Blueprint for *Pre-Calculus 11* Final Exam

Algebra and Number **General Outcome:** Develop algebraic reasoning and number sense.	
Specific Outcome: Demonstrate an understanding of the absolute value of real numbers. [R, V]	
1.1 Determine the distance of two real numbers of the form $\pm a$, $a \in R$, from 0 on a number line, and relate this to the absolute value of a ($\|a\|$).	
1.2 Determine the absolute value of a positive or negative real number.	MC #35 Procedural NR #36 Conceptual NR # 37 Procedural
1.3 Explain, using examples, how distance between two points on a number line can be expressed in terms of absolute value.	MC #39 Procedural
1.4 Determine the absolute value of a numerical expression.	MC #27 Conceptual
1.5 Compare and order the absolute values of real numbers in a given set.	MC #34 Conceptual
Specific Outcome: Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands. [CN, ME, PS, R, T]	
2.1 Compare and order radical expressions with numerical radicands in a given set.	
2.2 Express an entire radical with a numerical radicand as a mixed radical.	
2.3 Express a mixed radical with a numerical radicand as an entire radical.	
2.4 Perform one or more operations to simplify radical expressions with numerical or variable radicands.	MC #21 Problem Solving MC #22 Problem Solving MC #23 Procedural MC #24 Procedural MC #26 Conceptual
2.5 Rationalize the denominator of a rational expression with monomial or binomial denominators.	
2.6 Describe the relationship between rationalizing a binomial denominator of a rational expression and the product of the factors of a difference of squares expression.	
2.7 Explain, using examples, that $(-x)^2 = x^2$, $\sqrt{x^2} = \|x\|$ and $\sqrt{x^2} \neq \pm x$; e.g., $\sqrt{9} \neq \pm 3$.	
2.8 Identify the values of the variable for which a given radical expression is defined.	
2.9 Solve a problem that involves radical expressions.	MC #5 Procedural

Specific Outcome: Solve problems, using the cosine law and sine law, including the ambiguous case. [C, CN, PS, R, T]	
3.1 Determine any restrictions on values for the variable in a radical equation.	MC #29 Conceptual NR #33 Conceptual
3.2 Determine the roots of a radical equation algebraically, and explain the process used to solve the equation.	NR #30 Conceptual
3.3 Verify, by substitution, that the values determined in solving a radical equation algebraically are roots of the equation.	
3.4 Explain why some roots determined in solving a radical equation algebraically are extraneous.	
3.5 Solve problems by modelling a situation using a radical equation.	
Specific Outcome: Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [C, ME, R]	
4.1 Compare the strategies for writing equivalent forms of rational expressions to the strategies for writing equivalent forms of rational numbers.	MC #4 Procedural
4.2 Explain why a given value is non-permissible for a given rational expression.	
4.3 Determine the non-permissible values for a rational expression.	
4.4 Determine a rational expression that is equivalent to a given rational expression by multiplying the numerator and denominator by the same factor (limited to a monomial or a binomial), and state the non-permissible values of the equivalent rational expression.	
4.5 Simplify a rational expression.	
4.6 Explain why the non-permissible values of a given rational expression and its simplified form are the same.	MC #43 Conceptual
4.7 Identify and correct errors in a simplification of a rational expression, and explain the reasoning.	
Specific Outcome: Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). [CN, ME, R]	
5.1 Compare the strategies for performing a given operation on rational expressions to the strategies for performing the same operation on rational numbers.	
5.2 Determine the non-permissible values when performing operations on rational expressions.	MC #40 Conceptual
5.3 Determine, in simplified form, the sum or difference of rational expressions with the same denominator.	

5.4 Determine, in simplified form, the sum or difference of rational expressions in which the denominators are not the same and which may or may not contain common factors.	MC #29 Conceptual, Procedural
5.5 Determine, in simplified form, the product or quotient of rational expressions.	
5.6 Simplify an expression that involves two or more operations on rational expressions.	
Specific Outcome: Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials). [C, PS, R]	
6.1 Determine the non-permissible values for the variable in a rational equation.	
6.2 Determine the solution to a rational equation algebraically, and explain the process used to solve the equation.	MC #32 Procedural
6.3 Explain why a value obtained in solving a rational equation may not be a solution of the equation.	
6.4 Solve problems by modelling a situation using a rational equation.	MC #28 Problem Solving
Trigonometry **General Outcome:** Develop trigonometric reasoning.	
Specific Outcome: Demonstrate an understanding of angles in standard position [0° to 360°]. [R, V]	
1.1 Sketch an angle in standard position, given the measure of the angle.	
1.2 Determine the reference angle for an angle in standard position.	NR #8 Conceptual
1.3 Explain, using examples, how to determine the angles from 0° to 360° that have the same reference angle as a given angle.	
1.4 Illustrate, using examples, that any angle from 90° to 360° is the reflection in the x-axis and/or the y-axis of its reference angle.	
1.5 Determine the quadrant in which a given angle in standard position terminates.	MC #7 Procedural
1.6 Draw an angle in standard position given any point P(x, y) on the terminal arm of the angle.	
1.7 Illustrate, using examples, that the points P(x, y), P($-x$, y), P($-x$, $-y$) and P(x, $-y$) are points on the terminal sides of angles in standard position that have the same reference angle.	

Specific Outcome: Solve problems, using the three primary trigonometric ratios for angles from 0° to 360° in standard position. [C, ME, PS, R, T, V]	
2.1 Determine, using the Pythagorean theorem or the distance formula, the distance from the origin to a point P (x, y) on the terminal arm of an angle.	MC #24 Conceptual
2.2 Determine the value of sin θ, cos θ or tan θ, given any point P (x, y) on the terminal arm of angle θ.	
2.3 Determine, without the use of technology, the value of sin θ, cos θ or tan θ, given any point P (x, y) on the terminal arm of angle θ, where θ = 0º, 90º, 180º, 270º or 360º.	
2.4 Determine the sign of a given trigonometric ratio for a given angle, without the use of technology, and explain.	
2.5 Solve, for all values of θ, an equation of the form $\sin \theta = a$ or $\cos \theta = a$, where $-1 \leq a \leq 1$, and an equation of the form $\tan \theta = a$, where a is a real number.	MC #6 Procedural
2.6 Determine the exact value of the sine, cosine or tangent of a given angle with a reference angle of 30º, 45º or 60º.	
2.7 Describe patterns in and among the values of the sine, cosine and tangent ratios for angles from 0° to 360°.	
2.8 Sketch a diagram to represent a problem.	
2.9 Solve a contextual problem, using trigonometric ratios.	
Specific Outcome: Solve problems, using the cosine law and sine law, including the ambiguous case. [C, CN, PS, R, T]	
3.1 Sketch a diagram to represent a problem that involves a triangle without a right angle.	
3.2 Solve, using primary trigonometric ratios, a triangle that is not a right triangle.	
3.3 Explain the steps in a given proof of the sine law or cosine law.	
3.4 Sketch a diagram and solve a problem, using the cosine law.	MC #14 Conceptual, Procedural
3.5 Sketch a diagram and solve a problem, using the sine law.	MC #20 Conceptual, Procedural
3.6 Describe and explain situations in which a problem may have no solution, one solution or two solutions.	

Relations and Functions **General Outcome:** Develop algebraic and graphical reasoning through the study of relations.	
Specific Outcome: Factor polynomial expressions of the form: • $ax^2 + bx + c, a \neq 0$ • $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$ • $a(f(x))^2 + b(f(x)) + c, a \neq 0$ • $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$ where a, b and c are rational numbers. [CN, ME, R]	
1.1 Factor a given polynomial expression that requires the identification of common factors.	
1.2 Determine whether a given binomial is a factor for a given polynomial expression, and explain why or why not.	
1.3 Factor a given polynomial expression of the form: • $ax^2 + bx + c, a \neq 0$ • $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$.	
1.4 Factor a given polynomial expression that has a quadratic pattern, including: • $a(f(x))^2 + b(f(x)) + c, a \neq 0$ • $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$.	
Specific Outcome: Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. [C, PS, R, T, V]	
2.1 Create a table of values for $y = \|f(x)\|$, given a table of values for $y = f(x)$.	
2.2 Generalize a rule for writing absolute value functions in piecewise notation.	
2.3 Sketch the graph of $y = \|f(x)\|$; state the intercepts, domain and range; and explain the strategy used.	MC #49 Conceptual
2.4 Solve an absolute value equation graphically, with or without technology.	
2.5 Solve, algebraically, an equation with a single absolute value, and verify the solution.	MC #28 Conceptual
2.6 Explain why the absolute value equation $\|f(x)\| < 0$ has no solution.	
2.7 Determine and correct errors in a solution to an absolute value equation.	
2.8 Solve a problem that involves an absolute value function.	

Specific Outcome: Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the: • vertex • domain and range • direction of opening • axis of symmetry • x- and y-intercepts. [CN, R, T, V]	
3.1 Explain why a function given in the form $y = a(x - p)^2 + q$ is a quadratic function.	
3.2 Compare the graphs of a set of functions of the form $y = ax^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of a.	
3.3 Compare the graphs of a set of functions of the form $y = x^2 + q$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of q.	
3.4 Compare the graphs of a set of functions of the form $y = (x - p)^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of p.	
3.5 Determine the coordinates of the vertex for a quadratic function of the form $y = a(x - p)^2 + q$, and verify with or without technology.	
3.6 Generalize, using inductive reasoning, a rule for determining the coordinates of the vertex for quadratic functions of the form $y = a(x - p)^2 + q$.	
3.7 Sketch the graph of $y = a(x - p)^2 + q$, using transformations, and identify the vertex, domain and range, direction of opening, axis of symmetry and x- and y-intercepts.	MC #41 Procedural MC #42 Conceptual MC #43 Procedural
3.8 Explain, using examples, how the values of a and q may be used to determine whether a quadratic function has zero, one or two x-intercepts.	MC #46 Conceptual
3.9 Write a quadratic function in the form $y = a(x - p)^2 + q$ for a given graph or a set of characteristics of a graph.	
Specific Outcome: Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including: • vertex • domain and range • direction of opening • axis of symmetry • x- and y-intercepts and to solve problems. [CN, PS, R, T, V]	
4.1 Explain the reasoning for the process of completing the square as shown in a given example.	

4.2 Write a quadratic function given in the form $y = ax^2 + bx + c$ as a quadratic function in the form $y = a(x - p)^2 + q$ by completing the square.	MC #15 Procedural
4.3 Identify, explain and correct errors in an example of completing the square.	
4.4 Determine the characteristics of a quadratic function given in the form $y = ax^2 + bx + c$, and explain the strategy used.	MC #16 Conceptual
4.5 Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$.	
4.6 Verify, with or without technology, that a quadratic function in the form $y = ax^2 + bx + c$ represents the same function as a given quadratic function in the form $y = a(x - p)^2 + q$.	
4.7 Write a quadratic function that models a given situation, and explain any assumptions made.	MC #44 Problem Solving
4.8 Solve a problem, with or without technology, by analyzing a quadratic function.	MC #17 Conceptual
Specific Outcome: Solve problems that involve quadratic equations. [C, CN, PS, R, T, V]	
5.1 Explain, using examples, the relationship among the roots of a quadratic equation, the zeros of the corresponding quadratic function and the *x*-intercepts of the graph of the quadratic function.	MC #44 Procedural
5.2 Derive the quadratic formula, using deductive reasoning.	
5.3 Solve a quadratic equation of the form $ax^2 + bx + c = 0$ by using strategies such as: • determining square roots • factoring • completing the square • applying the quadratic formula • graphing its corresponding function.	NR #18 Conceptual NR # 19 Conceptual MC #45 Procedural MC #48 Conceptual
5.4 Select a method for solving a quadratic equation, justify the choice, and verify the solution.	
5.5 Explain, using examples, how the discriminant may be used to determine whether a quadratic equation has two, one or no real roots; and relate the number of zeros to the graph of the corresponding quadratic function.	MC #46 Problem Solving
5.6 Identify and correct errors in a solution to a quadratic equation.	
5.7 Solve a problem by: • analyzing a quadratic equation • determining and analyzing a quadratic equation.	

Specific Outcome: Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables. [CN, PS, R, T, V]	
6.1 Model a situation, using a system of linear-quadratic or quadratic-quadratic equations.	WR #4a), b) Procedural
6.2 Relate a system of linear-quadratic or quadratic-quadratic equations to the context of a given problem.	
6.3 Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations graphically, with technology.	
6.4 Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations algebraically.	WR #2a) Procedural
6.5 Explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations.	
6.6 Explain, using examples, why a system of linear-quadratic or quadratic-quadratic equations may have zero, one, two or an infinite number of solutions.	
6.7 Solve a problem that involves a system of linear-quadratic or quadratic-quadratic equations, and explain the strategy used.	WR #4c), d) Problem Solving
Specific Outcome: Solve problems that involve linear and quadratic inequalities in two variables. [C, PS, T, V]	
7.1 Explain, using examples, how test points can be used to determine the solution region that satisfies an inequality.	
7.2 Explain, using examples, when a solid or broken line should be used in the solution for an inequality.	
7.3 Sketch, with or without technology, the graph of a linear or quadratic inequality.	WR #1 Conceptual, Procedural WR #5a) Procedural
7.4 Solve a problem that involves a linear or quadratic inequality.	WR #2 Problem Solving WR #5b) Problem Solving
Specific Outcome: Solve problems that involve quadratic inequalities in one variable. [CN, PS, V]	
8.1 Determine the solution of a quadratic inequality in one variable, using strategies such as case analysis, graphing, roots and test points, or sign analysis; and explain the strategy used.	MC #26 Conceptual, Procedural
8.2 Represent and solve a problem that involves a quadratic inequality in one variable.	WR #3 Problem Solving
8.3 Interpret the solution to a problem that involves a quadratic inequality in one variable.	WR #2b) Conceptual

Specific Outcome: Analyze arithmetic sequences and series to solve problems. [CN, PS, R, T]	
9.1 Identify the assumption(s) made when defining an arithmetic sequence or series.	
9.2 Provide and justify an example of an arithmetic sequence.	MC #50, Procedural
9.3 Derive a rule for determining the general term of an arithmetic sequence.	MC #51 Procedural
9.4 Describe the relationship between arithmetic sequences and linear functions.	
9.5 Determine t_1, d, n or t_n in a problem that involves an arithmetic sequence.	
9.6 Derive a rule for determining the sum of n terms of an arithmetic series.	
9.7 Determine t_1, d, n or S_n in a problem that involves an arithmetic series.	MC #53 Procedural
9.8 Solve a problem that involves an arithmetic sequence or series.	MC #10 Conceptual MC #11 Conceptual MC #12 Conceptual MC #13 Conceptual
Specific Outcome: Analyze geometric sequences and series to solve problems. [PS, R, T]	
10.1 Identify assumptions made when identifying a geometric sequence or series.	
10.2 Provide and justify an example of a geometric sequence.	
10.3 Derive a rule for determining the general term of a geometric sequence.	
10.4 Determine t_1, r, n or t_n in a problem that involves a geometric sequence.	MC #2 Conceptual MC #3 Procedural
10.5 Derive a rule for determining the sum of n terms of a geometric series.	
10.6 Determine t_1, r, n or S_n in a problem that involves a geometric series.	MC #1 Procedural
10.7 Generalize, using inductive reasoning, a rule for determining the sum of an infinite geometric series.	
10.8 Explain why a geometric series is convergent or divergent.	
10.9 Solve a problem that involves a geometric sequence or series.	

Specific Outcome: Graph and analyze reciprocal functions (limited to the reciprocal of linear and quadratic functions). [CN, R, T, V]	
11.1 Compare the graph of $y = \frac{1}{f(x)}$ to the graph of $y = f(x)$.	WR #6 Conceptual WR #7 Procedural
11.2 Identify, given a function $f(x)$, values of x for which $y = \frac{1}{f(x)}$ will have vertical asymptotes; and describe their relationship to the non-permissible values of the related rational expression.	
11.3 Graph, with or without technology, $y = \frac{1}{f(x)}$, given $y = f(x)$ as a function or a graph, and explain the strategies used.	WR #8 Procedural
11.4 Graph, with or without technology, $y = f(x)$, given $y = \frac{1}{f(x)}$ as a function or a graph, and explain the strategies used.	