Chapter 9 Rational Functions

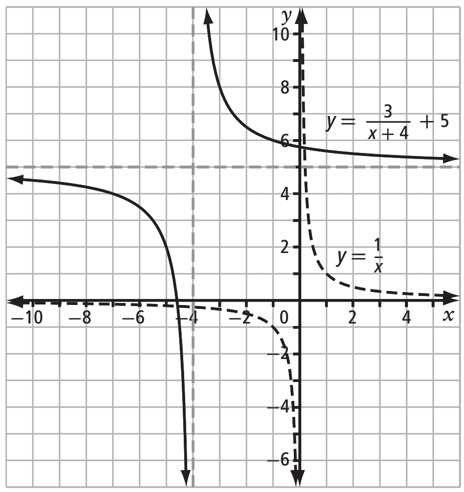
9.1 Exploring Rational Functions Using Transformations

KEY IDEAS

- Rational functions are functions of the form $y = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$.
- You can graph a rational function by creating a table of values and then graphing the points in the table. To create a table of values,
  – identify the non-permissible value(s)
  – write the non-permissible value in the middle row of the table
  – enter positive values above the non-permissible value and negative values below the non-permissible value
  – choose small and large values of x to give you a spread of values
- You can use what you know about the base function $y = \frac{1}{x}$ and transformations to graph equations of the form $y = \frac{a}{x-h} + k$.

  Example:
  For $y = \frac{3}{x+4} + 5$, the values of the parameters are

  $a = 3$, representing a vertical stretch by a factor of 3
  $h = 4$, representing a horizontal translation 4 units to the left
  $k = 5$, representing a vertical translation 5 units up
  vertical asymptote: $x = -4$
  horizontal asymptotes: $y = 5$



- Some equations of rational functions can be manipulated algebraically into the form $y = \frac{a}{x-h} + k$ by creating a common factor in the numerator and the denominator.

  Example:

  $$y = \frac{3x + 6}{x - 4}$$
  $$y = \frac{3x - 12 + 12 + 6}{x - 4}$$
  $$y = \frac{3x - 12 + 18}{x - 4}$$
  $$y = \frac{3(x - 4)}{x - 4} + \frac{18}{x - 4}$$
  $$y = \frac{18}{x - 4} + 3$$

Working Example 1: Graph a Rational Function Using a Table of Values

Graph $y = \frac{4}{x}$ using a table of values.

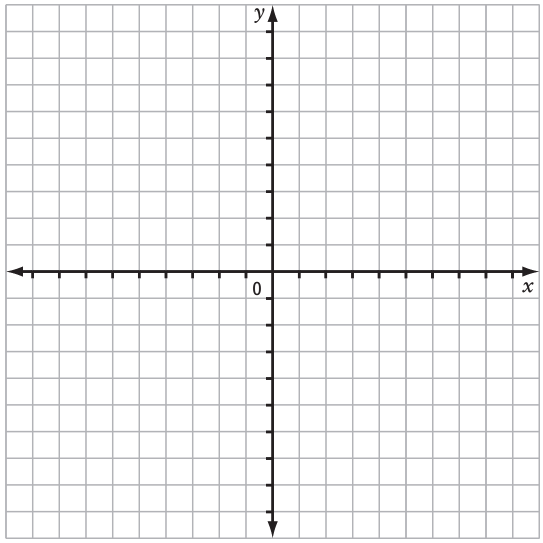
Solution

Begin by identifying any non-permissible values: what value(s) can x not equal? $x \neq$ ________.

Graphs of rational functions of the form $y = \frac{4}{x}$ approach asymptotes at $x =$ ________ and $y =$ ________. Plot the vertical and horizontal asymptotes on the grid below.

Create a table of values. Plot and connect the points from the table of values to generate the general shape of the graph. It is often easier to create a table of values if you rearrange the formula so that rather than being a quotient, it is a product of polynomials: $xy = 4$.

x	y
	undefined



Check your graph using your graphing calculator. How do the graphs compare?

Summarize the characteristics of the function using a table.

Characteristic	$y = \frac{4}{x}$
Non-permissible value	
Behaviour near non-permissible value	
End behaviour	
Domain	
Range	
Equation of vertical asymptote	
Equation of horizontal asymptote	

To see a similar example, see Example 1 on pages 432–434 of *Pre-Calculus 12*.

Working Example 2: Graph a Rational Function Using Transformations

Graph $y = \frac{3}{x-3} + 2$ using transformations.

Solution

Compare the function $y = \frac{3}{x-3} + 2$ to the form $y = \underline{\hspace{2cm}} + k$ to determine the value of the parameters. Then, describe the effect that each parameter has on the graph of $y = \frac{1}{x}$.

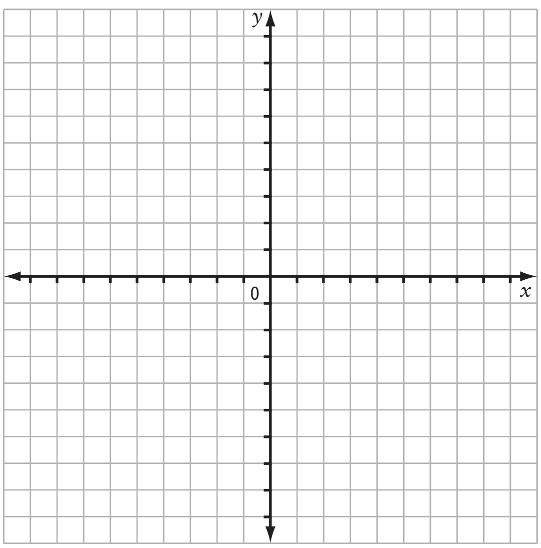
If the asymptotes of $y = \frac{1}{x}$ are $x = 0$ and $y = 0$, use the above transformation to determine the asymptotes of $y = \frac{3}{x-3} + 2$. Explain your reasoning.

Will the graph of $y = \frac{3}{x-3} + 2$ have an x-intercept or y-intercept? Explain how you know.

What are the x-intercept and y-intercept?

Which variable is set to 0 to find the x-intercept? the y-intercept?

Use all of the above information to graph $y = \frac{3}{x-3} + 2$.



Check your graph using your graphing calculator. How do the two graphs compare?

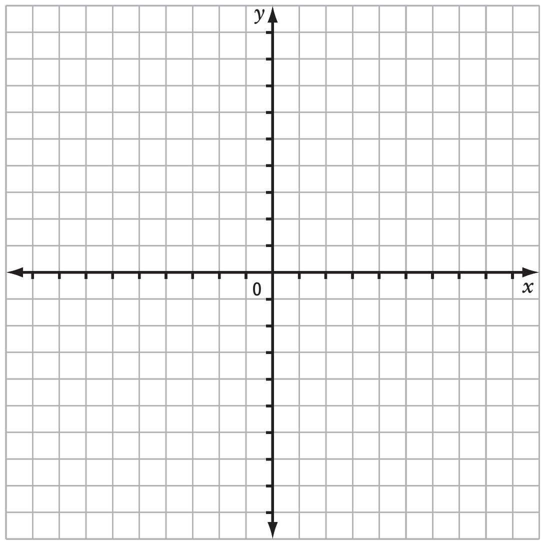
To see a similar example, see Example 2 on pages 434–435 of *Pre-Calculus 12*.

Working Example 3: Graph a Rational Function With Linear Expressions in the Numerator and the Denominator

Graph $y = \frac{4x + 2}{x - 1}$. Identify any asymptotes and intercepts.

Solution

Let $x = 0$. Solve for y to determine the y-intercept.



The y-intercept is at (0, ________).

Let $y = 0$. Solve for x to determine the x-intercept.

$$\mathbf{0} = \frac{4x + 2}{x - 1}$$

$$(\_\_\_\_\_\_\_\_)(0) = (\_\_\_\_\_\_\_\_)\frac{4x + 2}{x - 1}$$

$$\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$$

$$\_\_\_\_\_\_\_\_ = 4x$$

$$\_\_\_\_\_\_\_\_ = x$$

The x-intercept is at (________, 0).

Manipulate the equation of the function algebraically to obtain the form $y = \frac{a}{x - h} + k$.

$y = \frac{4x + 2}{x - 1}$

$y = \frac{4x - 4 + 4 + 2}{x - 1}$

$y =$

Why is 4 subtracted and added to the numerator?

Which parameters determine the vertical and horizontal asymptotes of the transformed function?

The parameters are $a =$ ________, $h =$ ________, and $k =$ ________. State the effect of each parameter on the graph of $y = \frac{1}{x}$. Then, use the information you have generated to sketch the transformed function on the grid above.

To see a similar example, see Example 3 on pages 435–437 of *Pre-Calculus 12*.

Check Your Understanding

Practise

1. Graph each function using a table of values. Identify the asymptotes.

   a) $y = \frac{6}{x}$

x	y

   b) $y = \frac{-4}{x}$

x	y

2. Graph each function using transformations. Label the asymptotes and intercepts.

   a) $y = \frac{2}{x + 1} - 3$

   b) $y = \frac{-3}{x - 2} + 1$

3. Graph each function using technology. Identify any asymptotes and intercepts. Sketch the graph.

   a) $y = \frac{2x + 1}{x - 2}$

   b) $y = \frac{-3x - 4}{x + 2}$

Apply

4. Match each graph with its equation.

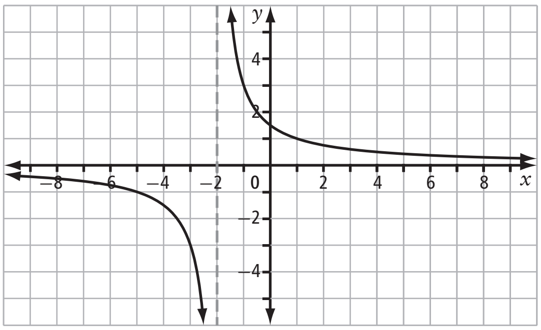
a) $y = \dfrac{3}{x - 2}$

b) $y = \dfrac{3}{x + 2}$

c) $y = \dfrac{3}{x} - 2$

d) $y = \dfrac{3}{x} + 2$

A



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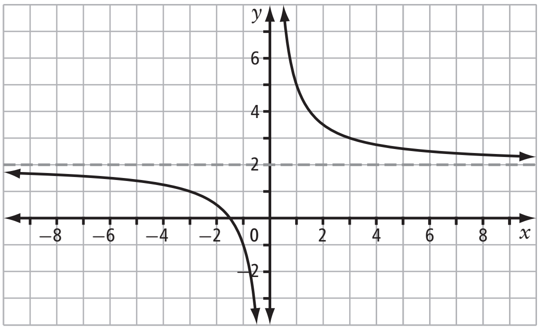

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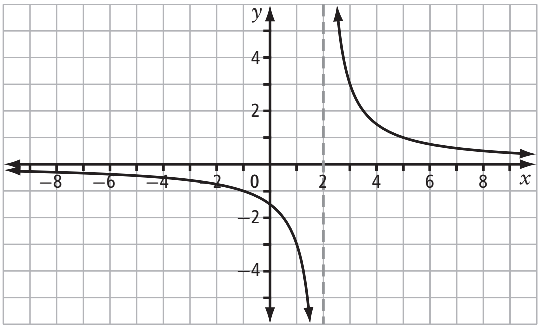

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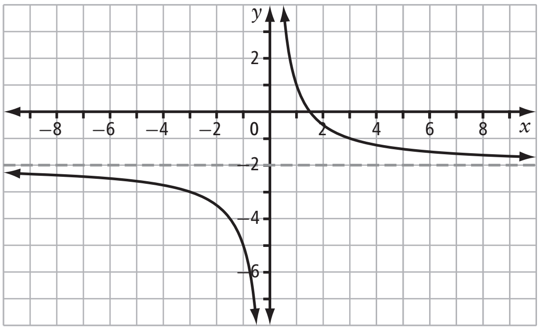
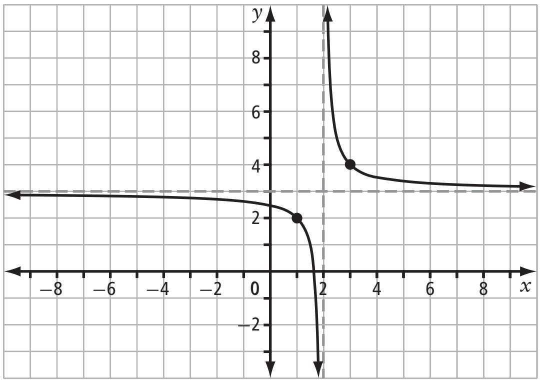


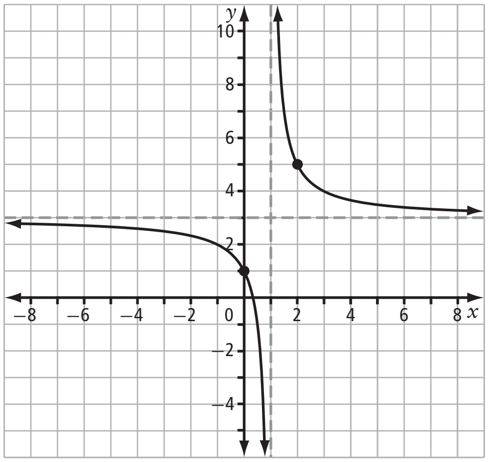
5. Write the equation of each function in the form of $y = \dfrac{a}{x - h} + k$.

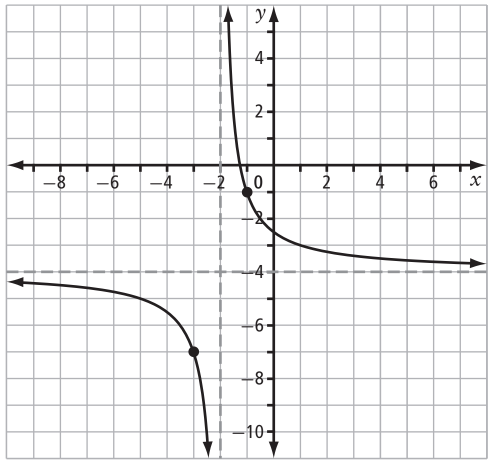
a)



For the graph of $y = \frac{1}{x}$, what is the relationship between the intersection of the asymptotes and the point (1, 1)? How can you use this knowledge to determine a vertical stretch?

The vertical asymptote provides the ________ parameter. The horizontal asymptote provides the ________ parameter.

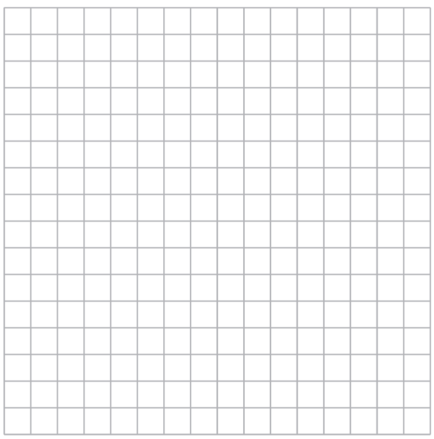
b) 

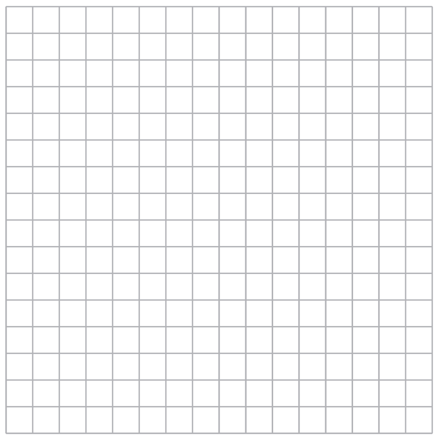
c) 

6. Write each equation in the form $y = \frac{a}{x - h} + k$. Then, graph the function using transformations. Indicate the asymptotes.

a) $y = \frac{7x - 23}{x - 4}$

b) $y = \frac{-5x - 1}{x + 2}$





Connect

7. **a)** Determine an equation of a rational function that has an asymptote at $x = -3$ and $y = -4$. Explain the rationale for your equation.

b) Sketch the graph of your function. Identify the asymptotes on your graph.

c) What is the domain and range of your function?

d) Is there another possible function with these asymptotes? Explain.

8. Describe the similarities and differences between graphing $y = \frac{2}{x-4} - 3$, $y = 2(x-4)^2 - 3$, and $y = 2\sqrt{x-4} - 3$ without technology.

9.2 Analysing Rational Functions

KEY IDEAS

Determining Asymptotes and Points of Discontinuity
The graph of a rational function may have an asymptote, a point of discontinuity, or both. To establish these important characteristics of a graph, begin by factoring the numerator and denominator fully.

- **Asymptotes: No Common Factors**

  If the numerator and denominator do not have a common factor, the function has an asymptote.
  - The vertical asymptotes are identified by the non-permissible values of the function.
  - For a function that can be rewritten in the form $y = \frac{a}{x - h} + k$, the k parameter identifies the horizontal asymptote.

Example: $y = \frac{x + 4}{x - 3}$

Since the non-permissible value is $x = 3$, the vertical asymptote is at $x = 3$.

$y = \frac{x + 4}{x - 3}$

$y = \frac{x - 3 + 3 + 4}{x - 3}$

$y = \frac{x - 3}{x - 3} + \frac{7}{x - 3}$

$y = \frac{7}{x - 3} + 1$

Since $k = 1$, the horizontal asymptote is at $y = 1$.

- **Points of Discontinuity: At Least One Common Factor**

  If the numerator and denominator have at least one common factor, there is at least one point of discontinuity in the graph.
  - Equate the common factor(s) to zero and solve for x to determine the x-coordinate of the point of discontinuity.
  - Substitute the x-value in the simplified expression to find the y-coordinate of the point of discontinuity.

Example: $y = \frac{(x - 4)(x + 2)}{x + 2}$

$x + 2 = 0$: the x-coordinate of the point of discontinuity is −2.

Substitute $x = -2$ into the simplified equation:

$y = x - 4$

$y = \mathbf{-2} - 4$

$y = -6$

point of discontinuity: (−2, −6)

- **Both Asymptote(s) and Point(s) of Discontinuity**

  If a rational expression remains after removing the common factor(s), there may be both a point of discontinuity and asymptotes.

Example:

$y = \frac{(x - 4)(x + 2)}{(x + 2)(x - 1)}$

$y = \frac{(x - 4)}{(x - 1)}$

- common factor: $x + 2$, so there is a point of discontinuity at (−2, 2)
- non-permissible value: $x = 1$, so the vertical asymptote is at $x = 1$
- simplified function can be rewritten as $y = -\frac{3}{x - 1} + 1$, so the horizontal asymptote is at $y = 1$

Working Example 1: Graph a Rational Function With a Point of Discontinuity

Sketch the graph of $f(x) = \frac{x^2 - 3x - 4}{x - 4}$.

Solution

Fully factor the numerator and denominator of the rational function.

There is a common factor, so the graph of the function has a ____________________.

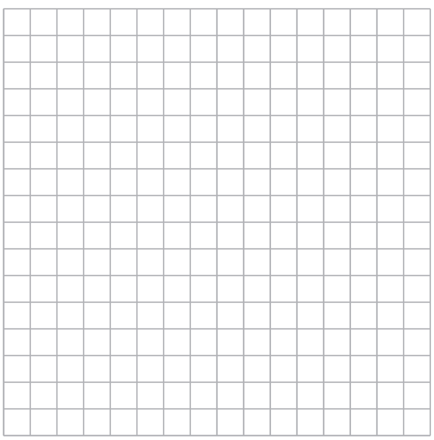
Simplify the rational function. What type of equation remains after the function is simplified?

Equate the common factor to zero and solve for x. Doing so identifies the ________-value of the ____________________.

Substitute the value of x into the simplified function and solve for y. Doing so identifies the ________-value of the point of discontinuity in the graph.

The point of discontinuity is ________.

Graph the rational function, labelling the point of discontinuity.



Does it matter if you graph the original equation or the simplified equation? Explain.

To see a similar example, see Example 1 on pages 447–448 of *Pre-Calculus 12*.

Working Example 2: Compare Points of Discontinuity and Asymptotes in Rational Functions

Compare the graphs of $f(x) = \frac{x^2 + 4x + 3}{x + 1}$ and $g(x) = \frac{x^2 - 4x + 3}{x + 1}$.

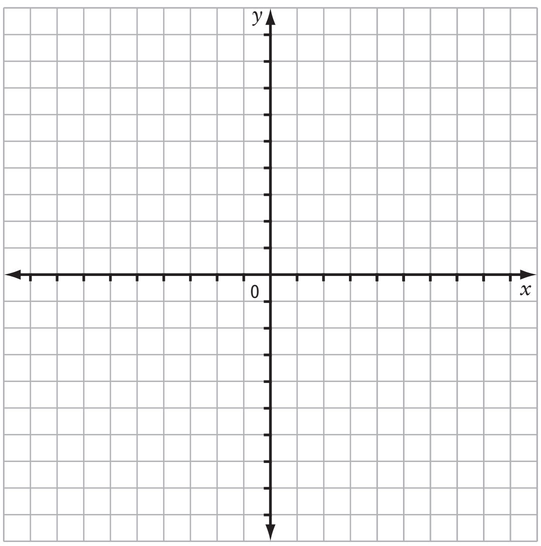
Solution

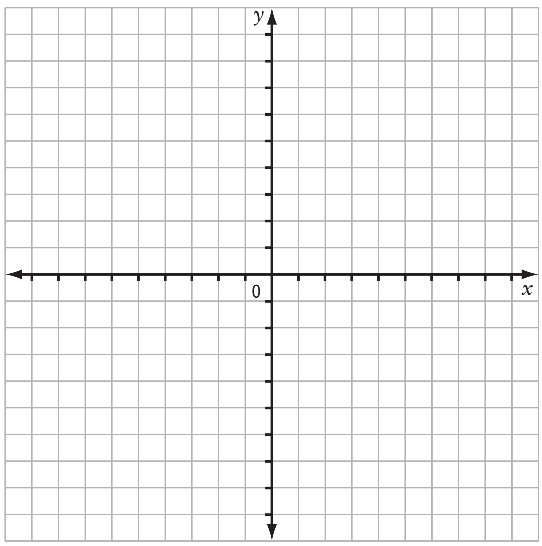
Fully factor the numerator and denominator of each rational function. Simplify the rational functions, if possible. How do the two simplified equations differ?

When simplified, $f(x)$ is a ______________ function. It has a(n) ______________.
(*point of discontinuity* or *asymptote*)

When simplified, $g(x)$ is a ______________ function. It has a(n) ______________.
(*point of discontinuity* or *asymptote*)

With the help of technology, sketch the graph of each function on the grids below. Draw and label any asymptotes that exist.





Both $f(x)$ and $g(x)$ have a non-permissible value at $x =$ ________. Describe what happens to each function as the graph approaches this non-permissible value.

To see a similar example, see Example 2 on pages 448–449 of *Pre-Calculus 12*.

Working Example 3: Sketch a Discontinuous Rational Function

Sketch the graph of $f(x) = \frac{x^2 + 2x - 8}{x^2 + 5x + 4}$. Label all important parts of the graph.

Solution

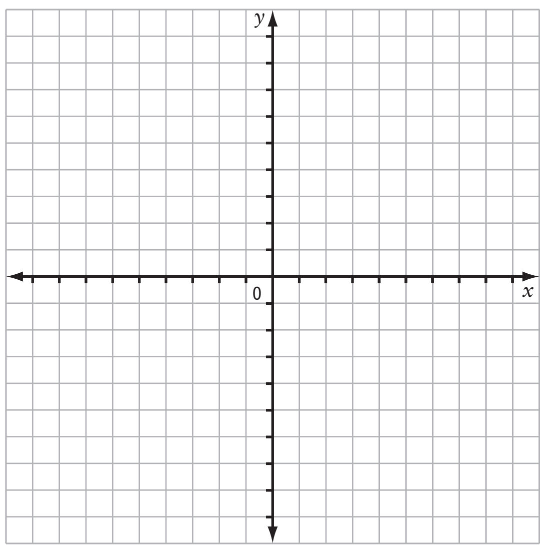
Fully factor the numerator and denominator of the rational function. Simplify the rational function.

Equate the common factor to zero and solve for x to establish the ______________ of the point of discontinuity. Substitute the value of x into the simplified function and solve for y to establish the ______________ of the point of discontinuity in the graph.

The point of discontinuity is at ______.

Find the x-intercept and y-intercept of the simplified function.

Find the horizontal and vertical asymptotes of the simplified function. Then, use the information you have generated to graph the general shape of the rational function. Label the point of discontinuity, the asymptotes, and the intercepts. Check your sketch using technology.



To see a similar example, see Graph 2 in Example 3 on page 449 of *Pre-Calculus 12*.

Check Your Understanding

Practise

1. Determine whether the following functions have points of discontinuity, vertical asymptotes, or both. Explain how you made your determination.

a) $f(x) = \dfrac{x + 5}{x + 4}$

b) $f(x) = \dfrac{x^2 - 5x + 6}{x - 3}$

c) $f(x) = \dfrac{x^2 - x - 12}{x^2 - 5x + 4}$

d) $f(x) = \dfrac{x^2 - 4x - 5}{x^2 - 5x + 6}$

2. For each function, predict the location of any points of discontinuity, vertical asymptotes, x-intercepts, and y-intercepts.

a) $f(x) = \dfrac{x + 1}{x - 4}$

b) $f(x) = \dfrac{x^2 + 7x + 12}{x + 3}$

c) $f(x) = \dfrac{x^2 - 7x + 10}{x^2 - 4x - 5}$

d) $f(x) = \dfrac{x^2 + 6x + 8}{x^2 - 5x - 6}$

3. Match each function with its graph.

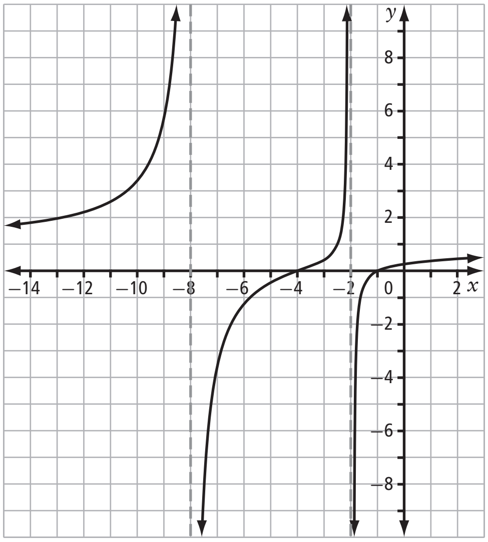
a) $f(x) = \dfrac{x + 4}{x + 8}$

b) $f(x) = \dfrac{x^2 + 12x + 32}{x + 8}$

c) $f(x) = \dfrac{x^2 + 12x + 32}{x^2 + 10x + 16}$

d) $f(x) = \dfrac{x^2 + 5x + 4}{x^2 + 10x + 16}$

A



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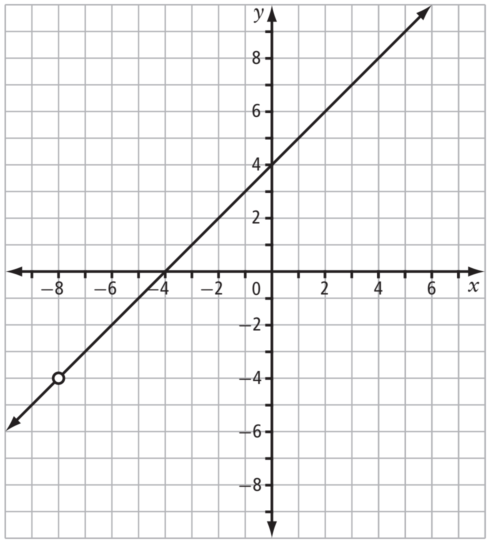

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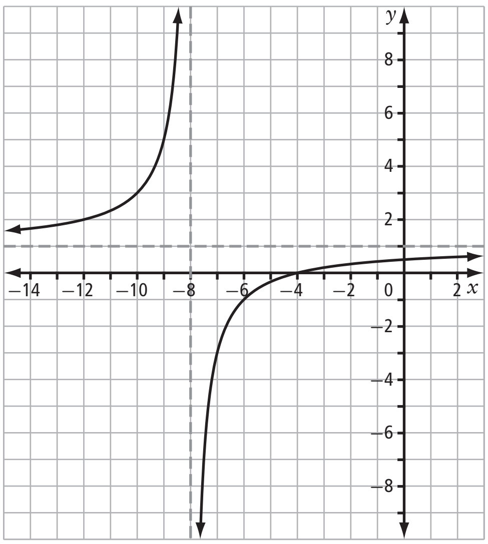

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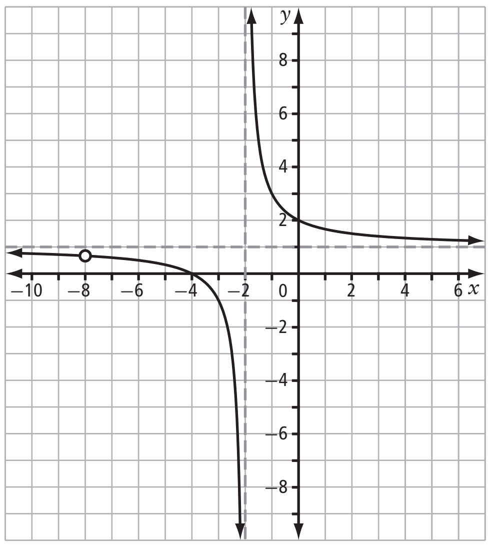
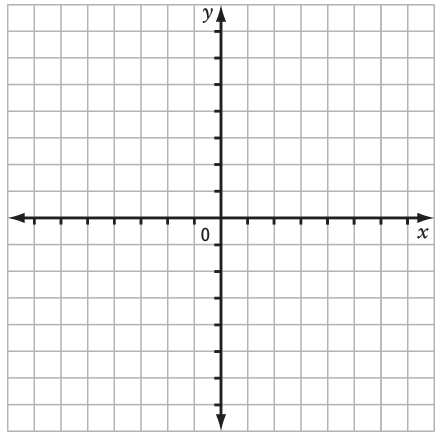
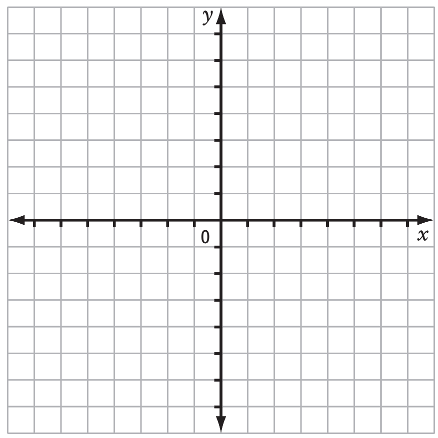


4. Identify any characteristics that are needed to sketch the graph. Then, sketch the function.

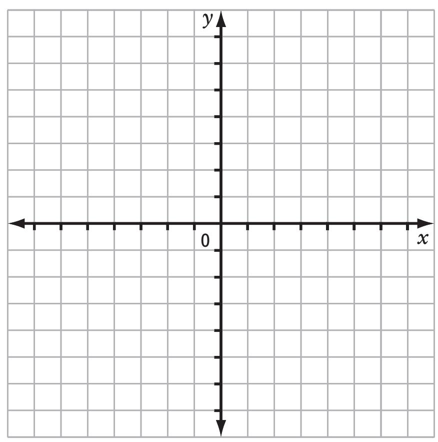
a) $f(x) = \dfrac{x + 3}{x - 2}$



b) $f(x) = \dfrac{x^2 - 5x + 6}{x - 2}$



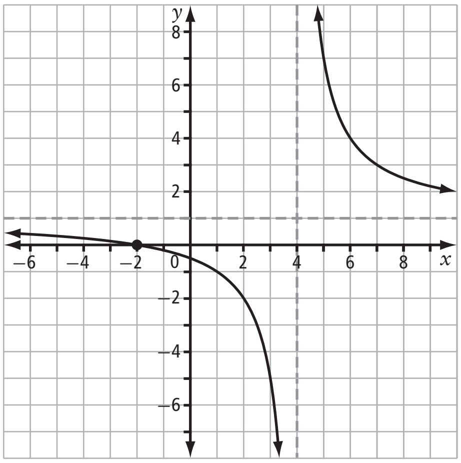
c) $f(x) = \dfrac{x^2 + 3x - 4}{x^2 + 2x - 8}$



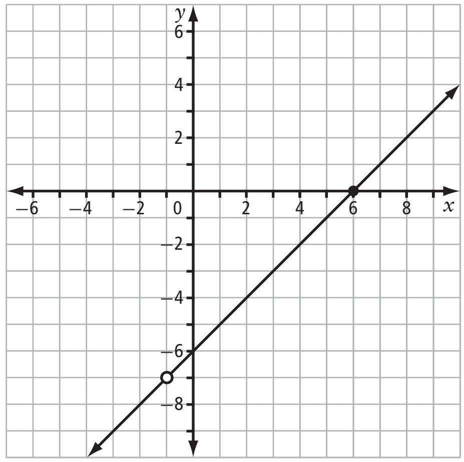
Apply

5. Write an equation for each rational function graphed below.

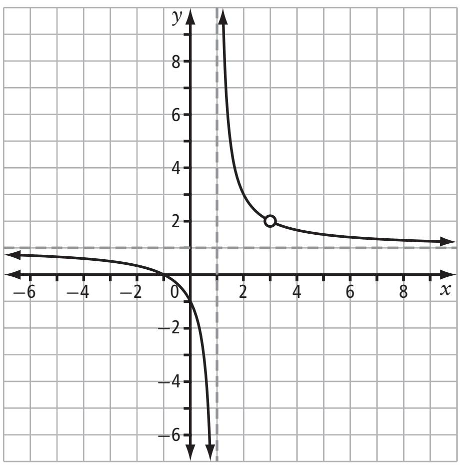
a)



b)



c)



6. Write an equation of a possible rational function with the following characteristics.

a) point of discontinuity: $(-4, -6)$
x-intercept: 2

b) point of discontinuity: $(3, 2)$
vertical asymptote: $x = 1$
x-intercept: -1

c) vertical asymptotes: $x = 4$ and $x = -3$
x-intercept: 0

d) vertical asymptote: $x = 7$
x-intercepts: -1 and -3

Connect

7. **a)** Where is the point of discontinuity on the graph of $y = \frac{2x^2 + 5x - 12}{2x - 3} - 5$?

b) Explain why the point of discontinuity is sometimes referred to as a "hole" in the graph. What does this mean?

8. Explain how you can determine whether a function has the following:

a) a point of discontinuity only

b) a vertical asymptote only

c) both a point of discontinuity and a vertical asymptote

9. Is it possible for a rational function to have neither a vertical asymptote nor a point of discontinuity? Explain.

9.3 Connecting Graphs and Rational Equations

KEY IDEAS

Solving Rational Equations
You can solve rational equations algebraically or graphically.

- **Algebraically**
  Solving algebraically determines the exact solution and any extraneous roots. To solve algebraically,
  – Equate to zero and list the restrictions.
  – Factor the numerator and denominator fully (if possible).
  – Multiply each term by the lowest common denominator to eliminate the fractions.
  – Solve for x.
  – Check the solution(s) against the restrictions.
  – Check the solution(s) in the original equation.

Example:

$$\frac{16}{x+6} = 4 - x$$

$$x + \frac{16}{x+6} - 4 = 0,\ x \neq -6$$

$$(x+6)\left(x + \frac{16}{x+6} - 4\right) = (x+6)(0)$$

$$(x+6)(x) + \left(\overset{1}{\cancel{x+6}}\right)\left(\frac{16}{\underset{1}{\cancel{x+6}}}\right) - (x+6)(4) = 0$$

$$x^2 + 6x + 16 - 4x - 24 = 0$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

roots: $x = -4$ and $x = 2$

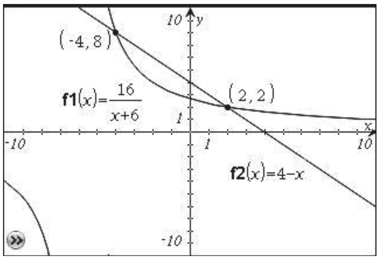
- **Graphically**
  There are two methods for solving equations graphically.

  Method 1: Use a System of Two Functions
  – Graph each side of the equation on the same set of axes.
  – The solution(s) will be the x-coordinate(s) of any point(s) of intersection.

  Method 2: Use a Single Function
  – Rearrange the equation so that one side is equal to zero.
  – Graph the corresponding function.
  – The solution(s) will be the x-intercept(s).

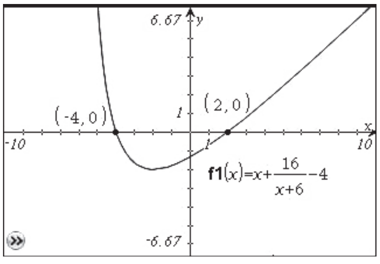
Example: $\frac{16}{x+6} = 4 - x$

Graph $y = \frac{16}{x+6}$ and $y = 4$ on the same axes.



The points of intersection are (−4, 8) and (2, 2), so the roots are $x = -4$ and $x = 2$.

Graph $y = x + \frac{16}{x+6} - 4$.



x-intercepts: $x = -4$ and $x = 2$

Working Example 1: Relate Roots and *x*-Intercepts

a) Find the solution to $\frac{3}{x} = 1 + \frac{x - 13}{6}$ algebraically.

b) Verify your solutions graphically.

Solution

$$\frac{3}{x} = 1 + \frac{x - 13}{6}$$

$0 = 1 + \frac{x - 13}{6} - \frac{\square}{\square}, x \neq 0$ State restrictions and equate to 0.

$0 = (\underline{\qquad\quad})(1) + (\underline{\qquad\quad})\left(\frac{x - 13}{6}\right) - (\underline{\qquad\quad})\left(\frac{\square}{\square}\right)$ Multiply by LCD.

$0 = (\underline{\qquad\quad}) + (\underline{\qquad\qquad\qquad}) - (\underline{\qquad\quad})$

$0 = \underline{\qquad\qquad\qquad\qquad\qquad}$ Combine like terms.

$0 = (\underline{\qquad\quad})(\underline{\qquad\quad})$ Factor.

$(\underline{\qquad\quad}) = 0$ or $(\underline{\qquad\quad}) = 0$

$x = \underline{\qquad\quad}$ $x = \underline{\qquad\quad}$ Solve.

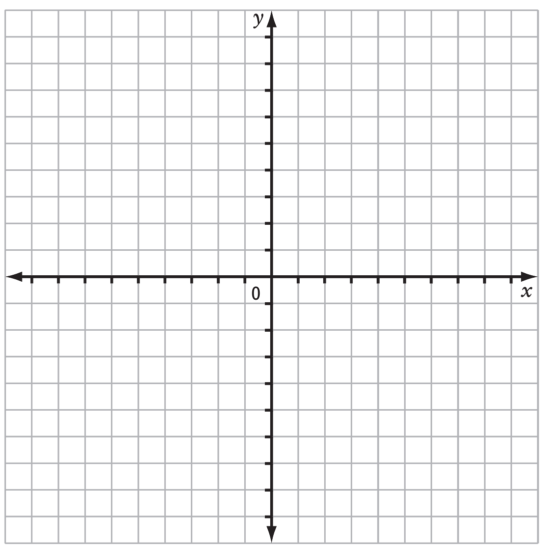
Is either value a non-permissible value? $\underline{\qquad\quad}$

Check by substitution:

b) Use technology to graph the original function using a single function or a system of two functions. Sketch the graph on the grid. Label the asymptote(s). If you used two functions, label the point(s) of intersection. If you used one function, label the *x*-intercept(s).

$x = \underline{\qquad\qquad\qquad}$

Compare the solution(s) obtained algebraically and graphically. What can you conclude?



To see a similar example, see Example 1 on pages 459–460 of *Pre-Calculus 12*.

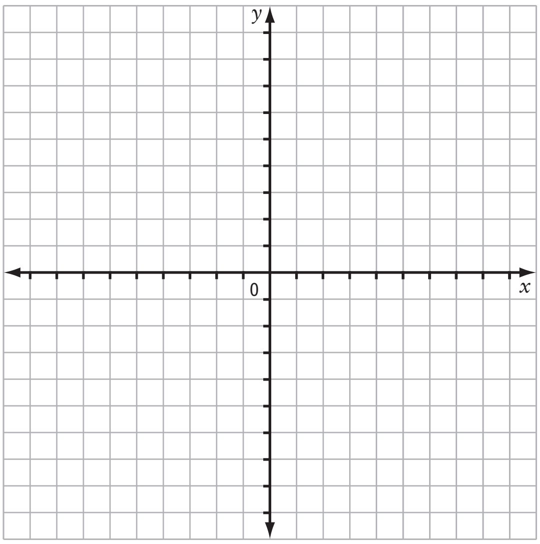
Working Example 2: Compare the Accuracy of Graphical and Algebraic Solutions

a) Solve $\frac{3}{2x} - \frac{2x}{x + 1} = -2$ graphically, using either one or two functions. (Use a different graphical method than the one you used in Working Example 1.)

b) Verify your solution(s) algebraically.

Solution

Use technology to graph the function, using either a single function or a system of two functions. Sketch the graph below. Label the asymptote(s) and the point(s) of intersection or the x-intercept(s).



b) Solve algebraically. Describe your solution strategy to the right of each step.

$\frac{3}{2x} - \frac{2x}{x + 1} = -2$

Compare the solutions you obtained graphically and algebraically. Which is more accurate?

To see a similar example, see Example 2 on pages 460–461 of *Pre-Calculus 12*.

Working Example 3: Solve a Rational Equation With an Extraneous Root

Solve $\frac{x}{x-1} - 2x = \frac{x+1}{2x-2}$ algebraically and graphically. Compare the solutions.

Solution

List the restriction(s) of the function.

Solve algebraically.

$$\frac{x}{x-1} - 2x = \frac{x+1}{2x-2}$$

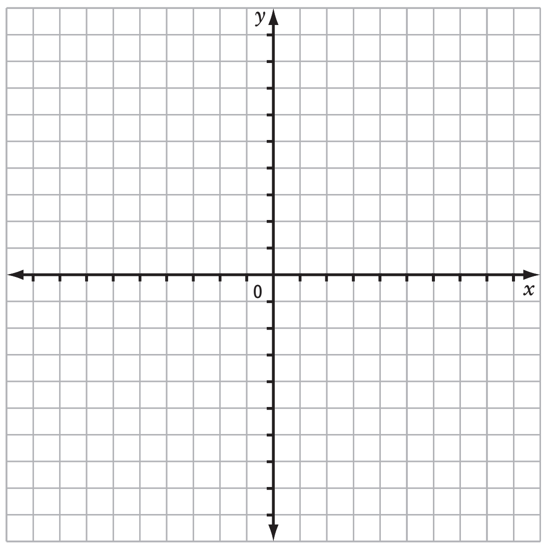
Check the solution(s) against the restriction(s).

Check your solution in the original equation.

Use technology to graph the function using either graphical method. Sketch the graph on the grid. Label the asymptote(s) and point(s) of intersection or x-intercept(s).

$x =$ ________________

Compare the solution you obtained algebraically to the one you obtained graphically. What do you notice?



To see a similar example, see Example 3 on pages 462–463 of *Pre-Calculus 12*.

Check Your Understanding

Practise

1. Solve each rational equation algebraically. Check your solutions.

a) $\frac{x + 1}{2} = \frac{3x + 7}{x}$

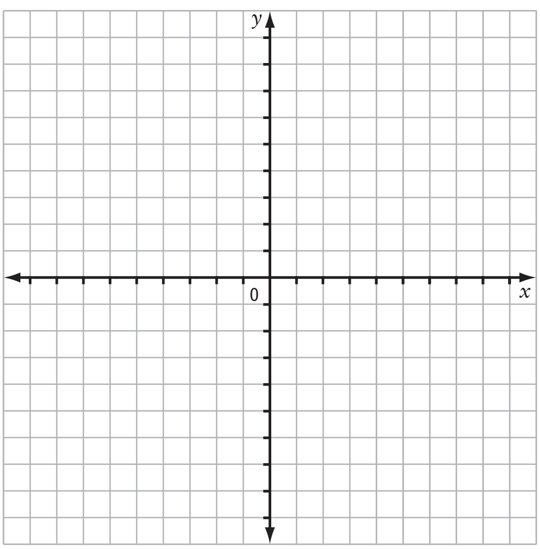
b) $\frac{6}{x} - \frac{9}{x - 1} = \frac{1}{4}$

c) $\frac{2x}{x + 3} + \frac{x}{x - 3} = \frac{18}{x^2 - 9}$

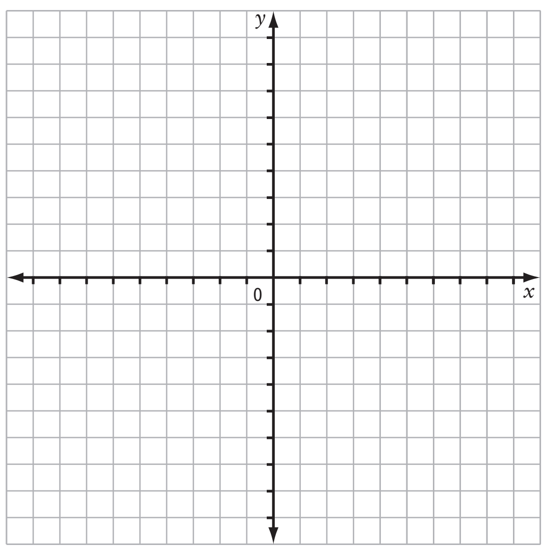
d) $\frac{3}{x^2 - 4} + \frac{3}{x + 2} = 2$

2. Solve each rational equation graphically. Sketch the graphs on the grids.

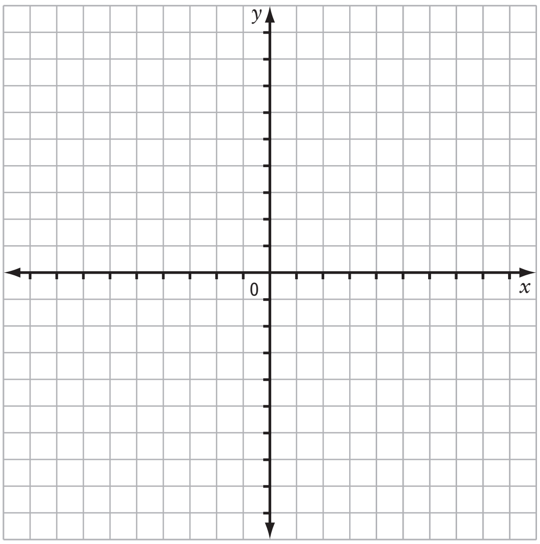
a) $\frac{4x}{3x - 2} + \frac{2x}{3x + 2} = 2$



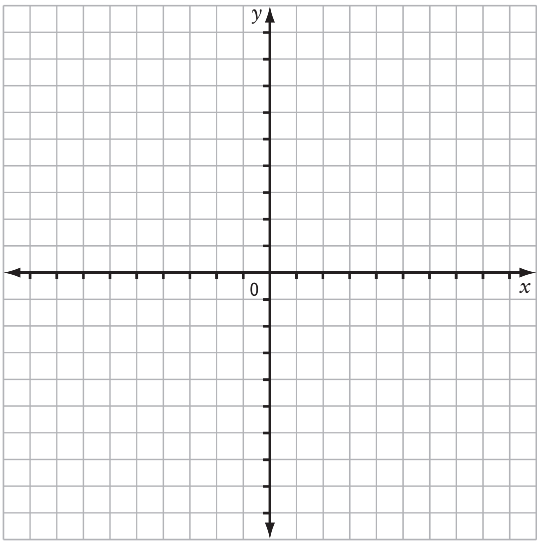
b) $\frac{2x - 3}{x - 3} - 2 = \frac{12}{x + 3}$



c) $\frac{2x-5}{x-1} - 2 = \frac{3}{x+2}$

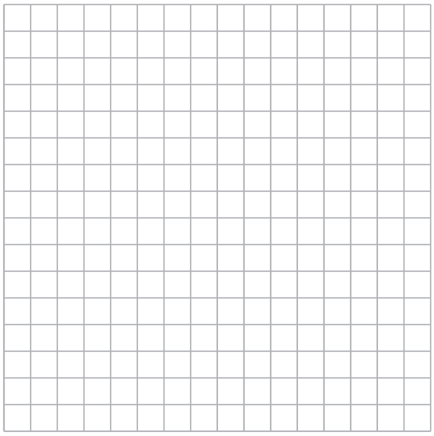


d) $\frac{5x}{x+2} + \frac{2}{x} = 5$

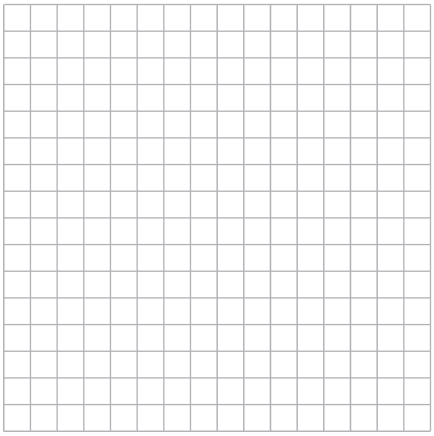


3. Solve each equation algebraically. Then, solve graphically using technology. Compare the solutions found using each method.

a) $\frac{x}{x+1} + \frac{5}{x-1} = 1$



b) $\frac{x^2-4}{x^2-1} = \frac{x}{x+3}$



Apply

4. Solve. State any extraneous root(s).

a) $\dfrac{1}{x-2} = \dfrac{x}{10-5x}$

b) $\dfrac{x^2}{x-1} + 2 = \dfrac{1}{x-1}$

c) $\dfrac{x}{x-2} + \dfrac{2}{x+3} = \dfrac{10}{x^2+x-6}$

d) $\dfrac{x^2}{x+2} = \dfrac{4}{x+2}$

5. Amber and Matteo are travelling separately from their home in Calgary to a wedding 400 km away. Amber leaves 1 h earlier than Matteo, but Matteo drives at an average speed 20 km/h faster than Amber. If they arrive at the wedding at the exact same time, what was the average speed at which each of them travelled?

a) Let x represent the time it takes Amber to travel to the wedding. Write an expression for the average speed that each person travels.

b) Write and solve an equation that represents the difference in their average speeds.

Connect

6. Explain why solving a rational equation graphically gives an approximate solution, while solving a rational equation algebraically gives an exact solution.

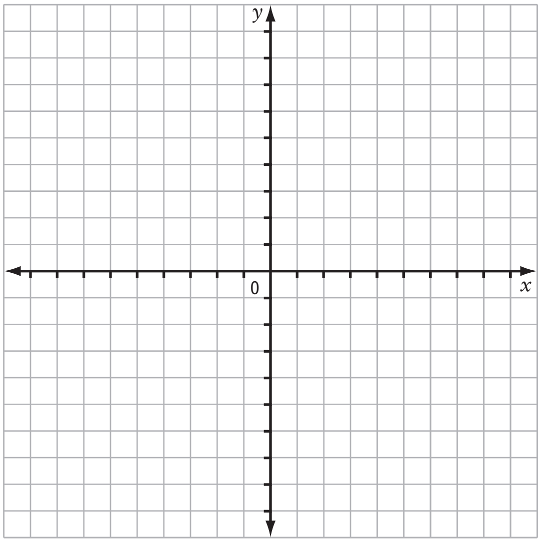
7. Explain why solving a rational equation graphically does not show extraneous roots.

Chapter 9 Review

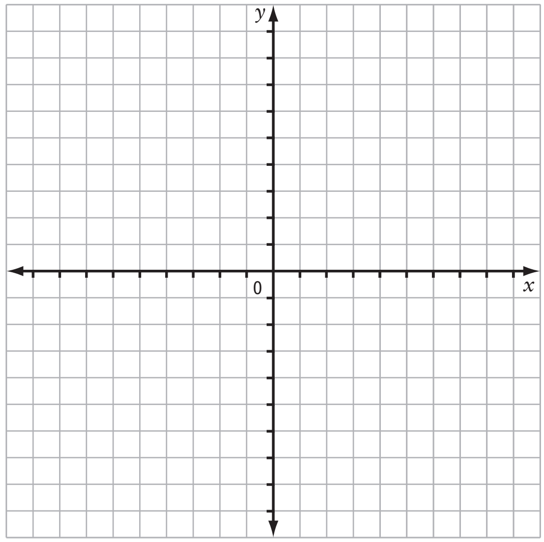
9.1 Exploring Rational Functions Using Transformations, pages 297–304

1. Graph each function using transformations. Label the important parts of the graph.

a) $y = \frac{3}{x - 4} + 2$

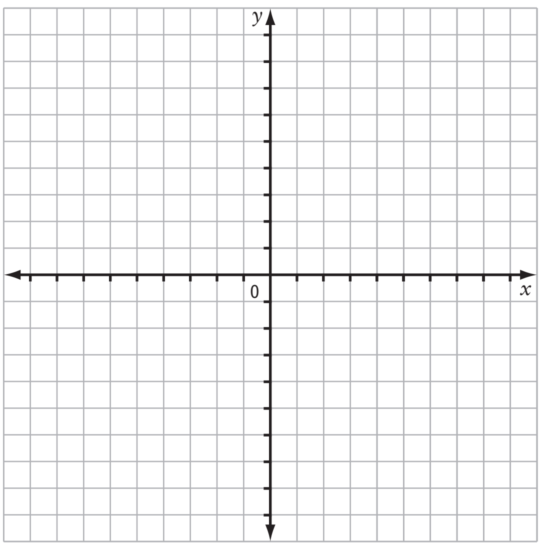


b) $y = \frac{7}{x - 1} - 2$

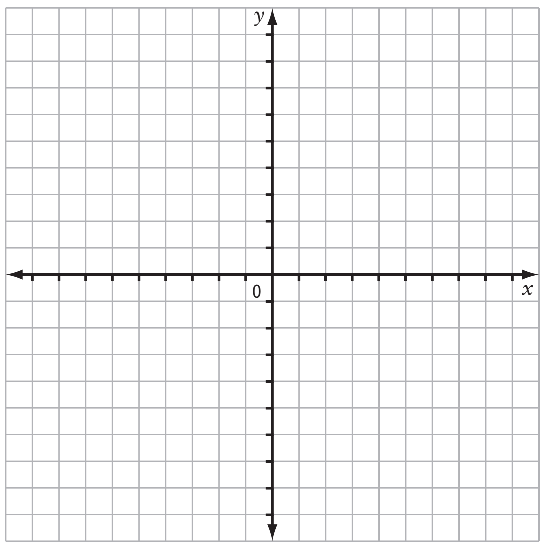


2. Graph the following functions without technology. Label all the important parts.

a) $f(x) = \frac{4x + 5}{x - 3}$



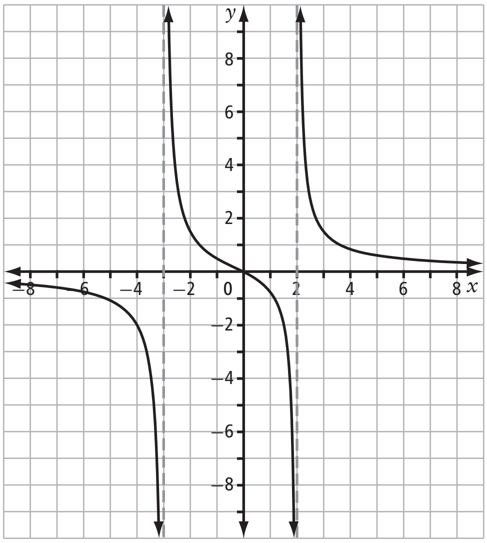
b) $f(x) = \frac{-2x + 5}{x - 3}$



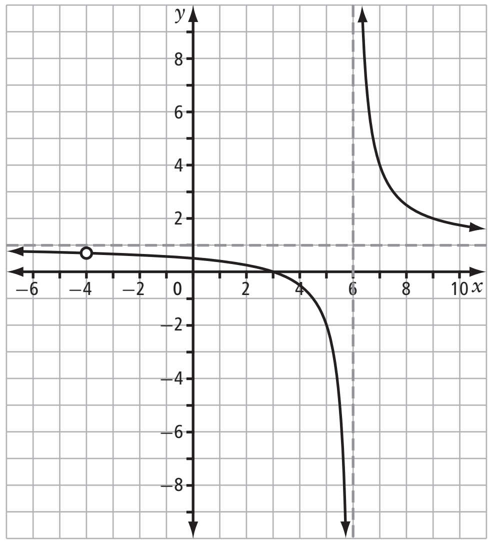
9.2 Analysing Rational Functions, pages 305–313

3. Match the graph of each rational function with the most appropriate equation. Give reasons for each choice.

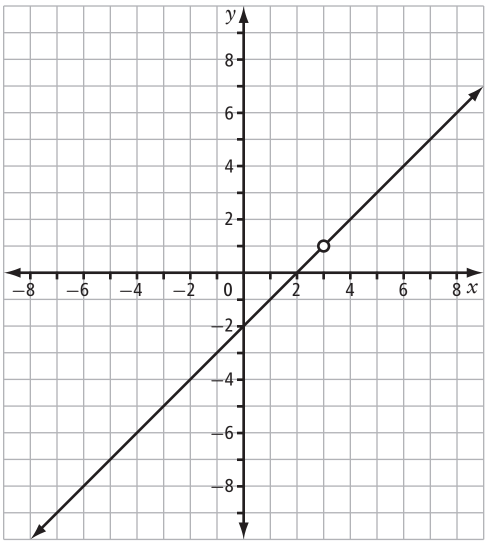
a)



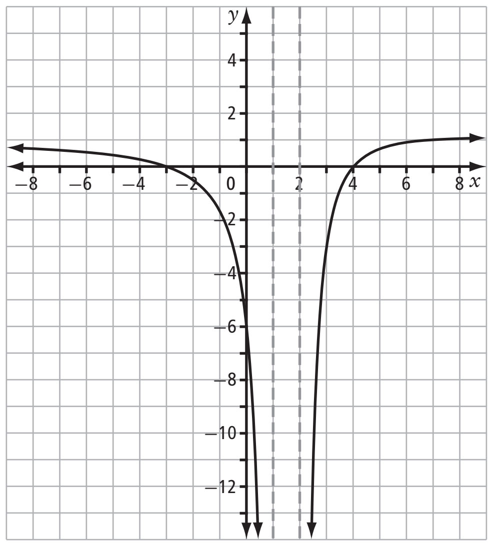
b)



c)



d)



A $f(x) = \dfrac{x^2 + x - 12}{x^2 - 2x - 24}$

B $g(x) = \dfrac{x^2 - x - 12}{x^2 - 3x + 2}$

C $h(x) = \dfrac{x^2 - 5x + 6}{x - 3}$

D $j(x) = \dfrac{3x}{x^2 + x - 6}$

4. For each function, predict the location of any points of discontinuity, vertical asymptotes, and intercepts.

a) $f(x) = \frac{2x + 1}{x + 5}$

b) $f(x) = \frac{x^2 - 8x + 12}{x - 2}$

9.3 Connecting Graphs and Rational Equations, pages 314–320

5. Solve each rational equation algebraically.

a) $\frac{3}{x} - \frac{6}{x - 2} = \frac{1}{4}$

b) $\frac{x - 2}{3} = \frac{2x - 4}{x}$

c) $\frac{x + 1}{x + 3} = \frac{x + 4}{x + 5}$

b) $\frac{x + 2}{x - 2} = \frac{2x + 4}{x + 1}$

6. Use technology to solve each rational equation graphically. Sketch and label a graph of the solution. Provide answers to the nearest tenth.

a) $\frac{4}{x} + \frac{3}{x + 1} = \frac{1}{2}$

$x =$ ________________

b) $\frac{3x - 1}{x + 4} + 3 = \frac{6}{x - 4}$

$x =$ ________________

Chapter 9 Skills Organizer

Think of all the material presented in Chapter 9. Compare and contrast solving rational equations algebraically and graphically. Use explanations and examples to complete the Venn diagram.