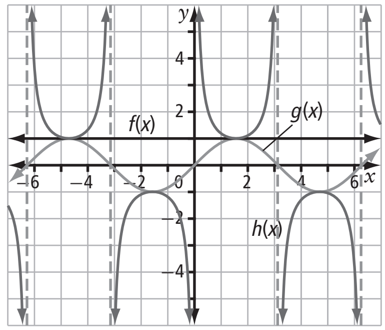
7. **a)** $g(x) = -x$ **b)** $g(x) = \dfrac{\sqrt{x^2-4}}{2-x}$, $x \leq -2$ and $x > 2$

8. **a)** $f(x) = -2x^3 - 12x^2$, $x \neq 0$

b) $f(x) = 2x^2 + 6x$, $x \neq 0$

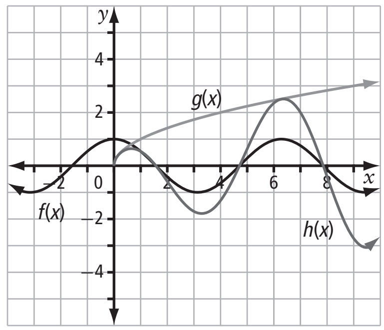
9. **a)** $h(x) = \dfrac{f(x)}{g(x)} = \dfrac{1}{\sin x}$

b)



restrictions on $h(x)$: $x \neq \pi n$, $n \in \mathrm{I}$

10. **a)**



b) domain: $\{x \mid x \geq 0, x \in \mathrm{R}\}$; range: $\{y \mid y \in \mathrm{R}\}$

11. **a)** $T(w) = 300 - 8w - 0.8w^2$

b) 223.2 h

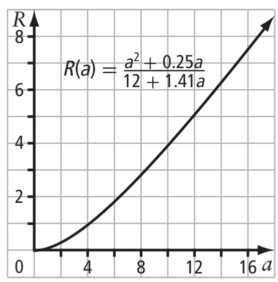
c) week 15; Example: I graphed the function $T(w) = 300 - 8w - 0.8w^2$ using graphing technology and determined the w-intercept in quadrant I. In other words, I determined the value of w when $T(w) = 0$.

d) domain: $\{w \mid 0 \leq w \leq 15, w \in \mathrm{W}\}$; range: $\{T \mid 0 \leq T \leq 300, T \in \mathrm{R}\}$

12. **a)** $R(a) = \dfrac{a^2 + 0.25a}{12 + 1.41a}$

b) 33.15 tonnes; 228.75 tonnes

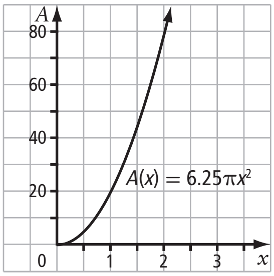
c)



domain: $\{a \mid a > 0, a \in \mathrm{R}\}$; range: $\{R \mid R > 0, R \in \mathrm{R}\}$

13. a) $A(x) = 6.25\pi x^2$

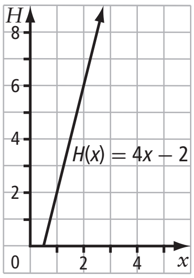
b)



domain: $\{x \mid x \geq 0, x \in \mathrm{R}\}$;
range: $\{A \mid A \geq 0, A \in \mathrm{R}\}$

c) $H(x) = 4x - 2$

d)



domain: $\{x \mid x \geq 0.5, x \in \mathrm{R}\}$;
range: $\{H \mid H \geq 0, H \in \mathrm{R}\}$

10.3 Composite Functions, pages 345–355

1. a) 0 **b)** 2 **c)** 6 **d)** 5

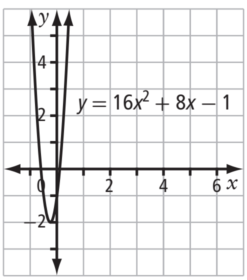
2. a) 23 **b)** −13 **c)** 62 **d)** 11

3. a) $(f \circ g)(x) = \sqrt{x^2 + 4}$; $(g \circ f)(x) = x + 4$

b) $(f \circ g)(x) = |-1 - x|$; $(g \circ f)(x) = 3 - |x - 4|$

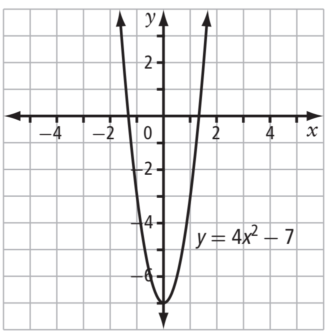
c) $(f \circ g)(x) = \dfrac{1}{x + 3}$; $(g \circ f)(x) = \dfrac{1}{x} + 3$

4. a)



domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq -2, y \in \mathrm{R}\}$

b)



domain: $\{x \mid x \in \mathrm{R}\}$; range: $\{y \mid y \geq -7, y \in \mathrm{R}\}$

5. a) $g(x) = x - 2$ **b)** $g(x) = x^2 - 2$ **c)** $g(x) = x - 5$

6. a) $(f \circ g)(x) = \sqrt{3x - 3}$; domain: $\{x \mid x \geq 1, x \in \mathrm{R}\}$

b) $(g \circ f)(x) = 3\sqrt{x - 4} + 1$; domain: $\{x \mid x \geq 4, x \in \mathrm{R}\}$

7. a) $f(g(x)) = x + 4$ **b)** $g(f(x)) = x + 4$

c) restriction on domain of $f(g(x))$: $x \neq 1$;
restriction on domain of $g(f(x))$: $x \neq \pm 1$

8. a) $h(k(x)) = k(h(x)) = \dfrac{1}{x^2}$ **b)** $x \neq 0$

9. a) $C(x(t)) = 787.5t + 900$

b) $C(x(t))$ represents the cost of manufacturing engines after t hours of production.

c) \$7200 **d)** 42 h

10. a) $R = p - 1200$ **b)** $D = 0.9p$

c) $(R \circ D)(p) = 0.9p - 1200$; The composite function represents the cost of the snowmobile when the dealer discount is computed before the factory rebate.

d) $(D \circ R)(p) = 0.9p - 1080$; The composite function represents the cost of the snowmobile when the factory rebate is subtracted before the dealer discount.

e) \$8475; \$8595; The lower cost is given by the composite function $(R \circ D)(p)$. Example: The price is lower when the \$1200 is subtracted after the dealer discount.

11. The sales representative's bonus is represented by $g(f(x)) = 0.5(x - 50\,000)$. Example: The bonus is computed after the \$50 000 is subtracted.

12. a) $f(x) = x^2$; $g(x) = 2x + 1$

b) $f(x) = \sqrt{x}$; $g(x) = 9 - x$

13. a) Example: $f(x) = 2x$, $g(x) = x - 3$, $h(x) = 5x + 1$

b) $((f \circ g) \circ h)(x) = 10x - 4$, $(f \circ (g \circ h))(x) = 10x - 4$; Therefore, the composition of functions does follow the associative property.

14. a) Example: $f(x) = x^2$, $g(x) = 3x - 2$

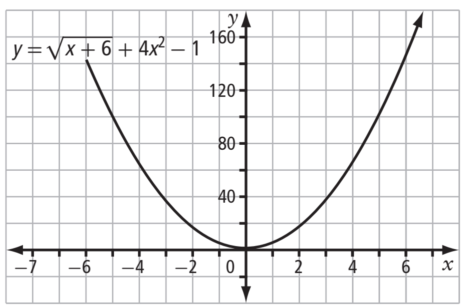
b) $(f \circ g)(x) = 9x^2 - 12x + 4$, $(g \circ f)(x) = 3x^2 - 2$; Therefore, the composition of functions does not follow the commutative property. There are no restrictions in this case.

Chapter 10 Review, pages 356–362

1. a) 6 **b)** −13

2. a) $h(x) = \sqrt{x + 6} + 4x^2 - 1$

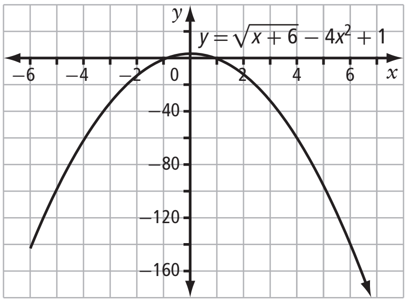
b)



c) domain: $\{x \mid x \geq -6, x \in R\}$; range: $\{y \mid y \geq 1.4, y \in R\}$

d) $k(x) = \sqrt{x + 6} - 4x^2 + 1$

e)

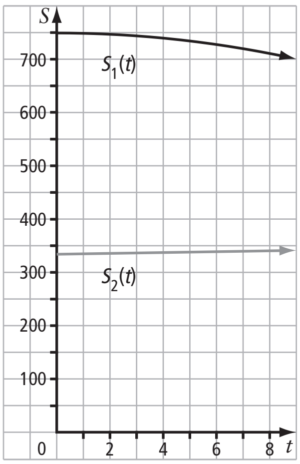


f) domain: $\{x \mid x \geq -6, x \in R\}$; range: $\{y \mid y \leq 3.5, y \in R\}$

3. a) $g(x) = -4x^2 + 11x - 3$

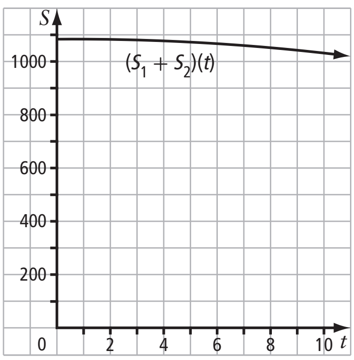
b) $g(x) = -2x - \sqrt{x} + 12$

4. a)



b) $(S_1 + S_2)(t) = 1085 - 0.6t^2 + 0.8t$

c)



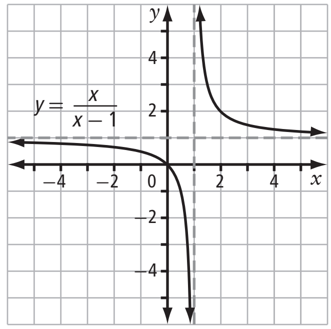
d) The total sales have been decreasing. Example: The combined sales in 2007 were \$1 061 200. The combined sales in 2012 were \$1 008 200.

5. a) $h(x) = -2x^3 + x^2 - 6x + 3$

b) $k(x) = \frac{x^2 + 3}{1 - 2x}, x \neq \frac{1}{2}$

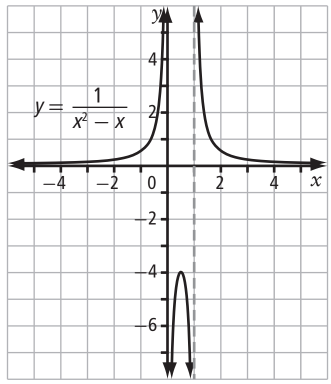
6. a) 1 **b)** 12 **c)** −3 **d)** $\frac{5}{9}$

7. a) $h(x) = \frac{x}{x - 1}$



domain: $\{x \mid x \neq 1, x \in R\}$

b) $k(x) = \frac{1}{x^2 - x}$

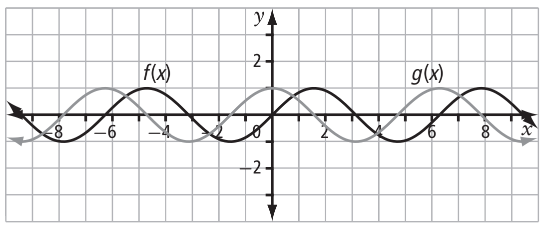


domain: $\{x \mid x \neq 0, 1, x \in R\}$

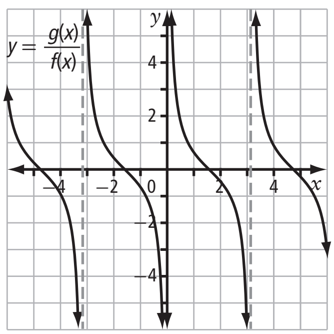
8. a) $g(x) = x - 1$ **b)** $g(x) = \sin x$

c) $g(x) = -x^2$

9. a)



b)

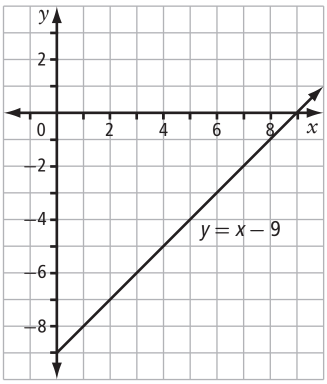


c) domain: $\{x \mid x \neq n\pi, n \in I, x \in R\}$; range: $\{y \mid y \in R\}$

d) $y = \cot x$

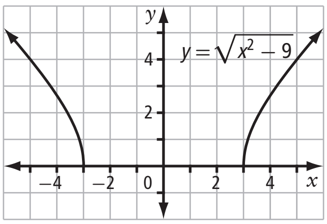
10. a) $(f \circ g)(x) = -x^2 - 2$ **b)** $(g \circ g)(x) = -x^4 + 2x^2$
c) -11 **d)** -8

11. a)



domain: $\{x \mid x \geq 0, x \in \mathrm{R}\}$;
range: $\{y \mid y \geq -9, y \in \mathrm{R}\}$

b)



domain: $\{x \mid x \leq -3 \text{ and } x \geq 3, x \in \mathrm{R}\}$;
range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$

12. a) $g(x) = 9 - x$
b) $g(x) = 7x - 2$
c) $g(x) = 2x - 5$

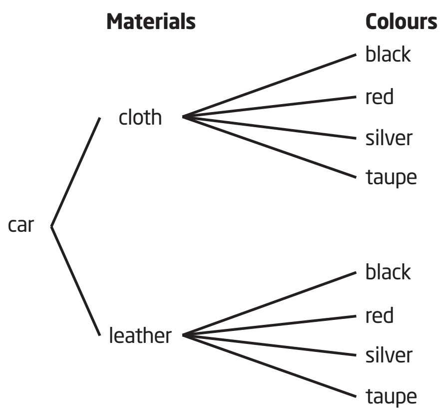
13. a) $d = 2r$ **b)** $V = 8r^3$ **c)** 3375 cm^3

14. a) $(C \circ n)(t) = -15t^2 + 4125t + 1195$
b) \$70 585
c) 24 h

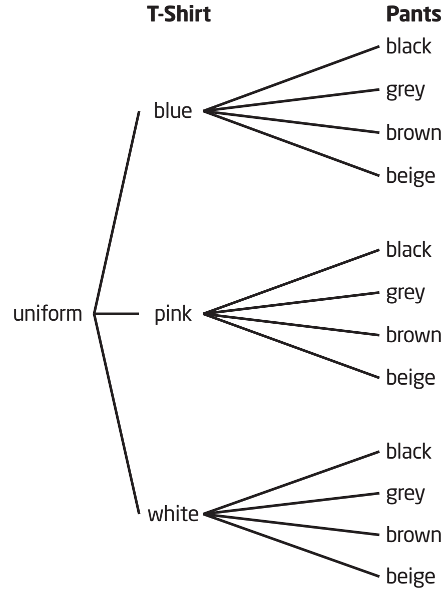
Chapter 11

11.1 Permutations, pages 364–373

1. a)



b)



2. a)

First	Second Digit
3	4
3	6
3	7
3	8
4	3
4	6
4	7
4	8
6	3
6	4
6	7
6	8
7	3
7	4
7	6
7	8
8	3
8	4
8	6
8	7

b)

Toronto to Calgary	Calgary to Vancouver
Plane	Bus
Plane	Plane
Plane	Train
Plane	Car
Train	Bus
Train	Plane
Train	Train
Train	Car

3. **a)** 42 **b)** 6 **c)** 79 833 600 **d)** 1680

4. **a)** Left Side $= 5! - 3!$
$= (5 \times 4 \times 3 \times 2!) - (3 \times 2!)$
$= (60)(2!) - (3)(2!)$
$= (60 - 3)(2!)$
$= 57(2!)$

Right Side $= (5! - 3!)$
$= 2!$

Left Side $\neq$ Right Side

b) Left Side $= 6! - 4!$
$= (6 \times 5 \times 4!) - (4!)$
$= (30)(4!) - (4!)$
$= (30 - 1)(4!)$
$= 29(4!)$

Right Side $= 29(4!)$

Left Side = Right Side

5. **a)** 19 600 **b)** 4320

c) 91 **d)** 3 628 800

6. **a)** $\frac{7!}{2!2!} = 1260$ **b)** $\frac{10!}{2!2!3!} = 151\,200$

c) $6! = 720$ **d)** $\frac{11!}{2!2!2!2!} = 2\,494\,800$

7. $7! = 5040$

8. **a)** $n = 7$ **b)** $r = 2$ **c)** $n = 10$ **d)** $r = 4$

9. **a)** 40 **b)** 72

10. **a)** $3(5!)2 = 720$

b) $(5!)(3!) = 720$ (count group of 3 boys as 1 : 1 + 4 girls = 5)

c) $(4!)(4!) = 576$ (count group of 4 girls as 1 : 1 + 3 boys = 4)

d) $3(5!)2 + 4(5!)3 = 720 + 1440 = 2160$

11. **a)** $7! = 5040$

b) $3(5!)2 = 720$

c) $4(5!)3 = 1440$

12. $13(9)(12)(8)(7) = 78\,624$

13. **a)** $\frac{13!}{3!3!2!} = 86\,486\,400$

b) $\frac{11!}{3!2!} = 3\,326\,400$ **c)** $\frac{11!}{3!2!} = 3\,326\,400$

d) $\frac{12!}{3!3!} = 13\,305\,600$ **e)** $8! = 40\,320$

14. **a)** 7 200 000 **b)** 18 000 **c)** 72 000

11.2 Combinations, pages 374–382

1. **a)** permutation; order matters

b) combination; order does not matter

c) combination; order does not matter

d) permutation; order matters

2. ${}_8P_5$ represents an arrangement of 8 objects taken 5 at a time, and order matters. ${}_8C_5$ represents a selection of 5 objects taken from 8, and order does not matter. ${}_8P_5 = 6720$; ${}_8C_5 = 56$

3. **a)** 21 **b)** 42 **c)** 84 **d)** 210

4. **a)** ${}_{12}C_8 = 495$ **b)** ${}_{12}P_6 = 665\,280$

5. **a)** ${}_5C_3 = 10$

b) ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE

c) ${}_5P_3 = 60$

d) There are $3! = 6$ times as many permutations as there are combinations.

6. **a)** $n = 6$ **b)** $n = 5$ **c)** $n = 5$ **d)** $n = 3$

7. **a)** case 1: single-digit numbers less than 800;
case 2: double-digit numbers less than 800;
case 3: triple-digit numbers less 800

b) case 1: 4 members are from grade 11;
case 2: 4 members are from grade 12

8. **a)** Left Side $= {}_{10}C_3$
$= \frac{10!}{(10-3)!3!}$
$= \frac{10!}{7!3!}$

Right Side $= {}_{10}C_7$
$= \frac{10!}{(10-7)!7!}$
$= \frac{10!}{3!7!}$

Left Side = Right Side

b) Left Side $= {}_{12}C_4$
$= \frac{12!}{(12-4)!4!}$
$= \frac{12 \times 11 \times 10 \times 9 \times 8!}{8!4!}$
$= \frac{12 \times 11 \times 10 \times 9}{4!}$

Right Side $= {}_{11}C_4 + {}_{11}C_3$
$= \frac{11!}{(11-4)!4!} + \frac{11!}{(11-3)!3!}$
$= \frac{11 \times 10 \times 9 \times 8 \times 7!}{7!4!}$
$+ \frac{11 \times 10 \times 9 \times 8!}{8!3!}$
$= \frac{11 \times 10 \times 9 \times 8}{4!} + \frac{11 \times 10 \times 9}{3!}$
$= \frac{(11 \times 10 \times 9 \times 8) + (11 \times 10 \times 9 \times 4)}{4!}$
$= \frac{(11 \times 10 \times 9)(8 + 4)}{4!}$
$= \frac{12 \times 11 \times 10 \times 9}{4!}$

Left Side = Right Side

9. a) ${}_{15}C_4 = 1365$
b) ${}_{7}C_1 \times {}_{8}C_3 = 392$
c) ${}_{8}C_2 \times {}_{7}C_2 + {}_{8}C_3 \times {}_{7}C_1 + {}_{8}C_4$
$= 28 \times 21 + 56 \times 7 + 70$
$= 1050$

10. a) ${}_{7}C_4 \times {}_{9}C_2 = 1260$
b) ${}_{7}C_4 \times {}_{9}C_2 + {}_{7}C_5 \times {}_{9}C_1 + {}_{7}C_6$
$= 35 \times 36 + 21 \times 9 + 7$
$= 1456$

11. Left Side $= 5({}_{n}C_5)$
$= \frac{5(n!)}{(n-5)!5!}$
$= \frac{n!}{(n-5)!4!}$
Right Side $= n({}_{n-1}C_4)$
$= \frac{n(n-1)!}{(n-5)!4!}$
$= \frac{n!}{(n-5)!4!}$
Left Side = Right Side

12. a) $n = 1$ b) $n = 4$

13. 2520

14. 2400

15. ${}_{11}C_8 = 165$

11.3 The Binomial Theorem, pages 383–389

1. a) 1 5 10 10 5 1
b) 1 7 21 35 35 21 7 1
c) 1 10 45 120 210 252 210 120 45 10 1

2. a) ${}_{3}C_0 \quad {}_{3}C_1 \quad {}_{3}C_2 \quad {}_{3}C_3$
b) ${}_{5}C_0 \quad {}_{5}C_1 \quad {}_{5}C_2 \quad {}_{5}C_3 \quad {}_{5}C_4 \quad {}_{5}C_5$
c) ${}_{6}C_0 \quad {}_{6}C_1 \quad {}_{6}C_2 \quad {}_{6}C_3 \quad {}_{6}C_4 \quad {}_{6}C_5 \quad {}_{6}C_6$

3. a) ${}_{4}C_3 = \frac{4!}{1!3!}$
b) ${}_{7}C_3 = \frac{7!}{4!3!}$
c) ${}_{9}C_6 = \frac{9!}{3!6!}$

4. a) 8 b) 7 c) 5 d) 6

5. a) $m^7 - 7m^6n + 21m^5n^2 - 35m^4n^3 + 35m^3n^4 - 21m^2n^5 + 7mn^6 - n^7$
b) $64x^6 + 192x^5 + 240x^4 + 160x^3 + 60x^2 + 12x + 1$
c) $a^{15} + 10a^{12} + 40a^9 + 80a^6 + 80a^3 + 32$

6. a) $x^6 - 12x^5 + 60x^4 - 160x^3 + 240x^2 - 192x + 64$
b) $a^4 - 8a^3b + 24a^2b^2 - 32ab^3 + 16b^4$
c) $1 + 6x^2 + 15x^4 + 20x^6 + 15x^8 + 6x^{10} + x^{12}$

7. a) $252a^5b^5$ b) $715x^9$ c) $210x^8$ d) $\frac{21}{4}x^{12}$
e) $1120x^4y^4$

8. a) $a^5 - 5a^3 + 10a - \frac{10}{a} + \frac{5}{a^3} - \frac{1}{a^5}$
b) $x^5 - 10x^2 + \frac{40}{x} - \frac{80}{x^4} + \frac{80}{x^7} - \frac{32}{x^{10}}$

9. a) $512a^9 - 2304a^7 + 4608a^5 - 5376a^3$
b) $256x^8 - 3072x^6 + 16\,128x^4 - 48\,384x^2$

10. a) $(x - y)^5$ b) $(x - 2)^6$

11. $t_{k+1} = {}_{7}C_k a^{14-2k}$

12. $-6x^4$

13. $x^7 + 6x^5 + 15x^3 + 20x + 15x^{-1} + 6x^{-3} + x^{-5}$

14. 1.1951

Chapter 11 Review, pages 390–392

1. a) 604 800 b) 840

2. a) $8! = 40\,320$ b) $\frac{6!}{3!} = 120$

3. a) $n = 5$ b) $r = 2$

4. To satisfy the requirements, the word must have a total of nine letters, with three of one kind and two of another kind. Example: excellent

5. a) $3(6!)2 = 4320$
b) $(6!)(3!) = 4320$
c) $(4!)(5!) = 2880$

6. a) permutation; order matters
b) combination; order does not matter

7. ${}_{9}P_6$ represents an arrangement of 9 objects taken 6 at a time, and order matters. ${}_{9}C_6$ represents a selection of 6 objects taken from 9, and order does not matter.

8. a) $n = 5$ b) $n = 7$

9. ${}_{13}C_1 \times {}_{13}C_1 \times {}_{13}C_1 \times {}_{13}C_1 = 28\,561$

10. a) ${}_{15}C_2 \times {}_{18}C_3 = 105 \times 816 = 85\,680$
b) ${}_{15}C_3 \times {}_{18}C_2 + {}_{15}C_2 \times {}_{18}C_3 + {}_{15}C_1 \times {}_{18}C_4$
$= 455 \times 153 + 105 \times 816 + 15 \times 3060$
$= 201\,195$

11. a) 1 2 1 b) 1 6 15 20 15 6 1

12. a) ${}_{4}C_0 \quad {}_{4}C_1 \quad {}_{4}C_2 \quad {}_{4}C_3 \quad {}_{4}C_4$
b) ${}_{8}C_0 \quad {}_{8}C_1 \quad {}_{8}C_2 \quad {}_{8}C_3 \quad {}_{8}C_4 \quad {}_{8}C_5 \quad {}_{8}C_6 \quad {}_{8}C_7 \quad {}_{8}C_8$

13. a) 5 terms; $16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4$
b) 6 terms; $a^{10} - 20a^8 + 160a^6 - 640a^4 + 1280a^2 - 1024$

14. a) $x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$
b) $64x^6 - 192x^4 + 240x^2 - 160 + 60x^{-2} - 12x^{-4} + x^{-6}$

15. a) $-71\,680a^3$ b) $12\,650a^{21}b^4$