Chapter 8 Systems of Equations

8.1 Solving Systems of Equations Graphically, pages 435 to 439

1. a) System A models the situation: to go off a ramp at different heights means two positive vertical intercepts, and in this system the launch angles are different, causing the bike with the lower trajectory to land sooner. System B is not correct because it shows both jumps starting from the same height. System C has one rider start from zero, which would mean no ramp. In System D, a steeper trajectory would mean being in the air longer but the rider is going at the same speed.

b) The rider was at the same height and at the same time after leaving the jump regardless of which ramp was chosen.

2. For $(0, -5)$: In $y = -x^2 + 4x - 5$:

Left Side	Right Side
$y = -5$	$-x^2 + 4x - 5$
	$= -(0)^2 + 4(0) - 5$
	$= -5$

Left Side = Right Side

In $y = x - 5$:

Left Side	Right Side
$y = -5$	$x - 5$
	$= 0 - 5$
	$= -5$

Left Side = Right Side

For $(3, -2)$: In $y = -x^2 + 4x - 5$:

Left Side	Right Side
$y = -2$	$-x^2 + 4x - 5$
	$= -(3)^2 + 4(3) - 5$
	$= -2$

Left Side = Right Side

In $y = x - 5$:

Left Side	Right Side
$y = -2$	$x - 5$
	$= 3 - 5$
	$= -2$

Left Side = Right Side

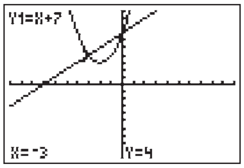
So, both solutions are verified.

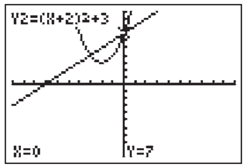
3. a) linear-quadratic; $(-4, 1)$ and $(-1, -2)$

b) quadratic-quadratic; no solution

c) linear-quadratic; $(1, -4)$

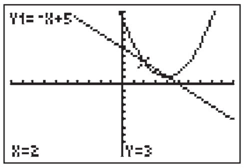
4. a)

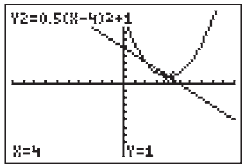




$(-3, 4)$ and $(0, 7)$

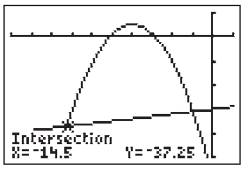
b)

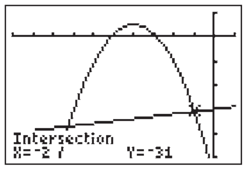




$(2, 3)$ and $(4, 1)$

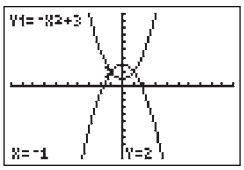
c)

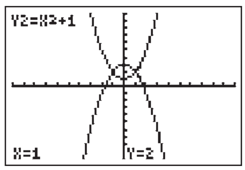




$(-14.5, -37.25)$ and $(-2, -31)$

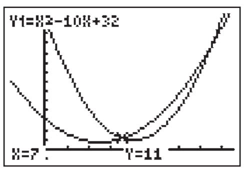
d)

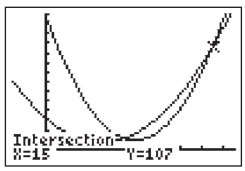




$(-1, 2)$ and $(1, 2)$

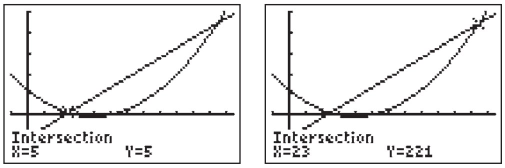
e)





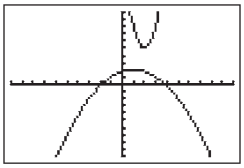
$(7, 11)$ and $(15, 107)$

5. a)



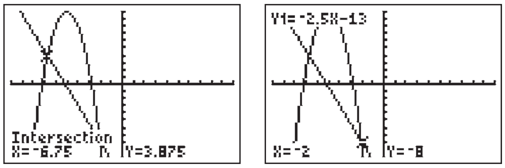
(5, 5) and (23, 221) or $d = 5$, $h = 5$ and $d = 23$, $h = 221$

b)



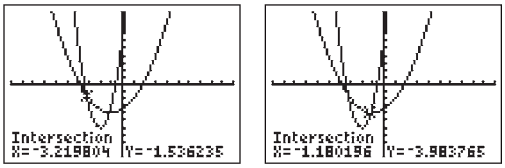
no solution

c)



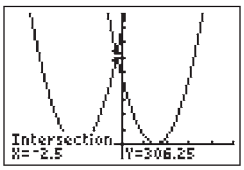
(−6.75, 3.875) and (−2, −8) or $v = -6.75$, $t = 3.875$ and $v = -2$, $t = -8$

d)



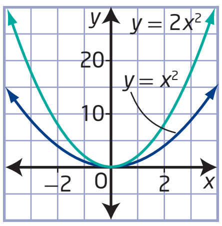
(−3.22, −1.54) and (−1.18, −3.98) or $n = -3.22$, $m = -1.54$ and $n = -1.18$, $m = -3.98$

e)



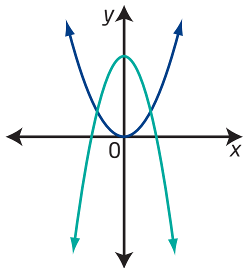
(−2.5, 306.25) or $h = -2.5$, $t = 306.25$

6. The two parabolas have the same vertex, but different values of a.
Example: $y = x^2$ and $y = 2x^2$.

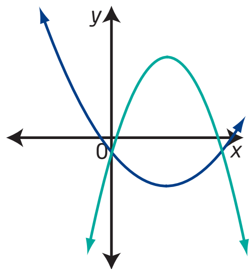


7. Examples:

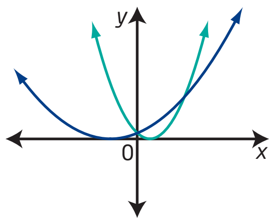
a)



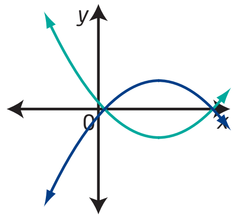
b)



c)



d)

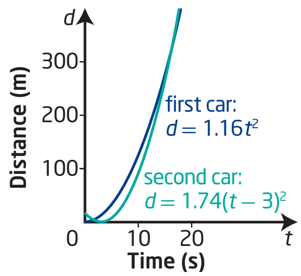


8. Examples:
a) $y = x - 3$ **b)** $y = -2$ **c)** $y = x - 1$

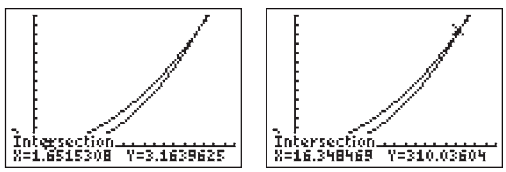
9. a) (100, 3800) and (1000, 8000)
b) When he makes and sells either 100 or 1000 shirts, Jonas makes no profit as costs equal revenue.
c) Example: (550, 15 500). This quantity (550 shirts) has the greatest difference between cost and revenue.

10. (0, 3.9) and (35.0, 3.725)

11. a) $d = 1.16t^2$ and $d = 1.74(t - 3)^2$
b) A suitable domain is $0 \leq t \leq 23$.



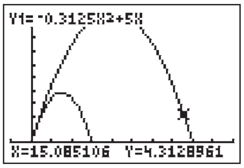
c)



(1.65, 3.16) and (16.35, 310.04) While (1.65, 3.16) is a graphical solution to the system, it is not a solution to the problem since the second car starts 3 s after the first car.

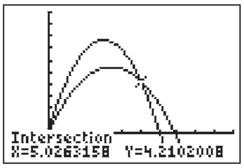
d) At 16.35 s after the first car starts, both cars have travelled the same distance.

12. a)



Both start at (0, 0), at the fountain, and they have one other point in common, approximately (0.2, 1.0). The tallest stream reaches higher and farther than the smaller stream.

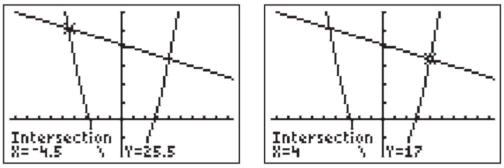
b)



They both start at (0, 0), but the second stream passes through the other fountain's spray 5.03 m from the fountain, at a height of 4.21 m.

13. a) Let x represent the smaller integer and y the larger integer. $x + y = 21$, $2x^2 - 15 = y$.

b)

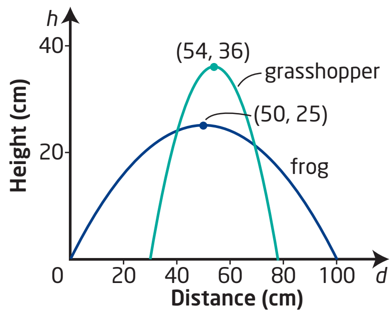


One point of intersection does not give integers. The two integers are 4 and 17.

14. a) The blue line and the parabola intersect at (2, 2). The green line and the parabola intersect at (−4.54, −2.16).

b) Example: There is one possible location to leave the jump and one location for the landing.

15. a)



b) Frog: $y = -0.01(x - 50)^2 + 25$

Grasshopper: $y = -0.0625(x - 54)^2 + 36$

c) (40.16, 24.03) and (69.36, 21.25)

d) These are the locations where the frog and grasshopper are at the same distance and height relative to the frog's starting point. If the frog does not catch the grasshopper at the first point, there is another opportunity. However, we do not know anything about time, i.e., the speed of either one, so the grasshopper may be gone.

16. a) (0, 0) and approximately (1.26, 1.59) Since 0 is a non-permissible value for x and y, the point (0, 0) is not a solution to this system.

b) 1.26 cm

c) $V = lwh$

$V = 1.26 \times 1.26 \times 1.26$

$V = 2.000\ 376$

So, the volume is very close to 2 cm^3.

d) If x represents the length of one side, then $V = x^3$. For a volume of 2 cm^3, $2 = x^3$. Then, $x = \sqrt[3]{2}$ or approximately 1.26. Menaechmus did not have a calculator to find roots.

17. Examples:

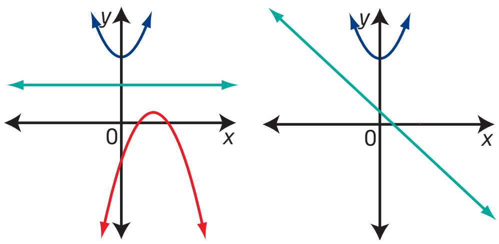
a) $y = x^2 + 1$ and $y = x + 3$

b) $y = x^2 + 1$ and $y = -(x - 1)^2 + 6$

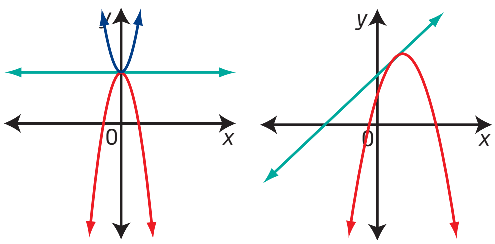
c) $y = x^2 + 1$ and $y = (x + 1)^2 - 2x$

18. Examples:

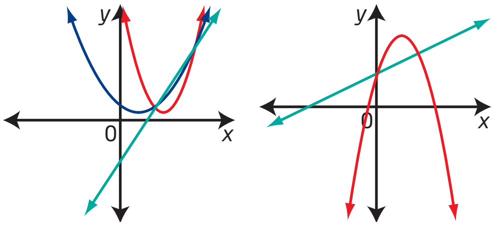
No solution: Two parabolas do not intersect and the line is between them, intersecting neither, or the parabolas are coincident and the line does not intersect them.



One solution: Two parabolas intersect once, with a line tangent to both curves, or the parabolas are coincident and the line is tangent.



Two solutions: Two parabolas intersect twice, with a line passing through both points of intersection, or the parabolas are coincident and the line passes through two points on them.



19. Example: Similarities: A different number of solutions are possible. It can be solved graphically or algebraically. Differences: Some systems involving quadratic equations cannot be solved by elimination. The systems in this section involve equations that are more difficult to solve.

20. a) Two solutions. The y-intercept of the line is above the vertex, and the parabola opens upward.

b) No solution. The parabola's vertex is at (0, 3) and it opens upward, while the line has y-intercept -5 and negative slope.

c) Two solutions. One vertex is directly above the other. The upper parabola has a smaller vertical stretch factor.

d) One solution. They share the same vertex. One opens upward, the other downward.

e) No solution. The first parabola has its vertex at (3, 1) and opens upward. The second parabola has it vertex at (3, -1) and opens downward.

f) An infinite number of solutions. When the first equation is expanded, it is exactly the same as the second equation.

8.2 Solving Systems of Equations Algebraically, pages 451 to 456

1. In $k + p = 12$:
Left Side
$k + p$
$= 5 + 7$
$= 12$
= Right Side

In $4k^2 - 2p = 86$:
Left Side
$4k^2 - 2p$
$= 4(5)^2 - 2(7)$
$= 86$
= Right Side

So, (5, 7) is a solution.

2. In $18w^2 - 16z^2 = -7$:

$$\begin{aligned}\text{Left Side} &= 18\left(\frac{1}{3}\right)^2 - 16\left(\frac{3}{4}\right)^2\\ &= 18\left(\frac{1}{9}\right) - 16\left(\frac{9}{16}\right)\\ &= 2 - 9\\ &= -7\\ &= \text{Right Side}\end{aligned}$$

In $144w^2 + 48z^2 = 43$:

$$\begin{aligned}\text{Left Side} &= 144\left(\frac{1}{3}\right)^2 + 48\left(\frac{3}{4}\right)^2\\ &= 16 + 27\\ &= 43\\ &= \text{Right Side}\end{aligned}$$

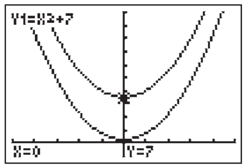
So, $\left(\frac{1}{3}, \frac{3}{4}\right)$ is a solution.

3. a) (-6, 38) and (2, 6) **b)** (0.5, 4.5)
c) (-2, 10) and (2, 30)
d) (-2.24, -1.94) and (2.24, 15.94)
e) no solution

4. a) $\left(-\frac{1}{2}, 4\right)$ and (3, 25)
b) (-0.5, 14.75) and (8, 19)
c) (-1.52, -2.33) and (1.52, 3.73)
d) (1.41, -4) and (-1.41, -4)
e) There are an infinite number of solutions.

5. a) (2.71, -1.37) and (0.25, 0.78)
b) (-2.41, 10.73) and (2.5, 10)
c) (0.5, 6.25) and (0.75, 9.3125)

6. a) They are both correct.
b) Graph $n = m^2 + 7$ and $n = m^2 + 0.5$ to see that there is no point of intersection.



7. a) Yes. Multiplying by (-1) and then adding is equivalent to subtraction.
b) Yes.
c) Example: Adding is easier for most people. Subtracting with negative signs can be error prone.

8. $m = 6$, $n = 40$

9. a) $7x + y + 13$ **b)** $5x^2 - x$
c) $60 = 7x + y + 13$ and $10y = 5x^2 - x$. Since the perimeter and the area are both based on the same dimensions, x and y must represent the same values. You can solve the system to find the actual dimensions.
d) (5, 12); the base is 24 m, the height is 10 m and the hypotenuse is 26 m.
e) A neat verification uses the Pythagorean Theorem: $24^2 + 10^2 = 676$ and $26^2 = 676$. Alternatively, in the context:
Perimeter = 24 + 10 + 26 = 60
Area = $\frac{1}{2}(24)(10) = 120$

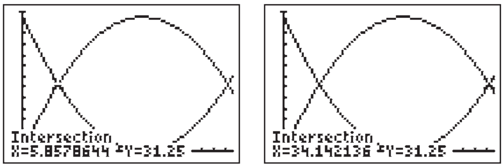
10. a) $x - y = -30$ and $y + 3 + x^2 = 189$
b) (12, 42) or (-13, 17)
c) For 12 and 42:
$12 - 42 = -30$ and
$42 + 3 + 12^2 = 189$
For -13 and 17: $-13 - 17 = -30$ and
$17 + 3 + (-13)^2 = 189$
So, both solutions check.

11. a) $C = 2\pi r$, $C = 3\pi r^2$

b) $r = \frac{2}{3}$, $C = \frac{4\pi}{3}$, $A = \frac{4\pi}{9}$

12. a) 5.86 m and 34.14 m **b)** 31.25 J

c)



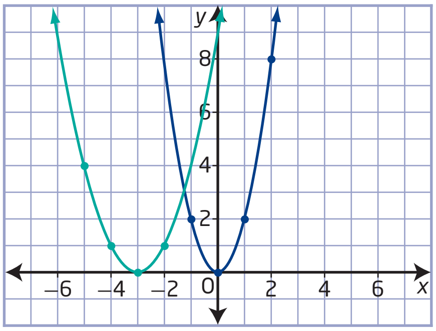
d) Find the sum of the values of E_k and E_p at several choices for d. Observe that the sum is constant, 62.5. This can be deduced from the graph because each is a reflection of the other in the horizontal line $y = 31.25$.

13. a) approximately 15.64 s

b) approximately 815.73 m

c)
$$\begin{aligned} h(t) &= -4.9t^2 + 2015 \\ &= -4.9(15.64)^2 + 2015 \\ &\approx 816.4 \end{aligned}$$
$$\begin{aligned} h(t) &= -10.5t + 980 \\ &= -10.5(15.64) + 980 \\ &\approx 815.78 \end{aligned}$$
The solution checks. Allowing for rounding errors, the height is about 816 m when the parachute is opened after 15.64 s.

14. a)



b) (−1.3, 3) **c)** $y = 2x^2$ and $y = (x + 3)^2$

d) (−1.24, 3.09) and (7.24, 104.91) Example: The estimate was close for one point, but did not get the other.

15. a) For the first fragment, substitute $v_0 = 60$, $\theta = 45°$, and $h_0 = 2500$:
$$h(x) = -\frac{4.9}{(v_0 \cos \theta)^2}x^2 + (\tan \theta)x + h_0$$
$$h(x) = -\frac{4.9}{(60 \cos 45°)^2}x^2 + (\tan 45°)x + 2500$$
$$h(x) \approx -0.003x^2 + x + 2500$$
For the second fragment, substitute $v_0 = 60$, $\theta = 60°$, and $h_0 = 2500$:
$$h(x) = -\frac{4.9}{(v_0 \cos \theta)^2}x^2 + (\tan \theta)x + h_0$$
$$h(x) = -\frac{4.9}{(60 \cos 60°)^2}x^2 + (\tan 60°)x + 2500$$
$$h(x) \approx -0.005x^2 + 1.732x + 2500$$

b) (0, 2500) and (366, 2464.13)

c) Example: This is where the fragments are at the same height and the same distance from the summit.

16. a) The solution for the system of equations will tell the horizontal distance from and the height above the base of the mountain, where the charge lands.

b) 150.21 m

17. a) 103 items **b)** \$377 125.92

18. a) (−3.11, 0.79), (3.11, 0.79) and (0, 16)

b) Example: 50 m²

19. a) (2, 6) **b)** $y = -\frac{1}{4}x + \frac{13}{2}$ **c)** 2.19 units

20. (2, 1.5) and (−1, 3)

21. $y = 0.5(x + 1)^2 - 4.5$ and $y = -x - 4$ or $y = 0.5(x + 1)^2 - 4.5$ and $y = 2x - 4$

22. Example: Graphing is relatively quick using a graphing calculator, but may be time-consuming and inaccurate using pencil and grid paper. Sometimes, rearranging the equation to enter into the calculator is a bit tricky. The algebraic methods will always give an exact answer and do not rely on having technology available. Some systems of equations may be faster to solve algebraically, especially if one variable is easily eliminated.

23. (−3.39, −0.70) and (−1.28, 4.92)

24. Example: Express the quadratic in vertex form, $y = (x - 2)^2 - 2$. This parabola has its minimum at (2, −2) and its y-intercept at 2. The linear function has its y-intercept at −2 and has a negative slope so it is never close to the parabola. Algebraically,
$$\begin{aligned} -\frac{1}{2}x - 2 &= x^2 - 4x + 2 \\ -x - 4 &= 2x^2 - 8x + 4 \\ 0 &= 2x^2 - 7x + 8 \end{aligned}$$
This quadratic equation has no real roots. Therefore, the graphs do not intersect.

25. Step 1: Example: In a standard viewing window, it looks like there are two solutions when $b > 0$, one solution when $b = 0$, and no solution when $b < 0$

Step 2: There are two solutions when $b > -\frac{1}{4}$, one solution when $b = -\frac{1}{4}$, and no solution when $b < -\frac{1}{4}$.

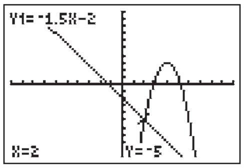
Steps 3 and 4: two solutions when $|m| > 2$, one solution when $m = \pm 2$, and no solution when $|m| < 2$

Step 5: For $m = 1$: two solutions when $b > 0$, one solution when $b = 0$, and no solution when $b < 0$; for $m = -1$: two solutions when $b < 0$, one solution when $b = 0$, and no solution when $b > 0$; two solutions when $|m| > 2b$, one solution when $m = \pm 2b$, and no solution when $|m| < 2b$

Chapter 8 Review, pages 457 to 458

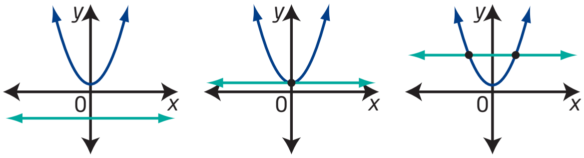
1. a) (2, −5)

b)

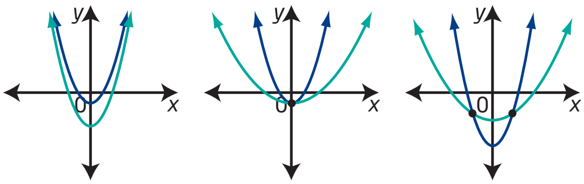


c) (6.75, −12.125)

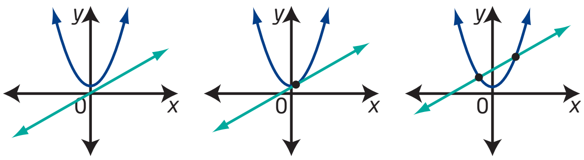
2. a) no solution, one solution, two solutions



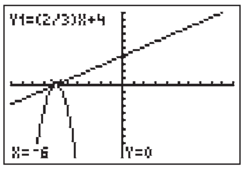
b) no solution, one solution, two solutions



c) no solution, one solution, two solutions

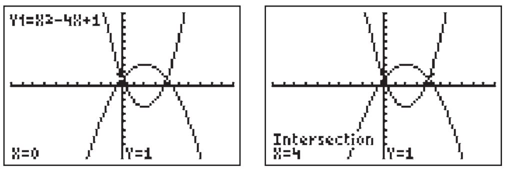


3. a)



(−6, 0)

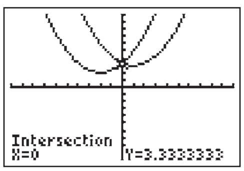
b)



(0, 1) and (4, 1)

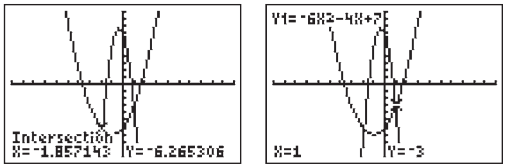
4. Example: Adam is not correct. For all values of x, $x^2 + 3$ is always 2 greater than $x^2 + 1$ and the two parabolas never intersect.

5. a)



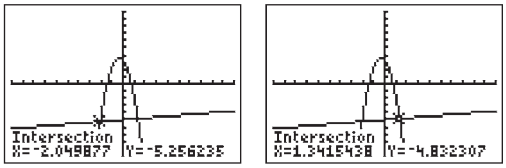
(0, 3.33) or $x = 0$, $p = 3\frac{1}{3}$

b)



(−1.86, −6.27) and (1, −3)

c)



(−2.05, −5.26) and (1.34, −4.83) or $d = -2.05$, $t = -5.26$ and $d = -1.34$, $t = -4.83$

6. a) road arch: $y = -\frac{3}{32}(x - 8)^2 + 6$

river arch: $y = -\frac{1}{18}(x - 24)^2 + 8$

b) (14.08, 2.53)

c) Example: the location and height of the support footing

7. a) Example: The first equation models the horizontal distance travelled and the height of the ball; it would follow a parabolic path that opens downward. The linear equation models the profile of the hill with a constant slope.

b) $d = 0$, $h = 0$ and $d = 14.44$, $h = 7.22$

c) Example: The point (0, 0) represents the starting point, where the ball was kicked. The point (14.44, 7.22) is where the ball would land on the hill. The coordinates give the horizontal distance and vertical distance from the point that the ball was kicked.

8. a) Example: (2, −3) and (6, 5)

b) (2, −3) and (6, 5)

9. The solution $\left(\frac{1}{2}, 1\right)$ is correct.

10. a) $\left(\frac{1}{2}, \frac{5}{2}\right)$ and $\left(-\frac{5}{3}, -4\right)$; substitution, because the first equation is already solved for p

b) (3.16, −13) and (−3.16, −13); elimination, because it is easy to make opposite coefficients for the y-terms

c) $\left(-\frac{2}{3}, -\frac{53}{9}\right)$ and (2, −1); elimination after clearing the fractions

d) (0.88, 0.45) and (−1.88, 3.22); substitution after isolating y in the second equation

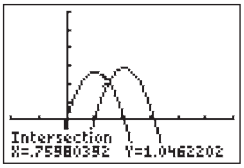
11. a) 0 m and 100 m **b)** 0 m and 10 m

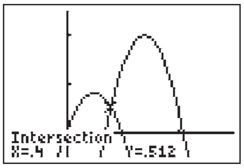
12. a) the time when both cultures have the same rate of increase of surface area

b) (0, 0) and (6.67, 0.02)

c) The point (0, 0) represents the starting point. In 6 h 40 min, the two cultures have the same rate of increase of surface area.

Chapter 8 Practice Test, pages 459 to 460

1. C
2. C
3. B
4. D
5. D
6. $n = -4$
7. a) $\left(\frac{3}{4}, -\frac{35}{16}\right)$ and $(-2, -7)$ **b)** $(1, -4)$
8. a)


$(0.76, 1.05)$
b) Example: At this time, 0.76 s after Sophie starts her jump, both dancers are at the same height above the ground.
9. a) perimeter: $8y = 4x + 28$
area: $6y + 3 = x^2 + 14x + 48$
b) The perimeter is 16 m. The area is 15 m^2.
10. a) $y = \frac{1}{3}x^2$ and $y = \frac{1}{2}(x - 1)^2$
b) $(5.45, 9.90)$ and $(0.55, 0.10)$
11. a)


b) Example: At this point, a horizontal distance of 0.4 cm and a vertical distance of 0.512 cm from the start of the jump, the second part of the jump begins.
12. A$(-3.52, 0)$, B$(7.52, 0)$, C$(6.03, 14.29)$
area = 78.88 square units