Answers

Chapter 1 Sequences and Series

1.1 Arithmetic Sequences, pages 16 to 21

1. a) arithmetic sequence: $t_1 = 16$, $d = 16$; next three terms: 96, 112, 128
b) not arithmetic
c) arithmetic sequence: $t_1 = -4$, $d = -3$; next three terms: -19, -22, -25
d) arithmetic sequence: $t_1 = 3$, $d = -3$; next three terms: -12, -15, -18

2. a) 5, 8, 11, 14 **b)** $-1, -5, -9, -13$
c) $4, \frac{21}{5}, \frac{22}{5}, \frac{23}{5}$ **d)** 1.25, 1.00, 0.75, 0.50

3. a) $t_1 = 11$ **b)** $t_7 = 29$ **c)** $t_{14} = 50$

4. a) 7, 11, 15, 19, 23; $t_1 = 7$, $d = 4$
b) $6, \frac{9}{2}, 3, \frac{3}{2}$; $t_1 = 6$, $d = -\frac{3}{2}$
c) 2, 4, 6, 8, 10; $t_1 = 2$, $d = 2$

5. a) 30 **b)** 82 **c)** 26 **d)** 17

6. a) $t_2 = 15$, $t_3 = 24$ **b)** $t_2 = 19$, $t_3 = 30$
c) $t_2 = 37$, $t_3 = 32$

7. a) 5, 8, 11, 14, 17 **b)** $t_n = 3n + 2$
c) $t_{50} = 152$, $t_{200} = 602$
d) The general term is a linear equation of the form $y = mx + b$, where $t_n = y$ and $n = x$. Therefore, $t_n = 3n + 2$ has a slope of 3.
e) The constant value of 2 in the general term is the y-intercept of 2.

8. A and C; both sequences have a natural-number value for n.

9. 5

10. $t_n = -3yn + 8y$; $t_{15} = -37y$

11. $x = -16$; first three terms: -78, -116, -154

12. $z = 2y - x$

13. a) $t_n = 6n + 4$ **b)** 58
c) 12

14. a) 0, 8, 16, 24
b) 32 players
c) $t_n = 8n - 8$
d) 12:16
e) Example: weather, all foursomes starting on time, etc.

15. 21 square inches

16. a) $t_n = 2n - 1$ **b)** 51st day
c) Susan continues the program until she accomplishes her goal.

17. a)

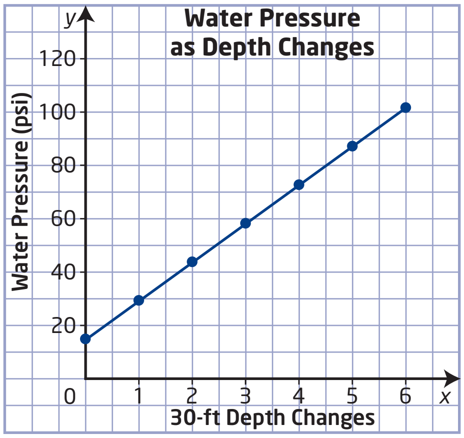
Carbon Atoms	1	2	3	4
Hydrogen Atoms	4	6	8	10

b) $t_n = 2n + 2$ or $H = 2C + 2$
c) 100 carbon atoms

18.

Multiples of	28	7	15
Between	1 and 1000	500 and 600	50 and 500
First Term, t_1	28	504	60
Common Difference, d	28	7	15
nth Term, t_n	980	595	495
General Term	$t_n = 28n$	$t_n = 7n + 497$	$t_n = 15n + 45$
Number of Terms	35	14	30

19. a) 14.7, 29.4, 44.1, 58.8; $t_n = 14.7n$, where n represents every increment of 30 ft in depth.
b) 490 psi at 1000 ft and 980 psi at 2000 ft
c)



d) 14.7 psi
e) 14.7
f) The y-intercept represents the first term of the sequence and the slope represents the common difference.

20. Other lengths are 6 cm, 12 cm, and 18 cm. Add the four terms to find the perimeter. Replace t_2 with $t_1 + d$, t_3 with $t_1 + 2d$, and t_4 with $t_1 + 3d$. Solve for d.

21. a) 4, 8, 12, 16, 20 **b)** $t_n = 4n$
c) 320 min

22. -29 beekeepers

23. 5.8 million carats. This value represents the increase of diamond carats mined each year.

24. 1696.5 m

25. a) 13:54, 13:59, 14:04, 14:09, 14:14; $t_1 = 13:54$, $d = 0:05$
b) $t_n = 0:05n + 13:49$
c) Assume that the arithmetic sequence of times continues.
d) 15:49

26. a) $d > 0$ **b)** $d < 0$
c) $d = 0$ **d)** t_1
e) t_n

27. Definition: An ordered list of terms in which the difference between consecutive terms is constant.
Common Difference: The difference between successive terms, $d = t_n - t_{n-1}$
Example: 12, 19, 26, …
Formula: $t_n = 7n + 5$

28. Step 1 The graph of an arithmetic sequence is always a straight line. The common difference is described by the slope of the graph. Since the common difference is always constant, the graph will be a straight line.
Step 2
a) Changing the value of the first term changes the y-intercept of the graph. The y-intercept increases as the value of the first term increases. The y-intercept decreases as the value of the first term decreases.
b) Yes, the graph keeps it shape. The slope stays the same.

Step 3
a) Changing the value of the common difference changes the slope of the graph.
b) As the common difference increases, the slope increases. As the common difference decreases, the slope decreases.

Step 4 The common difference is the slope.
Step 5 The slope of the graph represents the common difference of the general term of the sequence. The slope is the coefficient of the variable n in the general term of the sequence.

1.2 Arithmetic Series, pages 27 to 31

1. a) 493 **b)** 735
c) −1081 **d)** $\frac{301}{3} = 100.\overline{3}$

2. a) $t_1 = 1, d = 2, S_8 = 64$
b) $t_1 = 40, d = -5, S_{11} = 165$
c) $t_1 = \frac{1}{2}, d = 1, S_7 = 24.5$
d) $t_1 = -3.5, d = 2.25, S_6 = 12.75$

3. a) 344 **b)** 663
c) 195 **d)** 396
e) 133

4. a) 2 **b)** $\frac{500}{13} \approx 38.46$
c) 4 **d)** 41

5. a) 16 **b)** 10

6. a) $t_{10} = 50, S_{10} = 275$
b) $t_{10} = -17, S_{10} = -35$
c) $t_{10} = -46, S_{10} = -280$
d) $t_{10} = 7, S_{10} = 47.5$

7. a) 124 500 **b)** 82 665

8. 156 times

9. a) 2 **b)** 40 **c)** $\frac{n}{2}(1 + 3n)$

10. 8425

11. 3 + 10 + 17 + 24

12. a)
$S_n = \frac{n}{2}[2t_1 + (n - 1)d]$
$S_n = \frac{n}{2}[2(5) + (n - 1)10]$
$S_n = \frac{n}{2}[10 + 10n - 10]$
$S_n = \frac{n(10n)}{2}$
$S_n = \frac{10n^2}{2}$
$S_n = 5n^2$

b)
$S_{100} = \frac{100}{2}[2(5) + (100 - 1)10]$
$S_{100} = \frac{100}{2}[10 + 990]$
$S_{100} = \frac{100}{2}(1000)$
$S_{100} = 50\ 000$
$d(100) = 5(100)^2$
$d(100) = 5(10\ 000)$
$d(100) = 50\ 000$

13. 171

14. a) the number of handshakes between six people if they each shake hands once
b) 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9
c) 435
d) Example: The number of games played in a home and away series league for n teams.

15. a) $t_1 = 6.2, d = 1.2$
b) $t_{20} = 29$
c) $S_{20} = 352$

16. 173 cm

17. a) True. Example: 2 + 4 + 6 + 8 = 20, 4 + 8 + 12 + 16 = 40, 40 = 2 × 20
b) False. Example: 2 + 4 + 6 + 8 = 20, 2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 = 72, 72 ≠ 2 × 20
c) True. Example: Given the sequence 2, 4, 6, 8, multiplying each term by 5 gives 10, 20, 30, 40. Both sequences are arithmetic sequences.

18. a) 7 + 11 + 15 **b)** 250 **c)** 250
d)
$S_n = \frac{n}{2}[2t_1 + (n - 1)d]$
$S_n = \frac{n}{2}[2(7) + (n - 1)4]$
$S_n = \frac{n}{2}[14 + 4n - 4]$
$S_n = \frac{n}{2}[4n + 10]$
$S_n = n(2n + 5)$
$S_n = 2n^2 + 5n$

19. a) $240 + 250 + 260 + \cdots + 300$
b) $S_n = 235n + 5n^2$
c) 1890
d) Nathan will continue to remove an extra 10 bushels per hour.

20. $(-27) + (-22) + (-17)$

21. Jeanette and Pierre have used two different forms of the same formula. Jeanette has replaced t_n with $t_1 + (n - 1)d$.

22. a) 100
b) $S_{green} = 1 + 2 + 3 + \cdots + 10$
$S_{blue} = 0 + 1 + 2 + 3 + \cdots + 9$
$S_{total} = S_{green} + S_{blue}$
$S_{total} = \frac{10}{2}(1 + 10) + \frac{10}{2}(0 + 9)$
$S_{total} = 5(11) + 5(9)$
$S_{total} = 55 + 45$
$S_{total} = 100$

23. a) 55
b) The nth triangular number is represented by S_n.
$S_n = \frac{n}{2}[2t_1 + (n - 1)d]$
$S_n = \frac{n}{2}[2(1) + (n - 1)(1)]$
$S_n = \frac{n}{2}[2 + (n - 1)]$
$S_n = \frac{n}{2}(1 + n)$

1.3 Geometric Sequences, pages 39 to 45

1. a) geometric; $r = 2$; $t_n = 2^{n-1}$
b) not geometric
c) geometric; $r = -3$; $t_n = 3(-3)^{n-1}$
d) not geometric
e) geometric; $r = 1.5$; $t_n = 10(1.5)^{n-1}$
f) geometric; $r = 5$; $t_n = -1(5)^{n-1}$

2.

	Geometric Sequence	Common Ratio	6th Term	10th Term
a)	6, 18, 54, ...	3	1458	118 098
b)	1.28, 0.64, 0.32, ...	0.5	0.04	0.0025
c)	$\frac{1}{5}, \frac{3}{5}, \frac{9}{5}, \ldots$	3	$\frac{243}{5}$	$\frac{19\,683}{5}$

3. a) 2, 6, 18, 54 **b)** $-3, 12, -48, 192$
c) $4, -12, 36, -108$ **d)** $2, 1, \frac{1}{2}, \frac{1}{4}$

4. 18.9, 44.1, 102.9

5. a) $t_n = 3(2)^{n-1}$ **b)** $t_n = 192\left(-\frac{1}{4}\right)^{n-1}$
c) $t_n = \frac{5}{9}(3)^{n-1}$ **d)** $t_n = 4(2)^{n-1}$

6. a) 4 **b)** 7 **c)** 5
d) 6 **e)** 9 **f)** 8

7. 37

8. 16, 12, 9; $t_n = 16\left(\frac{3}{4}\right)^{n-1}$

9. a) $t_1 = 3$; $r = 0.75$
b) $t_n = 3(0.75)^{n-1}$
c) approximately 53.39 cm
d) 7

10. a) 95%
b) 100, 95, 90.25, 85.7375
c) 0.95
d) about 59.87%
e) After 27 washings, 25% of the original colour would remain in the jeans. Example: The geometric sequence continues for each washing.

11. 1.77

12. a) 1, 2, 4, 8, 16 **b)** $t_n = 1(2)^{n-1}$
c) 2^{29} or 536 870 912

13. a) 1.031 **b)** 216.3 cm
c) 56 jumps

14. a) 1, 2, 4, 8, 16, 32 **b)** $t_n = 1(2)^{n-1}$
c) 2^{25} or 33 554 432
d) All cells continue to double and all cells live.

15. 2.9%

16. 8 weeks

17. 65.2 m

18. 0.920

19. a) 76.0 mL **b)** 26 h

20. a)

Time, *d* (days)	Charge Level, *C* (%)
0	100
1	98
2	96.04
3	94.12

b) $t_n = 100(0.98)^{n-1}$
c) The formula in part b) includes the first term at $d = 0$ in the sequence. The formula $C = 100(0.98)^n$ does not consider the first term of the sequence.
d) 81.7%

21. a) 24.14 mm **b)** 1107.77 mm

22. Example: If a, b, c are terms of an arithmetic sequence, then $b - a = c - b$. If 6^a, 6^b, 6^c are terms of a geometric series, then $\frac{6^b}{6^a} = \frac{6^c}{6^b}$ and $6^{b-a} = 6^{c-b}$. Therefore, $b - a = c - b$. So, when 6^a, 6^b, 6^c form a geometric sequence, then a, b, c form an arithmetic sequence.

23. $\frac{5}{3}$; 9, 15, 25

24. a) 23.96 cm **b)** 19.02 cm
c) 2.13 cm **d)** 2.01 cm
e) 2.01, 1.90, 1.79; arithmetic; $d = -0.11$ cm

25. Mala's solution is correct. Since the aquarium loses 8% of the water every day, it maintains 92% of the water every day.

26.

	$\frac{1}{500}$			$\frac{50}{3}$				
	$\frac{1}{100}$	$\frac{1}{10}$	1	10				
	$\frac{1}{20}$		2	6	18	54		
$\frac{1}{16}$	$\frac{1}{4}$	1	4			9		
	$\frac{5}{4}$		8			$\frac{3}{2}$		
	$\frac{25}{4}$		16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
	$\frac{125}{4}$		32					
	$\frac{625}{4}$	100	64					

27. a) 0.86 cm^2 **b)** 1.72 cm^2
c) 3.43 cm^2 **d)** 109.88 cm^2

1.4 Geometric Series, pages 53 to 57

1. a) geometric; $r = 6$ **b)** geometric; $r = -\frac{1}{2}$
c) not geometric **d)** geometric; $r = 1.1$

2. a) $t_1 = 6$, $r = 1.5$, $S_{10} = \frac{174\,075}{256}$, $S_{10} \approx 679.98$
b) $t_1 = 18$, $r = -0.5$, $S_{12} = \frac{12\,285}{1024}$, $S_{12} \approx 12.00$
c) $t_1 = 2.1$, $r = 2$, $S_9 = \frac{10\,731}{10}$, $S_9 = 1073.10$
d) $t_1 = 0.3$, $r = 0.01$, $S_{12} = \frac{10}{33}$, $S_{12} \approx 0.30$

3. a) 12 276 **b)** $\frac{3280}{81}$
c) $-\frac{209\,715}{256}$ **d)** $\frac{36\,855}{256}$

4. a) 40.50 **b)** 0.96
c) 109 225 **d)** 39 063

5. a) 3 **b)** 295.7

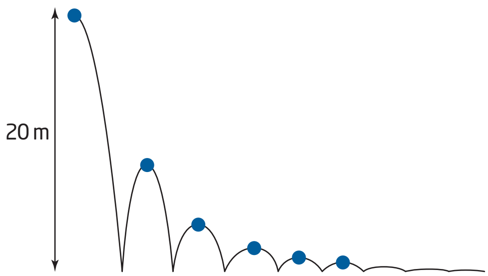
6. 7

7. a) 81 **b)** $81 + 27 + 9 + 3 + 1$

8. $t_2 = -\frac{81}{16}$; $S_6 = 7.8$

9. a) If the person in charge is included, the series is $1 + 4 + 16 + 64 + \cdots$. If the person in charge is not included, the series is $4 + 16 + 64 + \cdots$.
b) If the person in charge is included, the sum is 349 525. If the person in charge is not included, the sum is 1 398 100.

10. 46.4 m



11. 794.3 km

12. b)

Stage Number	Length of Each Line Segment	Number of Line Segments	Perimeter of Snowflake
1	1	3	3
2	$\frac{1}{3}$	12	4
3	$\frac{1}{9}$	48	$\frac{16}{3}$
4	$\frac{1}{27}$	192	$\frac{64}{9}$
5	$\frac{1}{81}$	768	$\frac{256}{27}$

c) length, $t_n = \left(\frac{1}{3}\right)^{n-1}$;
number of line segments, $t_n = 3(4)^{n-1}$;
perimeter, $t_n = 3\left(\frac{4}{3}\right)^{n-1}$

d) $\frac{1024}{81} \approx 12.64$

13. 98 739

14. 91 mm

15. a) 226.9 mg **b)** 227.3 mg

16. 8

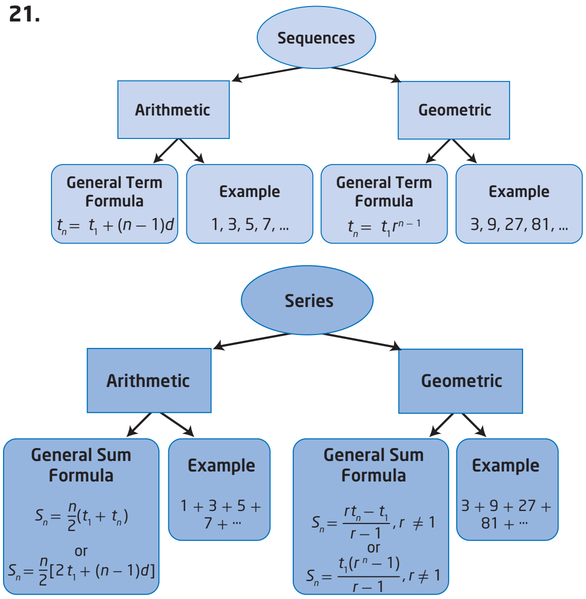
17. $\frac{58\,025}{48}$

18. $a = 5$, $b = 10$, $c = 20$ or $a = 20$, $b = 10$, $c = 5$

19. 15

20. $\frac{341}{4}\pi$

21.



22. Examples:
a) All butterflies produce the same number of eggs and all eggs hatch.
b) No. Tom determined the total number of butterflies from the first to fifth generations. He should have found the fifth term, which would determine the total number of butterflies in the fifth generation only.

c) This is a reasonable estimate, but it does include all butterflies up to the fifth generation, which is 6.42×10^7 more butterflies than those produced in the fifth generation.

d) Determine $t_5 = 1(400)^4$ or 2.56×10^{10}.

1.5 Infinite Geometric Series, pages 63 to 65

1. a) divergent **b)** convergent
c) convergent **d)** divergent
e) divergent

2. a) $\frac{32}{5}$ **b)** no sum
c) no sum **d)** 2
e) 2.5

3. a) $0.87 + 0.0087 + 0.000\,087 + \cdots$;
$S_\infty = \frac{87}{99}$ or $\frac{29}{33}$

b) $0.437 + 0.000\,437 + \cdots$; $S_\infty = \frac{437}{999}$

4. Yes. The sum of the infinite series representing 0.999… is equal to 1.

5. a) 15 **b)** $\frac{4}{5}$ or 0.8
c) 14

6. $t_1 = 27$; $27 + 18 + 12 + \cdots$

7. $r = \frac{2}{5}$; $-8 - \frac{16}{5} - \frac{32}{25} - \frac{64}{125} - \cdots$

8. a) 400 000 barrels of oil
b) Determining the lifetime production assumes the oil well continues to produce at the same rate for many months. This is an unreasonable assumption because 94% is a high rate to maintain.

9. $x = \frac{1}{4}$; $1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \cdots$

10. $r = \frac{1}{2}$

11. a) $-1 < x < 1$ **b)** $-3 < x < 3$
c) $-\frac{1}{2} < x < \frac{1}{2}$

12. 6 cm

13. 250 cm

14. No sum, since $r = 1.1 > 1$. Therefore, the series is divergent.

15. 48 m

16. a) approximately 170.86 cm
b) 300 cm

17. a) Rita
b) $r = -\frac{4}{3}$; therefore, $r < -1$, and the series is divergent.

18. 125 m

19. 72 cm

20. a) Example: $\frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \cdots + \left(\frac{4}{5}\right)^n$
and $\frac{1}{5} + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^3 + \cdots + \left(\frac{1}{5}\right)^n$

b) $S_\infty = \frac{t_1}{1-r} = \frac{\frac{4}{5}}{1-\frac{4}{5}} = \frac{\frac{4}{5}}{\frac{1}{5}} = 4$ and

$S_\infty = \frac{t_1}{1-r} = \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{4}$

21. Geometric series converge only when $-1 < r < 1$.

22. a) $S_n = -\frac{3}{8}n^2 + \frac{11}{8}n$

b) $S_n = \dfrac{\left(\frac{1}{4}\right)^n - 1}{-\frac{3}{4}}$

$S_n = -\frac{4}{3}\left(\frac{1}{4}\right)^n + \frac{4}{3}$

c) $S_\infty = \dfrac{1}{1-\frac{1}{4}}$

$S_\infty = \frac{4}{3}$

23. Step 3

n	1	2	3	4
Fraction of Paper	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$	$\frac{1}{256}$

Step 4 $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256}$, Example: $S_\infty = \frac{1}{3}$

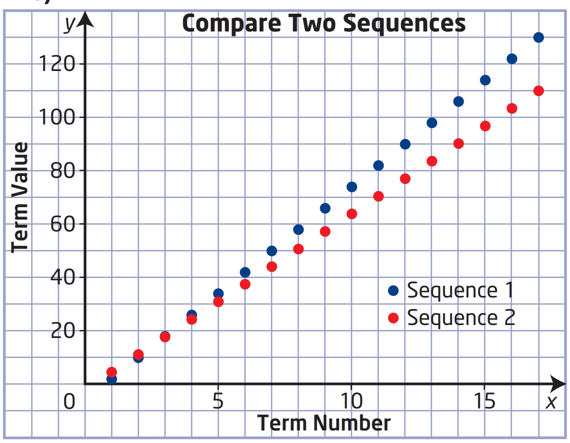
Chapter 1 Review, pages 66 to 68

1. a) arithmetic, $d = 4$ **b)** arithmetic, $d = -5$
c) not arithmetic **d)** not arithmetic

2. a) C **b)** D
c) E **d)** B
e) A

3. a) term, $n = 14$ **b)** not a term
c) term, $n = 54$ **d)** not a term

4. a) A
b)



In the graph, sequence 1 has a larger positive slope than sequence 2. The value of term 17 is greater in sequence 1 than in sequence 2.

5. $t_{10} = 41$

6. 306 cm

7. **a)** $S_{10} = 195$ **b)** $S_{12} = 285$
c) $S_{10} = -75$ **d)** $S_{20} = 3100$

8. $S_{40} = 3420$

9. **a)** 29 **b)** 225
c) 25 days

10. **a)** 61 **b)** 495

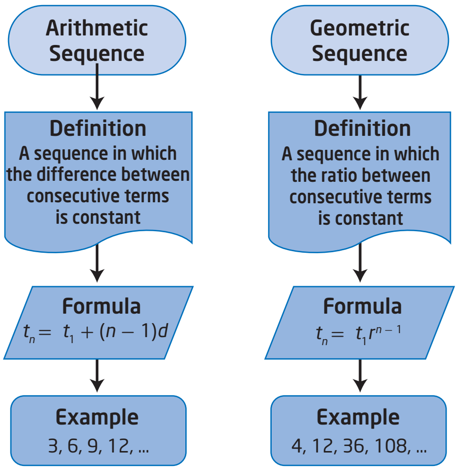
11. 1170

12. **a)** not geometric
b) geometric, $r = -2$, $t_1 = 1$, $t_n = (-2)^{n-1}$
c) geometric, $r = \frac{1}{2}$, $t_1 = 1$, $t_n = \left(\frac{1}{2}\right)^{n-1}$
d) not geometric

13. **a)** 7346 bacteria
b) $t_n = 5000(1.08)^n$

14. 2π cm or approximately 6.28 cm

15.



16. **a)** arithmetic **b)** geometric
c) geometric **d)** arithmetic
e) arithmetic **f)** geometric

17. **a)** $S_{10} = \frac{174\,075}{256}$, $S_{10} \approx 679.98$
b) $S_{12} = \frac{36\,855}{1024}$, $S_{12} \approx 35.99$
c) $S_{20} = \frac{20\,000}{3}$, $S_{20} \approx 6666.67$
d) $S_9 = \frac{436\,905}{4096}$, $S_9 \approx 106.67$

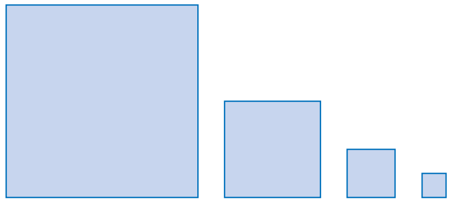
18. **a)** 19.1 mm **b)** 1.37 m

19. **a)** $S_\infty = 15$ **b)** $S_\infty = \frac{3}{4}$

20. **a)** convergent, $S_\infty = 16$
b) divergent
c) convergent, $S_\infty = -28$
d) convergent, $S_\infty = \frac{3}{2}$

21. **a)** $r = -0.4$
b) $S_1 = 7$, $S_2 = 4.2$, $S_3 = 5.32$, $S_4 = 4.872$, $S_5 = 5.0512$
c) 5
d) $S_\infty = 5$

22.



a) $1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}$. Yes. The areas form a geometric sequence. The common ratio is $\frac{1}{4}$.
b) $1\frac{21}{64}$ or 1.328 125 square units
c) $\frac{4}{3}$ square units

23. **a)** A series is geometric if there is a common ratio r such that $r \neq 1$.
An infinite geometric series converges if $-1 < r < 1$.
An infinite geometric series diverges if $r < -1$ or $r > 1$.
b) Example:
$4 + 2 + 1 + 0.5 + \cdots$; $S_\infty = 8$
$21 - 10.5 + 5.25 - 2.625 + \cdots$; $S_\infty = 14$

Chapter 1 Practice Test, pages 69 to 70

1. D

2. B

3. B

4. B

5. C

6. 11.62 cm

7. Arithmetic sequences form straight-line graphs, where the slope is the common difference of the sequence. Geometric sequences form curved graphs.

8. $A = 15$, $B = 9$

9. 0.7 km

10. **a)** 5, 36, 67, 98, 129, 160
b) $t_n = 31n - 26$
c) 5, 10, 20, 40, 80, 160
d) $t_n = 5(2)^{n-1}$

11. **a)** 17, 34, 51, 68, 85
b) $t_n = 17n$
c) 353 million years
d) Assume that the continents continue to separate at the same rate every year.

12. **a)** 30 s, 60 s, 90 s, 120 s, 150 s
b) arithmetic
c) 60 days
d) 915 min