Chapter 3 Quadratic Functions

3.1 Investigating Quadratic Functions in Vertex Form, pages 157 to 162

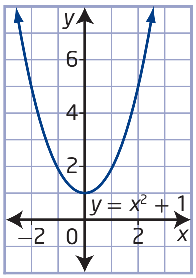
1. a) Since $a > 0$ in $f(x) = 7x^2$, the graph opens upward, has a minimum value, and has a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

b) Since $a > 0$ in $f(x) = \frac{1}{6}x^2$, the graph opens upward, has a minimum value, and has a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

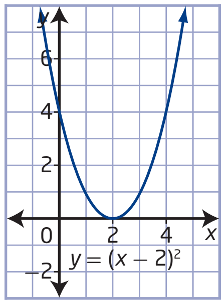
c) Since $a < 0$ in $f(x) = -4x^2$, the graph opens downward, has a maximum value, and has a range of $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

d) Since $a < 0$ in $f(x) = -0.2x^2$, the graph opens downward, has a maximum value, and has a range of $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

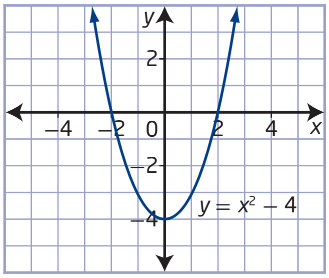
2. a) The shapes of the graphs are the same with the parabola of $y = x^2 + 1$ being one unit higher. vertex: (0, 1), axis of symmetry: $x = 0$, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 1, y \in \mathrm{R}\}$, no x-intercepts, y-intercept occurs at (0, 1)



b) The shapes of the graphs are the same with the parabola of $y = (x - 2)^2$ being two units to the right. vertex: (2, 0), axis of symmetry: $x = 2$, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$, x-intercept occurs at (2, 0), y-intercept occurs at (0, 4)

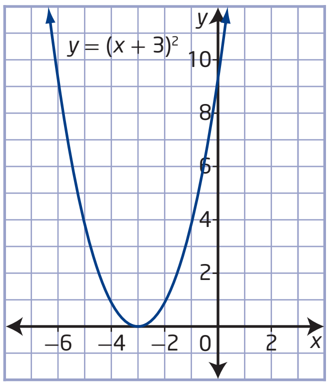


c) The shapes of the graphs are the same with the parabola of $y = x^2 - 4$ being four units lower.



vertex: (0, −4), axis of symmetry: $x = 0$, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq -4, y \in \mathrm{R}\}$, x-intercepts occur at (−2, 0) and (2, 0), y-intercept occurs at (0, −4)

d) The shapes of the graphs are the same with the parabola of $y = (x + 3)^2$ being three units to the left.



vertex: (−3, 0), axis of symmetry: $x = -3$, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$, x-intercept occurs at (−3, 0), y-intercept occurs at (0, 9)

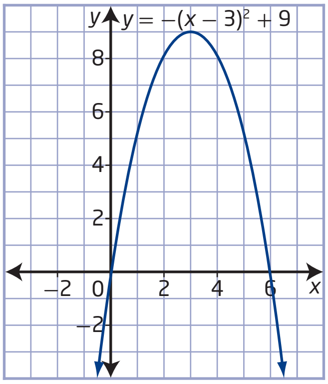
3. a) Given the graph of $y = x^2$, move the entire graph 5 units to the left and 11 units up.

b) Given the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of 3, making it narrower, reflect it in the x-axis so it opens downward, and move the entire new graph down 10 units.

c) Given the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of 5, making it narrower. Move the entire new graph 20 units to the left and 21 units down.

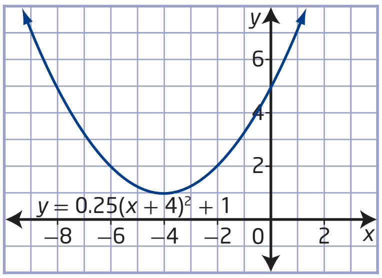
d) Given the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of $\frac{1}{8}$, making it wider, reflect it in the x-axis so it opens downward, and move the entire new graph 5.6 units to the right and 13.8 units up.

4. a)



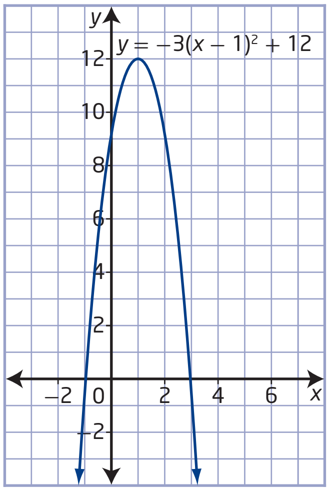
vertex: (3, 9), axis of symmetry: $x = 3$, opens downward, maximum value of 9, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq 9, y \in R\}$, x-intercepts occur at (0, 0) and (6, 0), y-intercept occurs at (0, 0)

b)



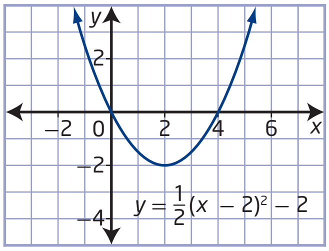
vertex: (−4, 1), axis of symmetry: $x = -4$, opens upward, minimum value of 1, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 1, y \in R\}$, no x-intercepts, y-intercept occurs at (0, 5)

c)



vertex: (1, 12), axis of symmetry: $x = 1$, opens downward, maximum value of 12, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq 12, y \in R\}$, x-intercepts occur at (−1, 0) and (3, 0), y-intercept occurs at (0, 9)

d)



vertex: (2, −2), axis of symmetry: $x = 2$, opens upward, minimum value of −2, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq -2, y \in R\}$, x-intercepts occur at (0, 0) and (4, 0), y-intercept occurs at (0, 0)

5. a) $y_1 = x^2$, $y_2 = 4x^2 + 2$, $y_3 = \frac{1}{2}x^2 - 2$, $y_4 = \frac{1}{4}x^2 - 4$

b) $y_1 = -x^2$, $y_2 = -4x^2 + 2$, $y_3 = -\frac{1}{2}x^2 - 2$, $y_4 = -\frac{1}{4}x^2 - 4$

c) $y_1 = (x + 4)^2$, $y_2 = 4(x + 4)^2 + 2$, $y_3 = \frac{1}{2}(x + 4)^2 - 2$, $y_4 = \frac{1}{4}(x + 4)^2 - 4$

d) $y_1 = x^2 - 2$, $y_2 = 4x^2$, $y_3 = \frac{1}{2}x^2 - 4$, $y_4 = \frac{1}{4}x^2 - 6$

6. For the function $f(x) = 5(x - 15)^2 - 100$, $a = 5$, $p = 15$, and $q = -100$.

a) The vertex is located at (p, q), or (15, −100).

b) The equation of the axis of symmetry is $x = p$, or $x = 15$.

c) Since $a > 0$, the graph opens upward.

d) Since $a > 0$, the graph has a minimum value of q, or -100.

e) The domain is $\{x \mid x \in R\}$. Since the function has a minimum value of -100, the range is $\{y \mid y \geq -100, y \in R\}$.

f) Since the graph has a minimum value of -100 and opens upward, there are two x-intercepts.

7. a) vertex: (0, 14), axis of symmetry: $x = 0$, opens downward, maximum value of 14, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq 14, y \in R\}$, two x-intercepts

b) vertex: $(-18, -8)$, axis of symmetry: $x = -18$, opens upward, minimum value of -8, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq -8, y \in R\}$, two x-intercepts

c) vertex: (7, 0), axis of symmetry: $x = 7$, opens upward, minimum value of 0, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 0, y \in R\}$, one x-intercept

d) vertex: $(-4, -36)$, axis of symmetry: $x = -4$, opens downward, maximum value of -36, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq -36, y \in R\}$, no x-intercepts

8. a) $y = (x + 3)^2 - 4$ **b)** $y = -2(x - 1)^2 + 12$

c) $y = \frac{1}{2}(x - 3)^2 + 1$ **d)** $y = -\frac{1}{4}(x + 3)^2 + 4$

9. a) $y = -\frac{1}{4}x^2$ **b)** $y = 3x^2 - 6$

c) $y = -4(x - 2)^2 + 5$ **d)** $y = \frac{1}{5}(x + 3)^2 - 10$

10. a) $(4, 16) \to (-1, 16) \to (-1, 24)$

b) $(4, 16) \to (4, 4) \to (4, -4)$

c) $(4, 16) \to (4, -16) \to (14, -16)$

d) $(4, 16) \to (4, 48) \to (4, 40)$

11. Starting with the graph of $y = x^2$, apply the change in width, which is a multiplication of the y-values by a factor of 5, reflect the graph in the x-axis, and then move the entire graph up 20 units.

12. Example: Quadratic functions will always have one y-intercept. Since the graphs always open upward or downward and have a domain of $\{x \mid x \in R\}$, the parabola will always cross the y-axis. The graphs must always have a value at $x = 0$ and therefore have one y-intercept.

13. a) $y = \frac{1}{30}x^2$

b) The new function could be $y = \frac{1}{30}(x - 30)^2 - 30$ or $y = \frac{1}{30}(x + 30)^2 - 30$. Both graphs have the same size and shape, but the new function has been transformed by a horizontal translation of 30 units to the right or to the left and a vertical translation of 30 units down to represent a point on the edge as the origin.

14. a) The vertex is located at (36, 20 000), it opens downward, and it has a change in width by a multiplication of the y-values by a factor of 2.5 of the graph $y = x^2$. The equation of the axis of symmetry is $x = 36$, and the graph has a maximum value of 20 000.

b) 36 times

c) 20 000 people

15. Examples: If the vertex is at the origin, the quadratic function will be $y = 0.03x^2$. If the edge of the rim is at the origin, the quadratic function will be $y = 0.03(x - 20)^2 - 12$.

16. a) Example: Placing the vertex at the origin, the quadratic function is $y = \frac{1}{294}x^2$ or $y \approx 0.0034x^2$.

b) Example: If the origin is at the top of the left tower, the quadratic function is $y = \frac{1}{294}(x - 84)^2 - 24$ or $y \approx 0.0034(x - 84)^2 - 24$. If the origin is at the top of the right tower, the quadratic function is $y = \frac{1}{294}(x + 84)^2 - 24$ or $y \approx 0.0034(x + 84)^2 - 24$.

c) 8.17 m; this is the same no matter which function is used.

17. $y = -\frac{9}{121}(x - 11)^2 + 9$

18. $y = -\frac{1}{40}(x - 60)^2 + 90$

19. Example: Adding q is done after squaring the x-value, so the transformation applies directly to the parabola $y = x^2$. The value of p is added or subtracted before squaring, so the shift is opposite to the sign in the bracket to get back to the original y-value for the graph of $y = x^2$.

20. a) $y = -\frac{7}{160\,000}(x - 8000)^2 + 10\,000$

b) domain: $\{x \mid 0 \leq x \leq 16\,000, x \in R\}$, range: $\{y \mid 7200 \leq y \leq 10\,000, y \in R\}$

21. a) Since the vertex is located at (6, 30), $p = 6$ and $q = 30$. Substituting these values into the vertex form of a quadratic function and using the coordinates of the given point, the function is $y = -1.5(x - 6)^2 + 30$.

b) Knowing that the x-intercepts are -21 and -5, the equation of the axis of symmetry must be $x = -13$. Then, the vertex is located at $(-13, -24)$. Substituting the coordinates of the vertex and one of the x-intercepts into the vertex form, the quadratic function is $y = 0.375(x + 13)^2 - 24$.

22. a) Examples: I chose $x = 8$ as the axis of symmetry, I choose the position of the hoop to be (1, 10), and I allowed the basketball to be released at various heights (6 ft, 7 ft, and 8 ft) from a distance of 16 ft from the hoop. For each scenario, substitute the coordinates of the release point into the function $y = a(x - 8)^2 + q$ to get an expression for q. Then, substitute the expression for q and the coordinates of the hoop into the function. My three functions are
$y = -\frac{4}{15}(x - 8)^2 + \frac{346}{15}$,
$y = -\frac{3}{15}(x - 8)^2 + \frac{297}{15}$, and
$y = -\frac{2}{15}(x - 8)^2 + \frac{248}{15}$.

b) Example: $y = -\frac{4}{15}(x - 8)^2 + \frac{346}{15}$ ensures that the ball passes easily through the hoop.

c) domain: $\{x \mid 0 \le x \le 16, x \in \mathrm{R}\}$,
range: $\left\{y \mid 0 \le y \le \frac{346}{15}, y \in \mathrm{R}\right\}$

23. $(m + p, an + q)$

24. Examples:

a) $f(x) = -2(x - 1)^2 + 3$

b) Plot the vertex (1, 3). Determine a point on the curve, say the y-intercept, which occurs at (0, 1). Determine that the corresponding point of (0, 1) is (2, 1). Plot these two additional points and complete the sketch of the parabola.

25. Example: You can determine the number of x-intercepts if you know the location of the vertex and the direction of opening. Visualize the general position and shape of the graph based on the values of a and q. Consider $f(x) = 0.5(x + 1)^2 - 3$, $g(x) = 2(x - 3)^2$, and $h(x) = -2(x + 3)^2 - 4$. For $f(x)$, the parabola opens upward and the vertex is below the x-axis, so the graph has two x-intercepts. For $g(x)$, the parabola opens upward and the vertex is on the x-axis, so the graph has one x-intercept. For $h(x)$, the parabola opens downward and the vertex is below the x-axis, so the graph has no x-intercepts.

26. Answers may vary.

3.2 Investigating Quadratic Functions in Standard Form, pages 174 to 179

1. a) This is a quadratic function, since it is a polynomial of degree two.

b) This is not a quadratic function, since it is a polynomial of degree one.

c) This is not a quadratic function. Once the expression is expanded, it is a polynomial of degree three.

d) This is a quadratic function. Once the expression is expanded, it is a polynomial of degree two.

2. a) The coordinates of the vertex are (−2, 2). The equation of the axis of symmetry is $x = -2$. The x-intercepts occur at (−3, 0) and (−1, 0), and the y-intercept occurs at (0, −6). The graph opens downward, so the graph has a maximum of 2 of when $x = -2$. The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \le 2, y \in \mathrm{R}\}$.

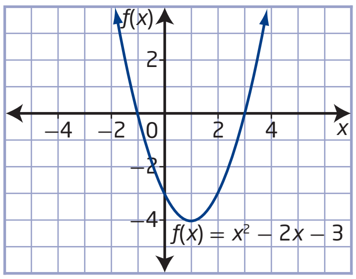
b) The coordinates of the vertex are (6, −4). The equation of the axis of symmetry is $x = 6$. The x-intercepts occur at (2, 0) and (10, 0), and the y-intercept occurs at (0, 5). The graph opens upward, so the graph has a minimum of −4 when $x = 6$. The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge -4, y \in \mathrm{R}\}$.

c) The coordinates of the vertex are (3, 0). The equation of the axis of symmetry is $x = 3$. The x-intercept occurs at (3, 0), and the y-intercept occurs at (0, 8). The graph opens upward, so the graph has a minimum of 0 when $x = 3$. The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \ge 0, y \in \mathrm{R}\}$.

3. a) $f(x) = -10x^2 + 50x$

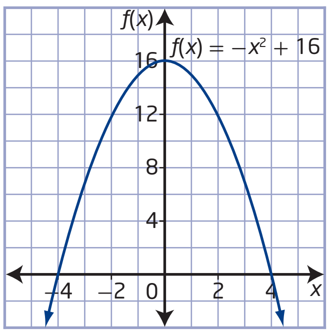
b) $f(x) = 15x^2 - 62x + 40$

4. a)

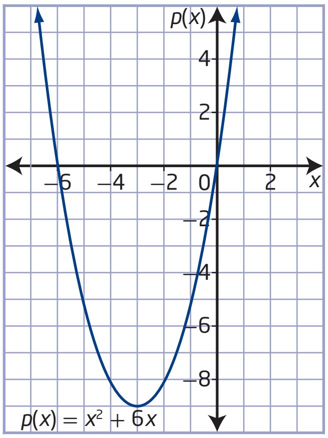


vertex is (1, −4); axis of symmetry is $x = 1$; opens upward; minimum value of −4 when $x = 1$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \ge -4, y \in \mathrm{R}\}$; x-intercepts occur at (−1, 0) and (3, 0), y-intercept occurs at (0, −3)

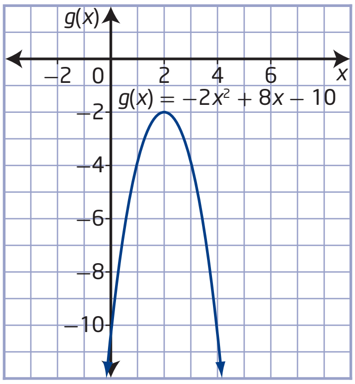
b)



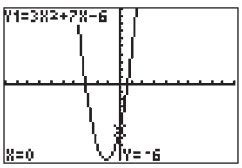
vertex is (0, 16); axis of symmetry is $x = 0$; opens downward; maximum value of 16 when $x = 0$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \leq 16, y \in \mathrm{R}\}$; x-intercepts occur at $(-4, 0)$ and $(4, 0)$, y-intercept occurs at (0, 16)

c) 

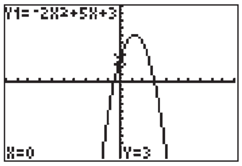
vertex is $(-3, -9)$; axis of symmetry is $x = -3$; opens upward; minimum value of -9 when $x = -3$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \geq -9, y \in \mathrm{R}\}$; x-intercepts occur at $(-6, 0)$ and (0, 0), y-intercept occurs at (0, 0)

d) 

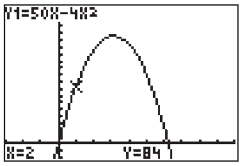
vertex is $(2, -2)$; axis of symmetry is $x = 2$; opens downward; maximum value of -2 when $x = 2$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \leq -2, y \in \mathrm{R}\}$; no x-intercepts, y-intercept occurs at $(0, -10)$

5. a) 

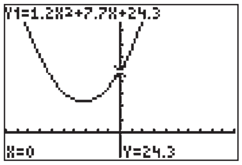
vertex is $(-1.2, -10.1)$; axis of symmetry is $x = -1.2$; opens upward; minimum value of -10.1 when $x = -1.2$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \geq -10.1, y \in \mathrm{R}\}$; x-intercepts occur at $(-3, 0)$ and (0.7, 0), y-intercept occurs at $(0, -6)$

b) 

vertex is (1.3, 6.1); axis of symmetry is $x = 1.3$; opens downward; maximum value of 6.1 when $x = 1.3$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \leq 6.1, y \in \mathrm{R}\}$; x-intercepts occur at $(-0.5, 0)$ and (3, 0), y-intercept occurs at (0, 3)

c) 

vertex is (6.3, 156.3); axis of symmetry is $x = 6.3$; opens downward; maximum value of 156.3 when $x = 6.3$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \leq 156.3, y \in \mathrm{R}\}$; x-intercepts occur at (0, 0) and (12.5, 0), y-intercept occurs at (0, 0)

d) 

vertex is $(-3.2, 11.9)$; axis of symmetry is $x = -3.2$; opens upward; minimum value of 11.9 when $x = -3.2$; domain is $\{x \mid x \in \mathrm{R}\}$, range is $\{y \mid y \geq 11.9, y \in \mathrm{R}\}$; no x-intercepts, y-intercept occurs at (0, 24.3)

6. a) $(-3, -7)$ **b)** $(2, -7)$ **c)** (4, 5)

7. a) 10 cm, h-intercept of the graph
b) 30 cm after 2 s, vertex of the parabola
c) approximately 4.4 s, t-intercept of the graph
d) domain: $\{t \mid 0 \leq t \leq 4.4, t \in \mathrm{R}\}$, range: $\{h \mid 0 \leq h \leq 30, h \in \mathrm{R}\}$
e) Example: No, siksik cannot stay in the air for 4.4 s in real life.

8. Examples:
a) Two; since the graph has a maximum value, it opens downward and would cross the x-axis at two different points. One x-intercept is negative and the other is positive.
b) Two; since the vertex is at (3, 1) and the graph passes through the point $(1, -3)$, it opens downward and crosses the x-axis at two different points. Both x-intercepts are positive.

c) Zero; since the graph has a minimum of 1 and opens upward, it will not cross the x-axis.

d) Two; since the graph has an axis of symmetry of $x = -1$ and passes through the x- and y-axes at (0, 0), the graph could open upward or downward and has another x-intercept at (−2, 0). One x-intercept is zero and the other is negative.

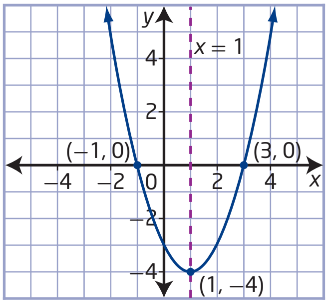
9. a) domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 68, y \in \mathrm{R}\}$

b) domain: $\{x \mid 0 \leq x \leq 4.06, x \in \mathrm{R}\}$, range: $\{y \mid 0 \leq y \leq 68, y \in \mathrm{R}\}$

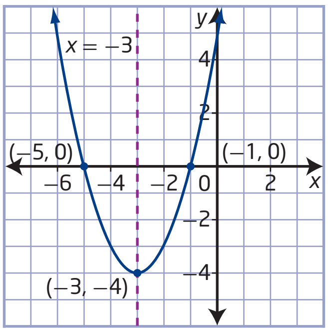
c) Example: The domain and range of algebraic functions may include all real values. For given real-world situations, the domain and range are determined by physical constraints such as time must be greater than or equal to zero and the height must be above ground, or greater than or equal to zero.

10. Examples:

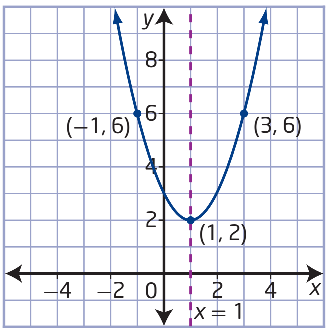
a)



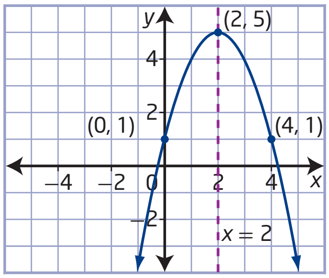
b)



c)

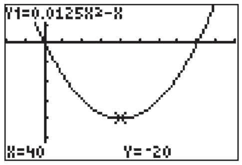


d)



11. a) $\{x \mid 0 \leq x \leq 80, x \in \mathrm{R}\}$

b)

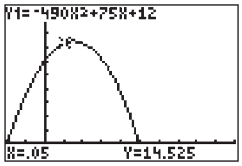


c) The maximum depth of the dish is 20 cm, which is the y-coordinate of the vertex (40, −20). This is not the maximum value of the function. Since the parabola opens upward, this the minimum value of the function.

d) $\{d \mid -20 \leq d \leq 0, d \in \mathrm{R}\}$

e) The depth is approximately 17.19 cm, 25 cm from the edge of the dish.

12. a)



b) The h-intercept represents the height of the log.

c) 0.1 s; 14.9 cm

d) 0.3 s

e) domain: $\{t \mid 0 \leq t \leq 0.3, t \in \mathrm{R}\}$, range: $\{h \mid 0 \leq h \leq 14.9, h \in \mathrm{R}\}$

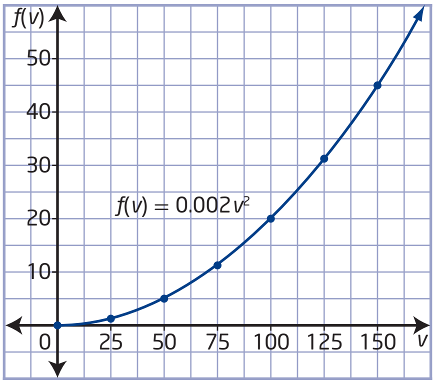
f) 14.5 cm

13. Examples:

a) $\{v \mid 0 \leq v \leq 150, v \in \mathrm{R}\}$

b)

v	f
0	0
25	1.25
50	5
75	11.25
100	20
125	31.25
150	45

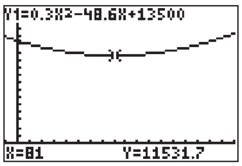


c) The graph is a smooth curve instead of a straight line. The table of values shows that the values of f are not increasing at a constant rate for equal increments in the value of v.

d) The values of the drag force increase by a value other than 2. When the speed of the vehicle doubles, the drag force quadruples.

e) The driver can use this information to improve gas consumption and fuel economy.

14. a)

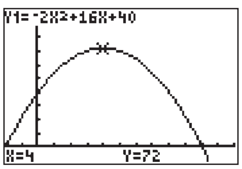


The coordinates of the vertex are (81, 11 532). The equation of the axis of symmetry is $x = 81$. There are no x-intercepts. The y-intercept occurs at (0, 13 500). The graph opens upward, so the graph has a minimum value of 11 532 when $x = 81$. The domain is $\{n \mid n \geq 0, n \in \mathrm{R}\}$. The range is $\{C \mid C \geq 11\,532, C \in \mathrm{R}\}$.

b) Example: The vertex represents the minimum cost of \$11 532 to produce 81 000 units. Since the vertex is above the n-axis, there are no n-intercepts, which means the cost of production is always greater than zero. The C-intercept represents the base production cost. The domain represents thousands of units produced, and the range represents the cost to produce those units.

15. a) $A = -2x^2 + 16x + 40$

b)



c) The values between the x-intercepts will produce a rectangle. The rectangle will have a width that is 2 greater than the value of x and a length that is 20 less 2 times the value of x.

d) The vertex indicates the maximum area of the rectangle.

e) domain: $\{x \mid -2 \leq x \leq 10, x \in \mathrm{R}\}$, range: $\{A \mid 0 \leq A \leq 72, A \in \mathrm{R}\}$; the domain represents the values for x that will produce dimensions of a rectangle. The range represents the possible values of the area of the rectangle.

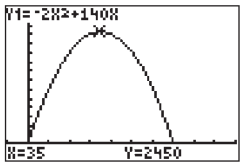
f) The function has both a maximum value and a minimum value for the area of the rectangle.

g) Example: No; the function will open downward and therefore will not have a minimum value for a domain of real numbers.

16. Example: No; the simplified version of the function is $f(x) = 3x + 1$. Since this is not a polynomial of degree two, it does not represent a quadratic function. The graph of the function $f(x) = 4x^2 - 3x + 2x(3 - 2x) + 1$ is a straight line.

17. a) $A = -2x^2 + 140x$; this is a quadratic function since it is a polynomial of degree two.

b)



c) (35, 2450); The vertex represents the maximum area of 2450 m² when the width is 35 m.

d) domain: $\{x \mid 0 \leq x \leq 70, x \in \mathrm{R}\}$, range: $\{A \mid 0 \leq A \leq 2450, A \in \mathrm{R}\}$
The domain represents the possible values of the width, and the range represents the possible values of the area.

e) The function has a maximum area (value) of 2450 m² and a minimum value of 0 m². Areas cannot have negative values.

f) Example: The quadratic function assumes that Maria will use all of the fencing to make the enclosure. It also assumes that any width from 0 m to 70 m is possible.

18. a) Diagram 4 Diagram 5 Diagram 6

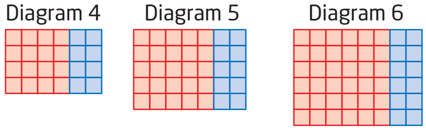


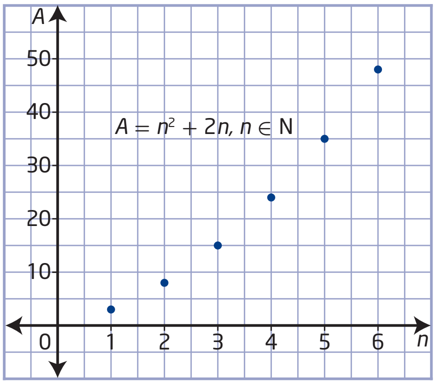
Diagram 4: 24 square units
Diagram 5: 35 square units
Diagram 6: 48 square units

b) $A = n^2 + 2n$

c) Quadratic; the function is a polynomial of degree two.

d) $\{n \mid n \geq 1, n \in \mathrm{N}\}$; The values of n are natural numbers. So, the function is discrete. Since the numbers of both diagrams and small squares are countable, the function is discrete.

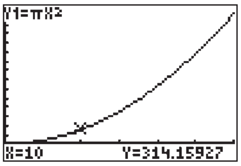
e)



19. a) $A = \pi r^2$

b) domain: $\{r \mid r \geq 0, r \in \mathrm{R}\}$, range: $\{A \mid A \geq 0, A \in \mathrm{R}\}$

c)



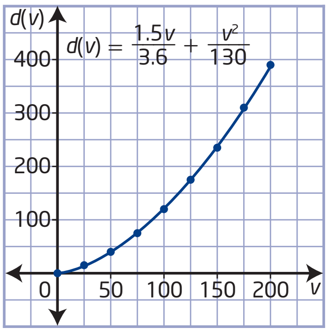
d) The x-intercept and the y-intercept occur at (0, 0). They represent the minimum values of the radius and the area.

e) Example: There is no axis of symmetry within the given domain and range.

20. a) $d(v) = \frac{1.5v}{3.6} + \frac{v^2}{130}$

b)

v	d
0	0
25	15
50	40
75	75
100	119
125	172
150	236
175	308
200	391

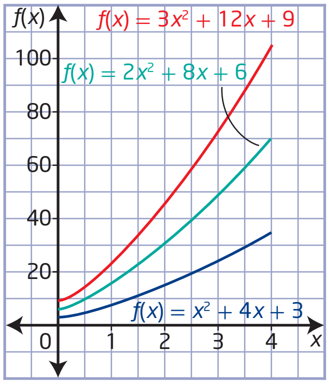


c) No; when v doubles from 25 km/h to 50 km/h, the stopping distance increases by a factor of $\frac{40}{15} = 2.67$, and when the velocity doubles from 50 km/h to 100 km/h, the stopping distance increases by a factor of $\frac{119}{40} = 2.98$. Therefore, the stopping distance increases by a factor greater than two.

d) Example: Using the graph or table, notice that as the speed increases the stopping distances increase by a factor greater than the increase in speed. Therefore, it is important for drivers to maintain greater distances between vehicles as the speed increases to allow for increasing stopping distances.

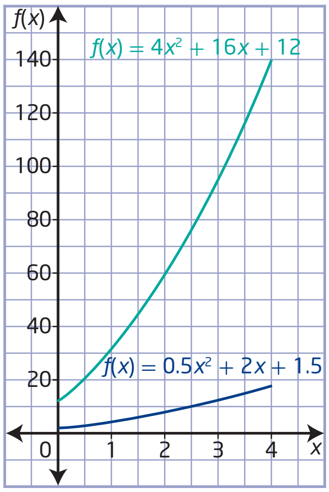
21. a) $f(x) = x^2 + 4x + 3$, $f(x) = 2x^2 + 8x + 6$, and $f(x) = 3x^2 + 12x + 9$

b)

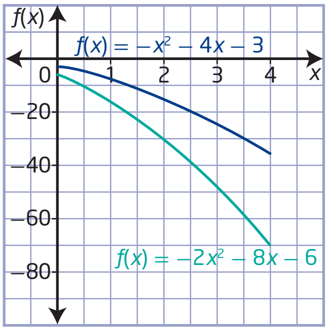


c) Example: The graphs have similar shapes, curving upward at a rate that is a multiple of the first graph. The values of y for each value of x are multiples of each other.

d) Example: If $k = 4$, the graph would start with a y-intercept 4 times as great as the first graph and increase with values of y that are 4 times as great as the values of y of the first function. If $k = 0.5$, the graph would start with a y-intercept $\frac{1}{2}$ of the original y-intercept and increase with values of y that are $\frac{1}{2}$ of the original values of y for each value of x.



e) Example: For negative values of k, the graph would be reflected in the x-axis, with a smooth decreasing curve. Each value of y would be a negative multiple of the original value of y for each value of x.



f) The graph is a line on the x-axis.

g) Example: Each member of the family of functions for $f(x) = k(x^2 + 4x + 3)$ has values of y that are multiples of the original function for each value of x.

22. Example: The value of a in the function $f(x) = ax^2 + bx + c$ indicates the steepness of the curved section of a function in that when $a > 0$, the curve will move up more steeply as a increases and when $-1 < a < 1$, the curve will move up more slowly the closer a is to 0. The sign of a is also similar in that if $a > 0$, then the graph curves up and when $a < 0$, the graph will curve down from the vertex. The value of a in the function $f(x) = ax + b$ indicates the exact steepness or slope of the line determined by the function, whereas the slope of the function $f(x) = ax^2 + bx + c$ changes as the value of x changes and is not a direct relationship for the entire graph.

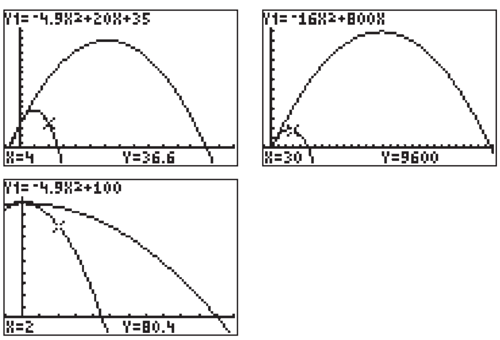
23. a) $b = 3$

b) $b = -3$ and $c = 1$

24. a)

Earth	Moon
$h(t) = -4.9t^2 + 20t + 35$	$h(t) = -0.815t^2 + 20t + 35$
$h(t) = -16t^2 + 800t$	$h(t) = -2.69t^2 + 800t$
$h(t) = -4.9t^2 + 100$	$h(t) = -0.815t^2 + 100$

b)



c) Example: The first two graphs have the same y-intercept at (0, 35). The second two graphs pass through the origin (0, 0). The last two graphs share the same y-intercept at (0, 100). Each pair of graphs share the same y-intercept and share the same constant term.

d) Example: Every projectile on the moon had a higher trajectory and stayed in the air for a longer period of time.

25. Examples:

a) $(2m, r)$; apply the definition of the axis of symmetry. The horizontal distance from the y-intercept to the x-coordinate of the vertex is $m - 0$, or m. So, one other point on the graph is $(m + m, r)$, or $(2m, r)$.

b) $(-2j, k)$; apply the definition of the axis of symmetry. The horizontal distance from the given point to the axis of symmetry is $4j - j$, or $3j$. So, one other point on the graph is $(j - 3j, k)$, or $(-2j, k)$.

c) $\left(\frac{s + t}{2}, d\right)$; apply the definitions of the axis of symmetry and the minimum value of a function. The x-coordinate of the vertex is halfway between the x-intercepts, or $\frac{s + t}{2}$. The y-coordinate of the vertex is the least value of the range, or d.

26. Example: The range and direction of opening are connected and help determine the location of the vertex. If $y \geq q$, then the graph will open upward. If $y \leq q$, then the graph will open downward. The range also determines the maximum or minimum value of the function and the y-coordinate of the vertex. The equation of the axis of symmetry determines the x-coordinate of the vertex. If the vertex is above the x-axis and the graph opens upward, there will be no x-intercepts. However, if it opens downward, there will be two x-intercepts. If the vertex is on the x-axis, there will be only one x-intercept.

27. Step 2 The y-intercept is determined by the value of c. The values of a and b do not affect its location.

Step 3 The axis of symmetry is affected by the values of a and b. As the value of a increases, the value of the axis of symmetry decreases. As the value of b increases, the value of the axis of symmetry increases.

Step 4 Increasing the value of a increases the steepness of the graph.

Step 5 Changing the values of a, b, and c affects the position of the vertex, the steepness of the graph, and whether the graph opens upward ($a > 0$) or downward ($a < 0$). a affects the steepness and determines the direction of opening. b and a affect the value of the axis of symmetry, with b having a greater effect. c determines the value of the y-intercept.

3.3 Completing the Square, pages 192 to 197

1. a) $x^2 + 6x + 9$; $(x + 3)^2$
b) $x^2 - 4x + 4$; $(x - 2)^2$
c) $x^2 + 14x + 49$; $(x + 7)^2$
d) $x^2 - 2x + 1$; $(x - 1)^2$

2. a) $y = (x + 4)^2 - 16$; $(-4, -16)$
b) $y = (x - 9)^2 - 140$; $(9, -140)$
c) $y = (x - 5)^2 + 6$; $(5, 6)$
d) $y = (x + 16)^2 - 376$; $(-16, -376)$

3. a) $y = 2(x - 3)^2 - 18$; working backward, $y = 2(x - 3)^2 - 18$ results in the original function, $y = 2x^2 - 12x$.
b) $y = 6(x + 2)^2 - 7$; working backward, $y = 6(x + 2)^2 - 7$ results in the original function, $y = 6x^2 + 24x + 17$.
c) $y = 10(x - 8)^2 - 560$; working backward, $y = 10(x - 8)^2 - 560$ results in the original function, $y = 10x^2 - 160x + 80$.
d) $y = 3(x + 7)^2 - 243$; working backward, $y = 3(x + 7)^2 - 243$ results in the original function, $y = 3x^2 + 42x - 96$.

4. a) $f(x) = -4(x - 2)^2 + 16$; working backward, $f(x) = -4(x - 2)^2 + 16$ results in the original function, $f(x) = -4x^2 + 16x$.
b) $f(x) = -20(x + 10)^2 + 1757$; working backward, $f(x) = -20(x + 10)^2 + 1757$ results in the original function, $f(x) = -20x^2 - 400x - 243$.
c) $f(x) = -(x + 21)^2 + 941$; working backward, $f(x) = -(x + 21)^2 + 941$ results in the original function, $f(x) = -x^2 - 42x + 500$.
d) $f(x) = -7(x - 13)^2 + 1113$; working backward, $f(x) = -7(x - 13)^2 + 1113$ results in the original function, $f(x) = -7x^2 + 182x - 70$.

5. Verify each part by expanding the vertex form of the function and comparing with the standard form and by graphing both forms of the function.

6. a) minimum value of -11 when $x = -3$
b) minimum value of -11 when $x = 2$
c) maximum value of 25 when $x = -5$
d) maximum value of 5 when $x = 2$

7. a) minimum value of $-\frac{13}{4}$
b) minimum value of $\frac{1}{2}$
c) maximum value of 47
d) minimum value of -1.92
e) maximum value of 18.95
f) maximum value of 1.205

8. a) $y = \left(x + \frac{3}{4}\right)^2 - \frac{121}{16}$
b) $y = -\left(x + \frac{3}{16}\right)^2 + \frac{9}{256}$
c) $y = 2\left(x - \frac{5}{24}\right)^2 + \frac{263}{288}$

9. a) $f(x) = -2(x - 3)^2 + 8$
b) Example: The vertex of the graph is $(3, 8)$. From the function $f(x) = -2(x - 3)^2 + 8$, $p = 3$ and $q = 8$. So, the vertex is $(3, 8)$.

10. a) maximum value of 62; domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq 62, y \in R\}$
b) Example: By changing the function to vertex form, it is possible to find the maximum value since the function opens down and $p = 62$. This also helps to determine the range of the function. The domain is all real numbers for non-restricted quadratic functions.

11. Example: By changing the function to vertex form, the vertex is $\left(\frac{13}{4}, -\frac{3}{4}\right)$ or $(3.25, -0.75)$.

12. a) There is an error in the second line of the solution. You need to add and subtract the square of half the coefficient of the x-term.
$y = x^2 + 8x + 30$
$y = (x^2 + 8x + 16 - 16) + 30$
$y = (x + 4)^2 + 14$
b) There is an error in the second line of the solution. You need to add and subtract the square of half the coefficient of the x-term. There is also an error in the last line. The factor of 2 disappeared.
$f(x) = 2x^2 - 9x - 55$
$f(x) = 2[x^2 - 4.5x + 5.0625 - 5.0625] - 55$
$f(x) = 2[(x^2 - 4.5x + 5.0625) - 5.0625] - 55$
$f(x) = 2[(x - 2.25)^2 - 5.0625] - 55$
$f(x) = 2(x - 2.25)^2 - 10.125 - 55$
$f(x) = 2(x - 2.25)^2 - 65.125$
c) There is an error in the third line of the solution. You need to add and subtract the square of half the coefficient of the x-term.
$y = 8x^2 + 16x - 13$
$y = 8[x^2 + 2x] - 13$
$y = 8[x^2 + 2x + 1 - 1] - 13$
$y = 8[(x^2 + 2x + 1) - 1] - 13$
$y = 8[(x + 1)^2 - 1] - 13$
$y = 8(x + 1)^2 - 8 - 13$
$y = 8(x + 1)^2 - 21$

d) There are two errors in the second line of the solution. You need to factor the leading coefficient from the first two terms and add and subtract the square of half the coefficient of the x-term. There is also an error in the last line. The -3 factor was not distributed correctly.
$f(x) = -3x^2 - 6x$
$f(x) = -3[x^2 + 2x + 1 - 1]$
$f(x) = -3[(x^2 + 2x + 1) - 1]$
$f(x) = -3[(x + 1)^2 - 1]$
$f(x) = -3(x + 1)^2 + 3$

13. 12 000 items

14. 9 m

15. a) 5.56 ft; 0.31 s after being shot
b) Example: Verify by graphing and finding the vertex or by changing the function to vertex form and using the values of p and q to find the maximum value and when it occurs.

16. a) Austin got $+12x$ when dividing $72x$ by -6 and should have gotten $-12x$. He also forgot to square the quantity $(x + 6)$. Otherwise his work was correct and his answer should be $y = -6(x - 6)^2 + 196$. Yuri got an answer of -216 when he multiplied -6 by -36. He should have gotten 216 to get the correct answer of $y = -6(x - 6)^2 + 196$.
b) Example: To verify an answer, either work backward to show the functions are equivalent or use technology to show the graphs of the functions are identical.

17. 18 cm

18. a) The maximum revenue is $151 250 when the ticket price is $55.
b) 2750 tickets
c) Example: Assume that the decrease in ticket prices determines the same increase in ticket sales as indicated by the survey.

19. a) $R(n) = -50n^2 + 1000n + 100\,800$, where R is the revenue of the sales and n is the number of $10 increases in price.
b) The maximum revenue is $105 800 when the bikes are sold for $460.
c) Example: Assume that the predictions of a decrease in sales for every increase in price holds true.

20. a) $P(n) = -0.1n^2 + n + 120$, where P is the production of peas, in kilograms, and n is the increase in plant rows.
b) The maximum production is 122.5 kg of peas when the farmer plants 35 rows of peas.
c) Example: Assume that the prediction holds true.

21. a) Answers may vary.
b) $A = -2w^2 + 90w$, where A is the area and w is the width.
c) 1012.5 m^2
d) Example: Verify the solution by graphing or changing the function to vertex form, where the vertex is (22.5, 1012.5).
e) Example: Assume that the measurements can be any real number.

22. The dimensions of the large field are 75 m by 150 m, and the dimensions of the small fields are 75 m by 50 m.

23. a) The two numbers are 14.5 and 14.5, and the maximum product is 210.25.
b) The two numbers are 6.5 and -6.5, and the minimum product is -42.25.

24. 8437.5 cm^2

25. $f(x) = -\frac{3}{4}\left(x - \frac{3}{4}\right)^2 + \frac{47}{64}$

26. a)
$y = ax^2 + bx + c$
$y = a\left(x^2 + \frac{b}{a}x\right) + c$
$y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b^2}{4a^2}\right)\right) + c$
$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{ab^2}{4a^2} + c$
$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4a^2c - ab^2}{4a^2}$
$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{a(4ac - b^2)}{4a^2}$
$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$
b) $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$
c) Example: This formula can be used to find the vertex of any quadratic function without using an algebraic method to change the function to vertex form.

27. a) (3, 4)
b) $f(x) = 2(x - 3)^2 + 4$, so the vertex is (3, 4).
c) $a = a$, $p = -\frac{b}{2a}$, and $q = \frac{4ac - b^2}{4a}$

28. a) $A = -\left(\frac{4 + \pi}{8}\right)w^2 + 3w$
b) maximum area of $\frac{18}{4 + \pi}$, or approximately 2.52 m^2, when the width is $\frac{12}{4 + \pi}$, or approximately 1.68 m
c) Verify by graphing and comparing the vertex values, $\left(\frac{12}{4 + \pi}, \frac{18}{4 + \pi}\right)$, or approximately (1.68, 2.52).
d) width: $\frac{12}{4 + \pi}$ or approximately 1.68 m,
length: $\frac{6}{4 + \pi}$ or approximately 0.84 m,
radius: $\frac{6}{4 + \pi}$ or approximately 0.84 m;
Answers may vary.

29. Examples:

a) The function is written in both forms; standard form is $f(x) = 4x^2 + 24$ and vertex form is $f(x) = 4(x + 0)^2 + 24$.

b) No, since it is already in completed square form.

30. Martine's first error was that she did not correctly factor -4 from $-4x^2 + 24x$. Instead of $y = -4(x^2 + 6x) + 5$, it should have been $y = -4(x^2 - 6x) + 5$. Her second error occurred when she completed the square. Instead of $y = -4(x^2 + 6x + 36 - 36) + 5$, it should have been $y = -4(x^2 - 6x + 9 - 9) + 5$. Her third error occurred when she factored $(x^2 + 6x + 36)$. This is not a perfect square trinomial and is not factorable. Her last error occurred when she expanded the expression $-4[(x + 6)^2 - 36] + 5$. It should be $-4(x - 3)^2 + 36 + 5$ not $-4(x + 6)^2 - 216 + 5$. The final answer is $y = -4(x - 3)^2 + 41$.

31. a) $R = -5x^2 + 50x + 1000$

b) By completing the square, you can determine the maximum revenue and price to charge to produce the maximum revenue, as well as predict the number of T-shirts that will sell.

c) Example: Assume that the market research holds true for all sales of T-shirts.

Chapter 3 Review, pages 198 to 200

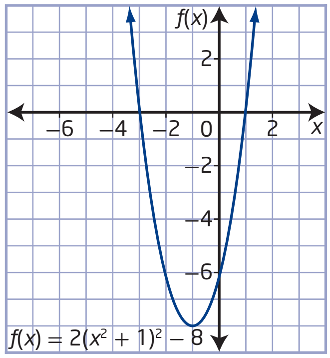
1. a) Given the graph of $f(x) = x^2$, move it 6 units to the left and 14 units down.
vertex: $(-6, -14)$, axis of symmetry: $x = -6$, opens upward, minimum value of -14, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq -14, y \in \mathrm{R}\}$

b) Given the graph of $f(x) = x^2$, change the width by multiplying the y-values by a factor of 2, reflect it in the x-axis, and move the entire graph up 19 units.
vertex: $(0, 19)$, axis of symmetry: $x = 0$, opens downward, maximum value of 19, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 19, y \in \mathrm{R}\}$

c) Given the graph of $f(x) = x^2$, change the width by multiplying the y-values by a factor of $\frac{1}{5}$, move the entire graph 10 units to the right and 100 units up.
vertex: $(10, 100)$, axis of symmetry: $x = 10$, opens upward, minimum value of 100, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 100, y \in \mathrm{R}\}$

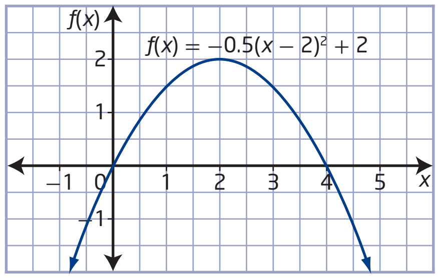
d) Given the graph of $f(x) = x^2$, change the width by multiplying the y-values by a factor of 6, reflect it in the x-axis, and move the entire graph 4 units to the right.
vertex: $(4, 0)$, axis of symmetry: $x = 4$, opens downward, maximum value of 0, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 0, y \in \mathrm{R}\}$

2. a)



vertex: $(-1, -8)$, axis of symmetry: $x = -1$, minimum value of -8, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq -8, y \in \mathrm{R}\}$, x-intercepts occur at $(-3, 0)$ and $(1, 0)$, y-intercept occurs at $(0, -6)$

b)



vertex: $(2, 2)$, axis of symmetry: $x = 2$, maximum value of 2, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 2, y \in \mathrm{R}\}$, x-intercepts occur at $(0, 0)$ and $(4, 0)$, y-intercept occurs at $(0, 0)$

3. Examples:

a) Yes. The vertex is $(5, 20)$, which is above the x-axis, and the parabola opens downward to produce two x-intercepts.

b) Yes. Since $y \geq 0$, the graph touches the x-axis at only one point and has one x-intercept.

c) Yes. The vertex of $(0, 9)$ is above the x-axis and the parabola opens upward, so the graph does not cross or touch the x-axis and has no x-intercepts.

d) No. It is not possible to determine if the graph opens upward to produce two x-intercepts or downward to produce no x-intercepts.

4. a) $y = -0.375x^2$ **b)** $y = 1.5(x - 8)^2$

c) $y = 3(x + 4)^2 + 12$

d) $y = -4(x - 4.5)^2 + 25$

5. a) $y = \frac{1}{4}(x + 3)^2 - 6$ **b)** $y = -2(x - 1)^2 + 5$

6. Example: Two possible functions for the mirror are $y = 0.0069(x - 90)^2 - 56$ and $y = 0.0069x^2$.

7. a) **i)** $y = \frac{22}{18\,769}x^2$ **ii)** $y = \frac{22}{18\,769}x^2 + 30$

iii) $y = \frac{22}{18\,769}(x - 137)^2 + 30$

b) Example: The function will change as the seasons change with the heat or cold changing the length of the cable and therefore the function.

8. $y = -\frac{8}{15}(x - 7.5)^2 + 30$ or $y \approx -0.53(x - 7.5)^2 + 30$

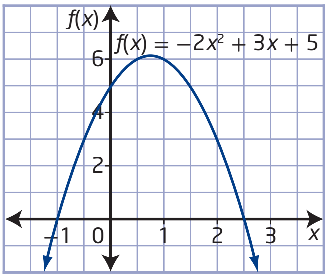
9. a) vertex: (2, 4), axis of symmetry: $x = 2$, maximum value of 4, opens downward, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \le 4, y \in \mathrm{R}\}$, x-intercepts occur at $(-2, 0)$ and $(6, 0)$, y-intercept occurs at (0, 3)

b) vertex: $(-4, 2)$, axis of symmetry: $x = -4$, maximum value of 2, opens upward, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \ge 2, y \in \mathrm{R}\}$, no x-intercepts, y-intercept occurs at (0, 10)

10. a) Expanding $y = 7(x + 3)^2 - 41$ gives $y = 7x^2 + 42x + 22$, which is a polynomial of degree two.

b) Expanding $y = (2x + 7)(10 - 3x)$ gives $y = -6x^2 - x + 70$, which is a polynomial of degree two.

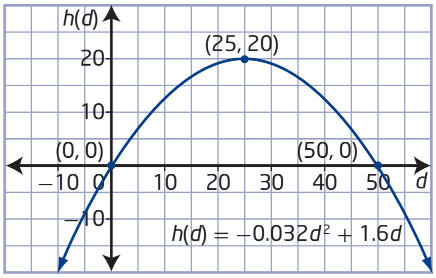
11. a)



vertex: (0.75, 6.125), axis of symmetry: $x = 0.75$, opens downward, maximum value of 6.125, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \le 6.125, y \in \mathrm{R}\}$, x-intercepts occur at $(-1, 0)$ and $(2.5, 0)$, y-intercept occurs at (0, 5)

b) Example: The vertex is the highest point on the curve. The axis of symmetry divides the graph in half and is defined by the x-coordinate of the vertex. Since $a < 0$, the graph opens downward. The maximum value is the y-coordinate of the vertex. The domain is all real numbers. The range is less than or equal to the maximum value, since the graph opens downward. The x-intercepts are where the graph crosses the x-axis, and the y-intercept is where the graph crosses the y-axis.

12. a)



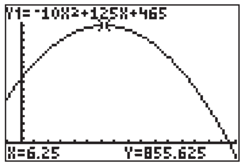
b) The maximum height of the ball is 20 m. The ball is 25 m downfield when it reaches its maximum height.

c) The ball lands downfield 50 m.

d) domain: $\{x \mid 0 \le x \le 50, x \in \mathrm{R}\}$, range: $\{y \mid 0 \le y \le 20, y \in \mathrm{R}\}$

13. a) $y = (5x + 15)(31 - 2x)$ or $y = -10x^2 + 125x + 465$

b)



c) The values between the x-intercepts will produce a rectangle.

d) Yes; the maximum value is 855.625; the minimum value is 0.

e) The vertex represents the maximum area and the value of x that produces the maximum area.

f) domain: $\{x \mid 0 \le x \le 15.5, x \in \mathrm{R}\}$, range: $\{y \mid 0 \le y \le 855.625, y \in \mathrm{R}\}$

14. a) $y = (x - 12)^2 - 134$

b) $y = 5(x + 4)^2 - 107$

c) $y = -2(x - 2)^2 + 8$

d) $y = -30(x + 1)^2 + 135$

15. vertex: $\left(\frac{5}{4}, -\frac{13}{4}\right)$, axis of symmetry: $x = \frac{5}{4}$, minimum value of $-\frac{13}{4}$, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\left\{y \mid y \ge -\frac{13}{4}, y \in \mathrm{R}\right\}$

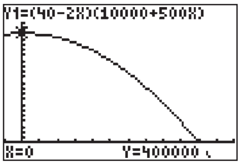
16. a) In the second line, the second term should have been $+3.5x$. In the third line, Amy found the square of half of 3.5 to be 12.25; it should have been 3.0625 and this term should be added and then subtracted. The solution should be

$y = -22x^2 - 77x + 132$
$y = -22(x^2 + 3.5x) + 132$
$y = -22(x^2 + 3.5x + 3.0625 - 3.0625) + 132$
$y = -22(x^2 + 3.5x + 3.0625) + 67.375 + 132$
$y = -22(x + 1.75)^2 + 199.375$

b) Verify by expanding the vertex form to standard form and by graphing both forms to see if they produce the same graph.

17. a) $R = (40 - 2x)(10\ 000 + 500x)$ or $R = -1000x^2 + 400\ 000$ where R is the revenue and x is the number of price decreases.

b) The maximum revenue is \$400 000 and the price is \$40 per coat.

c) 

d) The y-intercept represents the sales before changing the price. The x-intercepts indicate the number of price increases or decreases that will produce revenue.

e) domain: $\{x \mid -20 \leq x \leq 20, x \in \mathrm{R}\}$, range: $\{y \mid 0 \leq y \leq 400\ 000, y \in \mathrm{R}\}$

f) Example: Assume that a whole number of price increases can be used.

Chapter 3 Practice Test, pages 201 to 203

1. D

2. C

3. A

4. D

5. D

6. A

7. a) $y = (x - 9)^2 - 108$

b) $y = 3(x + 6)^2 - 95$

c) $y = -10(x + 2)^2 + 40$

8. a) vertex: $(-6, 4)$, axis of symmetry: $x = -6$, maximum value of 4, domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \leq 4, y \in \mathrm{R}\}$, x-intercepts occur at $(-8, 0)$ and $(-4, 0)$

b) $y = -(x + 6)^2 + 4$

9. a)

i) change in width by a multiplication of the y-values by a factor of 5

ii) vertical translation of 20 units down

iii) horizontal translation of 11 units to the left

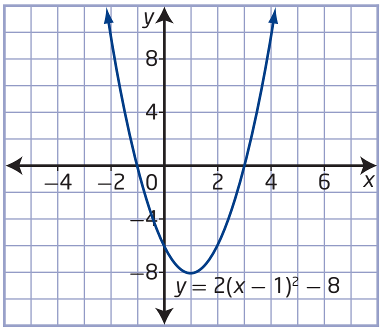
iv) change in width by a multiplication of the y-values by a factor of $\frac{1}{7}$ and a reflection in the x-axis

b) Examples:

i) The vertex of the functions in part a) ii) and iii) will be different as compared to $f(x) = x^2$ because the entire graph is translated. Instead of a vertex of (0, 0), the graph of the function in part a) ii) will be located at $(0, -20)$ and the vertex of the graph of the function in part a) iii) will be located at $(-11, 0)$.

ii) The axis of symmetry of the function in part a) iii) will be different as compared to $f(x) = x^2$ because the entire graph is translated horizontally. Instead of an axis of symmetry of $x = 0$, the graph of the function in part a) iii) will have an axis of symmetry of $x = -11$.

iii) The range of the functions in part a) ii) and iv) will be different as compared to $f(x) = x^2$ because the entire graph is either translated vertically or reflected in the x-axis. Instead of a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$, the function in part a) ii) will have a range of $\{y \mid y \geq -20, y \in \mathrm{R}\}$ and the function in part a) iv) will have a range of $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

10. 

Vertex	(1, −8)
Axis of Symmetry	$x = 1$
Direction of Opening	upward
Domain	$\{x \mid x \in \mathrm{R}\}$
Range	$\{y \mid y \geq -8, y \in \mathrm{R}\}$
x-Intercepts	−1 and 3
y-Intercept	−6

11. a) In the second line, the 2 was not factored out of the second term. In the third line, you need to add and subtract the square of half the coefficient of the x-term. The first three steps should be

$y = 2x^2 - 8x + 9$
$y = 2(x^2 - 4x) + 9$
$y = 2(x^2 - 4x + 4 - 4) + 9$

b) The rest of the process is shown.

$y = 2[(x^2 - 4x + 4) - 4] + 9$
$y = 2(x - 2)^2 - 8 + 9$
$y = 2(x - 2)^2 + 1$

c) The solution can be verified by expanding the vertex form to standard form or by graphing both functions to see that they coincide.

12. Examples:

a) The vertex form of the function $C(v) = 0.004v^2 - 0.62v + 30$ is $C(v) = 0.004(v - 77.5)^2 + 5.975$. The most efficient speed would be 77.5 km/h and will produce a fuel consumption of 5.975 L/100 km.

b) By completing the square and determining the vertex of the function, you can determine the most efficient fuel consumption and at what speed it occurs.

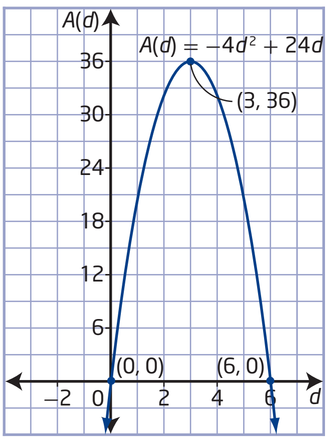
13. a) The maximum height of the flare is 191.406 25 m, 6.25 s after being shot.

b) Example: Complete the square to produce the vertex form and use the value of q to determine the maximum height and the value of p to determine when it occurs, or use the fact that the x-coordinate of the vertex of a quadratic function in standard form is $x = -\frac{b}{2a}$ and substitute this value into the function to find the corresponding y-coordinate, or graph the function to find the vertex.

14. a) $A(d) = -4d^2 + 24d$

b) Since the function is a polynomial of degree two, it satisfies the definition of a quadratic function.

c)



Example: By completing the square, determine the vertex, find the y-intercept and its corresponding point, plot the three points, and join them with a smooth curve.

d) (3, 36); the maximum area of 36 m^2 happens when the fence is extended to 3 m from the building.

e) domain: $\{d \mid 0 \le d \le 6, d \in \mathrm{R}\}$, range: $\{A \mid 0 \le A \le 36, A \in \mathrm{R}\}$; negative distance and area do not have meaning in this situation.

f) Yes; the maximum value is 36 when d is 3, and the minimum value is 0 when d is 0 or 6.

g) Example: Assume that any real-number distance can be used to build the fence.

15. a) $f(x) = -0.03x^2$

b) $f(x) = -0.03x^2 + 12$

c) $f(x) = -0.03(x + 20)^2 + 12$

d) $f(x) = -0.03(x - 28)^2 - 3$

16. a) $R = (2.25 - 0.05x)(120 + 8x)$

b) Expand and complete the square to get the vertex form of the function. A price of \$1.50 gives the maximum revenue of \$360.

c) Example: Assume that any whole number of price decreases can occur.