Chapter 5 Radical Expressions and Equations

5.1 Working With Radicals, pages 278 to 281

1.

Mixed Radical Form	Entire Radical Form
$4\sqrt{7}$	$\sqrt{112}$
$5\sqrt{2}$	$\sqrt{50}$
$-11\sqrt{8}$	$-\sqrt{968}$
$-10\sqrt{2}$	$-\sqrt{200}$

2. a) $2\sqrt{14}$ **b)** $15\sqrt{3}$
c) $2\sqrt[3]{3}$ **d)** $cd\sqrt{c}$

3. a) $6m^2\sqrt{2}$, $m \in R$ **b)** $2q\sqrt[3]{3q^2}$, $q \in R$
c) $-4st\sqrt[5]{5t}$, s, $t \in R$

4.

Mixed Radical Form	Entire Radical Form
$3n\sqrt{5}$	$\sqrt{45n^2}$, $n \geq 0$ or $-\sqrt{45n^2}$, $n < 0$
$-6\sqrt[3]{2}$	$\sqrt[3]{-432}$
$\frac{1}{2a}\sqrt[3]{7a}$	$\sqrt[3]{\frac{7}{8a^2}}$, $a \neq 0$
$4x\sqrt[3]{2x}$	$\sqrt[3]{128x^4}$

5. a) $15\sqrt{5}$ and $40\sqrt{5}$ **b)** $32z^4\sqrt{7}$ and $48z^2\sqrt{7}$
c) $-35\sqrt[4]{w^2}$ and $9w^2(\sqrt[4]{w^2})$
d) $6\sqrt[3]{2}$ and $18\sqrt[3]{2}$

6. a) $3\sqrt{6}$, $7\sqrt{2}$, 10
b) $-3\sqrt{2}$, -4, $-2\sqrt{\frac{7}{2}}$, $-2\sqrt{3}$
c) $\sqrt[3]{21}$, 2.8, $2\sqrt[3]{5}$, $3\sqrt[3]{2}$

7. Example: Technology could be used.

8. a) $4\sqrt{5}$ **b)** $10.4\sqrt{2} - 7$
c) $-4\sqrt[4]{11} + 14$ **d)** $-\frac{2}{3}\sqrt{6} + 2\sqrt{10}$

9. a) $12\sqrt{3}$ **b)** $6\sqrt{2} + 6\sqrt{7}$
c) $-28\sqrt{5} + 22.5$ **d)** $\frac{13}{4}\sqrt[3]{3} - 7\sqrt{11}$

10. a) $8a\sqrt{a}$, $a \geq 0$ **b)** $9\sqrt{2x} - \sqrt{x}$, $x \geq 0$
c) $2(r - 10)\sqrt[3]{5r}$, $r \in R$
d) $\frac{4w}{5} - 6\sqrt{2w}$, $w \geq 0$

11. $25.2\sqrt{3}$ m/s

12. $12\sqrt{2}$ cm

13. $12\sqrt[3]{3025}$ million kilometres

14. $2\sqrt{30}$ m/s $\approx$ 11 m/s

15. a) $2\sqrt{38}$ m **b)** $8\sqrt{19}$ m

16. $\sqrt{1575}$ mm², $15\sqrt{7}$ mm²

17. $7\sqrt{5}$ units

18. $14\sqrt{2}$ m

19. Brady is correct. The answer can be further simplified to $10y^2\sqrt{y}$.

20. $4\sqrt{58}$
Example: Simplify each radical to see which is not a like radical to $12\sqrt{6}$.

21. $\sqrt{2 - \sqrt{3}}$ m

22. $12\sqrt{2}$ cm

23. $5\sqrt{3}$ and $7\sqrt{3}$
It is an arithmetic sequence with a common difference of $2\sqrt{3}$.

24. a) $2\sqrt{75}$ and $108^{\frac{1}{2}}$ Example: Write the radicals in simplest form; then, add the two radicals with the greatest coefficients.

b) $2\sqrt{75}$ and $-3\sqrt{12}$ Example: Write the radicals in simplest form; then, subtract the radical with the least coefficient from the radical with the greatest coefficient.

25. a) Example: If $x = 3$,
$(-3)^2 = (-3)(-3)$
$(-3)^2 = 9$
$(-3)^2 = 3^2$

b) Example: If $x = 3$,
$\sqrt{3^2} = \sqrt{9}$
$\sqrt{9} = 3$
$\sqrt{3^2} \neq -3$

5.2 Multiplying and Dividing Radical Expressions, pages 289 to 293

1. a) $14\sqrt{15}$ **b)** -56 **c)** $4\sqrt[4]{15}$
d) $4x\sqrt{38x}$ **e)** $3y^3\left(\sqrt[3]{12y^2}\right)$ **f)** $\frac{3t^3}{2}\sqrt{6}$

2. a) $3\sqrt{11} - 4\sqrt{77}$
b) $-14\sqrt{10} - 6\sqrt{3} + \sqrt{26}$
c) $2y + \sqrt{y}$ **d)** $6z^2 - 5z^2\sqrt{3} + 2z\sqrt{3}$

3. a) $6\sqrt{2} + 12$ **b)** $1 - 9\sqrt{6}$
c) $\sqrt{15j} + 33\sqrt{5}$, $j \geq 0$ **d)** $3 - 16\sqrt[3]{4k}$

4. a) $8\sqrt{14} - 24\sqrt{7} + 2\sqrt{2} - 6$
b) -389
c) $-27 + 3\sqrt{5}$
d) $36\sqrt[3]{4} - 48\sqrt{13}(\sqrt[3]{2}) + 208$
e) $-4\sqrt{3} + 3\sqrt{30} - \sqrt{6} + 4\sqrt{2} - 6\sqrt{5} + 2$

5. a) $15c\sqrt{2} - 90\sqrt{c} + 2\sqrt{2c} - 12$, $c \geq 0$
b) $2 + 7\sqrt{5x} - 40x\sqrt{2x} - 140x^2\sqrt{10}$, $x \geq 0$
c) $258m - 144m\sqrt{3}$, $m \geq 0$
d) $20r\sqrt[3]{6r^2} + 30r\sqrt[3]{12r} - 16r\sqrt[3]{3} - 24\sqrt[3]{6r^2}$

6. a) $2\sqrt{2}$ **b)** -1
c) $3\sqrt{2}$ **d)** $\frac{9m\sqrt{35}}{7}$

7. a) $\frac{87\sqrt{11p}}{11}$ **b)** $\frac{6v^2\sqrt[3]{98}}{7}$

8. a) $2\sqrt{10}$ **b)** $\frac{-\sqrt{3m}}{m}$
c) $\frac{-\sqrt{15u}}{9u}$ **d)** $4\sqrt[3]{150t}$

9. a) $2\sqrt{3} - 1$; 11 **b)** $7 + \sqrt{11}$; 38
c) $8\sqrt{z} + 3\sqrt{7}$; $64z - 63$
d) $19\sqrt{h} - 4\sqrt{2h}$; $329h$

10. a) $10 + 5\sqrt{3}$ **b)** $\frac{-7\sqrt{3} + 28\sqrt{2}}{29}$
c) $\frac{\sqrt{35} + 2\sqrt{14}}{3}$ **d)** $\frac{-8 - \sqrt{39}}{5}$

11. a) $\frac{4r^2\sqrt{6} - 36r}{6r^2 - 81}$, $r \neq \frac{\pm 3\sqrt{6}}{2}$
b) $\frac{9\sqrt{2}}{2}$, $n > 0$
c) $\frac{16 + 4\sqrt{6t}}{8 - 3t}$, $t \neq \frac{8}{3}$, $t \geq 0$
d) $\frac{5\sqrt{30y} - 10\sqrt{3y}}{6}$, $y \geq 0$

12. $c^2 + 7c\sqrt{3c} + c^2\sqrt{c} + 7c^2\sqrt{3}$

13. a) When applying the distributive property, Malcolm distributed the 4 to both the whole number and the root. The 4 should only be distributed to the whole number. The correct answer is $12 + 8\sqrt{2}$.

b) Example: Verify using decimal approximations.
$\frac{4}{3 - 2\sqrt{2}} \approx 23.3137$
$12 + 8\sqrt{2} \approx 23.3137$

14. $\frac{\sqrt{5} + 1}{2}$

15. a) $T = \frac{\pi\sqrt{10L}}{5}$ **b)** $\frac{9\pi\sqrt{30}}{5}$ s

16. $860 + 172\sqrt{5}$ m

17. $-28 - 16\sqrt{3}$

18. a) $4\sqrt[3]{3}$ mm **b)** $2\sqrt[3]{6}$ mm **c)** $2\sqrt[3]{3} : \sqrt[3]{6}$

19. a) Lev forgot to switch the inequality sign when he divided by -5. The correct answer is $x < \frac{3}{5}$.

b) The square root of a negative number is not a real number.

c) Example: The expression cannot have a variable in the denominator or under the radical sign. $\frac{2x\sqrt{14}}{3\sqrt{5}}$

20. Olivia evaluated $\sqrt{25}$ as ± 5 in the third step. The final steps should be as follows:

$$\frac{\sqrt{3}(2c - 5c)}{3} = \frac{\sqrt{3}(-3c)}{3} = -c\sqrt{3}$$

21. 735 cm^3

22. 12 m^2

23. $\left(\frac{15\sqrt{3}}{2}, \frac{9\sqrt{2}}{2}\right)$

24. $\frac{25x^2 + 30x\sqrt{x} + 9x}{625x^2 - 450x + 81}$ or $\frac{x(25x + 30\sqrt{x} + 9)}{(25x - 9)^2}$

25. a) $-3 \pm \sqrt{6}$ **b)** -6 **c)** 3

d) Examples: The answer to part b) is the opposite value of the coefficient of the middle term. The answer to part c) is the value of the constant.

26. $\frac{(\sqrt[c]{a})(\sqrt[n-1]{r})}{r}$

27. $(15\sqrt{14} + 42\sqrt{7} + 245\sqrt{2} + 7\sqrt{2702})$ cm^2

28. Example: You cannot multiply or divide radical expressions with different indices, or algebraic expressions with different variables.

29. Examples: To rationalize the denominator you need to multiply the numerator and denominator by a conjugate. To factor a difference of squares, each factor is the conjugate of the other. If you factor $3a - 16$ as a difference of squares, the factors are $\sqrt{3a} - 4$ and $\sqrt{3a} + 4$. The factors form a conjugate pair.

30. **a)** 3 m

b) $h(t) = -5(t - 1)^2 + 8$; $t = \sqrt{\frac{8 - h}{5}} + 1$

c) $\frac{19 + 4\sqrt{10}}{4}$ m

Example: The snowboarder starts the jump at $t = 0$ and ends the jump at $t = \frac{5 + 2\sqrt{10}}{5}$. The snowboarder will be halfway at $t = \frac{5 + 2\sqrt{10}}{10}$. Substitute this value of t into the original equation to find the height at the halfway point.

31. Yes, they are. Example: using the quadratic formula

32. **a)** $\frac{\sqrt[3]{6V(V - 1)^2}}{V - 1}$

b) A volume greater than one will result in a real ratio.

33. **Step 1**

$y = \sqrt{x}$

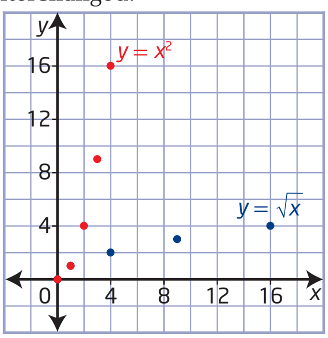
x	y
0	0
1	1
4	2
9	3
16	4

$y = x^2$

x	y
0	0
1	1
2	4
3	9
4	16

Step 2 Example: The values of x and y have been interchanged.

Step 3



Example: The restrictions on the radical function produce the right half of the parabola.

5.3 Radical Equations, pages 300 to 303

1. **a)** $3z$ **b)** $x - 4$
c) $4(x + 7)$ **d)** $16(9 - 2y)$

2. Example: Isolate the radical and square both sides. $x = 36$

3. **a)** $x = \frac{9}{2}$ **b)** $x = -2$ **c)** $x = -22$

4. **a)** $z = 25$ **b)** $y = 36$
c) $x = \frac{4}{3}$ **d)** $m = -\frac{49}{6}$

5. $k = -8$ is an extraneous root because if -8 is substituted for k, the result is a square root that equals a negative number, which cannot be true in the real-number system.

6. **a)** $n = 50$ **b)** no solution **c)** $x = -1$

7. **a)** $m = \pm 2\sqrt{7}$ **b)** $x = -16$, $x = 4$
c) $q = 2 + 2\sqrt{6}$ **d)** $n = 4$

8. **a)** $x = 10$ **b)** $x = -32$, $x = 2$
c) $d = 4$ **d)** $j = -\frac{2}{3}$

9. **a)** $k = 4$ **b)** $m = 0$
c) $j = 16$ **d)** $n = \frac{50 + 25\sqrt{3}}{2}$

10. **a)** $z = 6$ **b)** $y = 8$ **c)** $r = 5$ **d)** $x = 6$

11. The equation $\sqrt{x + 8} + 9 = 2$ has an extraneous root because simplifying it further to $\sqrt{x + 8} = -7$ has no solution.

12. Example: Jerry made a mistake when he squared both sides, because he squared each term on the right side rather than squaring $(x - 3)$. The right side should have been $(x - 3)^2 = x^2 - 6x + 9$, which gives $x = 8$ as the correct solution. Jerry should have listed the restriction following the first line: $x \geq -17$.

13. 11.1 m

14. **a)** $B \approx 6$ **b)** about 13.8 km/h

15. 1200 kg

16. $2 + \sqrt{n} = n$; $n = 4$

17. **a)** $v = \sqrt{19.6h}$, $h \geq 0$ **b)** 45.9 m
c) 34.3 m/s; A pump at 35 m/s will meet the requirements.

18. 6372.2 km

19. $a = \frac{3x - 4\sqrt{3x} + 4}{x}$

20. **a)** Example: $\sqrt{4a} = -8$
b) Example: $2 + \sqrt{x + 4} = x$

21. 2.9 m

22. 104 km

23. **a)** The maximum profit is $10 000 and it requires 100 employees.
b) $n = 100 \pm \sqrt{10\,000 - P}$
c) $P \leq 10\,000$
d) domain: $n \geq 0$, $n \in W$
range: $P \leq 10\,000$, $P \in W$

24. Example: Both types of equations may involve rearranging. Solving a radical involves squaring both sides; using the quadratic formula involves taking a square root.

25. Example: Extraneous roots may occur because squaring both sides and solving the quadratic equation may result in roots that do not satisfy the original equation.

26. **a)** 6.8% **b)** $P_f = P_i(r + 1)^3$
c) 320, 342, 365, 390
d) geometric sequence with $r = 1.068\ldots$

27. **Step 1**

1	$\sqrt{6+\sqrt{6}}$	2.906 800 603
2	$\sqrt{6+\sqrt{6+\sqrt{6}}}$	2.984 426 344
3	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}$	2.997 403 267
4	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}$	2.999 567 18
5	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}$	2.999 927 862
6	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}$	2.999 987 977
7	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}}$	2.999 997 996
8	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}}}$	2.999 999 666
9	$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}}}}}}$	2.999 999 944

Step 2 Example: 3.0

Step 3
$$x = \sqrt{6 + x},\ x \geq -6$$
$$x^2 = 6 + x$$
$$(x - 3)(x + 2) = 0$$
$$x = 3 \text{ or } x = -2$$

Step 4 The value of x must be positive because it is a square root.

Chapter 5 Review, pages 304 to 305

1. **a)** $\sqrt{320}$ **b)** $\sqrt[5]{-96}$
c) $\sqrt{63y^6}$ **d)** $\sqrt[3]{-108z^4}$
2. **a)** $6\sqrt{2}$ **b)** $6\sqrt{10}$
c) $3m\sqrt{3}$ **d)** $2xy^2(\sqrt[3]{10x^2})$
3. **a)** $\sqrt{13}$ **b)** $-4\sqrt{7}$ **c)** $\sqrt[3]{3}$
4. **a)** $-33x\sqrt{5x} + 14\sqrt{3x},\ x \geq 0$
b) $\frac{3}{10}\sqrt{11a} + 12a\sqrt{a},\ a \geq 0$
5. $3\sqrt{42}$ Example: Simplify each radical to see if it equals $8\sqrt{7}$.
6. $3\sqrt{7}$, 8, $\sqrt{65}$, $2\sqrt{17}$
7. **a)** $v = 13\sqrt{d}$ **b)** 48 km/h
8. $8\sqrt{6}$ km
9. **a)** false **b)** true **c)** false
10. **a)** $2\sqrt{3}$ **b)** $-30f^4\sqrt{3}$ **c)** $6\sqrt[4]{9}$
11. **a)** -1 **b)** $83 - 20\sqrt{6}$
c) $a^2 + 17a\sqrt{a} + 42a,\ a \geq 0$
12. Yes; they are conjugate pairs and the solutions to the quadratic equation.
13. **a)** $\frac{\sqrt{2}}{2}$ **b)** $\frac{-(\sqrt[3]{25})^2}{25}$ **c)** $\frac{-4a\sqrt{2}}{3}$
14. **a)** $\frac{-8 - 2\sqrt{3}}{13}$ **b)** $\frac{2\sqrt{35} + 7}{13}$
c) $\frac{12 - 6\sqrt{3m}}{4 - 3m},\ m \geq 0 \text{ and } m \neq \frac{4}{3}$
d) $\frac{a^2 + 2a\sqrt{b} + b}{a^2 - b},\ b \geq 0 \text{ and } b \neq a^2$
15. $4\sqrt{2} + 8\sqrt{5}$
16. **a)** $\frac{5\sqrt{6}}{18}$ **b)** $\frac{-2a^2\sqrt{2}}{3}$
17. $\frac{24 + 6\sqrt{2}}{7}$ units
18. **a)** radical defined for $x \geq 0$; solution: $x = 49$
b) radical defined for $x \leq 4$; no solution
c) radical defined for $x \geq 0$; solution: $x = 18$
d) radical defined for $x \geq 0$; solution: $x = 21$
19. **a)** restriction: $x \geq \frac{12}{7}$; solution: $x = \frac{9}{2}$
b) restriction: $y \geq 3$; solution: $y = 3$ and $y = 4$
c) restriction: $n \geq \frac{-25}{7}$; solution: $n = 8$
d) restriction: $0 \leq m \leq 24$; solution: $m = 12$
e) no restrictions; solution: $x = -21$
20. Example: Isolate the radical; then, square both sides. Expand and simplify. Solve the quadratic equation. $n = -3$ is an extraneous root because when it is substituted into the original equation a false statement is reached.
21. 33.6 m

Chapter 5 Practice Test, pages 306 to 307

1. B
2. D
3. C
4. D
5. B
6. C
7. $3\sqrt{11}$, $5\sqrt{6}$, $\sqrt{160}$, $9\sqrt{2}$
8. $\frac{-12n\sqrt{10} - 288n\sqrt{5}}{287}$
9. The radical is defined for $x \leq -\sqrt{5}$ and $x \geq \sqrt{5}$. The solution is $x = \frac{7}{3}$.
10. The solution is $\frac{102 + 6\sqrt{214}}{25}$. The extraneous root is $\frac{102 - 6\sqrt{214}}{25}$.
11. $15\sqrt{2}$
12. $9\sqrt{2}$ km
13. **a)** $\sqrt{6}$ **b)** $\sqrt{y - 3}$ **c)** $\sqrt[3]{49}$
14. \$6300
15. She is correct.
16. **a)** $\sqrt{1 + x^2}$ **b)** $2\sqrt{30}$ units
17. **a)** $R = \frac{P}{I^2}$ **b)** 400 Ω
18. **a)** $x = \sqrt{\frac{SA}{6}}$ **b)** $\frac{\sqrt{22}}{2}$ cm **c)** $\sqrt{2}$
19. **a)** $3713.15 = 3500(1 + i)^2$ **b)** 3%