Chapter 2 Trigonometry

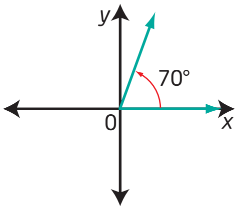
2.1 Angles in Standard Position, pages 83 to 87

1. a) No; the vertex is not at the origin.
b) Yes; the vertex is at the origin and the initial arm is on the *x*-axis.
c) No; the initial arm is not on the *x*-axis.
d) Yes; the vertex is at the origin and the initial arm is on the *x*-axis.

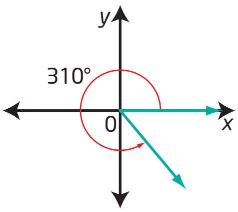
2. a) F **b)** C **c)** A
d) D **e)** B **f)** E

3. a) I **b)** IV **c)** III
d) I **e)** III **f)** II

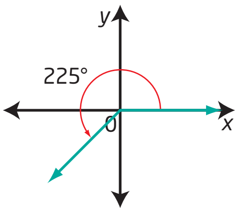
4. a)



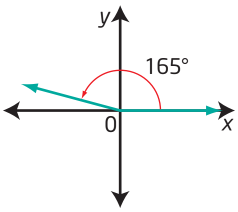
b)



c)



d)



5. a) 10° **b)** 15° **c)** 72° **d)** 35°

6. a) 135°, 225°, 315° **b)** 120°, 240°, 300°
c) 150°, 210°, 330° **d)** 105°, 255°, 285°

7. a) 288° **b)** 124° **c)** 198° **d)** 325°

8.

θ	sin θ	cos θ	tan θ
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

9. 159.6°

10. a) dogwood (−3.5, 2), white pine (3.5, −2), river birch (−3.5, −2)
b) red maple 30°, flowering dogwood 150°, river birch 210°, white pine 330°
c) 40 m

11. $50\sqrt{3}$ cm

12. a) A′(*x*, −*y*), A″(−*x*, *y*), A‴(−*x*, −*y*)
b) ∠A′OC = 360° − θ, ∠A″OC = 180° − θ, ∠A‴OB = 180° + θ

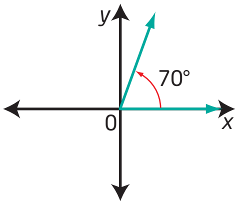
13. $(5\sqrt{3} - 5)$ m or $5(\sqrt{3} - 1)$ m

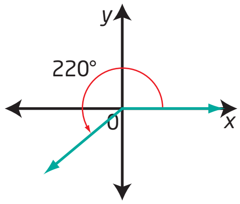
14. 252°

15. Cu (copper), Ag (silver), Au (gold), Uuu (unununium)

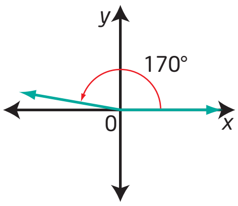
16. a) 216° **b)** 8 days **c)** 18 days

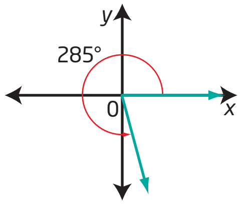
17. a) 70° **b)** 220°





c) 170° **d)** 285°





18. a)

Angle	Height (cm)
0°	12.0
15°	23.6
30°	34.5
45°	43.8
60°	51.0
75°	55.5
90°	57.0

b) A constant increase in the angle does not produce a constant increase in the height. There is no common difference between heights for each pair of angles; for example, 23.6 cm − 12 cm = 11.6 cm, 34.5 cm − 23.6 cm = 10.9 cm.
c) When θ extends beyond 90°, the heights decrease, with the height for 105° equal to the height for 75° and so on.

19. 45° and 135°

20. a) 19.56 m
b) **i)** 192° **ii)** 9.13 m

21. a) B **b)** D

22. $x^2 + y^2 = r^2$

23. a)

θ	20°	40°	60°	80°
sin θ	0.3420	0.6428	0.8660	0.9848
sin (180° − θ)	0.3420	0.6428	0.8660	0.9848
sin (180° + θ)	−0.3420	−0.6428	−0.8660	−0.9848
sin (360° − θ)	−0.3420	−0.6428	−0.8660	−0.9848

b) Each angle in standard position has the same reference angle, but the sine ratio differs in sign based on the quadrant location. The sine ratio is positive in quadrants I and II and negative in quadrants III and IV.

c) The ratios would be the same as those for the reference angle for cos θ and tan θ in quadrant I but may have different signs than sin θ in each of the other quadrants.

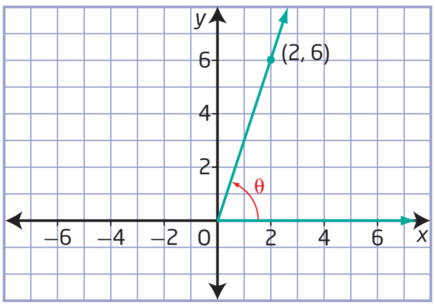
24. a) $\frac{3025\sqrt{3}}{16}$ ft

b) As the angle increases to 45° the distance increases and then decreases after 45°.

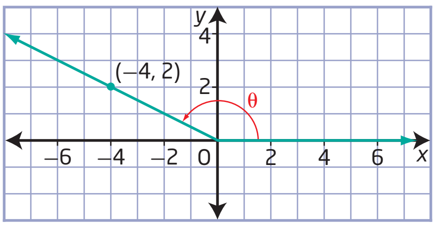
c) The greatest distance occurs with an angle of 45°. The product of cos θ and sin θ has a maximum value when θ = 45°.

2.2 Trigonometric Ratios of Any Angle, pages 96 to 99

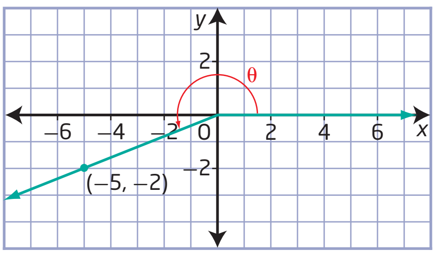
1. a)



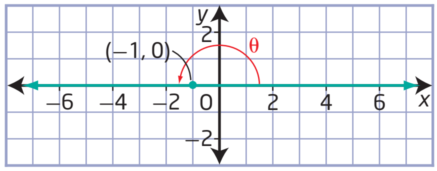
b)



c)



d)



2. a) $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$

b) $\sin 225° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$,

$\cos 225° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\tan 225° = 1$

c) $\sin 150° = \frac{1}{2}$, $\cos 150° = -\frac{\sqrt{3}}{2}$,

$\tan 150° = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

d) $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ is undefined

3. a) $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$

b) $\sin\theta = -\frac{5}{13}$, $\cos\theta = -\frac{12}{13}$, $\tan\theta = \frac{5}{12}$

c) $\sin\theta = -\frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = -\frac{15}{8}$

d) $\sin\theta = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\cos\theta = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$,

$\tan\theta = -1$

4. a) II b) I c) III d) IV

5. a) $\sin\theta = \frac{12}{13}$, $\cos\theta = -\frac{5}{13}$, $\tan\theta = -\frac{12}{5}$

b) $\sin\theta = -\frac{3}{\sqrt{34}}$ or $-\frac{3\sqrt{34}}{34}$,

$\cos\theta = \frac{5}{\sqrt{34}}$ or $\frac{5\sqrt{34}}{34}$, $\tan\theta = -\frac{3}{5}$

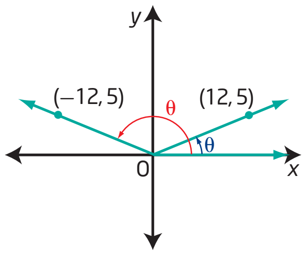
c) $\sin\theta = \frac{3}{\sqrt{45}}$ or $\frac{1}{\sqrt{5}}$, $\cos\theta = \frac{6}{\sqrt{45}}$ or $\frac{2}{\sqrt{5}}$,

$\tan\theta = \frac{1}{2}$

d) $\sin\theta = -\frac{5}{13}$, $\cos\theta = -\frac{12}{13}$, $\tan\theta = \frac{5}{12}$

6. a) positive b) positive
c) negative d) negative

7. a)



b) 23° or 157°

8. a) $\sin\theta = \frac{\sqrt{5}}{3}$, $\tan\theta = -\frac{\sqrt{5}}{2}$

b) $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$

c) $\sin\theta = -\frac{4}{\sqrt{41}}$ or $-\frac{4\sqrt{41}}{41}$,

$\cos\theta = \frac{5}{\sqrt{41}}$ or $\frac{5\sqrt{41}}{41}$

d) $\cos\theta = -\frac{2\sqrt{2}}{3}$, $\tan\theta = \frac{\sqrt{2}}{4}$

e) $\sin\theta = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$,

$\cos\theta = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

9. **a)** 60° and 300° **b)** 135° and 225°
c) 150° and 330° **d)** 240° and 300°
e) 60° and 240° **f)** 135° and 315°

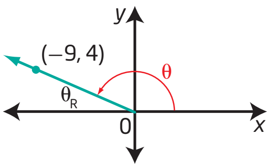
10.

θ	sin θ	cos θ	tan θ
0°	0	1	0
90°	1	0	undefined
180°	0	−1	0
270°	−1	0	undefined
360°	0	1	0

11. **a)** $x = -8$, $y = 6$, $r = 10$, $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$

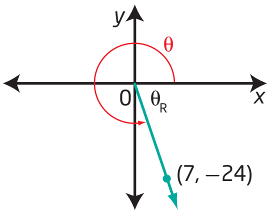
b) $x = 5$, $y = -12$, $r = 13$, $\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = -\frac{12}{5}$

12. **a)**



b) 24° **c)** 156°

13. **a)**



b) 74° **c)** 286°

14. **a)** $\sin\theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

b) $\sin\theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

c) $\sin\theta = \frac{2}{\sqrt{5}}$ or $\frac{2\sqrt{5}}{5}$

d) They all have the same sine ratio. This happens because the points P, Q, and R are collinear. They are on the same terminal arm.

15. **a)** 74° and 106°

b) $\sin\theta = \frac{24}{25}$, $\cos\theta = \pm\frac{7}{25}$, $\tan\theta = \pm\frac{24}{7}$

16. $\sin\theta = \frac{2\sqrt{6}}{5}$

17. $\sin 0° = 0$, $\cos 0° = 1$, $\tan 0° = 0$, $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ is undefined

18. **a)** True. θ_R for 151° is 29° and is in quadrant II. The sine ratio is positive in quadrants I and II.

b) True; both sin 225° and cos 135° have a reference angle of 45° and $\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$.

c) False; tan 135° is in quadrant II, where $\tan\theta < 0$, and tan 225° is in quadrant III, where $\tan\theta > 0$.

d) True; from the reference angles in a 30°-60°-90° triangle, $\sin 60° = \cos 330° = \frac{\sqrt{3}}{2}$.

e) True; the terminal arms lie on the axes, passing through P(0, −1) and P(−1, 0), respectively, so $\sin 270° = \cos 180° = -1$.

19.

θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined
120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135°	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	−1
150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
180°	0	−1	0
210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$
225°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	1
240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
270°	−1	0	undefined
300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$
315°	$-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	−1
330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$
360°	0	1	0

20. **a)** $\angle A = 45°$, $\angle B = 135°$, $\angle C = 225°$, $\angle D = 315°$

b) $A\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $B\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $C\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$, $D\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

21. a)

Angle	Sine	Cosine	Tangent
0°	0	1	0
15°	0.2588	0.9659	0.2679
30°	0.5	0.8660	0.5774
45°	0.7071	0.7071	1
60°	0.8660	0.5	1.7321
75°	0.9659	0.2588	3.7321
90°	1	0	undefined
105°	0.9659	−0.2588	−3.7321
120°	0.8660	−0.5	−1.7321
135°	0.7071	−0.7071	−1
150°	0.5	−0.8660	−0.5774
165°	0.2588	−0.9659	−0.2679
180°	0	−1	0

b) As θ increases from 0° to 180°, sin θ increases from a minimum of 0 to a maximum of 1 at 90° and then decreases to 0 again at 180°. $\sin\theta = \sin(180° - \theta)$. Cos θ decreases from a maximum of 1 at 0° and continues to decrease to a minimum value of −1 at 180°. $\cos\theta = -\cos(180° - \theta)$. Tan θ increases from 0 to being undefined at 90° then back to 0 again at 180°.

c) For $0° \le \theta \le 90°$, $\cos\theta = \sin(90° - \theta)$. For $90° \le \theta \le 180°$, $\cos\theta = -\sin(\theta - 90°)$.

d) Sine ratios are positive in quadrants I and II, and both the cosine and tangent ratios are positive in quadrant I and negative in quadrant II.

e) In quadrant III, the sine and cosine ratios are negative and the tangent ratios are positive. In quadrant IV, the cosine ratios are positive and the sine and tangent ratios are negative.

22. a) $\sin\theta = \dfrac{6}{\sqrt{37}}$ or $\dfrac{6\sqrt{37}}{37}$, $\cos\theta = \dfrac{1}{\sqrt{37}}$ or $\dfrac{\sqrt{37}}{37}$, $\tan\theta = 6$

b) $\dfrac{1}{20}$

23. As θ increases from 0° to 90°, x decreases from 12 to 0, y increases from 0 to 12, sin θ increases from 0 to 1, cos θ decreases from 1 to 0, and tan θ increases from 0 to undefined.

24. $\tan\theta = \dfrac{\sqrt{1 - a^2}}{a}$

25. Since ∠BOA is 60°, the coordinates of point A are $\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$. The coordinates of point B are (1, 0) and of point C are (−1, 0). Using the Pythagorean theorem $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$, $d_{AB} = 1$, $d_{BC} = 2$, and $d_{AC} = \sqrt{3}$.
Then, $AB^2 = 1$, $AC^2 = 3$, and $BC^2 = 4$.
So, $AB^2 + AC^2 = BC^2$.
The measures satisfy the Pythagorean Theorem, so △ABC is a right triangle and ∠CAB = 90°.
Alternatively, ∠CAB is inscribed in a semicircle and must be a right angle.
Hence, △CAB is a right triangle and the Pythagorean Theorem must hold true.

26. Reference angles can determine the trigonometric ratio of any angle in quadrant I. Adjust the signs of the trigonometric ratios for quadrants II, III, and IV, considering that the sine ratio is positive in quadrant II and negative in quadrants III and IV, the cosine ratio is positive in the quadrant IV but negative in quadrants II and III, and the tangent ratio is positive in quadrant III but negative in quadrants II and IV.

27. Use the reference triangle to identify the measure of the reference angle, and then adjust for the fact that P is in quadrant III. Since $\tan\theta_R = \dfrac{9}{5}$, you can find the reference angle to be 61°. Since the angle is in quadrant III, the angle is 180° + 61° or 241°.

28. Sine is the ratio of the opposite side to the hypotenuse. The hypotenuse is the same value, r, in all four quadrants. The opposite side, y, is positive in quadrants I and II and negative in quadrants III and IV. So, there will be exactly two sine ratios with the same positive values in quadrants I and II and two sine ratios with the same negative values in quadrants III and IV.

29. θ = 240°. Both the sine ratio and the cosine ratio are negative, so the terminal arm must be in quadrant III. The value of the reference angle when $\sin\theta_R = \dfrac{\sqrt{3}}{2}$ is 60°. The angle in quadrant III is 180° + 60° or 240°.

30. Step 4

a) As point A moves around the circle, the sine ratio increases from 0 to 1 in quadrant I, decreases from 1 to 0 in quadrant II, decreases from 0 to −1 in quadrant III, and increases from −1 to 0 in quadrant IV. The cosine ratio decreases from 1 to 0 in quadrant I, decreases from 0 to −1 in quadrant II, increases from −1 to 0 in quadrant III, and increases from 0 to 1 in quadrant IV. The tangent ratio increases from 0 to infinity in quadrant I, is undefined for an angle of 90°, increases from negative infinity to 0 in the second quadrant, increases from 0 to positive infinity in the third quadrant, is undefined for an angle of 270°, and increases from negative infinity to 0 in quadrant IV.

b) The sine and cosine ratios are the same when A is at approximately (3.5355, 3.5355) and (−3.5355, −3.5355). This corresponds to 45° and 225°.

c) The sine ratio is positive in quadrants I and II and negative in quadrants III and IV. The cosine ratio is positive in quadrant I, negative in quadrants II and III, and positive in quadrant IV. The tangent ratio is positive in quadrant I, negative in quadrant II, positive in quadrant III, and negative in quadrant IV.

d) When the sine ratio is divided by the cosine ratio, the result is the tangent ratio. This is true for all angles as A moves around the circle.

2.3 The Sine Law, pages 108 to 113

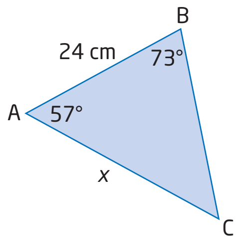
1. a) 8.9 **b)** 50.0
c) 8° **d)** 44°

2. a) 36.9 mm **b)** 50.4 m

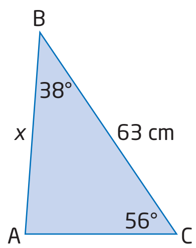
3. a) 53° **b)** 58°

4. a) ∠C = 86°, ∠A = 27°, a = 6.0 m or ∠C = 94°, ∠A = 19°, a = 4.2 m
b) ∠C = 54°, c = 40.7 m, a = 33.6 m
c) ∠B = 119°, c = 20.9 mm, a = 12.4 mm
d) ∠B = 71°, c = 19.4 cm, a = 16.5 cm

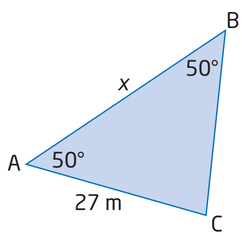
5. a) AC = 30.0 cm



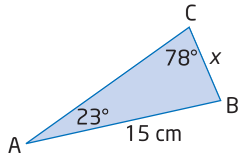
b) AB = 52.4 cm



c) AB = 34.7 m



d) BC = 6.0 cm



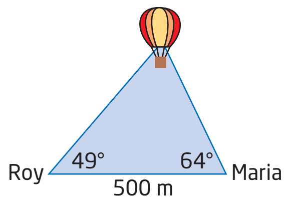
6. a) two solutions **b)** one solution
c) one solution **d)** no solutions

7. a) $a > b \sin A$, $a > h$, $b > h$
b) $a > b \sin A$, $a > h$, $a < b$
c) $a = b \sin A$, $a = h$
d) $a > b \sin A$, $a > h$, $a \geq b$

8. a) ∠A = 48°, ∠B = 101°, b = 7.4 cm or ∠A = 132°, ∠B = 17°, b = 2.2 cm
b) ∠P = 65°, ∠R = 72°, r = 20.9 cm or ∠P = 115°, ∠R = 22°, r = 8.2 cm
c) no solutions

9. a) $a \geq 120$ cm **b)** $a = 52.6$ cm
c) 52.6 cm < a < 120 cm
d) $a < 52.6$ cm

10. a)



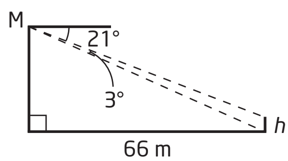
b) 409.9 m

11. 364.7 m

12. 41°

13. 4.5 m

14. a)



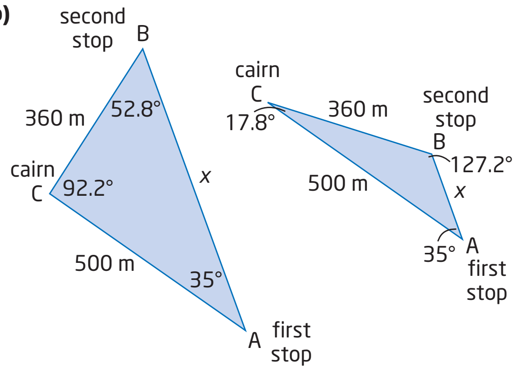
b) 4.1 m **c)** 72.2 m

15. a) 1.51 Å **b)** 0.0151 mm

16. least wingspan 9.1 m, greatest wingspan 9.3 m

17. a) Since $a < b$ (360 < 500) and $a > b \sin A$ (360 > 500 sin 35°), there are two possible solutions for the triangle.

b)



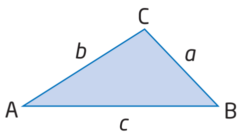
c) Armand's second stop could be either 191.9 m or 627.2 m from his first stop.

18. 911.6 m

19.

Statements	Reasons
$\sin C = \frac{h}{b}$ $\sin B = \frac{h}{c}$	sin B ratio in △ABD sin C ratio in △ACD
$h = b \sin C$ $h = c \sin B$	Solve each ratio for *h*.
$b \sin C = c \sin B$	Equivalence property or substitution
$\frac{\sin C}{c} = \frac{\sin B}{b}$	Divide both sides by *bc*.

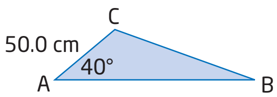
20.



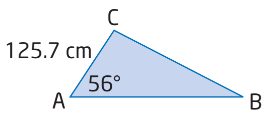
Given ∠A = ∠B, prove that side AC = BC, or $a = b$.
Using the sine law,
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
But ∠A = ∠B, so sin A = sin B.
Then, $\frac{a}{\sin A} = \frac{b}{\sin A}$.
So, $a = b$.

21. 14.1 km^2

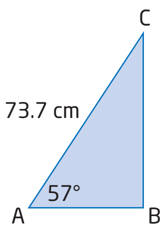
22. a) 32.1 cm < a < 50.0 cm



b) a < 104.2 cm



c) a = 61.8 cm



23. 166.7 m

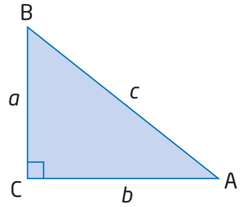
24. a) There is no known side opposite a known angle.

b) There is no known angle opposite a known side.

c) There is no known side opposite a known angle.

d) There is no known angle and only one known side.

25.



In △ABC,
$\sin A = \frac{a}{c}$ and $\sin B = \frac{b}{c}$
Thus, $c = \frac{a}{\sin A}$ and $c = \frac{b}{\sin B}$.
Then, $\frac{a}{\sin A} = \frac{b}{\sin B}$.
This is only true for a right triangle and does not show a proof for oblique triangles.

26. a) 12.9 cm

b) $(4\sqrt{5} + 4)$ cm or $4(\sqrt{5} + 1)$ cm

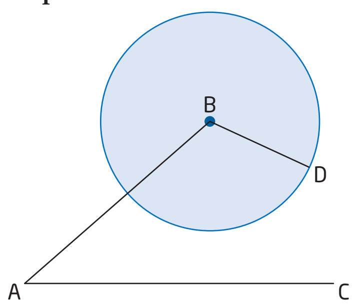
c) 4.9 cm

d) 3.1 cm

e) The spiral is created by connecting the 36° angle vertices for the reducing golden triangles.

27. Concept maps will vary.

28. Step 1

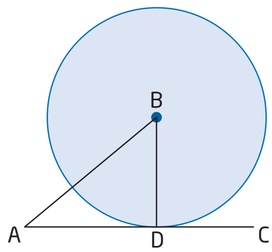


Step 2

a) No.

b) There are no triangles formed when BD is less than the distance from B to the line AC.

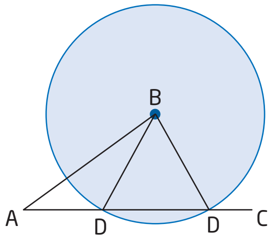
Step 3



a) Yes.

b) One triangle can be formed when BD equals the distance from B to the line AC.

Step 4



a) Yes.

b) Two triangles can be formed when BD is greater than the distance from B to the line AC.

Step 5

a) Yes.

b) One triangle is formed when BD is greater than the length AB.

Step 6 The conjectures will work so long as ∠A is an acute angle. The relationship changes when ∠A > 90°.

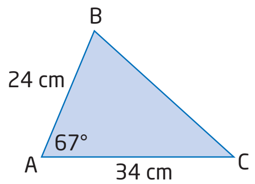
2.4 The Cosine Law, pages 119 to 125

1. a) 6.0 cm **b)** 21.0 mm **c)** 45.0 m

2. a) ∠J = 34° **b)** ∠L = 55°
c) ∠P = 137° **d)** ∠C = 139°

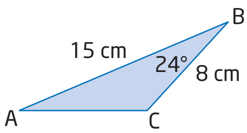
3. a) ∠Q = 62°, ∠R = 66°, p = 25.0 km
b) ∠S = 100°, ∠R = 33°, ∠T = 47°

4. a)



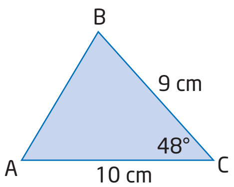
BC = 33.1 cm

b)



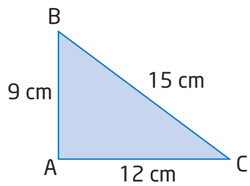
AC = 8.4 m

c)



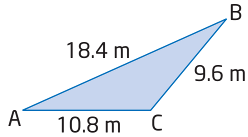
AB = 7.8 cm

d)



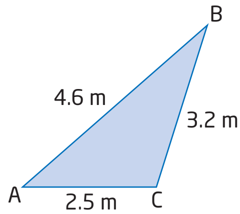
∠B = 53°

e)



∠A = 24°

f)



∠C = 107°

5. a) Use the cosine law because three sides are given (SSS). There is no given angle and opposite side to be able to use the sine law.
b) Use the sine law because two angles and an opposite side are given.
c) Use the cosine law to find the missing side length. Then, use the sine law to find the indicated angle.

6. a) 22.6 cm
b) 7.2 m

7. 53.4 cm

8. 2906 m

9. The angles between the buoys are 35°, 88°, and 57°.

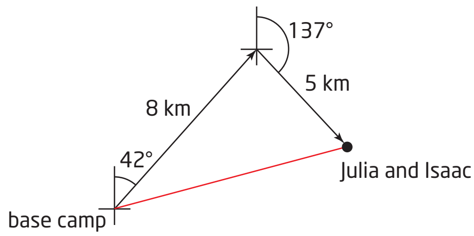
10. 4.2°

11. 22.4 km

12. 54.4 km

13. 458.5 cm

14. a)



b) 9.1 km
c) 255°

15. 9.7 m

16. Use the cosine law in each oblique triangle to find the measure of each obtuse angle. These three angles meet at a point and should sum to 360°. The three angles are 118°, 143°, and 99°. Since 118° + 143° + 99° = 360°, the side measures are accurate.

17. The interior angles of the bike frame are 73°, 62°, and 45°.

18. 98.48 m

19. 1546 km

20. 438.1 m

21. The interior angles of the building are 65°, 32°, and 83°.

22.

Statement	Reason
$c^2 = (a - x)^2 + h^2$	Use the Pythagorean Theorem in △ABD.
$c^2 = a^2 - 2ax + x^2 + h^2$	Expand the square of a binomial.
$b^2 = x^2 + h^2$	Use the Pythagorean Theorem in △ACD.
$c^2 = a^2 - 2ax + b^2$	Substitute b^2 for $x^2 + h^2$.
$\cos C = \frac{x}{b}$	Use the cosine ratio in △ACD.
$x = b \cos C$	Multiply both sides by b.
$c^2 = a^2 - 2ab \cos C + b^2$	Substitute $b \cos C$ for x in step 4.
$c^2 = a^2 + b^2 - 2ab \cos C$	Rearrange.

23. 36.2 km

24. No. The three given lengths cannot be arranged to form a triangle ($a^2 + b^2 < c^2$). When using the cosine law, the cosines of the angles are either greater than 1 or less than −1, which is impossible.

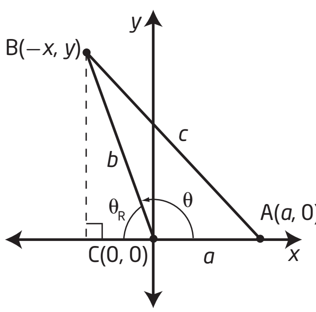
25. 21.2 cm

26. ∠ABC = 65°, ∠ACD = 97°

27. 596 km²

28. 2.1 m

29.



$\cos \theta_R = -\cos \theta = -\frac{x}{\sqrt{x^2 + y^2}}$

$b = \sqrt{x^2 + y^2}$

$c = \sqrt{(a + x)^2 + y^2}$

Prove that $c^2 = a^2 + b^2 - 2ab \cos C$:

Left Side $= \left(\sqrt{(a + x)^2 + y^2}\right)^2$

$= (a + x)^2 + y^2$

$= a^2 + 2ax + x^2 + y^2$

Right Side $= a^2 + \left(\sqrt{x^2 + y^2}\right)^2 - 2a\left(\sqrt{x^2 + y^2}\right)\left(-\frac{x}{\sqrt{x^2 + y^2}}\right)$

$= a^2 + x^2 + y^2 + 2ax$

$= a^2 + 2ax + x^2 + y^2$

Left Side = Right Side

Therefore, the cosine law is true.

30. 115.5 m

31. **a)** 228.05 cm²

b) 228.05 cm²

c) These methods give the same measure when ∠C = 90°.

d) Since cos 90° = 0, $2ab \cos 90° = 0$, so $a^2 + b^2 - 2ab \cos 90° = a^2 + b^2$. Therefore, c^2 can be found using the cosine law or the Pythagorean Theorem when there is a right triangle.

32.

Concept Summary for Solving a Triangle	
Given	**Begin by Using the Method of**
Right triangle	A
Two angles and any side	B
Three sides	C
Three angles	D
Two sides and the included angle	C
Two sides and the angle opposite one of them	B

33. **Step 2**

a) ∠A = 29°, ∠B = 104°, ∠C = 47°

b) The angles at each vertex of a square are 90°. Therefore,
360° = ∠ABC + 90° + ∠GBF + 90°
180° = ∠ABC + ∠GBF
∠GBF = 76°, ∠HCI = 133°, ∠DAE = 151°

c) GF = 6.4 cm, ED = 13.6 cm, HI = 11.1 cm

Step 3

a) For △HCI, the altitude from C to HI is 2.1 cm. For △AED, the altitude from A to DE is 1.6 cm. For △BGF, the altitude from B to GF is 3.6 cm. For △ABC, the altitude from B to AC is 2.9 cm.

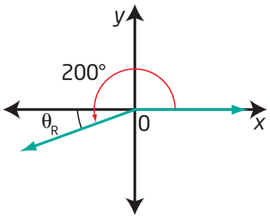
b) area of △ABC is 11.7 cm², area of △BGF is 11.7 cm², area of △AED is 11.7 cm², area of △HCI is 11.7 cm²

Step 4 All four triangles have the same area. Since you use reference angles to determine the altitudes, the product of $\frac{1}{2}bh$ will determine the same area for all triangles. This works for any triangle.

Chapter 2 Review, pages 126 to 128

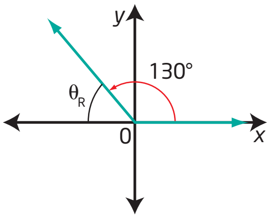
1. a) E **b)** D **c)** B **d)** A
e) F **f)** C **g)** G

2. a)



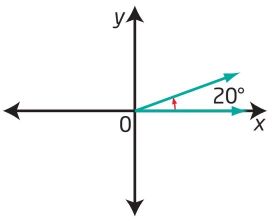
Quadrant III, $\theta_R = 20°$

b)



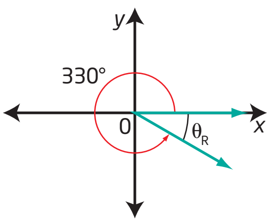
Quadrant II, $\theta_R = 50°$

c)



Quadrant I, $\theta_R = 20°$

d)



Quadrant IV, $\theta_R = 30°$

3. No. Reference angles are measured from the x-axis. The reference angle is 60°.

4. quadrant I: $\theta = 35°$, quadrant II: $\theta = 180° - 35°$ or 145°, quadrant III: $\theta = 180° + 35°$ or 215°, quadrant IV: $\theta = 360° - 35°$ or 325°

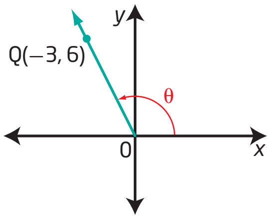
5. a) $\sin 225° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$,
$\cos 225° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\tan 225° = 1$

b) $\sin 120° = \frac{\sqrt{3}}{2}$, $\cos 120° = -\frac{1}{2}$,
$\tan 120° = -\sqrt{3}$

c) $\sin 330° = -\frac{1}{2}$, $\cos 330° = \frac{\sqrt{3}}{2}$,
$\tan 330° = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

d) $\sin 135° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$,
$\cos 135° = -\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$, $\tan 135° = -1$

6. a)



b) $\sqrt{45}$ or $3\sqrt{5}$

c) $\sin\theta = \frac{6}{\sqrt{45}}$ or $\frac{2\sqrt{5}}{5}$,
$\cos\theta = -\frac{3}{\sqrt{45}}$ or $-\frac{\sqrt{5}}{5}$, $\tan\theta = -2$

d) 117°

7. (2, 5), (−2, 5), (−2, −5)

8. a) $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90°$ is undefined

b) $\sin 180° = 0$, $\cos 180° = -1$, $\tan 180° = 0$

9. a) $\cos\theta = -\frac{4}{5}$, $\tan\theta = \frac{3}{4}$

b) $\sin\theta = -\frac{\sqrt{8}}{3}$ or $-\frac{2\sqrt{2}}{3}$,
$\tan\theta = -\sqrt{8}$ or $-2\sqrt{2}$

c) $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

10. a) 130° or 310° **b)** 200° or 340°
c) 70° or 290°

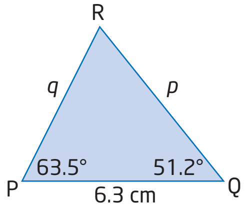
11. a) Yes; there is a known angle (180° − 18° − 114° = 48°) and a known opposite side (3 cm), plus another known angle.

b) Yes; there is a known angle (90°) and opposite side (32 cm), plus one other known side.

c) No; there is no known angle or opposite side.

12. a) $\angle C = 57°$, $c = 36.9$ mm
b) $\angle A = 78°$, $\angle B = 60°$

13.

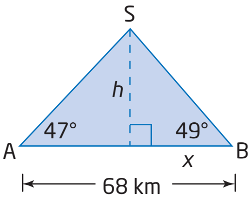


$\angle R = 65.3°$, $q = 5.4$ cm, $p = 6.2$ cm

14. 2.8 km

15. a) Ship B, 50.0 km

b)



Use $\tan 49° = \frac{h}{x}$ and $\tan 47° = \frac{h}{68 - x}$.

Solve $x \tan 49° = (68 - x) \tan 47°$.

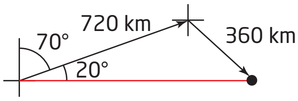
$x = 32.8$ km

Then, use $\cos 49° = \frac{32.8}{BS}$ and $\cos 47° = \frac{35.2}{AS}$ to find BS and AS.

AS = 51.6 km, BS = 50.0 km

16. no solutions if $a < b \sin A$, one solution if $a = b \sin A$ or if $a \geq b$, and two solutions if $b > a > b \sin A$

17. a)



b) 47° E of S

c) 939.2 km

18. a) The three sides do not meet to form a triangle since $4 + 2 < 7$.

b) $\angle A + \angle C > 180°$

c) Sides a and c lie on top of side b, so no triangle is formed.

d) $\angle A + \angle B + \angle C < 180°$

19. a) sine law; there is a known angle and a known opposite side plus another known angle

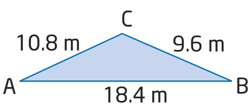
b) cosine law; there is a known SAS (side-angle-side)

20. a) $a = 29.1$ cm

b) $\angle B = 57°$

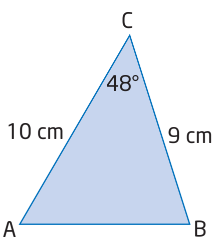
21. 170.5 yd

22. a)



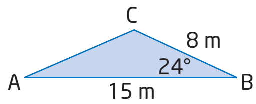
$\angle A = 24°$

b)



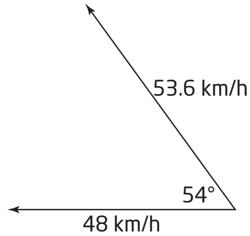
AB = 7.8 cm

c)



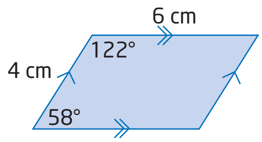
$\angle A = 23°$, $\angle C = 133°$, AC = 8.4 m

23. a)



b) 185.6 km

24. a)



b) 8.8 cm and 5.2 cm

Chapter 2 Practice Test, pages 129 to 130

1. A

2. A

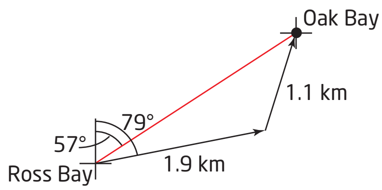
3. C

4. B

5. C

6. −6

7. a)



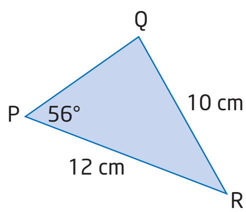
b) 2.6 km

8. a) two

b) $\angle B = 53°$, $\angle C = 97°$, $c = 19.9$ or $\angle B = 127°$, $\angle C = 23°$, $c = 7.8$

9. $\angle R = 17°$

10. a)



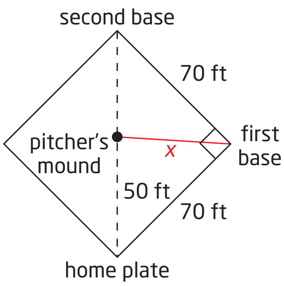
b) $\angle R = 40°$, $\angle Q = 84°$, $r = 7.8$ cm or $\angle R = 28°$, $\angle Q = 96°$, $r = 5.7$ cm

11. 5.2 cm

12. a) 44° **b)** 56° **c)** 1.7 m

13. quadrant I: $\theta = \theta_R$, quadrant II: $\theta = 180° - \theta_R$, quadrant III: $\theta = 180° + \theta_R$, quadrant IV: $\theta = 360° - \theta_R$

14. a)



b) $a^2 + b^2 = c^2$
$70^2 + 70^2 = c^2$
$c = 99$
Second base to pitcher's mound is 99 − 50 or 49 ft.
Distance from first base to pitcher's mound is $x^2 = 50^2 + 70^2 - 2(50)(70) \cos 45°$ or 49.5 ft.

15. Use the sine law when the given information includes a known angle and a known opposite side, plus one other known side or angle. Use the cosine law when given oblique triangles with known SSS or SAS.

16. patio triangle: 38°, 25°, 2.5 m; shrubs triangle: 55°, 2.7 m, 3.0 m

17. 3.1 km

Cumulative Review, Chapters 1–2, pages 133 to 135

1. a) A **b)** D **c)** E **d)** C **e)** B

2. a) geometric, $r = \frac{2}{3}$; $\frac{16}{3}, \frac{32}{9}, \frac{64}{27}$

b) arithmetic, $d = -3$; 5, 2, −1

c) arithmetic, $d = 5$; −1, 4, 9

d) geometric, $r = -2$; 48, −96, 192

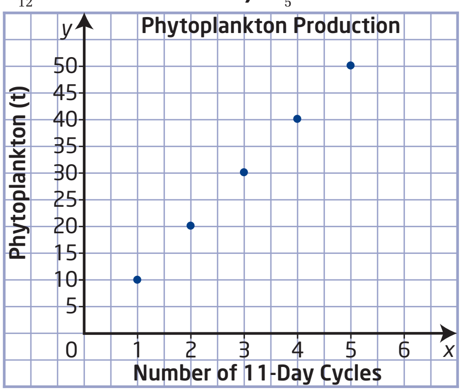
3. a) $t_n = -3n + 21$

b) $t_n = \frac{3}{2}n - \frac{1}{2}$

4. $t_n = 2(-2)^{n-1} \Rightarrow t_{20} = -2^{20}$ or −1 048 576

5. a) $S_{12} = 174$ **b)** $S_5 = 484$

6. a)



b) $t_n = 10n$

c) The general term is a linear equation with a slope of 10.

7. 201 m

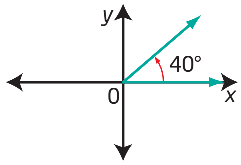
8. a) $r = 0.1$, $S_\infty = 1$

b) Answers will vary.

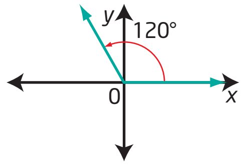
9. $2\sqrt{5}$

10. $\sin \theta = \frac{8}{17}$, $\cos \theta = \frac{15}{17}$, $\tan \theta = \frac{8}{15}$

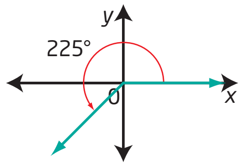
11. a) 40°



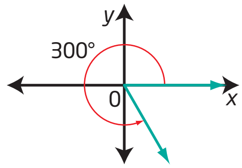
b) 60°



c) 45°

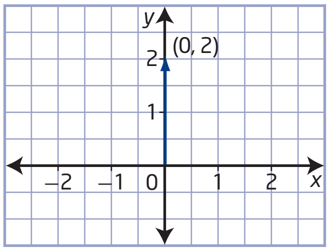


d) 60°



12. a) 90°

b)



c) $\sin \theta = 1$, $\cos \theta = 0$, $\tan \theta$ is undefined

13. a) $\sin 405° = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$

b) $\cos 330° = \frac{\sqrt{3}}{2}$

c) $\tan 225° = 1$

d) $\cos 180° = -1$

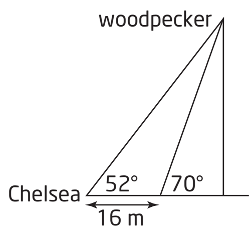
e) $\tan 150° = = -\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$

f) $\sin 270° = -1$

14. The bear is 8.9 km from station A and 7.4 km from station B.

15. 9.4°

16. a)



b) 40.8 m

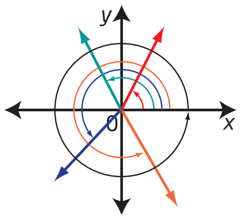
17. 134.4°

Unit 1 Test, pages 136 to 137

1. B

2. C

3. D

4. C
5. D
6. \$0.15 per cup
7. 45°
8. 300°
9. 2775
10. a) 5 b) −6
c) $t_n = 5n - 11$ d) $S_{10} = 165$
11. \$14 880.35
12. 4 km
13. a) 64, 32, 16, 8, … b) $t_n = 64\left(\frac{1}{2}\right)^{n-1}$
c) 63 games
14. a)


b) 60, 120, 180, 240, 300, 360
c) $t_n = 60n$
15. a) 58° b) 5.3 m
16. 38°