Chapter 6 Rational Expressions and Equations

6.1 Rational Expressions, pages 317 to 321

1. a) 18 **b)** $14x$ **c)** 7
d) $4x - 12$ **e)** 8 **f)** $y + 2$

2. a) Divide both by pq.
b) Multiply both by $(x - 4)$.
c) Divide both by $(m - 3)$.
d) Multiply both by $(y^2 + y)$.

3. a) 0 **b)** 1 **c)** -5
d) none **e)** ± 1 **f)** none

4. The following values are non-permissible because they would make the denominator zero, and division by zero is not defined.
a) 4 **b)** 0 **c)** $-2, 4$
d) $-3, 1$ **e)** 0 **f)** $\frac{4}{3}, -\frac{5}{2}$

5. a) $r \neq 0$ **b)** $t \neq \pm 1$
c) $x \neq 2$ **d)** $g \neq 0, \pm 3$

6. a) $\frac{2}{3}$; $c \neq 0, 5$ **b)** $\frac{3(2w + 3)}{2(3w + 2)}$; $w \neq -\frac{2}{3}, 0$
c) $\frac{x + 7}{2x - 1}$; $x \neq \frac{1}{2}, 7$ **d)** $-\frac{1}{2}$; $a \neq -2, 3$

7. a) x^2 is not a factor.
b) Factor the denominator. Set each factor equal to zero and solve. $x \neq -3, 1$
c) Factor the numerator and denominator. Determine the non-permissible values. Divide like factors. $\frac{x + 1}{x + 3}$

8. a) $\frac{3r}{2p}$, $r \neq 0, p \neq 0$ **b)** $-\frac{3}{5}$, $x \neq 2$
c) $\frac{b - 4}{2(b - 6)}$, $b \neq \pm 6$ **d)** $\frac{k + 3}{2(k - 3)}$, $k \neq -\frac{5}{2}, 3$
e) -1, $x \neq 4$ **f)** $\frac{5(x + y)}{x - y}$, $x \neq y$

9. Sometimes true. The statement is not true when $x = 3$.

10. There may have been another factor that divided out. For example: $\frac{y(y + 3)}{(y - 6)(y + 3)}$

11. yes, provided the non-permissible value, $x \neq 5$, is discussed

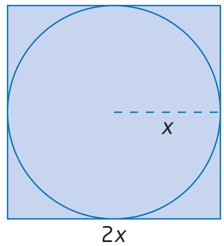
12. Examples: $\frac{x^2 + 2x + 1}{x^2 + 3x + 2}$, $\frac{x^2 + 4x + 4}{x^2 + 5x + 6}$, $\frac{2x^2 + 5x + 2}{3x^2 + 7x + 2}$
Write a rational expression in simplest form, and multiply both the numerator and the denominator by the same factor. For example, the first expression was obtained as follows: $\frac{x + 1}{x + 2} = \frac{(x + 1)(x + 1)}{(x + 2)(x + 1)}$.

13. Shali divided the term 2 in the numerator and the denominator. You may only divide by factors. The correct solution is the second step, $\frac{g + 2}{2}$.

14. Example: $\frac{2p}{p^2 + p - 2}$

15. a) $\frac{2n^2 + 11n + 12}{2n^2 - 32}$ **b)** $\frac{2n + 3}{2(n - 4)}$, $n \neq \pm 4$

16. a)



b) $\frac{\pi x^2}{4x^2}$
c) $x \neq 0$
d) $\frac{\pi}{4}$
e) 79%

17. a) The non-permissible value, -2, does not make sense in the context as the mass cannot be -2 kg.
b) $p = 0$ **c)** 900 kg

18. a) $\frac{50}{q}$, $q \neq 0$ **b)** $\frac{100}{p - 4}$, $p \neq 4$

19. a) \$620 **b)** $\frac{350 + 9n}{n}$
c) \$20.67

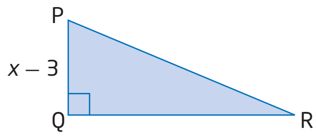
20. a) No; she divided by the term, 5, not a factor.
b) Example: If $m = 5$ then $\frac{5}{10} \neq \frac{1}{6}$.

21. a) Multiply by $\frac{5}{5}$. **b)** Multiply by $\frac{x - 2}{x - 2}$.

22. a) $\frac{4x - 8}{12}$ **b)** $\frac{3x - 6}{9}$
c) $\frac{2x^2 + x - 10}{6x + 15}$

23. a) $\frac{25b}{5b}$ **b)** $\frac{4a^2bx + 4a^2b}{12a^2b}$
c) $\frac{2b - 2a}{-14x}$

24. a)



b) $2(x + 2)$ **c)** $x \neq 3$

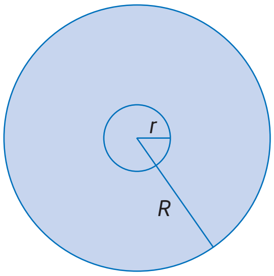
25. a) $\frac{(2x - 1)(3x + 1)}{(3x + 1)(3x - 1)} = \frac{(2x - 1)}{(3x - 1)}$, $x \neq \pm\frac{1}{3}$
b) In the last step: $\frac{n + 3}{-n} = \frac{-n - 3}{n}$, $n \neq 0, \frac{5}{2}$

26. a) $\frac{x + 6}{x + 3}$, $x \neq \pm 3$
b) $(2x - 7)(2x - 5)$, $x \neq -3$
c) $\frac{(x - 3)(x + 2)}{(x + 3)(x - 2)}$, $x \neq -3, -1, 2$
d) $\frac{(x + 5)(x + 3)}{3}$, $x \neq \pm 1$

27. $6x^2 + \frac{19}{2}x + 2$, $x \neq \frac{1}{4}, \frac{3}{2}$

28. a) Lt

b)



$\pi(R - r)(R + r)$

c) $L = \frac{\pi(R + r)(R - r)}{t}$, $t > 0$, $R > r$, and t, R, and r should be expressed in the same units.

29. Examples:

a) $\frac{2}{(x + 2)(x - 5)}$

b) $\frac{x^2 + 3x}{x^2 + 2x - 3}$; the given expression has a non-permissible value of -1. Multiply the numerator and denominator by a factor, $x + 3$, that has a non-permissible value of -3.

30. a) Example: if $y = 7$,

$\frac{y - 3}{4}$ and $\frac{2y^2 - 5y - 3}{8y + 4}$

$= \frac{7 - 3}{4}$ $\quad = \frac{2(7^2) - 5(7) - 3}{8(7) + 4}$

$= 1$ $\quad = \frac{60}{60}$

$= 1$

b) $\frac{2y^2 - 5y - 3}{8y + 4} = \frac{(2y + 1)(y - 3)}{4(2y + 1)} = \frac{y - 3}{4}$

c) The algebraic approach, in part b), proves that the expressions are equivalent for all values of y, except the non-permissible value.

31. a) $m = \frac{p - 8}{p + 1}$

b) Any value $-1 < p < 8$ will give a negative slope. Example: If $p = 0$, $m = \frac{-8}{1}$.

c) If $p = -1$, then the expression is undefined, and the line is vertical.

32. Example: $\frac{12}{15} = \frac{(3)(4)}{(3)(5)} = \frac{4}{5}$,

$$\frac{x^2 - 4}{x^2 + 5x + 6} = \frac{(x + 2)(x - 2)}{(x + 3)(x + 2)}$$
$$= \frac{(x - 2)}{(x + 3)}, x \neq -3, -2$$

6.2 Multiplying and Dividing Rational Expressions, pages 327 to 330

1. a) $9m$, $c \neq 0$, $f \neq 0$, $m \neq 0$

b) $\frac{a - 5}{5(a - 1)}$, $a \neq -5, 1$, $a \neq b$

c) $\frac{4(y - 7)}{(2y - 3)(y - 1)}$, $y \neq -3, 1, \pm\frac{3}{2}$

2. a) $\frac{d - 10}{4}$, $d \neq -10$ **b)** $\frac{a - 1}{a - 3}$, $a \neq \pm 3, -1$

c) $\frac{1}{2}$, $z \neq 4, \pm\frac{5}{2}$

d) $\frac{p + 1}{3}$, $p \neq -3, 1, \frac{3}{2}, \frac{1}{2}$

3. a) $\frac{t}{2}$ **b)** $\frac{3}{2x - 1}$

c) $\frac{y - 3}{8}$ **d)** $\frac{p - 3}{2p - 3}$

4. a) $s \neq 0$, $t \neq 0$ **b)** $r \neq \pm 7, 0$

c) $n \neq \pm 1$

5. $x - 3$, $x \neq -3$

6. $\frac{y}{y + 3}$, $y \neq \pm 3, 0$

7. a) $\frac{3 - p}{p - 3} = \frac{-1(p - 3)}{p - 3} = -1$, $p \neq 3$

b) $\frac{7k - 1}{3k} \times \frac{1}{1 - 7k}$

$= \frac{7k - 1}{3k} \times \frac{1}{-1(7k - 1)}$

$= \frac{-1}{3k}$ or $-\frac{1}{3k}$, $k \neq 0, \frac{1}{7}$

8. a) $\frac{w - 2}{3}$, $w \neq -2, -\frac{3}{2}$

b) $\frac{v^2}{v + 3}$, $v \neq 0, -3, 5,$

c) $\frac{-1(3x - 1)}{x + 5}$, $x \neq -5, 2, -\frac{1}{3}$

d) $\frac{-2}{y - 2}$, $y \neq \pm 1, 2, -\frac{1}{2}, \frac{3}{4}$

9. -3 and -2 are the non-permissible values of the original denominators, and -1 is the non-permissible value when the reciprocal of the divisor is created.

10. $\frac{n^2 - 4}{n + 1} \div (n - 2)$; $\frac{n + 2}{n + 1}$, $n \neq -1, 2$

11. a) $\frac{(x - 3)}{5}(60) = 12x - 36$ metres

b) $900 \div \frac{600}{n + 1} = \frac{3n + 3}{2}$ kilometres per hour, $n \neq -1$

c) $\frac{x^2 + 2x + 1}{(2x - 3)(x + 1)} = \frac{x + 1}{2x - 3}$ metres, $x \neq \frac{3}{2}, -1$

12. They are reciprocals of each other. This is always true. The divisor and dividend are interchanged.

13. Example:

$$1 \text{ yd}\left(\frac{3 \text{ ft}}{1 \text{ yd}}\right)\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right)\left(\frac{2.54 \text{ cm}}{1 \text{ in.}}\right) = 91.44 \text{ cm}$$

14. a) Tessa took the reciprocal of the dividend, not the divisor.

b) $= \frac{(c + 6)(c - 6)}{2c} \times \frac{8c^2}{c + 6}$

$= 4c(c - 6)$

$= 4c^2 - 24c$, $c \neq 0, -6$

c) The correct answer is the reciprocal of Tessa's answer. Taking reciprocals of either factor produces reciprocal answers.

15. $(x^2 - 9) \div \frac{x^2 - 2x - 3}{x + 1} = x + 3$; $x \neq 3, x \neq -1$

16. $\left(\frac{1}{2}\right)\left(\frac{x + 2}{x - 8}\right)\left(\frac{x^2 - 7x - 8}{x^2 - 4}\right)$; $\frac{x + 1}{2(x - 2)}$, $x \neq \pm 2, 8$

17. a) $K = \frac{Pw}{2h}$, $m \neq 0, w \neq 0, h \neq 0$

b) $y = \frac{2\pi r}{x}$, $d \neq 0, x \neq 0, r \neq 0$

c) $a = vw, w \neq 0$

18. $2(n - 4)$, n $\neq -4, 1, 4$

19. a) Yes; when the two binomial factors are multiplied, you get the expression $x^2 - 5$.

b) $\frac{x + \sqrt{7}}{x - \sqrt{3}}$

c) $x + \sqrt{7}$; it is the same.

20. a) approximately 290 m

b) $\frac{(x + 3)^2}{4g(x - 5)^2}$ metres

21. Agree. Example: $\left(\frac{2}{3}\right)\left(\frac{1}{5}\right) = \frac{(2)(1)}{(3)(5)} = \frac{2}{15}$,

and $\frac{2}{3} \div \frac{1}{5} = \left(\frac{2}{3}\right)\left(\frac{5}{1}\right) = \frac{10}{3}$

$$\frac{(x + 2)}{(x + 3)} \times \frac{(x + 1)}{(x + 3)} = \frac{(x + 2)(x + 1)}{(x + 3)(x + 3)} = \frac{x^2 + 3x + 2}{x^2 + 6x + 9}, x \neq -3$$

$$\frac{(x + 2)}{(x + 3)} \div \frac{(x + 1)}{(x + 3)} = \frac{(x + 2)}{(x + 3)} \times \frac{(x + 3)}{(x + 1)} = \frac{(x + 2)}{(x + 1)}, x \neq -3, -1$$

22. a) $\frac{p + 2}{4 - p}$ b) $\frac{p - 4}{p + 2}$

23. a) $\tan B = \frac{b}{a}$ b) $\frac{\frac{b}{c}}{\frac{a}{c}} = \frac{b}{a}$

c) They are the same; $\tan B = \frac{\sin B}{\cos B}$.

6.3 Adding and Subtracting Rational Expressions, pages 336 to 340

1. a) $\frac{7x}{6}$ b) $\frac{10}{x}$, $x \neq 0$

c) $\frac{4t + 4}{5}$ or $\frac{4(t + 1)}{5}$ d) m, $m \neq -1$

e) $a + 3$, $a \neq 4$

2. $$\frac{3x - 7}{9} + \frac{6x + 7}{9} = \frac{3x - 7 + 6x + 7}{9} = \frac{9x}{9} = x$$

3. a) $\frac{-4x + 13}{(x - 3)(x + 1)}$, $x \neq -1, 3$

b) $\frac{3x(x + 6)}{(x - 2)(x + 10)(x + 2)}$, $x \neq -10, \pm 2$

4. a) 24, 12; LCD = 12

b) $50a^3y^3$, $10a^2y^2$; LCD = $10a^2y^2$

c) $(9 - x^2)(3 + x)$, $9 - x^2$; LCD = $9 - x^2$ or $(3 - x)(3 + x)$

5. a) $\frac{11}{15a}$, $a \neq 0$ b) $\frac{x + 9}{6x}$, $x \neq 0$

c) $\frac{2(10x - 3)}{5x}$, $x \neq 0$

d) $\frac{(2z - 3x)(2z + 3x)}{xyz}$, $x \neq 0, y \neq 0, z \neq 0$

e) $\frac{4st + t^2 - 4}{10t^3}$, $t \neq 0$

f) $\frac{6bxy^2 - 2ax + a^2b^2y}{a^2b^2y}$, $a \neq 0, b \neq 0, y \neq 0$

6. a) $\frac{-5x + 18}{(x + 2)(x - 2)}$, $x \neq \pm 2$

b) $\frac{3x - 11}{(x - 4)(x + 3)}$, $x \neq -3, 4$

c) $\frac{2x(x - 4)}{(x - 2)(x + 2)}$, $x \neq \pm 2$

d) $\frac{3}{y}$, $y \neq -1, 0$

e) $\frac{-3(5h + 9)}{(h + 3)(h + 3)(h - 3)}$, $h \neq \pm 3$

f) $\frac{(2x - 3)(x + 2)}{x(x - 2)(x - 1)(x + 3)}$, $x \neq -3, 0, 1, 2$

7. a) $\frac{2(x^2 - 3x + 5)}{(x - 5)(x + 5)}$, $x \neq \pm 5, \frac{1}{2}$

b) $\frac{-x + 4}{(x - 2)(x + 3)}$, $x \neq -3, 0, 2, 8$

c) $\frac{n + 8}{(n - 4)(n - 2)}$, $n \neq 2, 3, 4$

d) $\frac{w + 9}{(w + 3)(w + 4)}$, $w \neq -2, -3, -4$

8. In the third line, multiplying by -7 should give $-7x + 14$. Also, she has forgotten to list the non-permissible values.

$$= \frac{6x + 12 + 4 - 7x + 14}{(x - 2)(x + 2)} = \frac{-x + 30}{(x - 2)(x + 2)}, x \neq \pm 2$$

9. Yes. Factor -1 from the numerator to create $-1(x - 5)$. Then, the expression simplifies to $\frac{-1}{x + 5}$.

10. a) $\frac{2x}{x + 3}$, $x \neq 0, \pm 3$

b) $\frac{3(t + 6)}{2(t - 3)}$, $t \neq -6, -2, 0, 3$

c) $\frac{3m}{m + 3}$, $m \neq 0, -\frac{3}{2}, -3$

d) $\frac{x}{x - 2}$, $x \neq \pm 4, 2$

11. a) $\dfrac{\frac{AD}{B}+C}{D} = \left(\dfrac{AD+CB}{B}\right) \div D$
$= \left(\dfrac{AD+CB}{B}\right)\left(\dfrac{1}{D}\right)$
$= \dfrac{AD+CB}{BD}$
$= \dfrac{AD}{BD} + \dfrac{CB}{BD}$
$= \dfrac{A}{B} + \dfrac{C}{D}$

b) $\left[\dfrac{\left(\frac{AB}{D}+C\right)D}{F} + E\right]F = \left(\dfrac{AB}{D}+C\right)D + EF$
$= AB + CD + EF$

12. $\dfrac{\sqrt{5x^2-2x+1}}{4}$

13. a) $\dfrac{200}{m}$ tells the expected number of weeks to gain 200 kg; $\dfrac{200}{m+4}$ tells the number of weeks to gain 200 kg when the calf is on the healthy growth program.

b) $\dfrac{200}{m} - \dfrac{200}{m+4}$

c) $\dfrac{800}{m(m+4)}$, $m \neq 0, -4$; yes, the expressions are equivalent.

14. a) $\dfrac{200}{n}$ minutes

b) $\left(\dfrac{200}{n} + \dfrac{500}{n} + \dfrac{1000}{n}\right)$ minutes

c) $\dfrac{1700}{n}$ minutes; the time it would take to type all three assignments

d) $\left(\dfrac{200}{n} + \dfrac{500}{n-5} + \dfrac{1000}{n-10}\right) - \dfrac{1700}{n}$
$= \dfrac{12\,500n - 75\,000}{n(n-5)(n-10)}$

15. a) $\dfrac{2x^2+13}{(x-4)(x+5)}$, $x \neq -5, -2, 0, 3, 4$

b) $\dfrac{-9}{(x-1)(x+2)}$, $x \neq -3, -2, 0, 1, \frac{1}{2}$

c) $\dfrac{3(1-4x)}{(x+5)(x-4)}$, $x \neq -5, -2, 0, 3, 4$

d) $\dfrac{15}{(x+6)(x+3)}$, $x \neq 0, -2, -3, -6, -\frac{1}{2}$

16. $\left(\dfrac{20}{x} + \dfrac{16}{x-2}\right)$ hours

17. Example: In a three-person relay, Barry ran the first 12 km at a constant rate. Jim ran the second leg of 8 km at a rate 3 km/h faster, and Al ran the last leg of 5 km at a rate 2 km/h slower than Barry. The total time for the relay would be $\left(\dfrac{12}{x} + \dfrac{8}{x+3} + \dfrac{5}{x-2}\right)$ hours.

18. a) Incorrect: $\dfrac{a}{b} - \dfrac{b}{a} = \dfrac{a^2-b^2}{ab}$. Find the LCD first, do not just combine pieces.

b) Incorrect: $\dfrac{ca+cb}{c+cd} = \dfrac{a+b}{1+d}$. Factor c from the numerator and from the denominator, remembering that $c(1) = c$.

c) Incorrect: $\dfrac{a}{4} - \dfrac{6-b}{4} = \dfrac{a-6+b}{4}$. Distribute the subtraction to both terms in the numerator of the second rational expression by first putting the numerator in brackets.

d) Incorrect: $\dfrac{1}{1-\frac{a}{b}} = \dfrac{b}{b-a}$. Simplify the denominator first, and then divide.

e) Incorrect: $\dfrac{1}{a-b} = \dfrac{-1}{b-a}$. Multiplying both numerator and denominator by -1, which is the same as multiplying the whole expression by 1, changes every term to its opposite.

19. a) Agree. Each term in the numerator is divided by the denominator, and then can be simplified.

b) Disagree. If Keander was given the rational expression $\dfrac{3x-7}{x}$, there are multiple original expressions that he could come up with, for example $\dfrac{2x-1}{x} + \dfrac{x-6}{x}$ or $\dfrac{x^2-x+11}{x} - \dfrac{x^2-4x+18}{x}$.

20. a) $\dfrac{12}{13}\ \Omega$

b) $\dfrac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}$

c) $\dfrac{12}{13}\ \Omega$

d) the simplified form from part b), because with it you do not need to find the LCD first

21. Example:

Arithmetic:	Algebra:
If $\frac{2}{3} = \frac{6}{9}$, then $\frac{2}{3} = \frac{2-6}{3-9}$ $= \frac{-4}{-6}$ $= \frac{2}{3}$	If $\frac{x}{2} = \frac{3x}{6}$, then $\frac{x}{2} = \frac{x-3x}{2-6}$ $= \frac{-2x}{-4}$ $= \frac{x}{2}$

22. a) $\dfrac{-2p+9}{2(p-3)}$, $p \neq 3$

b) $\dfrac{3}{0}$; the slope is undefined when $p = 3$, so this is a vertical line through A and B.

c) The slope is negative.

d) When $p = 4$, the slope is positive; from $p = 5$ to $p = 10$ the slope is always negative.

23. 3

24. Examples: $\frac{2}{5} + \frac{1}{5} = \frac{2+1}{5} = \frac{3}{5}$ and

$\frac{2}{5} + \frac{1}{3} = \frac{2(3) + 1(5)}{15} = \frac{11}{15}$

$\frac{2}{x} + \frac{1}{x} = \frac{2+1}{x} = \frac{3}{x}$ and

$\frac{2}{x} + \frac{1}{y} = \frac{2(y) + 1(x)}{xy} = \frac{2y + x}{xy}$

25. a) The student's suggestion is correct.

Example: find the average of $\frac{1}{2}$ and $\frac{3}{4}$.

$$\left(\frac{1}{2} + \frac{3}{4}\right) \div 2 = \left(\frac{2+3}{4}\right) \times \left(\frac{1}{2}\right)$$
$$= \frac{5}{8}$$

Halfway between $\frac{1}{2}$ and $\frac{3}{4}$, or $\frac{4}{8}$ and $\frac{6}{8}$, is $\frac{5}{8}$.

b) $\frac{13}{4a}$, $a \neq 0$

26. Yes. Example: $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ and $\frac{1}{2} + \frac{1}{3} = \frac{1}{\frac{6}{5}} = \frac{5}{6}$

$\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy}$ and $\frac{1}{x} + \frac{1}{y} = \frac{1}{\frac{xy}{x+y}} = \frac{x+y}{xy}$

27. a) $\frac{1}{u} + \frac{1}{v} = \frac{u+v}{uv}$ b) 5.93 cm

c) $f = \frac{uv}{u+v}$

28. **Step 3** Yes

Step 4 a) $A = 2, B = 1$

b) $A = 3, B = 3$

Step 5 Always:

$$\frac{3}{x-4} + \frac{-2}{x-1} = \frac{3(x-1) + -2(x-4)}{(x-4)(x-1)}$$
$$= \frac{x+5}{(x-4)(x-1)}$$

6.4 Rational Equations, pages 348 to 351

1. a) $4(x-1) - 3(2x-5) = 5 + 2x$

b) $2(2x+3) + 1(x+5) = 7$

c) $4x - 5(x-3) = 2(x+3)(x-3)$

2. a) $f = -1$ b) $y = 6, y \neq 0$

c) $w = 12, w \neq 3, 6$

3. a) $t = 2$ or $t = 6, t \neq 0$ b) $c = 2, c \neq \pm 3$

c) $d = -2$ or $d = 3, d \neq -4, 1$

d) $x = 3, x \neq \pm 1$

4. No. The solution is not a permissible value.

5. a) $\frac{3-x}{x^2} - \frac{2}{x}, \frac{3-3x}{x^2}, x > 0$

b) $\frac{3-x}{x^2} \times \frac{2}{x}, \frac{6-2x}{x^3}, x > 0$

c) $x = \frac{1}{2}$

6. a) $b = 3.44$ or $b = 16.56$

b) $c = -3.54$ or $c = 2.54$

7. $l = 15(\sqrt{5} + 1)$, 48.5 cm

8. The numbers are 5 and 20.

9. The numbers are 3 and 4.

10. 30 students

11. The integers are 5 and 6.

12. a) Less than 2 min. There is more water going in at once.

b)

	Time to Fill Tub (min)	Fraction Filled in 1 min	Fraction Filled in x minutes
Cold Tap	2	$\frac{1}{2}$	$\frac{x}{2}$
Hot Tap	3	$\frac{1}{3}$	$\frac{x}{3}$
Both Taps	x	$\frac{1}{x}$	1

c) $\frac{x}{2} + \frac{x}{3} = 1$ d) 1.2 min

13. 6 h

14. a)

	Distance (km)	Rate (km/h)	Time (h)
Downstream	18	$x + 3$	$\frac{18}{x+3}$
Upstream	8	$x - 3$	$\frac{8}{x-3}$

b) $\frac{18}{x+3} = \frac{8}{x-3}$ c) 7.8 km/h

d) $x \neq \pm 3$

15. 28.8 h

16. 5.7 km/h

17. about 50 km/h west of Swift Current, and 60 km/h east of Swift Current

18. about 3.5 km/h

19.

	Reading Rate in Pages per Day	Number of Pages Read	Number of Days
First Half	x	259	$\frac{259}{x}$
Second Half	$x + 12$	259	$\frac{259}{x+12}$

about 20 pages per day for the first half of the book

20. a) 2 L   b) 4.5 L

21. $a = \pm\frac{1}{3}$

22. a) $\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{x}, x = \frac{2ab}{a+b}$

b) 4 and 12, or -6 and 2

23. a) $\frac{1}{x} - \frac{1}{y} = a$ or $\frac{1}{x} - \frac{1}{y} = a$

First method:

$$y - x = axy$$
$$y = axy + x$$
$$y = x(ay + 1)$$
$$\frac{y}{ay+1} = x$$

Second method:

$$\frac{y-x}{xy} = a$$
$$y - x = axy$$
$$y = axy + x$$
$$y = x(ay + 1)$$
$$\frac{y}{ay+1} = x$$

In both, $x \neq 0, y \neq 0, ay \neq -1$.

b) $\frac{2d - gt^2}{2t} = v_0,\ t \neq 0$

c) $n = \frac{Ir}{E - IR},\ n \neq 0,\ R \neq -\frac{r}{n},\ E \neq Ir,\ I \neq 0$

24. a) Rational expressions combine operations and variables in one or more terms. Rational equations involve rational expressions and an equal sign.

Example: $\frac{1}{x} + \frac{1}{y}$ is a rational expression, which can be simplified but not solved.

$\frac{1}{x} + \frac{1}{2x} = 5$ is a rational equation that can be solved.

b) Multiply each term by the LCD. Then, divide common factors.

$$\frac{5}{x} - \frac{1}{x-1} = \frac{1}{x-1}$$

$$\cancel{x}(x-1)\left(\frac{5}{\cancel{x}}\right) - x\cancel{(x-1)}\left(\frac{1}{\cancel{x-1}}\right) = x\cancel{(x-1)}\left(\frac{1}{\cancel{x-1}}\right)$$

Simplify the remaining factors by multiplying. Solve the resulting linear equation.

$$(x-1)(5) - x(1) = x(1)$$
$$5x - 5 - x = x$$
$$3x = 5$$
$$x = \frac{5}{3}$$

c) Example: Add the second term on the left to both sides, to give $\frac{5}{x} = \frac{2}{x-1}$.

25. a) 5.5 pages per minute

b) and **c)** Answers may vary.

26. a) 46

b) $\frac{45}{50}$ is 90%, so $\frac{10(40) + 5(x)}{15} = 45$. For this equation to be true, you would need 55 on each of the remaining quizzes, which is not possible.

27. a) The third line should be

$$2x + 2 - 3x^2 + 3 = 5x^2 - 5x$$
$$0 = 8x^2 - 7x - 5$$

b) $\frac{7 \pm \sqrt{209}}{16}$

c) $x = 1.34$ or $x = -0.47$

Chapter 6 Review, pages 352 to 354

1. a) 0. It creates an expression that is undefined.

b) Example: Some rational expressions have non-permissible values.

For $\frac{2}{x-3}$, x may not take on the value 3.

2. Agree. Example: There are an unlimited number of ways of creating equivalent expressions by multiplying the numerator and denominator by the same term; because you are actually multiplying by 1 $\left(\frac{X}{X} = 1\right)$.

3. a) $y \neq 0$ **b)** $x \neq -1$ **c)** none

d) $a \neq -2, 3$ **e)** $m \neq -1, \frac{3}{2}$ **f)** $t \neq \pm 2$

4. a) $-6;\ s \neq 0$ **b)** $-1;\ x \neq \frac{3}{5}$ **c)** $-\frac{1}{4};\ b \neq 2$

5. a) $\frac{2x^2 - 6x}{10x}$ **b)** $\frac{1}{x+3}$

c) $\frac{3c - 6d}{9f}$ **d)** $\frac{m^2 - 3m - 4}{m^2 - 16}$

6. a) Factor the denominator(s), set each factor equal to zero, and solve.

Example: Since $\frac{m-4}{m^2-9} = \frac{m-4}{(m+3)(m-3)}$, the non-permissible values are ±3.

b) **i)** $x - 5,\ x \neq -\frac{2}{3}$ **ii)** $\frac{a}{a+3},\ a \neq \pm 3$

iii) $-\frac{3}{4},\ x \neq y$ **iv)** $\frac{9x-2}{2},\ x \neq \frac{2}{9}$

7. a) $x + 1$

b) $x \neq 1$, as this would make a width of 0, and $x \neq -1$, as this would make a length of 0.

8. Example: The same processes are used for rational expressions as for fractions. Multiplying involves finding the product of the numerators and then the product of the denominators. To divide, you multiply by the reciprocal of the divisor. The differences are that rational expressions involve variables and may have non-permissible values.

$\left(\frac{1}{2}\right)\left(\frac{3}{5}\right) = \frac{(1)(3)}{(2)(5)}$ $= \frac{3}{10}$	$\frac{x+2}{2} \times \frac{x+3}{5} = \frac{(x+2)(x+3)}{(2)(5)}$ $= \frac{x^2 + 5x + 6}{10}$
$\frac{3}{4} \div \frac{1}{2} = \left(\frac{3}{4}\right)\left(\frac{2}{1}\right)$ $= \frac{3}{2}$	$\frac{x+2}{4} \div \frac{x+1}{2} = \frac{x+2}{4} \times \frac{2}{x+1}$ $= \frac{x+2}{2(x+1)},\ x \neq -1$

9. a) $\frac{5q}{2r},\ r \neq 0,\ p \neq 0$ **b)** $\frac{m^2}{4t^3},\ m \neq 0,\ t \neq 0$

c) $\frac{3}{2},\ a \neq -b$

d) $\frac{2(x-2)(x+5)}{(x^2+25)},\ x \neq -2, 0$

e) $1,\ d \neq -3, -2, -1$

f) $\frac{-(y-8)(y+5)}{(y-1)},\ y \neq \pm 1, 5, 9$

10. a) $8t$ **b)** $\frac{1}{b},\ a \neq 0,\ b \neq 0$

c) $\frac{-1}{5(x+y)},\ x \neq \pm y$ **d)** $\frac{3}{a+3},\ a \neq \pm 3$

e) $\frac{1}{x+1},\ x \neq -2, -1, 0, \pm\frac{2}{3}$

f) $\frac{-(x+2)}{3},\ x \neq 2$

11. a) $\frac{m}{2},\ m \neq 0$ **b)** $\frac{x-1}{x},\ x \neq -3, -2, 0, 2$

c) $\frac{1}{6},\ a \neq \pm 3, 4$ **d)** $\frac{1}{5},\ x \neq 3, -\frac{4}{3}, -4$

12. x centimetres

13. a) $10x$

b) $(x-2)(x+1)$

Example: The advantage is that less simplifying needs to be done.

14. a) $\frac{m+3}{5}$ **b)** $\frac{m}{x}, x \neq 0$

c) $1, x \neq -y$ **d)** -1

e) $\frac{1}{x-y}, x \neq \pm y$

15. a) $\frac{5x}{12}$ **b)** $1, y \neq 0$

c) $\frac{9x+34}{(x+3)(x-3)}, x \neq \pm 3$

d) $\frac{a}{(a+3)(a-2)}, a \neq -3, 2$

e) $1, a \neq \pm b$

f) $\frac{2x^2-6x-3}{(x+1)(2x-3)(2x+3)}, x \neq -1, \pm\frac{3}{2}$

16. a) $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$

b) Left Side $= \frac{1}{a} + \frac{1}{b}$

$= \frac{b}{ab} + \frac{a}{ab}$

$= \frac{a+b}{ab}$

$=$ Right Side

17. Exam mark, $d = \frac{a+b+c}{3}$;

Final mark $= \left(\frac{1}{2}\right)\left(\frac{a+b+c}{3}\right) + \left(\frac{1}{2}\right)d$

$= \frac{a+b+c+3d}{6}$

Example: $\frac{60+70+80}{3} = d$

$\frac{60+70+80+3(70)}{6} = 70$

18. a) **i)** the amount that Beth spends per chair; \$10 more per chair than planned

ii) the amount that Helen spends per chair; \$10 less per chair than planned

iii) the number of chairs Helen bought

iv) the number of chairs Beth bought

v) the total number of chairs purchased by the two sisters

b) $\frac{450c-500}{c^2-100}$ or $\frac{50(9c-10)}{(c-10)(c+10)}, c \neq \pm 10$

19. Example: When solving a rational equation, you multiply all terms by the LCD to eliminate the denominators. In addition and subtraction of rational expressions, you use an LCD to simplify by grouping terms over one denominator.

Add or subtract.	Solve.
$\frac{x}{3} + \frac{x}{2}$ $= \frac{2x}{6} + \frac{3x}{6}$ $= \frac{5x}{6}$	$\frac{x}{3} + \frac{x}{2} = 5$ $2x + 3x = 30$ $5x = 30$ $x = 6$

20. a) $s = -9, s \neq -3$

b) $x = -4$ or $x = -1, x \neq 1, -\frac{2}{3}$

c) $z = 1, z \neq 0$

d) $m = 1$ or $m = -\frac{21}{2}, m \neq \pm 3$

e) no solution, $x \neq 3$

f) $x = \frac{\pm\sqrt{6}}{2}, x \neq 0, -\frac{1}{2}$

g) $x = -5$ or $x = 1, x \neq -2, 3$

21. The numbers are 4 and 8.

22. Elaine would take 7.5 h.

23. a) $\frac{160}{x} + 36 + \frac{160}{x+0.7} = 150$

b) $570x^2 - 1201x - 560 = 0, x = 2.5$ m/s.

c) The rate of ascent is 9 km/h.

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1. D

2. B

3. A

4. A

5. D

6. $x \neq -3, -1, 3, \frac{5}{3}$

7. $k = -1$

8. $\frac{5y-2}{6}, y \neq 2$

9. Let x represent the time for the smaller auger to fill the bin.

$\frac{6}{x} + \frac{6}{x-5} = 1$

10. Example: For both you use an LCD. When solving, you multiply by the LCD to eliminate the denominators, while in addition and subtraction of rational expressions, you use the LCD to group terms over a single denominator.

Add or subtract.	Solve.
$\frac{x}{4} - \frac{x}{7}$ $= \frac{7x}{28} - \frac{4x}{28}$ $= \frac{3x}{28}$	$\frac{x}{5} + \frac{x}{3} = 16$ $15\left(\frac{x}{5}\right) + 15\left(\frac{x}{3}\right) = 15(16)$ $3x + 5x = 240$ $8x = 240$ $x = 30$

11. $x = 4; x \neq -2, 3$

12. $\frac{5x+3}{5x} - \frac{2x-1}{2x} = \frac{2x-1}{2x} - \frac{3-x}{x}; x = 2.3$

13. The speed in calm air is 372 km/h.