Chapter 4 Quadratic Equations

4.1 Graphical Solutions of Quadratic Equations, pages 215 to 217

1. a) 1 **b)** 2 **c)** 0 **d)** 2

2. a) 0 **b)** -1 and -4
c) none **d)** -3 and 8

3. a) $x = -3$, $x = 8$ **b)** $r = -3$, $r = 0$
c) no real solutions **d)** $x = 3$, $x = -2$
e) $z = 2$ **f)** no real solutions

4. a) $n \approx -3.2$, $n \approx 3.2$ **b)** $x = -4$, $x = 1$
c) $w = 1$, $w = 3$ **d)** $d = -8$, $d = -2$
e) $v \approx -4.7$, $v \approx -1.3$ **f)** $m = 3$, $m = 7$

5. 60 yd

6. a) $-x^2 + 9x - 20 = 0$ or $x^2 - 9x + 20 = 0$
b) 4 and 5

7. a) $x^2 + 2x - 168 = 0$
b) $x = 12$ and $x = 14$ or $x = -12$ and $x = -14$

8. a) Example: Solving the equation leads to the distance from the firefighter that the water hits the ground. The negative solution is not part of this situation.
b) 12.2 m
c) Example: Assume that aiming the hose higher would not reach farther. Assume that wind does not affect the path of the water.

9. a) Example: Solving the equation leads to the time that the fireworks hit the ground. The negative solution is not part of the situation.
b) 6.1 s

10. a) $-0.75d^2 + 0.9d + 1.5 = 0$ **b)** 2.1 m

11. a) $-2d^2 + 3d + 10 = 0$ **b)** 3.1 m

12. a) first arch: $x = 0$ and $x = 84$, second arch: $x = 84$ and $x = 168$, third arch: $x = 168$ and $x = 252$
b) The zeros represent where the arches reach down to the bridge deck.
c) 252 m

13. a) $k = 9$ **b)** $k < 9$ **c)** $k > 9$

14. a) 64 ft

b) The relationship between the height, radius, and span of the arch stays the same. Input the measures in metres and solve.

15. about 2.4 s

16. For the value of the function to change from negative to positive, it must cross the x-axis and therefore there must be an x-intercept between the two values of x.

17. The other x-intercept would have to be 4.

18. The x-coordinate of the vertex is halfway between the two roots. So, it is at 2. You can then substitute $x = 2$ into the equation to find the minimum value of -16.

4.2 Factoring Quadratic Equations, pages 229 to 233

1. a) $(x + 2)(x + 5)$ **b)** $5(z + 2)(z + 6)$
c) $0.2(d - 4)(d - 7)$

2. a) $(y - 1)(3y + 7)$ **b)** $(4k - 5)(2k + 1)$
c) $0.2(2m - 3)(m + 3)$

3. a) $(x + 5)(x - 4)$ **b)** $(x - 6)^2$
c) $\frac{1}{4}(x + 2)(x + 6)$ **d)** $2(x + 3)^2$

4. a) $(2y + 3x)(2y - 3x)$
b) $(0.6p + 0.7q)(0.6p - 0.7q)$
c) $\left(\frac{1}{2}s + \frac{3}{5}t\right)\left(\frac{1}{2}s - \frac{3}{5}t\right)$
d) $(0.4t + 4s)(0.4t - 4s)$

5. a) $(x + 8)(x - 5)$
b) $(2x^2 - 8x + 9)(3x^2 - 12x + 11)$
c) $(-4)(8j)$

6. a) $(10b)(10b - 7)$
b) $16(x^2 - x + 1)(x^2 + x + 1)$
c) $(10y^3 - x)(10y^3 + x)$

7. a) $x = -3, x = -4$ **b)** $x = 2, x = -\frac{1}{2}$
c) $x = -7, x = 8$ **d)** $x = 0, x = -5$
e) $x = -\frac{1}{3}, x = \frac{4}{5}$ **f)** $x = 4, x = \frac{7}{2}$

8. a) $n = -2, n = 2$ **b)** $x = -4, x = -1$
c) $w = -9, x = -\frac{1}{3}$ **d)** $y = \frac{5}{4}, y = \frac{3}{2}$
e) $d = -\frac{3}{2}, d = -1$ **f)** $x = \frac{3}{2}$

9. a) 0 and 5 **b)** $-\frac{8}{9}$ and 1
c) -5 and -3 **d)** $-\frac{21}{5}$ and $\frac{21}{5}$
e) -5 and 7 **f)** $\frac{7}{2}$

10. a) -6 and 7 **b)** -10 and 3
c) -7 and 3 **d)** $-\frac{1}{3}$ and $\frac{3}{2}$
e) -5 and 2 **f)** -3 and $\frac{1}{2}$

11. a) $(x + 10)(2x - 3) = 54$ **b)** 3.5 cm

12. a) 1 s and 5 s

b) Assume that the mass of the fish does not affect the speed at which the osprey flies after catching the fish. This may not be a reasonable assumption for a large fish.

13. a) $150t - 5t^2 = 0$ **b)** 30 s

14. 8 and 10 or 0 and -2

15. 15 cm

16. 3 s; this seems a very long time considering the ball went up only 39 ft.

17. a) 1 cm

b) 7 cm by 5 cm

18. a) No; $(x - 5)$ is not a factor of the expression $x^2 - 5x - 36$, since $x = 5$ does not satisfy the equation $x^2 - 5x - 36 = 0$.

b) Yes; $(x + 3)$ is a factor of the expression $x^2 - 2x - 15$, since $x = -3$ satisfies the equation $x^2 - 2x - 15 = 0$.

c) No; $(4x + 1)$ is not a factor of the expression $6x^2 + 11x + 4$, since $x = -\frac{1}{4}$ does not satisfy the equation $6x^2 + 11x + 4 = 0$.

d) Yes; $(2x - 1)$ is a factor of the expression $4x^2 + 4x - 3$, since $x = \frac{1}{2}$ satisfies the equation $4x^2 + 4x - 3 = 0$.

19. a) $-\frac{1}{2}$ and 2 **b)** -4 and 3

20. 20 cm and 21 cm

21. 8 m and 15 m

22. a) $x(x - 7) = 690$ **b)** 30 cm by 23 cm

23. 5 m

24. 5 m

25. $P = \frac{1}{2}d(v_1 + v_2)(v_1 - v_2)$

26. No; the factor $6x - 4$ still has a common factor of 2.

27. a) $6(z - 1)(2z + 5)$
b) $4(2m^2 - 8 - 3n)(2m^2 - 8 + 3n)$
c) $\frac{1}{36}(2y - 3x)^2$
d) $7\left(w - \frac{5}{3}\right)(5w + 1)$

28. $4(3x + 5y)$ centimetres

29. The shop will make a profit after 4 years.

30. a) $x^2 - 9 = 0$ **b)** $x^2 - 4x + 4 = 0$
c) $3x^2 - 14x + 8 = 0$
d) $10x^2 - x - 3 = 0$

31. Example: $x^2 - x + 1 = 0$

32. a) Instead of evaluating $81 - 36$, use the difference of squares pattern to rewrite the expression as $(9 - 6)(9 + 6)$ and then simplify. You can use this method when a question asks you to subtract a square number from a square number.

b) Examples:

$$\begin{aligned}144 - 25 &= (12 - 5)(12 + 5)\\ &= (7)(17)\\ &= 119\\ 256 - 49 &= (16 - 7)(16 + 7)\\ &= (9)(23)\\ &= 207\end{aligned}$$

4.3 Solving Quadratic Equations by Completing the Square, pages 240 to 243

1. a) $c = \frac{1}{4}$ **b)** $c = \frac{25}{4}$
c) $c = 0.0625$ **d)** $c = 0.01$
e) $c = \frac{225}{4}$ **f)** $c = \frac{81}{4}$

2. a) $(x + 2)^2 = 2$ **b)** $(x + 2)^2 = \frac{17}{3}$
c) $(x - 3)^2 = -1$

3. a) $(x - 6)^2 - 27 = 0$ **b)** $5(x - 2)^2 - 21 = 0$
c) $-2\left(x - \frac{1}{4}\right)^2 - \frac{7}{8} = 0$
d) $0.5(x + 2.1)^2 + 1.395 = 0$
e) $-1.2(x + 2.125)^2 - 1.981\ 25 = 0$
f) $\frac{1}{2}(x + 3)^2 - \frac{21}{2} = 0$

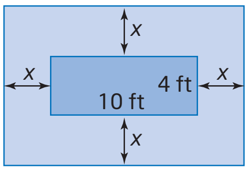
4. a) $x = \pm 8$ **b)** $s = \pm 2$
c) $t = \pm 6$ **d)** $y = \pm\sqrt{11}$

5. a) $x = 1, x = 5$ **b)** $x = -5, x = 1$
c) $d = -\frac{3}{2}, d = \frac{1}{2}$ **d)** $h = \frac{3 \pm \sqrt{7}}{4}$
e) $s = \frac{-12 \pm \sqrt{3}}{2}$ **f)** $x = -4 \pm 3\sqrt{2}$

6. a) $x = -5 \pm \sqrt{21}$ **b)** $x = 4 \pm \sqrt{3}$
c) $x = -1 \pm \sqrt{\frac{2}{3}}$ or $\frac{-3 \pm \sqrt{6}}{3}$
d) $x = 1 \pm \sqrt{\frac{5}{2}}$ or $\frac{2 \pm \sqrt{10}}{2}$
e) $x = -3 \pm \sqrt{13}$ **f)** $x = 4 \pm 2\sqrt{7}$

7. a) $x = 8.5, x = -0.5$ **b)** $x = -0.8, x = 2.1$
c) $x = 12.8, x = -0.8$ **d)** $x = -7.7, x = 7.1$
e) $x = -2.6, x = 1.1$ **f)** $x = -7.8, x = -0.2$

8. a)



b) $4x^2 + 28x - 40 = 0$
c) 12.4 ft by 6.4 ft

9. a) $-0.02d^2 + 0.4d + 1 = 0$
b) 22.2 m

10. 200.5 m

11. 6 in. by 9 in.

12. 53.7 m

13. a) $x^2 - 7 = 0$ **b)** $x^2 - 2x - 2 = 0$
c) $4x^2 - 20x + 14 = 0$ or $2x^2 - 10x + 7 = 0$

14. a) $x = -1 \pm \sqrt{k + 1}$ **b)** $x = \frac{1 \pm \sqrt{k^2 + 1}}{k}$
c) $x = \frac{k \pm \sqrt{k^2 + 4}}{2}$

15. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ No. Some will result in a negative in the radical, which means the solution(s) are not real.

16. a) $n = 43$ **b)** $n = 39$

17. a) $12^2 = 4^2 + x^2 - 2(4)(x) \cos (60°)$
b) 13.5 m

18. Example: In the first equation, you must take the square root to isolate or solve for x. This creates the $\pm$ situation. In the second equation, $\sqrt{9}$ is already present, which means the principle or positive square root only.

19. Example: Allison did all of her work on one side of the equation; Riley worked on both sides. Both end up at the same solution but by different paths.

20. Example:
- Completing the square requires operations with rational numbers, which could lead to arithmetic errors.
- Graphing the corresponding function using technology is very easy. Without technology, the manual graph could take a longer amount of time.
- Factoring should be the quickest of the methods.

All of the methods lead to the same answers.

21. a) Example: $y = 2(x - 1)^2 - 3$, $0 = 2x^2 - 4x - 1$
b) Example: $y = 2(x + 2)^2$, $0 = 2x^2 + 8x + 8$
c) Example: $y = 3(x - 2)^2 + 1$, $0 = 3x^2 - 12x + 13$

4.4 The Quadratic Formula, pages 254 to 257

1. a) two distinct real roots
b) two distinct real roots
c) two distinct real roots
d) one distinct real root
e) no real roots
f) one distinct real root

2. a) 2 **b)** 2 **c)** 1
d) 1 **e)** 0 **f)** 2

3. a) $x = -3, x = -\frac{3}{7}$ **b)** $p = \frac{3 \pm 3\sqrt{2}}{2}$
c) $q = \frac{-5 \pm \sqrt{37}}{6}$ **d)** $m = \frac{-2 \pm 3\sqrt{2}}{2}$
e) $j = \frac{7 \pm \sqrt{17}}{4}$ **f)** $g = -\frac{3}{4}$

4. a) $z = -4.28, z = -0.39$
b) $c = -0.13, c = 1.88$
c) $u = 0.13, u = 3.07$
d) $b = -1.41, b = -0.09$
e) $w = -0.15, w = 4.65$
f) $k = -0.27, k = 3.10$

5. a) $x = \frac{-3 \pm \sqrt{6}}{3}$, -0.18 and -1.82

b) $h = \frac{-1 \pm \sqrt{73}}{12}$, -0.80 and 0.63

c) $m = \frac{-0.3 \pm \sqrt{0.17}}{0.4}$, -1.78 and 0.28

d) $y = \frac{3 \pm \sqrt{2}}{2}$, 0.79 and 2.21

e) $x = \frac{1 \pm \sqrt{57}}{14}$, -0.47 and 0.61

f) $z = \frac{3 \pm \sqrt{7}}{2}$, 0.18 and 2.82

6. Example: Some are easily solved so they do not require the use of the quadratic formula. $x^2 - 9 = 0$

7. a) $n = -1 \pm \sqrt{3}$; complete the square
b) $y = 3$; factor
c) $u = \pm 2\sqrt{2}$; square root
d) $x = \frac{1 \pm \sqrt{19}}{3}$; quadratic formula
e) no real roots; graphing

8. 5 m by 20 m or 10 m by 10 m

9. 0.89 m

10. $1 \pm \sqrt{23}$, -3.80 and 5.80

11. 5 m

12. a) $(30 - 2x)(12 - 2x) = 208$
b) 2 in.
c) 8 in. by 26 in. by 2 in.

13. a) 68.8 km/h **b)** 95.2 km/h
c) 131.2 km/h

14. a) 4.2 ppm **b)** 3.4 years

15. \$155, 130 jackets

16. 169.4 m

17. $b = 13$, $x = \frac{3}{2}$

18. 2.2 cm

19. a) $\left(-3 + 3\sqrt{5}\right)$ m **b)** $\left(-45 + 27\sqrt{5}\right)$ m^2

20. 3.5 h

21. Error in Line 1: The $-b$ would make the first number $-(-7) = 7$.
Error in Line 2: $-4(-3)(2) = +24$ not -24.
The correct solution is $x = \frac{-7 \pm \sqrt{73}}{6}$.

22. a) $x = -1$ and $x = 4$
b) Example: The axis of symmetry is halfway between the roots. $\frac{-1 + 4}{2} = \frac{3}{2}$. Therefore, the equation of the axis of symmetry is $x = \frac{3}{2}$.

23. Example: If the quadratic is easily factored, then factoring is faster. If it is not easily factored, then using the quadratic formula will yield exact answers. Graphing with technology is a quick way of finding out if there are real solutions.

24. Answers may vary.

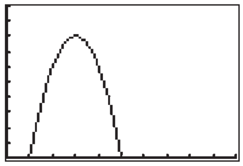
Chapter 4 Review, pages 258 to 260

1. a) $x = -6$, $x = -2$ **b)** $x = -1$, $x = 5$
c) $x = -2$, $x = -\frac{4}{3}$ **d)** $x = -3$, $x = 0$
e) $x = -5$, $x = 5$

2. D

3. Example: The graph cannot cross over or touch the x-axis.

4. a) Example:



b) 1000 key rings or 5000 key rings produce no profit or loss because the value of P is 0 then.

5. a) -1 and 6 **b)** 6 m

6. a) $(x - 1)(4x - 9)$ **b)** $\frac{1}{2}(x + 1)(x - 4)$
c) $(3v + 10)(v + 2)$
d) $(3a^2 - 12 + 35b)(3a^2 - 12 - 35b)$

7. a) $x = -7$, $x = -3$ **b)** $m = -10$, $m = 2$
c) $p = -3$, $p = \frac{2}{5}$ **d)** $z = \frac{1}{2}$, $z = 3$

8. a) $g = 3$, $g = -\frac{1}{2}$ **b)** $y = \frac{1}{2}$, $y = \frac{5}{4}$
c) $k = \frac{3}{5}$ **d)** $x = -\frac{3}{2}$, $x = 6$

9. a) Example: $0 = x^2 - 5x + 6$
b) Example: $0 = x^2 + 6x + 5$
c) Example: $0 = 2x^2 + 5x - 12$

10. 6 s

11. a) $V = 15(x)(x + 2)$ **b)** $2145 = 15x(x + 2)$
c) 11 m by 13 m

12. $x = -4$ and $x = 6$. Example: Factoring is fairly easy and exact.

13. a) $k = 4$ **b)** $k = \frac{9}{4}$

14. a) $x = \pm 7$ **b)** $x = 2$, $x = -8$
c) $x = 5 \pm 2\sqrt{6}$ **d)** $x = \frac{3 \pm \sqrt{5}}{3}$

15. a) $x = 4 \pm \sqrt{\frac{29}{2}}$ or $\frac{8 \pm \sqrt{58}}{2}$
b) $y = -2 \pm \sqrt{\frac{19}{5}}$ or $\frac{-10 \pm \sqrt{95}}{5}$
c) no real solutions

16. 68.5 s

17. a) $0 = -\frac{1}{2}d^2 + 2d + 1$ **b)** 4.4 m

18. a) two distinct real roots
b) one distinct real root
c) no real roots
d) two distinct real roots

19. a) $x = -\frac{5}{3}$, $x = 1$ **b)** $x = \frac{-7 \pm \sqrt{29}}{10}$
c) $x = \frac{2 \pm \sqrt{7}}{3}$ **d)** $x = -\frac{9}{5}$

20. a) $0 = -2x^2 + 6x + 1$ **b)** 3.2 m

21. a) $3.7 - 0.05x$ **b)** $2480 + 40x$

c) $R = -2x^2 + 24x + 9176$

d) 5 or 7

22.

Algebraic Steps	Explanations
$ax^2 + bx = -c$	Subtract c from both sides.
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	Divide both sides by a.
$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$	Complete the square.
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	Factor the perfect square trinomial.
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	Take the square root of both sides.
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Solve for x.

Chapter 4 Practice Test, pages 261 to 262

1. C

2. B

3. D

4. B

5. B

6. a) $x = 3, x = 1$ **b)** $x = -\frac{3}{2}, x = 5$

c) $x = -3, x = 1$

7. $x = \frac{-5 \pm \sqrt{37}}{6}$

8. $x = -2 \pm \sqrt{11}$

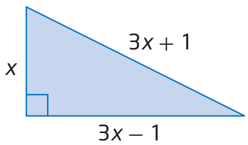
9. a) one distinct real root

b) two distinct real roots

c) no real roots

d) two distinct real roots

10. a)



b) $x^2 + (3x - 1)^2 = (3x + 1)^2$

c) 12 cm, 35 cm, and 37 cm

11. a) 3.8 s

b) 35 m

c) Example: Choose graphing with technology so you can see the path and know which points correspond to the situation.

12. 5 cm

13. 22 cm by 28 cm

14. a) $(9 + 2x)(6 + 2x) = 108$ or $4x^2 + 30x - 54 = 0$

b) $x = 1.5$
Example: Factoring is the most efficient strategy.

c) 42 m

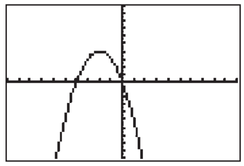
Cumulative Review, Chapters 3–4, pages 264 to 265

1. a) C **b)** A **c)** D **d)** B

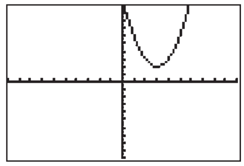
2. a) not quadratic **b)** quadratic

c) not quadratic **d)** quadratic

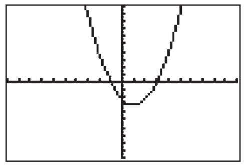
3. a) Example:



b) Example:



c) Example:



4. a) vertex: $(-4, -3)$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq -3, y \in R\}$, axis of symmetry: $x = -4$, x-intercepts occur at approximately $(-5.7, 0)$ and $(-2.3, 0)$, y-intercept occurs at $(0, 13)$

b) vertex: $(2, 1)$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq 1, y \in R\}$, axis of symmetry: $x = 2$, x-intercepts occur at $(1, 0)$ and $(3, 0)$, y-intercept occurs at $(0, -3)$

c) vertex: $(0, -6)$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \leq -6, y \in R\}$, axis of symmetry: $x = 0$, no x-intercepts, y-intercept occurs at $(0, -6)$

d) vertex: $(-8, 6)$, domain: $\{x \mid x \in R\}$, range: $\{y \mid y \geq 6, y \in R\}$, axis of symmetry: $x = -8$, no x-intercepts, y-intercept occurs at $(0, 38)$

5. a) $y = (x - 5)^2 - 7$; the shapes of the graphs are the same with the parabola of $y = (x - 5)^2 - 7$ being translated 5 units to the right and 7 units down.

b) $y = -(x - 2)^2 - 3$; the shapes of the graphs are the same with the parabola of $y = -(x - 2)^2 - 3$ being reflected in the x-axis and translated 2 units to the right and 3 units down.

c) $y = 3(x - 1)^2 + 2$; the shape of the graph of $y = 3(x - 1)^2 + 2$ is narrower by a multiplication of the y-values by a factor of 3 and translated 1 unit to the right and 2 units up.

d) $y = \frac{1}{4}(x + 8)^2 + 4$; the shape of the graph of $y = \frac{1}{4}(x + 8)^2 + 4$ is wider by a multiplication of the y-values by a factor of $\frac{1}{4}$ and translated 8 units to the left and 4 units up.

6. a) 22 m **b)** 2 m **c)** 4 s

7. In order: roots, zeros, x-intercepts

8. a) $(3x + 4)(3x - 2)$ **b)** $(4r - 9s)(4r + 9s)$
c) $(x + 3)(2x + 9)$ **d)** $(xy + 4)(xy - 9)$
e) $5(a + b)(13a + b)$ **f)** $(11r + 20)(11r - 20)$

9. 7, 8, 9 or −9, −8, −7

10. 15 seats per row, 19 rows

11. 3.5 m

12. Example: Dallas did not divide the 2 out of the −12 in the first line or multiply the 36 by 2 and thus add 72 to the right side instead of 36 in line two. Doug made a sign error on the −12 in the first line. He should have calculated 200 as the value in the radical, not 80. When he simplified, he took $\sqrt{80}$ divided by 4 to get $\sqrt{20}$, which is not correct.
The correct answer is $3 \pm \frac{5}{\sqrt{2}}$ or $\frac{6 \pm 5\sqrt{2}}{2}$.

13. a) Example: square root, $x = \pm\sqrt{2}$
b) Example: factor, $m = 2$ and $m = 13$
c) Example: factor, $s = -5$ and $s = 7$
d) Example: use quadratic formula, $x = -\frac{1}{16}$ and $x = 3$

14. a) two distinct real roots
b) one distinct real root
c) no real roots

15. a) $85 = x^2 + (x + 1)^2$
b) Example: factoring, $x = -7$ and $x = 6$
c) The top is 7-in. by 7-in. and the bottom is 6-in. by 6-in.
d) Example: Negative lengths are not possible.

Unit 2 Test, pages 266 to 267

1. A
2. D
3. D
4. B
5. B
6. 76
7. \$900
8. 0.18
9. a) 53.5 cm **b)** 75.7 cm **c)** No
10. a) 47.5 m **b)** 6.1 s
11. 12 cm by 12 cm
12. a) $3x^2 + 6x - 672 = 0$
b) $x = -16$ and $x = 14$
c) 14 in., 15 in., and 16 in.
d) Negative lengths are not possible.