Quadratic Functions

Opener

***Pre-Calculus 11*, pages 140–141**

Suggested Timing
20–30 min

Blackline Masters
BLM 3–2 Chapter 3 Prerequisite Skills
BLM U2–1 Unit 2 Project Checklist

Key Terms
quadratic function
parabola
vertex (of a parabola)
minimum value
maximum value
axis of symmetry
vertex form (of a quadratic function)
standard form (of a quadratic function
completing the square

What's Ahead

In Chapter 3, students explore quadratic functions. They investigate the shape of graphs of quadratic functions in general, as well as in different applied contexts. Students learn new terminology such as *vertex form* (of a quadratic equation) and apply terms that are familiar, such as *maximum* and *minimum value*, *intercepts*, *domain*, and *range* to identify features of graphs of quadratic functions. Students learn how to work with equations for quadratic functions in both vertex and standard form. They learn the process of completing the square and use it to convert quadratic equations in standard form to vertex form. Students write quadratic functions to model situations and analyse quadratic equations using a variety of methods.

Planning Notes

Begin Chapter 3 by having students look at the three digital photos on pages 140 and 141. Ask them to describe how the number of pixels affects the appearance of each photo. Clarify the meaning of the terms *pixel* and *megapixel*. Ask students who are knowledgeable about digital cameras to estimate the capabilities of digital cameras they have seen or used in terms of number of pixels. Ask how these numbers of pixels might compare to a professional photographer's equipment. Have students generate other possible dimensions that would give a 1-megapixel image if the dimensions of the image were any rectangular shape (not only a square).

Consider having students work in pairs to construct a table that shows the relationship between the side lengths of a square image and the total number of pixels. Students might use this to investigate the pattern in the values and observe that the pattern is not linear. They might use a table similar to this one.

Side Length of Image	Total Number of Pixels
1000	1 000 000
2000	
3000	
x	

Use leading questions such as the following to promote discussion:
- What pattern do you observe in the total number of pixels as the side length of the image increases?
- How can you tell that this does not represent a linear function?
- What function could you write that shows the relationship between side length and the total number of pixels?

Clarify the term *quadratic* and explain to students that throughout this chapter, they will build their knowledge of quadratic functions.

As students progress through the chapter, have them record the Key Terms and develop their own definitions. They can refer to the definitions in the student resource. You might have students explain their understanding of the Key Terms to a classmate.

As a class, highlight the careers related to quadratic functions throughout the chapter, and have students explain how quadratic functions are used in each career.

Unit Project

You might take the opportunity to discuss the Unit 2 project described in the Unit 2 opener.

Throughout the chapter, there are several Project Corner boxes that provide information about quadratic functions in everyday situations. These features are not mandatory but are recommended because they provide some background for the Unit 2 project.

If you are going to develop a project rubric with the class, you may want to start now. See pages 168 and 169 in this Teacher's Resource for information on working with students to develop a class rubric.

Chapter Summary

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. If they have used Foldables™ before, you may wish to have them select a style they found useful to keep their notes in for Chapter 3. Ask students to describe other methods of summarizing information. For example, many students may have used different types of graphic organizers, such as a mind map, concept map, spider map, Frayer model, and KWL chart. Ask students which one(s) might be useful in this chapter.

Encourage students to use a summary method of their choice. Allowing personal choice in this way will increase student ownership in their work. It may also encourage some students to experiment with different summary techniques.

Give students time to develop the summary method they have chosen. Ask them to include some method of keeping track of what they need to work on. Explain the advantage of doing this.

Meeting Student Needs

- Consider having students complete the questions on **BLM 3–2 Chapter 3 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- Have students recall what they know about functions, function notation, and graphing linear functions. Have them reactivate their knowledge of characteristics of graphs of linear relations, such as intercepts, slope, domain, and range. You might have students create a poster that features the characteristics of a graph of a linear relation, such as intercepts, slope, domain, and range. Display the poster in the classroom for reference.
- You may wish to post the student learning outcomes for the entire chapter in the classroom, colour-coding the outcomes by section in the chapter. Ensure that students understand the outcomes as written. Provide students with their own copy. They can then refer to the outcomes as they work through the chapter. This will help them to self-assess their progress and to identify areas of weakness.
- In advance, gather visuals illustrating quadratic functions. Use the visuals to help students explain how an understanding of quadratic functions is needed in different occupations.
- Hand out to students **BLM U2–1 Unit 2 Project Checklist**, which provides a list of all the requirements for the Unit 2 project.

ELL

- Encourage students to create their own math vocabulary dictionary for the Key Terms using written descriptions, diagrams, and examples.

Enrichment

- Students may wish to brainstorm and research careers that involve quadratic functions. For example, they might research how scientists and engineers apply quadratic functions.
- Encourage students to list examples in which wind resistance plays a role in performance. The examples might range from sports to aircraft design. (Students might mention Olympic speed skating suits, sports cars, bobsleds, transport trucks, and submarines.)
- Invite students to research pixels and the computer monitor. They may find the related Web Link at the end of this section useful. Have them present their findings to the class.

Gifted

- Challenge students to consider the fact that drag on an object travelling through air increases as the speed of the object increases. Ask them to predict if the relationship between drag and speed is linear or quadratic, and explain their thinking. (Students should indicate an understanding of the fundamental difference between linear and quadratic relationships. A sample response might be: The relationship between drag and speed is quadratic because the force of the air more than doubles when the speed doubles, since the amount of air that applies the drag to the object increases by length and width as well as depth, thus producing more than a linear increase.

Career Link

Consider taking the opportunity to describe some examples of how engineers apply principles of mathematics and science to research, design, and develop products, such as a space-plane. For instance, engineers use their knowledge to improve on existing technology, such as the efficiency of a space vehicle. You may wish to have students who are interested in learning more about engineering research the career, including the training and qualifications required, and employment opportunities. Have students present their findings orally and explain how this career connects to the chapter.

Web Link

For information about a career in engineering and the related educational requirements, go to www.mhrprecalc11.ca and follow the links.

For information about careers related to quadratic functions such as civil engineers, electricians, web developers, set designers, and automotive designers, go to www.mhrprecalc11.ca and follow the links.

For information about digital imaging, go to www.mhrprecalc11.ca and follow the links.

Investigating Quadratic Functions in Vertex Form

3.1

***Pre-Calculus 11*, pages 142–162**

Suggested Timing
120–180 min

Materials
- grid paper
- ruler
- graphing calculator
- graphing software (optional)

Blackline Masters
Master 2 Centimetre Grid Paper
Master 3 0.5 Centimetre Grid Paper
BLM 3–3 Chapter 3 Warm-Up
BLM 3–4 Section 3.1 Extra Practice

Mathematical Processes
- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcomes
RF3 Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the:
- vertex
- domain and range
- direction of opening
- axis of symmetry
- x- and y-intercepts.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2a), b), 3a), c), 4, 6, 7, 8a)–c), 9, 11, 21, 24, 25
Typical	#2, 4, 5, 7, 9, 12–14, 16, 18, one of 17, 19, or 20, 24–26
Extension/Enrichment	#7, 12–14, 18, 21–26

Planning Notes

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce prerequisite skills needed for this section. If you have posted the outcomes, refer to the outcomes for this section.

As a class, have students consider the situation shown in the photograph and graph. Use prompts such as the following to promote a class discussion:
- Explain the different starting points for the three curves.
- Do the three curves have the same shape? How can you tell?
- How do the equations of the three functions compare with each other?

You might mention that jet- and rocket-powered vehicles have been used to achieve speeds of over 1000 km/h on the Bonneville Salt Flats.

Investigate Graphs of Quadratic Functions in Vertex Form

Consider having students work in pairs for this investigation, but record their own response for each question. Decide whether students will use grid paper and create graphs manually or use graphing calculators. Alternatively, allow them to choose the method they are most comfortable with. After students complete each part, you might have them compare their findings about the effect of changing each parameter in the vertex form of the function with another student pair.

For Part A, as students begin to see the effect of a on the graph, ask them to confirm their ideas using other values for a. Some students may not discover the effect of changing a when $a > 1$ versus when $0 < a < 1$, or when $a < 0$. Help students make these discoveries by asking guiding questions such as the following:
- Did you try a range of possible values for a? If so, which values did you try?
- What other values might you try?
- Is there more than one type of change that happens as a result of changes in a? What are the changes that occur?

As students complete Part B, encourage them to be specific in describing the changes in the graph of each function. Prompt them to identify whether the change that results from the value of q is horizontal or vertical.

For Part C, students may find it challenging to determine the effect of changing p in the graph. Some confusion may arise with the sign of p. Students could make a table to organize the functions they use, the value of p in each case, and the result in the graph.

For Reflect and Respond #9, coach students by asking questions such as the following:

- What is the value of p for the function $f(x) = (x - 5)^2$? Is it 5 or -5?
- What is the value of p for the function $f(x) = (x + 7)^2$?

As a class, have students compare their responses to Reflect and Respond #3, 6, and 9.

Meeting Student Needs

- For the opener, you might have students speculate about the possibility of powering a snow machine, ATV, or boat using hydrogen fuel cells. Ask about the advantages and disadvantages of doing so.
- If time is a constraint, divide students into groups. Assign each group to investigate one of the parameters. Have groups model and explain their findings.
- You may wish to reactivate students' knowledge of transformations prior to beginning this section. Ask students:
  – Where have you worked with transformations before?
  – What types of transformations have you done?
  Consider projecting an applet about quadratic transformations and highlight the changes that can take place, such as translations, reflections, and rotations.
- Tell students to pay particular attention to the transformations on the graph as the values of a, p, and q change.
- For the investigation, some students may find it helpful to use larger-scale grid paper.
- For the investigation, have students highlight the sketch of $f(x) = x^2$ and then compare each sketch and equation to the sketch of $f(x) = x^2$. Encourage them to write an observation about each equation once it is sketched.
- Visual learners may benefit from sketching each graph using a different colour in order to make the changes clear.
- At the end of the investigation, have students form groups of three and have each member take on a different role: a, q, or p. Have them take turns explaining how the value they represent transforms the standard graph of $f(x) = x^2$.

ELL

- Some students may not be familiar with the terms *alternative-fuel vehicles*, and *hydrogen powered fuel cell*. Clarify the terms and encourage students to visit the related Web Link on page 142 in the student resource.

Common Errors

- For Part C, some students may not readily understand the effect of p in terms of the function values.

R_x You might encourage students to compare the y-coordinates of points for the corresponding x-values. Ask them to compare the height of the curves at any point.

- Some students may not understand the signs of the values of p in Part C.

R_x Encourage students to describe the changes in terms of specific functions rather than in terms of the general function $f(x) = (x - p)^2$. At a later stage, students can move toward working in general terms.

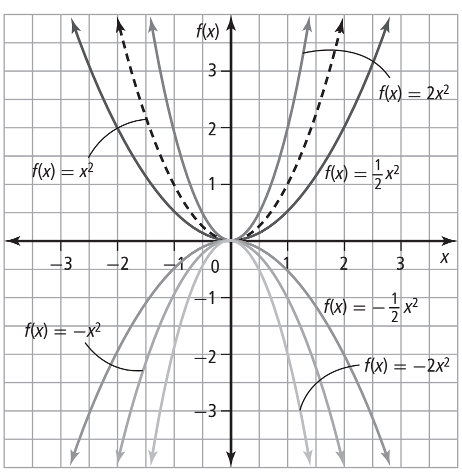
Web Link

For an applet that students can use to explore transformations of quadratic functions in vertex form, go to www.mhrprecalc11.ca and follow the links.

Answers

Investigate Graphs of Quadratic Functions in Vertex Form

1. a)



b) The graph of $f(x) = 2x^2$ is narrower than the graph of $f(x) = x^2$. The graph of $f(x) = \frac{1}{2}x^2$ is wider than the graph of $f(x) = x^2$. The graph of $f(x) = -x^2$ is the same size and shape as the graph of $f(x) = x^2$, but it is reflected in the x-axis. The graph of $f(x) = -2x^2$ is narrower than the graph of $f(x) = x^2$, and it is reflected in the x-axis. The graph of $f(x) = -\frac{1}{2}x^2$ is wider than the graph of $f(x) = x^2$, and it is reflected in the x-axis.

c) Parameter a determines the shape and the orientation of the parabola.

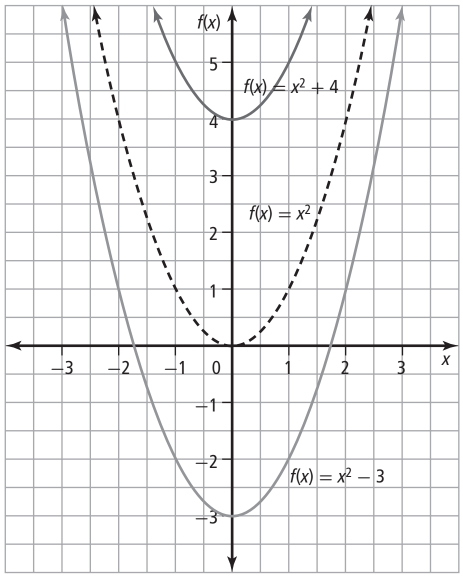
3. a) When a is a positive number greater than 1, the graph opens upward and is narrower compared to the graph of $f(x) = x^2$. As the value of a increases, the graph becomes narrower.

b) When a is a positive number less than 1, the graph opens upward and is wider compared to the graph of $f(x) = x^2$. As the value of a decreases, the graph becomes wider.

c) Compared to the graph of $f(x) = x^2$, when a is a negative number, the graph opens downward and is narrower or wider depending on if a is greater or less than -1, respectively.

Answers

4. a)



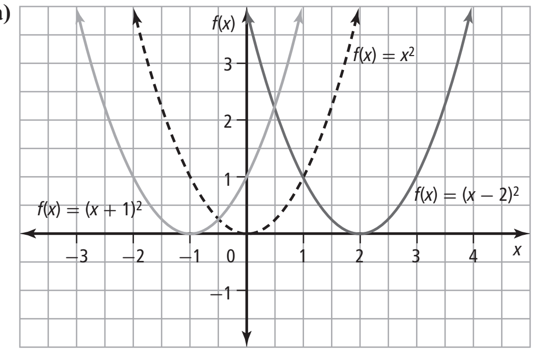
b) The graph of $f(x) = x^2 + 4$ is translated 4 units up relative to the graph of $f(x) = x^2$. The graph of $f(x) = x^2 - 3$ is translated 3 units down relative to the graph of $f(x) = x^2$.

c) Parameter q translates the graph vertically q units compared to the graph of $f(x) = x^2$.

6. a) When q is a positive number, the graph is translated upward q units relative to $f(x) = x^2$.

b) When q is a negative number, the graph is translated downward $|q|$ units relative to $f(x) = x^2$.

7. a)



b) The graph of $f(x) = (x - 2)^2$ is translated 2 units to the right relative to the graph of $f(x) = x^2$. The graph of $f(x) = (x + 1)^2$ is translated 1 unit to the left relative to $f(x) = x^2$.

b) Parameter p translates the graph horizontally p units compared to the graph of $f(x) = x^2$.

9. a) When the value of p is positive, the graph is translated to the right $|p|$ units relative to $f(x) = x^2$.

b) When the value of p is negative, the graph is translated to the left p units relative to $f(x) = x^2$.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions with a partner or individually. Listen as students discuss what they learned during the investigation. Encourage them to generalize and reach a conclusion about their findings.	• Students may benefit from creating a summative chart that considers each parameter and how it affects the quadratic function. Consider completing the chart as a class and posting it in the classroom. • Some students may find it beneficial to do a SMART Board activity in which they manipulate the arms of the quadratic and its vertex.

Link the Ideas

As a class, define *quadratic function*, *parabola*, *vertex*, *minimum value*, *maximum value*, and *axis of symmetry*. Have students write their own definition for each term and encourage them to sketch a parabola and label it. Ask students to identify things that might be parabolic in shape. Help them recall that *degree* refers to the highest-degree term in a polynomial. For example, the polynomial $6x^2 + 2x$ has a degree of 2.

Alternatively, have students explain each term to a classmate and refer to the graphs they produced in the investigation to help with their explanation. Or, you might use technology to graph and project the functions shown in the Link the Ideas and ask students to use correct mathematical terminology in describing the features of the graphs. If you do this, you might ask students the following questions:

- How does the shape and orientation of a graph determine whether it has a maximum or a minimum value?
- How can you identify the axis of symmetry on a graph? (You might prompt students about how to write the equation of a vertical line.)

Example 1

In this example, students sketch graphs of quadratic functions using two methods: using transformations and using points and symmetry. Begin by having students identify the characteristics of each graph. Encourage students to look for connections between the changes in the graph and the corresponding changes in the function $f(x) = x^2$. You might use prompts such as the following:

- What do the values of a, p, and q in the equation tell you about the characteristics of the graph?

- What effect do the values of a, p, and q have on the graph of the function relative to the graph of $f(x) = x^2$?

As students work through part a) Method 1, encourage them to make the changes one at a time. You might ask:

- How are the y-values affected when $a = 2$? $a = -\frac{1}{4}$?
- How are p and q related to the direction of the translation and the location of the vertex?

For part a) Method 2, ask students to work through the solution using a different x-coordinate and then determine the corresponding point equidistant from the axis of symmetry. Have them plot these two points and complete the sketch of the parabola. Encourage students to work through the solution to part b) using points and symmetry before comparing their answer with the given solution in the student resource.

As students work on the Your Turn, encourage them to use a different method for each question. When combining transformations, reinforce the importance of working step by step, and of describing each change before sketching the graph of the function. You might suggest sketching each intermediate step before arriving at the final graph. Have students compare their solution with a classmate. Ask them to compare the methods and decide which method they prefer and explain why.

Example 2

In this example, students determine a quadratic function given its graph. Students may be able to identify the values of p and q more readily than the value of a. The example models two methods: determining a by substituting a point and by comparing with the graph of $f(x) = x^2$. It is important for students to work through both methods.

As students work through Method 1, you might ask:

- How do you know that you can substitute a point on the graph for x and y in the equation of the function?
- Would it matter which point you used? Explain.

You may wish to have students use another point to determine the value of a, to help them realize that any point can be used as long as it is on the graph of the function. The x- and y-coordinates in the equation make up all of the points located on the graph.

As they work through Method 2, you might ask:

- Which points on each curve correspond to each other?
- How do the vertical distances in relation to the vertex compare relative to those on the graph for $f(x) = x^2$?
- How is the value of a related to the vertical distances of the points on the graph?

- What effect does a negative value for a have on the graph?

You may wish to discuss the *vertical stretch* and how it affects the appearance of the curve. The parabola will appear wider if the y-coordinates are multiplied by a constant between -1 and 1. Similarly, it will appear narrower if the y-coordinates are multiplied by a constant that is less than -1 or greater than 1.

Example 3

In this example, students determine the number of x-intercepts for a function without generating the graph of the function. They do this by considering the effect of a and q on the graph. Help students understand the logic used in this example by asking questions such as the following:

- What can you tell about the graph of a quadratic function if you know the value of a? the value of q?
- How does knowing the direction of opening and the location of the vertex help you determine the number of x-intercepts?

Consider asking students to generate their own example of a function for each of zero, one, and two x-intercepts.

As students work on the Your Turn, remind them that they need only visualize the graph or draw a rough sketch. However, tell them to clearly explain their answer for each quadratic function.

Example 4

In this example, students write the equation of a quadratic function for a given situation. You may wish to begin by asking students to identify the part of the bridge that has the parabolic shape. (Point out the parabolic-shaped main cable that runs from the top of one tower to the other.) Encourage them to sketch and label the cable and superimpose the coordinate axes on the sketch. Assist students to work through the solution by asking questions such as the following:

- Why is the low point of the cables the simplest point to use for the origin?
- How can you use a point on a graph to determine the value of a?
- Is there more than one point that could be used to determine a? Explain.
- When substituting into the equation $f(x) = ax^2$, where does 44 come from?

Some students may suggest that the value of a can be determined by considering vertical heights. If so, encourage them to explain how they would do this.

After students understand how to generate the equation of the function, you may wish to have them consider the function for different locations of the origin. For instance, have students consider the origin placed at the water's surface directly below the lowest point of the cables. Ask students:

- What is the vertex? (0, 67)
- What is the quadratic function?
  ($f(x) = 0.000\ 79x^2 + 67$ or $f(x) = \frac{11}{13\,924}x^2 + 67$; this represents a vertical translation) This may help students connect to their understanding of vertical translations.

Have students consider the origin placed at water level at the base of one tower. Ask students:

- For the tower on the left, what is the vertex? (236, 67)
- What is the quadratic function?
  ($f(x) = 0.000\ 79(x - 236)^2 + 67$ or
  $f(x) = \frac{11}{13\,924}(x - 236)^2 + 67$)
- For the tower on the right, what is the vertex? (−236, 67)
- What is the quadratic function?
  ($f(x) = 0.000\ 79(x + 236)^2 + 67$)

In the case of either tower, students should identify that this represents a horizontal and a vertical translation. Asking students to consider other locations of the origin may help them connect their understanding of translations of quadratic functions with contextual situations.

You might then ask students if the answer to part b) would change if a different equation of the function were used. They may need to be prompted that in the case of a different equation, the x-coordinate may need to change:

<table>
<tr><td colspan="2">If students use $f(x) = \frac{11}{13\,924}x^2$ or $f(x) = \frac{11}{13\,924}x^2 + 67$ to determine the height, a point 90 m from one tower is the same as (236 − 90) or 146 for x. Substitute 146 for x and determine the value of y.</td><td>If students use $f(x) = \frac{11}{13\,924}(x - 236)^2 + 67$ to determine the height, the origin is located at the base of one tower so they can use 90 for x.</td></tr>
<tr><td>$f(x) = \frac{11}{13\,924}x^2$
$f(x) = \frac{11}{13\,924}(146)^2$
$f(x) \approx 16.8$</td><td>$f(x) = \frac{11}{13\,924}x^2 + 67$
$f(x) = \frac{11}{13\,924}(146)^2 + 67$
$f(x) \approx 83.8$</td><td rowspan="2">$f(x) = \frac{11}{13\,924}(x - 236)^2 + 67$
$f(x) = \frac{11}{13\,924}(90 - 236)^2 + 67$
$f(x) = \frac{11}{13\,924}(-146)^2 + 67$
$f(x) \approx 83.8$
Since this function's vertex is at water level, the value that results is the overall height of the cables above water, or 83.8 m.</td></tr>
<tr><td>This represents 16.8 m above the low point in the cables, which are 67 m above water. The height is approximately 67 + 16.8 or 83.8 m.</td><td>Since this function has its vertex at water level, the resulting value is the overall height of the cables above water, or 83.8 m.</td></tr>
</table>

Following this process should help students realize that regardless of which function is used, the solution should be the same assuming that the appropriate value of x is used.

Have students complete the Your Turn and compare their solution with a classmate's. You might ask student pairs to generate equations for the function using different locations for the origin.

Key Ideas

Have students record their own summary of the Key Ideas. Have them store the summary of the Key Ideas for each section in the same location for review purposes.

Meeting Student Needs

- For Link the Ideas, invite students to create and display a poster that illustrates the Key Terms. Students can refer to this poster when working through the examples. Alternatively, you might post a diagram illustrating the key parts of a quadratic function: vertex (p, q), axis of symmetry ($x = p$), x-intercept, y-intercept, domain, and range. Highlight the axis of symmetry, one point, and the point created when it is reflected by the axis of symmetry. Include the equation of the graph in vertex form. Referring to the poster, discuss situations that would involve a maximum value or a minimum value.
- For Link the Ideas, have students rejoin their group of three and their role as either a, p, or q. Have each student read silently the section of the text that pertains to their role. After a few minutes, have each student present the key ideas from their section to the other two group members. When finished, have each student write a short summary. Consider having students work in their groups to work through the section on combining transformations.
- As students work through Example 1 part a) Method 1, encourage them to make the changes one at a time as follows: apply transformations represented by the parameter a before translations represented by the parameters p and q.
- For Example 2, consider sketching the graphs of $f(x) = x^2$ and another equation on the same axes, but using different colours. Project the graphs on an overhead or computer projector. Ensure that the units are spaced far enough apart so that the vertical distances can easily be seen. Encourage students to determine the distance between the points on the graph and state the translation.

- Consider having students create a presentation highlighting the key concepts in this section. The presentation could be a song, a video clip, a dramatization, or other format that you have approved. Students might then add to the presentation as they work through sections 3.2 and 3.3. Consider using this project for assessment purposes.

ELL

- Check that English language learners understand the effect of each parameter and the accompanying graph in the Link the Ideas. Consider displaying the graphs on the board and referring to the related graph as you talk through each explanation.
- Ensure that students add the following terms to their math vocabulary dictionary: *quadratic function*, *vertex form*, *transformation*, *translation*, *reflection*, *rotation*, *orientation*, *translates vertically*, *translates horizontally*, *parabola*, *axis of symmetry*, *vertex*, *minimum value*, and *maximum value*. Encourage them to include a description, diagram, and/or example for each term.
- For Example 4, use the photograph on page 154 in the student resource to help clarify the context and explain the terms *towers* and *cables*.

Common Errors

- Some students may be confused about why the horizontal translation seems to work opposite to what they think. For instance, they may wonder why a function such as $f(x) = (x - 3)^2$ is translated in the positive direction, while $f(x) = (x + 3)^2$ is translated in the negative direction.

R_x Encourage students to think about how the values of x need to change *before* substitution in order to obtain the same values for y. Help them understand, for example, that in the function $f(x) = (x + 3)^2$, all values of x need to be 3 units less to obtain the same y-values, which translates the graph to the left.

- Some students may find it difficult to work with combinations of transformations.

R_x Encourage students to consider the changes in the graph and corresponding changes in the equation of the function one at a time.

- When choosing a point on a graph to substitute into an equation, some students may have difficulty identifying points that have integral values or points that are easy to work with.

R_x Encourage students to locate a point on the curve that also intersects the grid lines. Then suggest that students check nearby intersections of grid lines and determine a point that seems most likely to have coordinates that are integers.

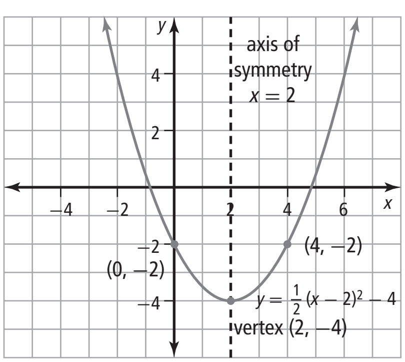
Web Link

For an applet to investigate changes in quadratic functions in vertex form, go to www.mhrprecalc11.ca and follow the links.

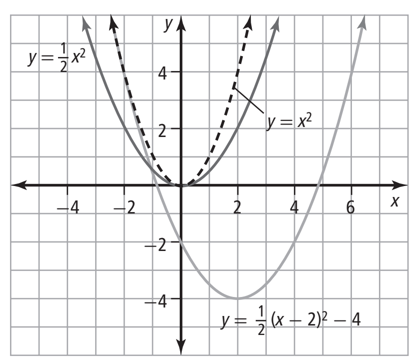
Answers

Example 1: Your Turn

a) The vertex is located at $(2, -4)$. The domain is $\{x \mid x \in \mathrm{R}\}$. The range is $\{y \mid y \geq -4, y \in \mathrm{R}\}$. The graph opens upward since $a > 0$. The equation of the axis of symmetry is $x = 2$.
To sketch the graph using points and symmetry, plot the coordinates of the vertex $(2, -4)$ and the axis of symmetry at $x = 2$. Use substitution to determine the coordinates of $(4, -2)$. When $x = 4$, $y = \frac{1}{2}(4 - 2)^2 - 4 = -2$. Then select its corresponding point equidistant from the axis of symmetry at $(0, -2)$. Plot these two additional points and complete the sketch of the parabola.



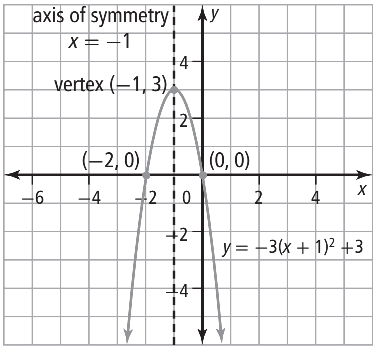
To sketch the graph using transformations, given the graph of $y = x^2$, first apply the change in width by a factor of $\frac{1}{2}$. Then translate the graph 2 units to the right and 4 units downward.



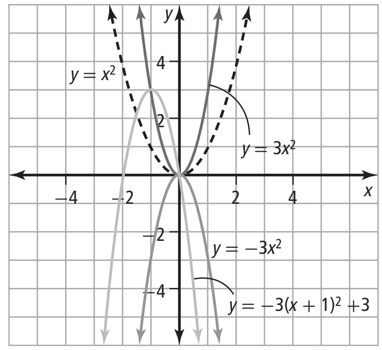
b) The vertex is located at $(-1, 3)$. The domain is $\{x \mid x \in \mathrm{R}\}$. The range is $\{y \mid y \leq 3, y \in \mathrm{R}\}$. The graph opens downward since $a < 0$. The equation of the axis of symmetry is $x = -1$.
To sketch the graph using points and symmetry, plot the coordinates of the vertex $(-1, 3)$ and the axis of symmetry at $x = -1$.
Use substitution to determine the coordinates of $(0, 0)$.

Answers

When $x = 0$, $y = -3(0 + 1)^2 + 3 = 0$. Then select its corresponding point equidistant from the axis of symmetry at $(-2, 0)$. Plot these two additional points and complete the sketch of the parabola.



To sketch the graph using transformations, given the graph of $y = x^2$, first apply the change in width by a factor of 3 and then reflect in the x-axis. Then translate the graph 1 unit to the left and 3 units upward.



Example 2: Your Turn

a) $f(x) = -\frac{1}{2}(x + 3)^2$ **b)** $f(x) = 4(x - 2)^2 + 1$

Example 3: Your Turn

a) 2 x-intercepts **b)** 1 x-intercept **c)** 0 x-intercepts

Example 4: Your Turn

a) $f(x) = -\frac{27}{2450}x^2 + 216$ **b)** 126.73 cm or $\frac{6210}{49}$ cm or $126\frac{36}{49}$ cm

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to refer to Example 1 when describing the graph of each quadratic function. Coach students through the equation in part a) by asking them to explain the role of the values of a, p, and q in any quadratic equation. Then, ask them to identify the specific values of a, p, and q in the given function and explain what the values tell about the graph. • Allow students to choose the method they use for sketching the graph. Encourage more than one approach.
Example 2 Have students do the Your Turn related to Example 2.	• This example demonstrates two methods for determining the quadratic function. For part a), you might have students choose the method of their choice and compare solutions with a classmate who used a different method. • Check that the graphs are accurately labelled and that students use mathematical terminology correctly when developing the quadratic function in vertex form.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to use their own strategy. Some students may benefit from drawing a rough sketch of the graph for each quadratic function first. • Encourage students to refer to the table on pages 153 and 154 in the student resource to help them. • Have students explain the difference between the questions that have brackets and those that do not have brackets. Ask them what effect the brackets have.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to draw a sketch of the archway and label its dimensions before developing the quadratic function. • Ask students what the vertex should be and explain why. Help them realize that the quadratic function will be of the form $f(x) = ax^2$. Ask students to verbalize what values to substitute.

Check Your Understanding

Practise

These questions allow students to build their understanding of quadratic equations in vertex form. Consider allowing students to complete these questions individually or with a partner. After they complete a question, remind students to check the answer. In cases where there is a difference between their answer and the one in the answer key, encourage them to get help from a classmate or from you.

Encourage students to make connections between descriptions, graphs, and equations of quadratic functions. You might use the following prompts about transformations:

- How does each type of transformation affect the graph of a function?
- Which transformation(s) affects the location of the vertex?
- Which transformation(s) affects the axis of symmetry?
- Which transformation(s) affects the range?

For each part of #9, encourage students to draw a rough sketch of the function. For both #8 and 9, direct students to refer back to Example 2. Encourage students to check their work by substituting a point from a given graph into the related quadratic function they developed.

Apply

Many of these questions provide opportunities for students to apply what they know about quadratic functions to real-life situations. Allow students to work in pairs to solve at least some of the problems.

For #10, students need to track the location of a point throughout a series of transformations. Encourage them to explain their strategies to a classmate as they work on the solution. You may wish to remind students that multiplying by a number, c, results in a vertical stretch by a factor of c.

The quadratic function for #11 is not in a familiar form. Provide coaching to students who need it by asking the following question:

- How could you write this quadratic function in vertex form?

The situation in #14 might prompt a class discussion about how viewing a commercial affects a person's desire to buy a particular product. You might have student pairs discuss why the graph in this situation has the shape that it does.

For #13, 15 to 18, and 21, students write equations of quadratic functions. Encourage them to draw a rough sketch of each graph and label the coordinates of known points to help them determine the quadratic function. For #13 and 16, students need to consider various locations of the origin. Direct students to refer to Example 4 for guidance. Assist students who would benefit from coaching by posing questions such as the following:

- If the origin is at the vertex, what are the values of p and q?
- How can you determine the value of a if you know the coordinates of a point on the graph?
- How is the quadratic function different if the origin is not located at the vertex?

Students may not realize that the choice of origin will affect the quadratic function they write for #15. Have them explain their rationale for the location they chose before writing the quadratic function.

For #17, 18, and 20, students need to ignore friction due to air resistance. In these questions, students are given the location of the vertex, but both a horizontal and a vertical translation are involved. You might ask students:

- How can you determine the values of p and q?
- How can you determine the value of a?

For #19, students need to think critically about why horizontal translations seem to occur opposite to what they might expect. You might coach students by asking:

- For each case, in what order are the operations applied?
- If you add 4 *after* squaring a value, how does that affect the y-values for the corresponding x-values?
- If you add 4 *before* squaring a value, how do the x-values need to be different in order to get the corresponding y-values?

For #21b), students may benefit from coaching to locate the vertex. Ask students:

- How can you determine the location of the axis of symmetry if you know the x-intercepts?
- How does knowing the axis of symmetry help you develop the quadratic function?

Extend

For #22, encourage students to draw their own set of axes on grid paper in order to experiment with different trajectories. They may not realize that they need to consider only a few points on the path (i.e., the starting point, a point through the hoop, and a vertex that allows the path to work).

Create Connections

Have students work individually to develop a quadratic function for #24 and explain their personal strategy, and then compare their response with that of a classmate. Have them identify the similarities and differences in their personal strategies.

For #25, tell students to be specific when describing how they can determine the number of x-intercepts given a function in vertex form. Encourage them to think about all the possible cases.

For #26, which is a Mini Lab, students build on their understanding of quadratic functions by creating an illustration using quadratic and/or linear functions with restricted domains. Show students how to restrict a domain using their calculator. Some students may use only quadratic functions; others may use both quadratic and linear functions. Encourage them to use variety in their designs. Students who want to use linear functions may need a reminder of how to write equations of linear functions. Some students may ask to use other types of functions that they know about. Encourage them to include other types of functions if they can convey their understanding of the connection between the equation and the related graph for each one.

Project Corner

The Project Corner box provides information about physical objects that appear to have a parabolic shape. Encourage students to choose an object that might be modelled using quadratic functions. Have students brainstorm some possibilities. You may wish to have students investigate the difference between a parabolic curve and a catenary. Encourage them to explore the related Web Link at the end of this section.

Meeting Student Needs

- Encourage students to draw a rough sketch of a graph whenever appropriate to help them gain a visual understanding of the problem.
- For the Project Corner, encourage students to research activities such as hunting and fishing that may involve applications of quadratic functions. For instance, students might consider activities such as setting snares and using hunting weapons, such as bows and arrows. Alternatively, some students may be interested in researching games involving projectiles at sports or cultural events. They might research traditional games such as those involving slingshots, hatchet throwing, snowsnake, or longball. Students may find the related Web Link at the end of this section a helpful starting point.
- Have students refer to their own list of learning outcomes and assess their progress. Tell them to highlight in yellow each outcome addressed in this section. Once they have completed the Check Your Understanding questions, have them use a different colour to highlight the outcomes that they have no problem with. Have students identify the outcomes that remain highlighted in yellow and require more work. Provide any needed coaching.
- Provide **BLM 3–4 Section 3.1 Extra Practice** to students who would benefit from more practice.

ELL

- For #13, use the related Did You Know? to clarify the term *parabolic mirror* and explain its connection to the Olympic Games.
- For #20, use the related diagram and the Did You Know? to help clarify the terms *weightlessness*, *zero-g*, *manoeuvre*, *inverted parabolic arc*, and *two-g*.
- Some students may not be familiar with the following terms: *satellite dishes*, *headlights*, *spotlights*, *pedestrian*, *nozzle*, and *parabolic trajectory*. Use a combination of descriptions, visuals, and examples to assist in student understanding.

Enrichment

- Students may wish to research applications of parabolic functions such as the one shown in #13.
- Ask students to speculate how and why the following is possible: A pottery wheel spinning produces a circular shape in the clay and at the same time produces a parabolic shape. (There are two forces acting on the clay. Centrifugal force pushes the clay along the x-axis and the force of gravity attracts the clay along the y-axis. As the clay is spun farther from the centre of the spin, the centrifugal force component increases, whereas the force of gravity remains the same. This produces a vertical shape that is parabolic.)

Gifted

- For #17, challenge students to develop a sports question related to their cultural heritage.
- The shape of a wing that produces optimal aerodynamic lift is a parabola. The Wright brothers determined that the best place for the maximum chord was one quarter of the distance down the leading edge of the wing. Ask students to speculate what these criteria do to the quadratics involved. (The chord placement produces two parabolic portions on top of the wing. The leading edge has a greater change in slope than the edge that follows.)

- Challenge students to describe why a mathematical model of airflow over wings can be used to improve airplane design. Ask them to include an explanation of the connection to quadratic functions. (Students should show that quadratics can be used to produce functions that imitate the airflow over wings. This allows designers to experiment with changes in shape and understand the impact without actually building the wings. Designers can use this information to develop computer-generated designs that improve performance far beyond the trial-and-error methods of the pre-computer era.)

Web Link

For information about telescopes as an application of quadratic functions, go to www.mhrprecalc11.ca and follow the links.

For information about traditional games of Aboriginal peoples, go to www.mhrprecalc11.ca and follow the links

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 2a), b), 3a), c), 4, 6, 7, 8a) to c), 9, 11, and 21. Students who have no problems with these questions can go on to the remaining questions.	• You may wish to orally review the effect of *a* on a quadratic function in vertex form. Then complete #1 orally as a class. • You might complete #2, 3, and 7 orally as a class. The class discussion may benefit students who need reinforcement about the effect of each parameter on a quadratic function. • Reinforce the importance of labelling key points on sketches of graphs to clearly show understanding. • Encourage students to sketch a graph using the given information for each part of #9. The graph may help students determine the quadratic function and verify, for example, whether *a* is positive or negative. • For #21, it may be helpful for students to write the coordinates for the points and sketch a graph using the coordinates. Remind them to use the form $f(x) = ax^2$ as a starting point and compare it to the quadratic function they develop. Encourage them to graph $f(x) = ax^2$ and then graph the new quadratic function on the same graph. This will provide a visual image of the effects of the changes in the parameters.
Assessment *as* Learning	
Create Connections Have all students complete #24 and 25.	• For #24b), encourage students to explain their personal strategy for sketching a function to a classmate before writing their response. • For #25, some students may benefit from a visual of the graph for each number of *x*-intercepts before providing a rationale for determining the *x*-intercepts. Encourage them to draw a sketch of each possibility: two intercepts, one intercept, and zero intercepts, and then use the visuals to help them answer the question. Consider collecting students' responses to these questions and checking for understanding. Use the feedback to provide coaching to students, as needed.

Investigating Quadratic Functions in Standard Form

3.2

***Pre-Calculus 11*, pages 163–179**

Suggested Timing
100–120 min

Materials
- grid paper
- ruler
- calculator or graphing calculator
- graphing software (optional)
- 100-cm length of string (optional)

Blackline Masters
Master 2 Centimetre Grid Paper
Master 0.5 Centimetre Grid Paper
BLM 3–3 Chapter 3 Warm-Up
BLM 3–5 Section 3.2 Extra Practice
TM 3–1 How to Do Page 168 Example 2 Using TI-Nspire™
TM 3–2 How to Do Page 168 Example 2 Using TI-83/84
TM 3–3 How to Do Page 168 Example 2a) Using Microsoft® *Excel*
TM 3–4 How to Do Page 168 Example 2a) Using *The Geometer's Sketchpad®*
TM 3–5 How to Do Page 168 Example 2a) Using *GeoGebra*
TM 3–6 How to Do Page 174 #5a) Using TI-Nspire™
TM 3–7 How to Do Page 174 #5a) Using TI-83/84
TM 3–8 How to Do Page 179 #27 Using *The Geometer's Sketchpad®*
TM 3–9 How to Do Page 179 #27 Using *GeoGebra*

Mathematical Processes
- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcomes
RF4 Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including:
- vertex
- domain and range
- direction of opening
- axis of symmetry
- x- and y-intercepts

and to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 4a), b), 5a), b), 7, 9, 10, 12, 13, 26, 27
Typical	#1–8, 11, 12, four of 14–21, 26, 27
Extension/Enrichment	#15, 16, 20–26

Planning Notes

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce prerequisite skills needed for this section. If you have posted the outcomes, refer to the outcomes for this section.

As a class, read and talk about the situation of kicking a football into the air. Ask students questions such as the following:
- What might a sketch of the path of the ball after it leaves the punter's foot look like?
- How would the shape of the ball's path be affected by how the punter kicks the ball?

Explain that the path of a football after it is kicked into the air can be represented with a quadratic function. Check that your students connect with this idea at some level. Tell them they will explore this situation in this section.

Investigate Quadratic Functions in Standard Form

The purpose of this investigation is for students to develop an understanding of real-life situations that can be represented with quadratic functions. Students have had extensive experience in prior years working with linear functions, but this may be their first exposure to a function that is non-linear. In Part A of the investigation, students create graphs to represent the motion of a kicked football. You may wish to mention that the effects of air resistance are ignored for the purpose of this investigation. In Part B, they explore quadratic functions in standard form, and describe how the parameters a, b, and c affect the function. The objective of both parts of the investigation is for students to consider the characteristics of graphs of a quadratic function. A second objective is for students to observe a maximum or minimum value in a function, which their prior studies of linear functions did not necessarily allow them to do. Have students work in pairs to complete Part A. They will need grid paper.

As you circulate, students consider how they might create different paths for #2. Ask students:
- How high did the ball go? How far away from the kicker was it at that point?
- How far away was the ball at the end of its motion?
- What can you change to create different paths?
- What dimensions are appropriate for your graph?

When completed, invite student groups to share their response to the Reflect and Respond questions with the class.

Have students work individually or in pairs to complete Part B. They will need grid paper and graphing technology. Before they begin, direct students to the definition for *standard form*.

For #9, students may benefit from coaching to deal with the leading negative sign. Ask students:

- How does the leading negative sign affect the values?
- What two operations are involved here?
- In what order do you need to perform them?
- What do you notice about all the $f(x)$-values?
- How is the leading negative sign connected to the shape of the graph?

When completed, invite students to share their responses to the Reflect and Respond questions with the class.

Meeting Student Needs

- As an alternative to introduce the section, take students either to the gymnasium or outdoors and allow them to kick a football with different trajectories. It is not necessary to take any measurements; the activity allows students to visualize the motion of a projectile. Ask students:
  - – Is there an ultimate kick for both height and distance?
  - – What factor(s) would influence such a kick?
- You might invite a kicker from a football team to explain the complexities of his/her position. What factors are considered for each kick? What produces the ultimate kick?
- Allow students who may prefer to use graphing calculators for Part B to do so. Consider developing a handout with step-by-step instructions for graphing a quadratic function using a graphing calculator. This will allow students to quickly enter the different equations in the Investigate and then focus on the graphs. They can explore the process of graphing at a later time.

ELL

- Clarify the meaning of *hang time*. Explain that this term can also be applied to an athlete jumping or to a ball in the air in another sport. Ask students for examples of hang time that they are familiar with. Check that they understand what modelling the height of a ball as a function of horizontal distance downfield means. You might sketch the path of a ball through the air and mark off the horizontal distance along its path.
- Ensure that students add *standard form* (of a quadratic function) to their vocabulary dictionary. Encourage them to include a description, diagram, and/or example.

Enrichment

- Invite students to conduct a virtual experiment that involves throwing a pumpkin, tracking its trajectory, and fitting a quadratic function to the curve. Direct students to the Web Link at the end of this section.
- Invite students to use an online simulation of projectile motion and trajectories to explore how variables such as initial velocity, angle of the launch, and mass of an object affect the graph of a quadratic function.

Gifted

- Challenge students to research siege-warfare tactics used in medieval times, such as building and deploying trebuchets. Have them sketch a graph of a trebuchet in motion (e.g., velocity as a function of time) and explain the meaning of the graph to the class. They may find the related Web Link at the end of this section helpful.

Common Errors

- Some students may calculate values for $f(x) = ax^2 + 4x + 5$ incorrectly, either because they do not understand how to do this or they use a calculator incorrectly.

R$_x$ Remind students that the result is always positive when squaring a number, regardless of whether the number is positive or negative. Demonstrate using brackets with negative numbers when using a calculator.

- Some students may calculate values for $f(x) = -x^2 + bx + 5$ incorrectly for different reasons. These reasons include incorrectly substituting values, multiplying the leading negative sign before squaring, or using their calculator incorrectly (not using brackets, if necessary).

R$_x$ Remind students of the correct order of operations. Explain that $-x^2$ is similar to $-1x^2$ or $-1(x)^2$, and that they need to square the value of x before applying the leading negative sign.

Web Link

For an applet that traces the path of a thrown pumpkin, go to www.mhrprecalc11.ca and follow the links. You will need to download the GeoGebra application first. Students use sliders to determine a quadratic function in vertex form that approximates the path of the pumpkin as closely as possible.

For online simulations of projectile motion and trajectories, go to www.mhrprecalc11.ca and follow the links. The applets allow users to change variables, such as initial velocity, the angle of the launch, and the mass of the object.

Web Link

For a guided investigation about the characteristics of graphs of quadratic functions, go to www.mhrprecalc11.ca and follow the links. Using a ball toss simulator, users explore the relationship between the ball's height and its time in the air.

For information about the trebuchet and a graph showing velocity in relation to time, go to www.mhrprecalc11.ca and follow the links.

Answers

Investigate Quadratic Functions in Standard Forms

1. The football is located at different heights above the ground at different horizontal distances.

3. All sketches of the flight path curve upward and show increases in height until the football reaches the top of its path. Then the sketches curve downward and show decreases in height as the football travels toward the ground. Students should observe similar shapes in other students' graphs.

4. All graphs are smooth curves that show an increase in the football's height until the maximum height and then a decrease in height until the football reaches the ground (x-axis).

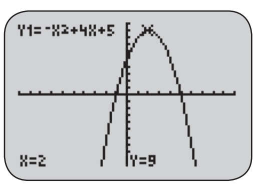
5. The highest point on the graph is the maximum and the lowest points on the graph are the minimums.

6. The graphs are symmetrical about a line drawn from the maximum point down to the x-axis.

7. The domain is $x \geq 0$ and less than or equal to the horizontal distance of the football. The range is $y \geq 0$ and less than or equal to the maximum height of the football.

8. The domain has a maximum value equal to the horizontal distance that the football travels. The range has a maximum value equal to the maximum height of the football.

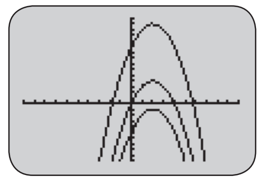
9.



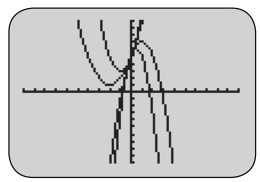
10. The graph is symmetrical about a line drawn at $x = 2$ from the maximum point downward.

11. The maximum y-value is 9. There is no minimum y-value as the graph continues downward.

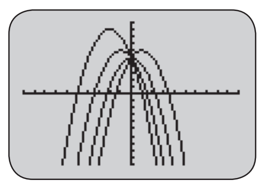
12.



13.



14.



15. Example: Parameter c determines the y-intercept.

16. Example: Parameter a determines the shape and whether the graph opens upward (positive a) or downward (negative a).

17. Example: Parameter b influences the position of the graph.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions with a partner or individually. Listen as students discuss what they learned during the investigation. Encourage them to generalize and reach a conclusion about their findings.	• Encourage student pairs to use the terms *quadratic function*, *standard form*, *symmetry*, *maximum value*, and *minimum value* as they work through the Investigate. • Students may benefit from working with another student pair to compare their graphs. • Students may benefit from reactivating their knowledge of the terms *maximum*, *minimum*, *symmetry*, *domain*, and *range* before completing Reflect and Respond #5 to 7. • For Part B, encourage students to draw the graphs on one sheet of paper to help them observe the similarities and differences.

Link the Ideas

Give students enough time to work through this section, which will clarify and reinforce the conclusions they were able to draw about the effects of a, b, and c in quadratic functions during the investigation.

Begin by having students consider the definition of *transformation* and the types of transformations that they are familiar with. As they consider the effect of each parameter, encourage them to look for connections between the changes in the graph and the equation of each function relative to $f(x) = x^2$. You might ask questions such as the following:

- How does the graph of each function (for each parameter) compare to the graph of $f(x) = x^2$?
- What relationship do you observe between the parameter and the location of the corresponding graph?
- What is the effect of parameter a in $f(x) = ax^2$ on the graph of $f(x) = x^2$?
- What is the effect of parameter b in $f(x) = x^2 + bx$ on the graph of $f(x) = x^2$?
- What is the effect of parameter c in $f(x) = x^2 + c$ on the graph of $f(x) = x^2$?

Encourage students to record their own summary of the Link the Ideas.

Example 1

This example gives students an opportunity to use and reinforce the terms introduced in this section. In addition, students identify characteristics of graphs of quadratic functions in standard form and look for connections between the values of the various characteristics. You might use prompts such as the following to promote student thinking and discussion:

- How can you tell from the equation whether the graph opens upward or downward?
- How is the sign of a in the function $f(x) = ax^2 + bx + c$ related to the direction of opening?
- What connection is there between the equation of the axis of symmetry and the coordinates of the vertex?
- How could you determine the x-coordinate of the vertex just from the x-intercepts?
- If you knew only the location of the vertex and the direction of opening, what other information could you determine?
- How are the locations of the axis of symmetry and the x-intercepts connected?
- How do you determine the x-intercepts and the y-intercepts of a function?
- How can you identify the y-intercept from the equation in standard form?
- Why is the domain the set of all real numbers when only some x- and y-intercepts are plotted on the graph?
- How is the range related to the direction of opening?
- How is the range related to whether the function has a maximum or minimum value?

Have students complete the Your Turn individually and compare their solution with that of a classmate. As students work, encourage them to use correct notation when identifying the characteristics of each graph. Students should write an equation for the axis of symmetry instead of identifying only the x-value, identify the coordinates of the point for intercepts rather than only the value, and use proper set notation for domain and range.

Example 2

This example allows students to analyse a quadratic function by focussing on the meaning of the various features of its related graph. The equation of the quadratic function representing a real-world situation is given. Many students will interpret the graph as a representation of the actual path the frog takes. Reinforce that the function compares the height of the frog to the time that elapses after it jumps (not height to horizontal distance). In order to help students understand how the various characteristics of the graph relate to the situation, use coaching questions such as the following:

- How is the shape of the graph connected to the situation?
- What do the coordinates of the vertex represent?
- What are the units of measure of h and t?
- Which quadrant on a coordinate grid represents this situation? Why are the other three quadrants not needed?
- Why is the domain not all real numbers in this situation?
- This function has a maximum value. Does it have a minimum value? Explain.

For part b), you might show how to determine the y-intercept of the function by substituting 0 for t in $h(t) = -490t^2 + 150t + 25$.

$$h(t) = -490t^2 + 150t + 25$$
$$h(0) = -490(0)^2 + 150(0) + 25$$
$$h(0) = 0 + 0 + 25$$
$$h(0) = 25$$

When 0 is substituted into the function, all terms except the constant term have a value of zero. The y-intercept of $h(t) = -490t^2 + 150t + 25$ is equal to the value of the constant term, 25.

For part e), you might explain that the function in this situation applies only for times from the moment the frog jumps at 0 s to when it contacts the water at approximately 0.4 s. During this time interval, the highest the frog goes is approximately 36.5 cm and the lowest is 0 cm.

This example models using a graphing calculator and a spreadsheet to solve the problem. You might walk through the solution using graphing software as a class. Consider inviting students to use graphing software after they have worked through the solution using one of the methods shown in the student resource. You may wish to have students work through the entire example using **TM 3–1 How to Do Page 168 Example 2 Using TI-Nspire™** or **TM 3–2 How to Do Page 168 Example 2 Using TI-83/84**. Alternatively, you may wish to refer students to **TM 3–3 How to Do Page 168 Example 2a) Using Microsoft® *Excel***, **TM 3–4 How to Do Page 168 Example 2a) Using *The Geometer's Sketchpad*®** or **TM 3–5 How to Do Page 168 Example 2a) Using *GeoGebra*** to work through part a) only. These masters show students how to create a graph of a quadratic function using technology.

Have students complete the Your Turn individually and compare solutions with a classmate. Encourage students to be specific and fully explain the connection between the characteristics of the graph and the situation.

Example 3

This example allows students to consider how a quadratic function can be used to model a real-world situation. Many students may not realize that the area of a rectangle of a fixed perimeter changes as the dimensions are altered. To help them visualize this, you might use a 100-cm length of string and have two volunteers hold the string in the shape of a rectangle using their hands. Ask them to move their hands to change the dimensions and then have the class observe how the area changes.

In order to clarify understanding, students may benefit from walking through the process of generating the equation step by step. Consider using prompts such as the following:
- How do you calculate the area of a rectangle?
- If the perimeter is a fixed value, how are the length and width related?
- Why do you eliminate one variable in order to create a function?
- How could you eliminate the width instead of the length? What would be the resulting function?
- Why do the area values increase from zero and then decrease back to zero?
- Although 0 and 50 are theoretically possible, can these values actually be used? Explain.

For part b), discuss that when the width is 25 m, the area is at its maximum value of 25×25, or 625 m^2. The maximum area occurs when the length and width are the same, that is, the special case in which the rectangle is a square.

You may wish to have students use a graphing calculator to work through the solution.

Key Ideas

Before reading the Key Ideas, ask students to identify the characteristics of graphs of quadratic functions. Encourage them to draw their own graph and label it using appropriate mathematical terminology. Then, have them compare their summary with the Key Ideas. Have them revise their summary to reflect any concepts they missed.

You might use the Key Ideas to help build the chapter review. Have students store their summary for each section in the same location in order to create a reference for review purposes. Consider allowing students to use their summary for the chapter test.

Meeting Student Needs

- For the worked examples, consider dividing students into pairs or small groups and give each group at least one graphing calculator and grid paper. As they work through the examples, have them create their own sketch of each graph. Encourage discussion of the key points within each group. Then, have students work through the Key Ideas as a whole class.
- You might have students use the software described in the Web Link at the end of this section to generate different quadratic functions. Have students work individually or as a class. As a class, consider using a SMART Board or a projector and a computer with Internet access to demonstrate the applet.
- For Key Ideas, students may benefit from creating a poster illustrating the characteristics of a graph of a quadratic function in standard form. Display the poster in the classroom to serve as a visual reference.

ELL

- For Example 2: Your Turn, check that students understand the terms *diver*, *springboard*, and *velocity*.
- For Example 3, use the photograph in the student resource to help explain the terms *rancher*, *fencing*, and *corral*. For the Your Turn, clarify *stroller parking*.

Enrichment

- Invite students to explore free online graphing software and use software of their choice to work through the solution for Example 2.

Gifted

- Challenge students to create an applet that allows them to match a curve to any three given points. This will provide an opportunity for students to discover that just as only one straight line passes through any two given points, only one parabola passes through any three given points. They might use free software called *GeoGebra* to do this.

Common Errors

- Some students may not write an equation for the axis of symmetry, and may provide only the x-value.

R_x Reinforce how to identify horizontal or vertical lines using an equation. Ask them to consider the coordinates of points anywhere on a vertical line and help them realize that the x-coordinate does not change; it is the y-intercept that changes. This justifies the form of the equation of a vertical line.

- Some students may not realize that the maximum or minimum value of a function refers to the y-value.

R_x Remind students that the value of a function refers to the y-value. Help them realize that the y-values increase (or decrease) until they reach the maximum (or minimum), but then begin to decrease (or increase).

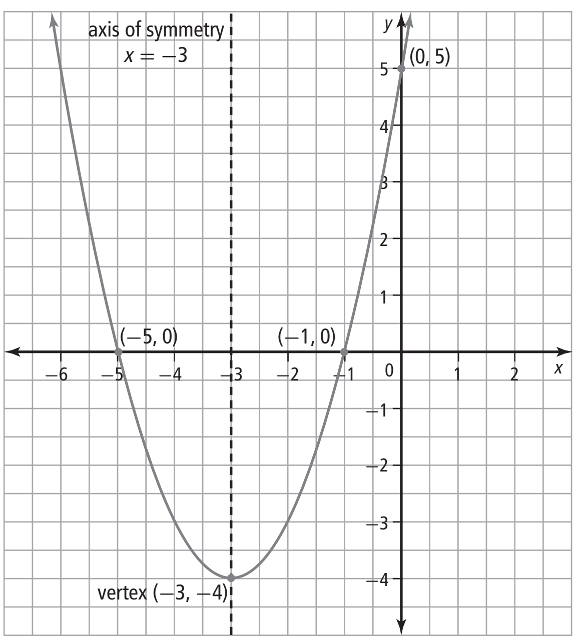
Web Link

For an applet that generates different quadratic functions, go to www.mhrprecalc11.ca and follow the links. You will need to download the GeoGebra application first. The user enters the axis of symmetry, the vertex, and some ordered pairs from the table of values and then clicks on Draw Graph to check how close the estimates are.

Answers

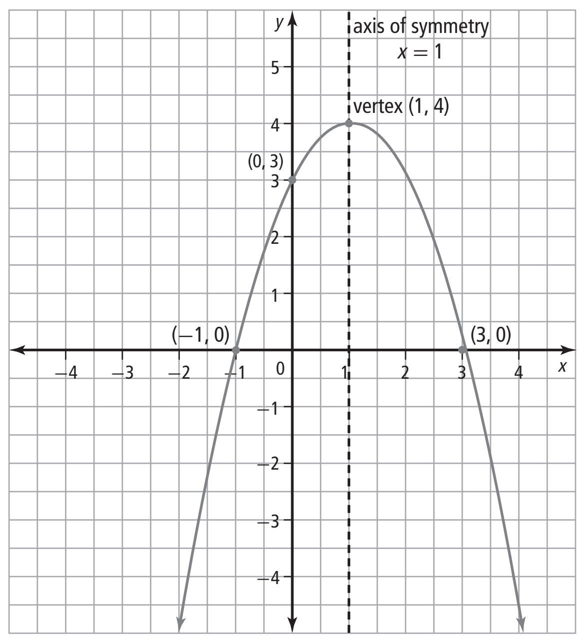
Example 1: Your Turn

a) $y = x^2 + 6x + 5$



The graph opens upward. The coordinates of the vertex are $(-3, -4)$. The graph has a minimum value of -4 when x is -3. The equation of the axis of symmetry is $x = -3$. The x-intercepts occur at $(-5, 0)$ and $(-1, 0)$, and have values of -5 and -1 respectively. The y-intercept occurs at $(0, 5)$ and has a value of 5. The domain is the set of all real numbers or $\{x \mid x \in R\}$. The range is the set of real numbers where y is greater than or equal to -4, or $\{y \mid y \geq -4, y \in R\}$.

b) $y = -x^2 + 2x + 3$

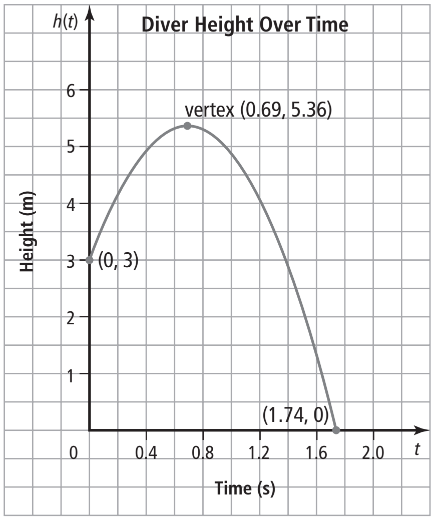


The graph opens downward. The coordinates of the vertex are $(1, 4)$. The graph has a maximum value of 4 when x is 1. The equation of the axis of symmetry is $x = 1$. The x-intercepts occur at $(-1, 0)$ and $(3, 0)$ and have values of -1 and 3 respectively. The y-intercept occurs at $(0, 3)$ and has a value of 3. The domain is the set of all real numbers or $\{x \mid x \in R\}$. The range is the set of real numbers where y is less than or equal to 4, or $\{y \mid y \leq 4, y \in R\}$.

Answers

Example 2: Your Turn

a)



b) height of the diving board

c) The diver achieves a maximum height of 5.36 m, 0.69 s after leaving the board.

d) The diver contacts the water 1.74 s after leaving the diving board.

e) domain $\{t \mid 0 \le t \le 1.74, t \in \mathrm{R}\}$; range $\{h \mid 0 \le h \le 5.36, h \in \mathrm{R}\}$

f) $h(0.6) = -4.9(0.6)^2 + 6.8(0.6) + 3 = 5.32$. The diver has a height of 5.32 m, 0.6 s after leaving the board.

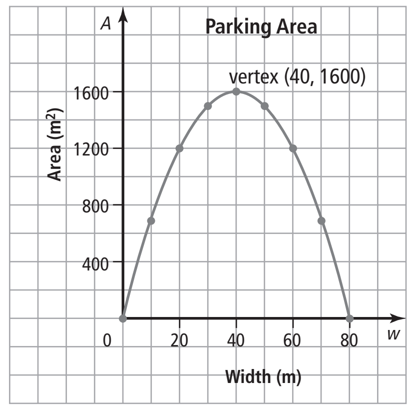
Example 3: Your Turn

a) Since $P = 2l + 2w$ and $P = 160$ m, then $l = 80 - w$ and $A = -w^2 + 80w$.

$$\begin{aligned} 160 &= 2l + 2w \\ 160 &= 2(l + w) \\ 80 &= l + w \\ l &= 80 - w \end{aligned} \qquad \begin{aligned} A &= lw \\ A &= (80 - w)w \\ A &= 80w - w^2 \end{aligned}$$

b) Use $x = \frac{-b}{2a}$ to find the x-coordinate of the vertex. Then, substitute that value into the function to find the y-coordinate of the vertex. The vertex is located at (40, 1600). The vertex represents the maximum area of 1600 m^2 when the dimensions of the stroller park are 40 m by 40 m.

c)



d) domain $\{w \mid 0 \le w \le 80, w \in \mathrm{R}\}$; range $\{A \mid 0 \le A \le 1600, A \in \mathrm{R}\}$

e) Assumptions include:
- All 160 m of rope is used to rope off the stroller park.
- Any length or width from 0 m to 80 m is possible.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Reinforce the terms associated with the characteristics of a quadratic function by having students verbally explain the meaning and how to determine each one before solving the Your Turn.
Example 2 Have students do the Your Turn related to Example 2.	• Some students may benefit from recalling the steps in using technology to solve the problem. • Ensure that students can explain why the part of the graph to the left of the *y*-axis is not included in the solution.
Example 3 Have students do the Your Turn related to Example 3.	• Some students may benefit from working in pairs to use technology. • Some students may benefit from recalling the steps in using technology to solve the problem. • Encourage students to recall using the distributive property to simplify polynomial expressions.

Check Your Understanding

Practise

The practice questions allow students to reinforce their understanding of the characteristics of graphs of quadratic functions. Encourage students to use correct mathematical terminology when identifying the characteristics. Students can complete these questions individually or with a partner. After they complete a question, remind students to check the answer. In cases where there is a difference between their answer and the one in the answer key, encourage them to get help from a classmate or from you.

For #3, students may benefit from recalling how to write a function in standard form. Use prompts such as the following:

- What form of the function is standard form?
- What algebraic operations could you use to change the given function?

For #5, students may need to be reminded of various ways to sketch a graph, such as setting up a table of values using a spreadsheet, and using a graphing calculator. Students may benefit from help to set the dimensions of the graph when using technology. Coach them to first change the horizontal dimension so that it is appropriate for seeing both x-intercepts, and then change the vertical dimension in order to see the vertex.

For #7, instruct students to be specific when describing how the various characteristics of the graph connect with the situation.

For students who find #8 challenging, encourage them to sketch a graph of each quadratic function using the given information, and then use the sketch to answer the question.

For #9, students make the connection between the domain and range of a function with no context and the restriction on the domain and range of a function with a context. You might ask them to graph the function and then highlight the part of the graph that applies when considered in the context of a ball kicked into the air.

Apply

These questions provide opportunities for students to use their understanding of quadratic functions in different situations. Consider allowing students to work in pairs to solve some of the problems. Alternatively, ask students to work independently on one or two problems at a time and then compare their solutions with a classmate.

Consider advising students to add the given points/characteristics to the grid one at a time before trying to sketch a function for each part of #10.

For #11a), students may not realize that they can determine the domain of the function without seeing the graph by considering the given information. Ask students:

- What does x represent in the situation?
- What are some possible values of x given the dimensions of the dish?

Students need to recognize the domain in order to sketch a graph that shows both x-intercepts. They may find it helpful to use technology for solving the problem.

To complete #12, encourage students to use technology. They may not initially realize that the times involved are small in value and may need to adjust the dimensions several times before creating a graph that shows all of the required information.

The situation in #13 involves the force of drag on a moving object. Help students understand drag force by asking questions such as the following:

- How does drag force (or wind resistance) change as speed increases?
- What does the graph show about the relationship between speed and drag force?

For part e), students should understand that as the speed of a vehicle increases, the force of drag increases in a quadratic rather than a linear fashion. Consequently, the force of drag can be quite significant at high speeds and make a vehicle less fuel efficient.

For #14, students may benefit from coaching to set up appropriate dimensions for the graph. Encourage them to use technology. Use prompts such as the following:

- How can you use the constant in the function to help plan dimensions for the graph?
- Why are there no horizontal intercepts in this situation? What would it mean if there were?

For each of #15 and 17, students write a quadratic function for area. Encourage students to draw their own diagram and label the length and width. You might provide coaching using prompts such as the following:

- What is the formula for the area of a rectangle?
- What equation involving perimeter relates length and width?
- How can you use the equation to eliminate one variable so that you have a function of width only?

For #16, students may choose to use their algebraic skills to convert the given form of the function to standard form. You might point out that they could graph the function in its given form to see the nature of the graph that results. They might then compare this graph to one in standard form.

For #18, you may need to remind students about the difference between discrete and continuous values. Use prompts such as the following:
- What types of values are possible for *n*? Is any value possible or only a certain type?
- What are the possible values for the range?
- How might you express the domain and range in this situation?

Students may not realize that the function in the situation for #19 is one that they are already familiar with. Ask students:
- How is the area of a circle related to its radius?
- What formula for the area of a circle do you know?
- What quadrant of a grid would contain the graph for this situation?

For #20, students consider stopping distance for a vehicle, which depends on reaction time and braking distance. Some students may find the formula challenging at first, but once they substitute the given values, they may find the resulting quadratic function to be simpler. Some students may be familiar with using unit analysis to check the use of units in a formula and be concerned that the units for each substituted value involve different units of measurement for distance and time (i.e., metres, kilometres, hours, seconds). Let them know that the constant values involved in the formula have accounted for the conversion factors between units. For part d), encourage students to discuss with a classmate what the pattern in stopping distances shows about safe driving. You might use prompts such as the following:
- How do stopping distances at high speeds compare to stopping distances at low speeds?
- Does reaction time increase at higher speeds?

Extend

These questions require students to extend their knowledge of quadratic functions by using new and previously learned skills and processes to solve problems.

For #21, students may benefit from using graphing software that allows them to create a variable with a slider for the value of *k*. Doing so lets students dynamically observe the effect of *k*. You might have students explore other families of functions, such as adding a constant *k* to a function in contrast to multiplying a constant *k*. Consider allowing students to work in pairs to solve the problem.

For #22, students consider slope and compare the effect of the leading coefficient in quadratic functions and linear functions. Consider using prompts such as the following:
- What effect does *a* being positive or negative have on the graph of a quadratic function? a linear function?
- For positive values of *a*, what happens to the graphs of quadratic functions as the value of *a* increases or decreases? How does this compare with what happens to linear functions?

Students need to realize that they can solve #23a) by substituting the given point and solving for *b*. In #23b), students may not realize that they can use each point to create an equation that involves *b* and *c*, and that the two equations can be solved simultaneously as a linear system. Consider using prompts such as the following:
- You know that you can solve an equation that has only one variable. Can you solve an equation that has two variables? In this latter case, how many equations do you need in order to solve for two variables?
- What methods can you use to solve a system of two linear equations?

Direct students to use the related Did You Know? to help solve #24. They should recognize that in the second situation the initial height is zero, and in the third situation the initial velocity is zero. Have students compare the graphs for each situation on Earth and the moon. Allow students to work in small groups to solve the problem.

Students may find it helpful to sketch a set of axes without a scale for each situation in #25.

Create Connections

For #26, students consider the relationships between the various characteristics of graphs of quadratic functions. You might use the following prompts to coach students:
- If you know only the vertex and direction of opening of a quadratic function, what other information can you identify?
- If you know the *x*-intercepts and maximum or minimum value of a quadratic function, what other information can you identify?

For #27, which is a Mini Lab, students might work on their own or in pairs. Have students use graphing or geometry software that allows them to create sliders for the coefficients *a*, *b*, and *c*. Let students know that the purpose is not necessarily to find answers, but rather to investigate, observe, and make connections that they may not have made earlier in the section. It is not critical for students to make all of the same discoveries. Encourage them to use mathematical terminology appropriately as they describe how each coefficient affects the graph.

Some students will already know what effect c has on the graph and/or how a positive or negative value of a causes the graph to open upward or downward. Whether or not students have made these connections earlier, the Mini Lab activity provides an opportunity for them to strengthen their understanding by seeing these connections in a dynamic way.

Students may have difficulty in seeing how b affects the graph. Step 4 of the Mini Lab might help them note a difference in how a and b affect the graph.

Meeting Student Needs

- As students work on #5, you may wish to have them use **TM 3–6 How to Do Page 174 #5a) Using TI-Nspire™** or **TM 3–7 How to Do Page 174 #5a) Using TI-83/84.**
- As students work on #27, you may wish to provide **TM 3–8 How to Do Page 179 #27 Using *The Geometer's Sketchpad*®** or **TM 3–9 How to Do Page 179 #27 Using *GeoGebra*.**
- Have students create two graphic organizers: one for $y = ax^2 + bx + c$ and one for $y = a(x - p)^2 + q$. Have students extend branches from each equation, and write a summary for the values of a, b, and c or a, p, and q. Invite them to add any other pertinent information.
- You might challenge students who have completed #24 about projectiles to consider what the three coefficients a, b, and c (from #27) represent in the context of projectile motion. Have them consider how changes to these coefficients affect the motion of a projectile, and how these changes would be reflected in the graph of projectile motion.
- Consider assigning two quadratic equations to each student pair. One quadratic should open upward; the other should open downward. Have students sketch the quadratic functions for these equations. For each sketch, have them label the vertex, axis of symmetry, x-intercepts, y-intercept, and minimum or maximum value. Ask them to determine the relationships between the values of a and b and the vertex and axis of symmetry, and the relationship between the value of c and the y-intercept.
- Invite students who developed a presentation about Key Ideas in section 3.1 to incorporate content from the section 3.2 Key Ideas at this time.
- Have students refer to their own list of learning outcomes for the chapter and assess their progress. Tell them to highlight in yellow each outcome addressed in this section. Once they have completed the Check Your Understanding questions, have them use a different colour to highlight the outcomes that they have no problem with. Have students identify the outcomes that remain highlighted in yellow and require more work. Provide any needed coaching.
- Provide **BLM 3–5 Section 3.2 Extra Practice** to students who would benefit from more practice.

ELL

- Have students record unfamiliar terms in their math vocabulary dictionary using a combination of written descriptions, visuals, and examples.
- For #11, use the photograph to clarify what a *satellite dish antenna* is.
- For #12, use the Did You Know? to clarify what a jumping spider is.
- For #13, use the Did You Know? to clarify the term *newton*.
- For #24, explain that a projectile is any object that travels through the air that has no capacity for self-propulsion. As long as any force such as wind resistance can be ignored, the only force acting on a projectile after its release is gravity. Use the Did You Know? to clarify the term *acceleration due to gravity*.

Enrichment

- Challenge students to investigate how all three terms in a quadratic function written in standard form contribute to the value of the function as the value of x changes. Consider the function $f(x) = -x^2 + 1000x + 100\ 000$. This function is a quadratic with three terms. The value of the function is the sum of the terms. Depending on the value of x, one term or another may be the one that contributes most to the value of the function. Create a spreadsheet to investigate the values of each of the three terms separately as well as the value of the function.
  - Which term is the most significant in the function's value for small positive values of x?
  - As x is increased from 0, is the same term always the most significant in contributing to the value of the function, or is there a change? Explain.
  - What happens if the spreadsheet is filled down for thousands of rows or more? Which term ends up becoming the most significant, or *dominant* term for larger and larger values of x?
  - Explain why you think this pattern occurs.

  The intent of this question is for students to realize that for smaller values of x, each term contributes to the value of the function, but that for greater values of x, the x-term and the constant term become less and less significant.

- Have students make connections between quadratic functions and linear functions. Looking at first differences can be used to determine whether a given set of data represents a quadratic function. The first differences for a table of values are the differences between consecutive values of y for evenly spaced values of x.
  – Copy and complete the table to determine the first differences for the linear function $f(x) = 3x - 2$. What pattern do you notice?

x	y	First Differences
−3	−11	
−2	−8	−8 − (−11) = 3
−1		
0		
1		
2		
3		

  – Analyse the first differences for $f(x) = -2x + 7$ and at least one other linear function of your choice.
  – What seems to be true about first differences for linear functions?
  – The second differences are the differences between the first differences. Copy and complete the table for the function $f(x) = x^2$.

x	y	First Differences	Second Differences
−3	9		
−2	4	4 − 9 = −5	
−1	1	1 − 4 = −3	−3 − (−5) = 2
0			
1			
2			
3			

  – Analyse the first and second differences for the functions $f(x) = x^2 + 2x + 5$, $f(x) = 2x^2 + 4x - 3$, and at least one other quadratic function of your choice.
  – What seems to be true about first differences for a quadratic function?
  – What seems to be true about the second differences?
  – How could you use first and second differences to decide if a given set of points represents a quadratic function, a linear function, or neither.
- Encourage students to explore applications of quadratic functions that are of personal interest. They may find the related Web Link at the end of this section useful. Ask them to report their findings to the class.
- Challenge students to explore one of the following topics:
  – Research the design of the perfect stage and audience seating area for a music concert.
  – Explore the shape of a satellite dish to understand the optimum placement of the receiver within the parabolic shape.

  Have students present their findings to the class using a format of their choice.
- The path of a projectile, such as a kicked soccer ball, is theoretically a parabola. Challenge students to sketch the predicted path of the soccer ball and explain what factors might influence the experimental performance of a kicked ball compared to the theoretical path. (Since air resistance slows down the horizontal speed of the ball during its flight, the path of the ball is not parabolic. The shape of the curve before the maximum height is relatively elongated because the horizontal distance covered per time segment is greater due to the greater speed. As the ball slows, the horizontal distance covered becomes less, and thus the curve becomes steeper.)

Gifted

- The formula for the force of drag F_d is proportional to $-\frac{1}{2}v^2$. Have students brainstorm what other factors enter into the relationship that are reflected in the drag equation developed from the proportion. Have them research Lord Rayleigh for an explanation or check out the related Web Link at the end of this section. (Other factors include the density of fluid and the shape of the object.)
- Challenge students to explore vertices and intercepts of parabolas in standard form, vector form, and factored form. They may find the related Web Link at the end of this section interesting.

Common Errors

- Some students may not know how to choose appropriate dimensions for graphs of contextual situations.

R_x Encourage students to consider what values are possible for the given situation. They may need a reminder that negative values are not possible in many situations. If using technology, remind students that they can try one set of dimensions as a starting point and then adjust as necessary.

Web Link

For online graphing software, go to www.mhrprecalc11.ca and follow the links.

For an applet that allows students to investigate changes in the graphs of quadratic functions in standard form, go to www.mhrprecalc11.ca and follow the links.

For information about the history of quadratic functions, go to www.mhrprecalc11.ca and follow the links.

For information about wind resistance, go to www.mhrprecalc11.ca and follow the links.

For applets set up to show vertex and intercepts of parabolas in standard form, vector form, and factored form, go to www.mhrprecalc11.ca and follow the links. You will need to download the GeoGebra application first.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1 to 3, 4a), b), 5a), b), 7, 9, 10, 12, and 13. Students who have no problems with these questions can go on to the remaining questions.	• For #1, have students orally explain what makes a function quadratic before proceeding. • For #2, ensure that students can identify each given characteristic of the graph of a quadratic function. Encourage students to refer to their summary of the Key Ideas. Provide additional coaching to students who need it. • For #3, have students refer to their definition of standard form. • For #4, circulate and check that students can sketch the graph and determine each given characteristic on the graph. Encourage students to refer to the worked examples if they are unsure. Provide additional coaching to students who need it. • For #7, encourage students to refer to Example 3 to help them solve a contextual problem. Have them discuss the solution for #7e) as a class. • For all questions, encourage students to refer to their notes and posters displayed in the classroom for the mathematical terminology related to quadratic functions. Reinforce the importance of using terminology correctly. • Consider using #10 for assessment *for* learning purposes since it requires students to show their understanding of all the characteristics of a graph of a quadratic function. Have students compare the solution with a classmate or take up the solution as a class before having students move on to #12 and 13.
Assessment *as* Learning	
Create Connections Have all students complete #26 and 27.	• For #26, some students may benefit from sketching a graph for a quadratic function and using their sketch to help explain the connections between the various characteristics. Remind students to address each given term. • For #27, it may be beneficial to have student pairs present the results of the Mini Lab on the board. Encourage them to use a sketch for each step to illustrate their findings. Have students look for similarities and differences in each other's results. Working in pairs, have students identify the characteristics of the graph of a quadratic function.

Completing the Square

3.3

***Pre-Calculus 11*, pages 180–197**

Suggested Timing

100–150 min

Materials

- grid paper
- ruler
- graphing calculator
- graphing software (optional)
- algebra tiles

Blackline Masters

Master 2 Centimetre Grid Paper
Master 3 0.5 Centimetre Grid Paper
Master 4 Algebra Tiles (Positive Tiles)
Master 5 Algebra Tiles (Negative Tiles)
BLM 3–3 Chapter 3 Warm-Up
BLM 3–6 Section 3.3 Extra Practice

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcomes

RF4 Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including:
- vertex
- domain and range
- direction of opening
- axis of symmetry
- x- and y-intercepts

and to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), b), 2a), b), 3, 4a), b), 5a), d), 7a)–c), 8a), b), 9, 12, 14, 29, 30
Typical	#1, 3, 5–12, two of 13, 16, 17, three of 18–24, 29, 30
Extension/Enrichment	#12, 17, 23–31

Planning Notes

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce prerequisite skills needed for this section. If you have posted the outcomes, refer to the outcomes for this section.

As a class, read the information about the craft fair. Ask students why it is important for a seller to think carefully about the optimum price for sale items at a fundraiser. Invite students to describe any experience they may have had with deciding on prices for items at a fundraiser.

Investigate Completing the Square

In Part A of this investigation, students use quadratic functions that model how increasing the price of a sale item affects sales.

Students need grid paper or graphing technology to complete Part A. For #1, students complete the table that models how revenue changes as prices increase. Some students may benefit from coaching to understand how the values in the table are related. Use prompts such as the following:

- How can you calculate revenue if you know the price of an item and the number sold?
- How do you expect the revenue to change as the price increases?
- What price gives the maximum revenue? How do you know?

For #4, some students may not recall that a function in vertex form gives the maximum value, and when this value occurs, without graphing. Ask students which form of a function gives the vertex without graphing.

For Reflect and Respond #6a), have students consider what a linear function would mean if it were used to model the situation. (Revenue continues to increase as price increases. The same number of people buy regardless of price). For #6b), students may benefit from some coaching as they consider the assumptions that are being made in using this model. Ask students:

- Is price the only factor that might affect total sales? What other factors might affect sales?
- What might be different this year compared to previous years, other than the sale price? How would these factors affect sales?

When completed, invite students to explain their response to Reflect and Respond #6 to the class.

Before students begin Part B, have them refer back to the quadratic functions in Part A. Reinforce that despite the difference in appearance of the two forms, the forms represent the same function. Ask students how they can convert from vertex to standard form and from standard to vertex form.

In Part B of this investigation, students use algebra tiles to model the relationship between perfect square trinomials and binomial squares. Provide students with algebra tiles or **Master 4 Algebra Tiles (Positive Tiles)** and **Master 5 Algebra Tiles (Negative Tiles)**.

For #8 and 9, encourage students to repeat the activity in #7 until they understand the pattern. Some students may not realize that positive tiles are needed to complete the square, even when the x-term has a negative coefficient. Coach students by asking them:

- What are the dimensions of this incomplete square (as you refer to a particular incomplete square) when modelled with tiles?
- To make the dimensions match the square, what type of tile do you need to complete the square? Does this tile seem opposite to what you might have predicted? Explain.

As students begin to make sense of the pattern, you might ask them to explain how they would model an incomplete square such as $x^2 + 24x$ or $x^2 + 32x$. It will benefit students if they can internalize the pattern in perfect square trinomials before learning the process of completing the square in the worked examples.

Invite volunteers to explain their response to Reflect and Respond #11 and 12 to the class. You might challenge students to explain how they think their work during the investigation relates to what they will do in this section. You might hint that one purpose of this section is to learn how to change the form of quadratic functions.

Meeting Student Needs

- For the opener, ask students to suggest situations that they know about in which a maximum or minimum value may be needed.
- For Part A, some students may benefit from using base 10 blocks and money manipulatives as a concrete model. Have students use two groups: one group has 14 blocks; the other group has 14 groups of $\$40 = \$560$. Record the total revenue. Remove one block and put 13 groups of \$45 in the other. Record total revenue. Remove one more block and put 12 groups of \$50 in the other. Record total revenue. Remove one block and put 11 groups of \$55 in the other. Record total revenue. Have students repeat the process until they observe a pattern. Then have them complete the investigation.
- It may be helpful for students to refer back to the definition of standard form in section 3.2 and then compare standard form and vertex form. Consider asking them to convert a function in vertex form to standard form as shown in the section 3.2 Link the Ideas. Help students realize that the forms represent the same function.
- For Part B, provide pairs of students with algebra tiles. Ensure that students take turns to use the algebra tiles.
- Have students participate in a whole class discussion of the learning gained from the investigation. Have students write three to five sentences summarizing the key ideas in their math journal.

ELL

- Use the Did You Know? on page 181 in the student resource to help explain the meaning of *mukluk*. You may wish to point out that specific designs and the materials used to make mukluks are unique to each region.

Common Errors

- Some students may use negative tiles to complete an incomplete square when the x-tiles are negative.

R$_x$ Have students identify the relationship between the dimensions of a square and the area of the square. Remind them that although it looks like a complete square regardless of whether positive or negative tiles are used, it is necessary that the dimensions of the sides correspond to the tiles.

Answers

Investigate Completing the Square

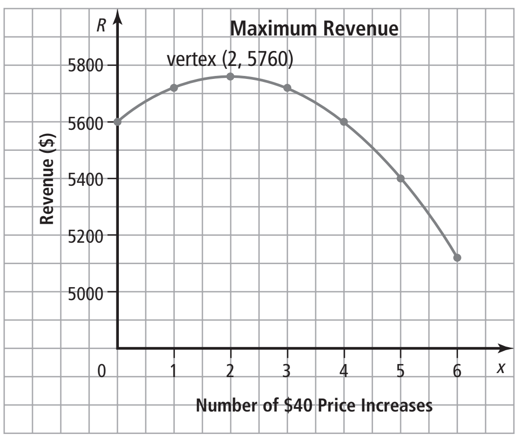
1.

Number of Mukluks Sold	Cost Per Mukluk ($)	Revenue, $R(x)$ ($)
16	320	5120
15	360	5400
14	400	5600
13	440	5720
12	480	5760
11	520	5720

2. Revenue increases to a maximum of \$5760 after two price increases; then revenue decreases. As the price increases, the sales decrease. Therefore, at some point the product of (price)(number of pairs sold) begins to decrease due to diminishing sales.

3. a) $(400 + 40x)$ **b)** $(14 - x)$ **c)** $R(x) = (14 - x)(400 + 40x)$
d) $R(x) = -40x^2 + 160x + 5600$

4. a)



b) \$5760
c) \$480

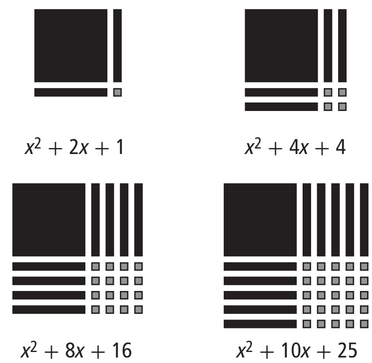
5. a) The two forms represent the same function.
b) $R(x) = -40(x - 2)^2 + 5760$; the quadratic function in vertex form gives the coordinates of the vertex.

6. a) Example: Quadratic functions always have a maximum or minimum value. A quadratic function is a good model because it demonstrates that increasing the price increases revenue to a maximum revenue and then decreases revenue due to lost sales. Linear functions do not have maximum or minimum values for all real values of x and y.

b) This model assumes there is a relationship between the price and the number of products sold. Actual sales can be affected by other factors. For example, in times of extreme cold, more people might be motivated to spend money on mukluks.

7. b) 9 unit tiles **c)** $x^2 + 6x + 9$ **d)** $(x + 3)^2$

8. a) $x^2 + 2x + 1$
$x^2 + 4x + 4$
$x^2 + 8x + 16$
$x^2 + 10x + 25$



b) There is a relationship between the number of x-tiles and the number of unit tiles added. The number of tiles added is equal to $\left(\frac{\#\text{ of }x\text{-tiles}}{2}\right)^2$.

9. $x^2 - 2x + 1$
$x^2 - 4x + 4$
$x^2 - 6x + 9$
$x^2 - 8x + 16$
$x^2 - 10x + 25$
The same relationship exists as for #8a). The number of tiles added is equal to $\left(\frac{\#\text{ of }x\text{-tiles}}{2}\right)^2$.

10. a) 256; $x^2 + 32x + 256$ **b)** $(x + 16)^2$

11. a) The number of tiles added is equal to $\left(\frac{\#\text{ of }x\text{-tiles}}{2}\right)^2$.
b) No. The same number of tiles is added for positive and negative x- tiles, because positive and negative values squared are always positive.
c) Yes. The resulting value added would be a rational number.

12. $\blacktriangle = \left(\frac{\blacksquare}{2}\right)^2$; $\bullet = \left(\frac{\blacksquare}{2}\right)$

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions with a partner or individually. Listen as students discuss what they learned during the investigation. Encourage them to generalize and reach a conclusion about their findings.	• It may benefit students if you display the graph for Part A on the board for a class discussion of the results. You might do Part A as a class but then allow time for students to work individually or in pairs to complete #6. In the follow-up discussion, ensure students clearly understand the differences between a linear graph and a quadratic graph. • For Part B, consider having students work in pairs but record their own response to all questions. • For #7, emphasize that the model must be square. • It is important that students are able to model expressions using a variety of tools and symbols. Encourage students who understand the square model to think of a different way to model an x or x^2.

Link the Ideas

Have students recall the two forms of quadratic functions that they know. Ask them if they know how to convert from one to the other. Students may not realize that they already know how to convert from vertex form to standard form by using algebra skills learned in previous math courses. Explain that they will need to apply the idea of completing the square in order to convert from standard form to vertex form. Help students realize that a function in vertex form allows them to easily determine the maximum or minimum value of a function and the x-value at which it occurs.

Have students work in pairs to read the information about completing the square and then work through the example that follows it. Ask them to try to explain to their partner the reasoning for each step in the example.

Example 1

In this example, students use the process of completing the square to convert equations from standard form to vertex form. The example has three parts: in part a) $a = 1$, in part b) $a > 0$ and $a \neq 1$, and in part c) $a < 0$.

Assign student pairs to work through the example and encourage them to discuss the reasoning for each step in the process with each other. For parts a) and b), Method 1 allows students to use algebra tiles to make connections with the understanding that they developed during Part B of the investigation. Then, have students use Method 2, which involves using algebra. Strongly encourage students to respond to the questions in green typeface for the solutions in both methods. You might have students compare the steps in the two methods; this may help them build their understanding of the reasoning behind the process.

Once students have a grasp of the two methods shown for parts a) and b), have them proceed to work through part c), which involves a negative leading coefficient. Some students may benefit from coaching through part c). Use questions such as:

- When a negative value is factored out of an expression, what happens to the signs of the terms?
- How does a leading coefficient that is negative affect the process of completing the square?

Encourage students to show their steps clearly as they do the Your Turn. Have them compare their solution with a classmate's and then explain their steps for one question to each other.

Example 2

In this example, students encounter a quadratic function that involves using fractions or decimals in the process of completing the square.

You might ask students why the process cannot be completed using integers, as in Example 1. Have them compare the two methods shown for solving part a). Ask students if they find it easier to complete the square using fractions or decimals, and explain why.

For part b), students consider how they can verify that two different forms of a function represent the same function. Method 1 involves working backward to convert the vertex form back to standard form. Method 2 involves graphing both forms of the function and then checking that the two graphs are identical. Ask students:

- Explain why each method works to verify that the standard form is equivalent to the vertex form.
- Which method do you think is more reliable to use for checking your work?

Have students complete the Your Turn using fractions or decimals for part a). For part b), have students use the method of their choice. Have students compare their solution with a classmate who may have used a different method.

Example 3

In this example, students consider how to use completing the square as a strategy to analyse a quadratic function. They need to recognize that completing the square allows them to change a quadratic function from standard form to vertex form, and helps them determine the vertex of a graph of the function. Help students come to this understanding by asking:

- What characteristics of a graph can you determine from a quadratic function in standard form? in vertex form?
- Why might it be useful to convert the quadratic function to vertex form?

In parts b) and c), students look at how they can quickly determine the x-coordinate of the vertex using the values of a and b. The example does not present a full proof that the equation of the axis of symmetry is $x = -\frac{b}{2a}$, but you may wish to have students try to do this. Note that Extend #27 relates to this idea. Ask students:

- How might you use the fact that the axis of symmetry and vertex occur at $x = -\frac{b}{2a}$ as a method to verify your work?
- Once you know the x-coordinate of the vertex, how could you find the maximum value?

Assign students to complete the Your Turn. Consider having them complete the square first to determine the vertex and then verify the result in several different ways. You might invite volunteers to present their solution to the class.

Example 4

In this example, students write a quadratic model function and use it to determine the maximum possible revenue for a real-life situation. Have students recall the investigation to help them understand this example. Encourage students to read carefully and consider each step one at a time.

For part c), you may wish to remind students to adjust the increments in the table to identify the maximum or minimum value.

The Your Turn gives students a chance to work through a similar problem to help reinforce their understanding. Encourage them to show the steps and reasoning clearly. Have students compare their solution with a classmate and explain the steps and reasoning to each other.

Key Ideas

Encourage students to develop their own example of completing the square and describe each step in their own words before looking at the Key Ideas. You might ask them to explain in their own words why it is helpful to convert a function from standard form to vertex form.

Then, have students add any missing ideas to their summary of the Key Ideas. Have them store the summary for each section of the chapter in the same location for review purposes.

Meeting Student Needs

- For the Link the Ideas, some students may benefit from checking that both forms in the given example represent the same quadratic function. The following are two possible methods they could use:
- For Example 1, have students work with algebra tiles as you demonstrate using algebra tiles projected on an overhead or a SMART Board.
- Consider developing templates of an example for students to fill in as you demonstrate completing the square for the example on the board. One template is needed for $1x^2$ questions and another template is needed for questions in which the leading coefficient, a, is not 1. The templates might consist of blanks in which a is factored, b is divided by 2 and squared, the value found is added and subtracted, and brackets are used to regroup. Students may benefit from filling in the blanks using the templates for the first worked examples. One step at a time, remove parts of the template until students can complete the square on their own.
- Some students may wish to use virtual algebra tiles to complete Example 1 parts a) and b). They may find the related Web Link at the end of this section helpful.
- For Example 1 part a), consider giving the following instruction for completing the square algebraically: To determine the value that completes the square, mentally divide the coefficient of the middle term by 2 and square it.
- For Example 2 part a), some students may prefer the following method for converting to vertex form:

$y = 4x^2 - 28x - 23$

$y = (4x^2 - 28x + 49) - 49 - 23$ Set the pattern.

If a and b are the coefficients of x^2 and x, add (and subtract) $a\left(\frac{b}{2a}\right)^2$, which gives the value 49.

$y = 4(x^2 - 7x + 12.25) - 72$ Remove a factor of 4 from the first three terms.

3.5 is half of 7

$(-3.5)^2 = 12.25$

$y = 4(x - 3.5)^2 - 72$

- For Example 3, have students write the steps for determining the vertex of a quadratic function using completing the square. Post this information in the classroom for students to reference as they complete the Check Your Understanding questions.

Substitute *x*-value(s):

Substitute one or more values for x, such as $x = 2$.

$y = x^2 - 8x + 5$	$y = (x - 4)^2 - 11$
$y = (2)^2 - 8(2) + 5$	$y = (2 - 4)^2 - 11$
$y = 4 - 16 + 5$	$y = (-2)^2 - 11$
$y = -7$	$y = 4 - 11$
	$y = -7$

Substitution of $x = 2$ gives the same y-value using both forms. Other values of x can also be used to confirm that the two forms are equivalent.

Work backward:

Expand and simplify the vertex form of the equation to get the original standard form.

$y = (x - 4)^2 - 11$

$y = (x^2 - 8x + 16) - 11$

$y = x^2 - 8x + 16 - 11$

$y = x^2 - 8x + 5$

- For Example 4, have small groups of students work through the given solution. If students find it difficult to generate the equation of the function algebraically, suggest they try using a spreadsheet first.
- Invite students who developed a presentation in sections 3.1 and 3.2 to incorporate content from the section 3.3 Key Ideas at this time.

ELL

- Ensure that students add the terms *completing the square* and *function table* to their math vocabulary dictionary. Encourage them to include a description, diagram, and/or example.

Web Link

For virtual algebra tiles, go to www.mhrprecalc11.ca and follow the links.

For an applet that demonstrates solving quadratic functions by completing the square, go to www.mhrprecalc11.ca and follow the links. Click on Show Steps to view the complete solution.

Answers

Example 1: Your Turn

a) $y = (x + 4)^2 - 23$ **b)** $y = 2(x - 5)^2 - 50$ **c)** $y = -3(x + 3)^2 + 3$

Example 2: Your Turn

a) $y = -3\left(x + \frac{9}{2}\right)^2 + \frac{295}{4}$

b) The two forms are equivalent.

$$y = -3\left(x + \frac{9}{2}\right)^2 + \frac{295}{4}$$
$$y = -3\left(x + \frac{9}{2}\right)\left(x + \frac{9}{2}\right) + \frac{295}{4}$$
$$y = -3\left(x^2 + 9x + \frac{81}{4}\right) + \frac{295}{4}$$
$$y = -3x^2 - 27x - \frac{243}{4} + \frac{295}{4}$$
$$y = -3x^2 - 27x + \frac{52}{4}$$
$$y = -3x^2 - 27x + 13$$

Example 3: Your Turn

a) $y = 3(x + 5)^2 - 34$. The vertex of the graph is $(-5, -34)$.

b) $x = \frac{-b}{2a} = \frac{-30}{(2)(3)} = -5$

$y = 3(-5)^2 + 30(-5) + 41 = -34$

The vertex is $(-5, -34)$, which is the same as in part a).

Example 4: Your Turn

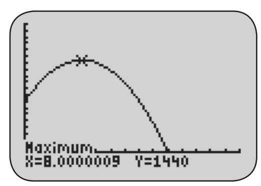
a) Let n represent the number of \$2 price increases, and thus price will be $(8 + 2n)$. The number of bottles sold will be $(100 - 5n)$.
Revenue = (price)(number of items sold)

$R = (8 + 2n)(100 - 5n)$

$R = -10n^2 + 160n + 800$

b) By completing the square, $R = -10n^2 + 160n + 800$ becomes $R = -10(n - 8)^2 + 1440$. The maximum revenue is \$1440 when there are 8 price increases and the price of the water bottles is \$24.

c) Verify the solution using technology by graphing the function expressed in standard form. The vertex is (8, 1440).



d) The problem assumes that price affects the number of sales and, therefore, the revenue. Other factors that may affect sales include quality of the product, safety of the materials used in the production of the product, weather, and motivation of people to buy water bottles at any price.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Some students may have a preference for a method to convert from standard form to vertex form. Encourage them to use their preferred method but ensure that they try the two methods shown and can explain their understanding of each one. • Emphasize the importance of the term *square* when using tiles. • Remind students that where two tiles meet, the colour is determined by the positive or negative value of the product (same colours meeting results in a positive tile; different colours meeting results in a negative tile.)
Example 2 Have students do the Your Turn related to Example 2.	• Some students may find it difficult to complete the square in part a). Coach students through the method of their choice as shown in the worked example. • Some students may benefit from writing the steps of the process of completing the square before attempting the Your Turn. • For part b), encourage students to use a different method to verify that the two forms are equivalent. • If students are using technology, ensure that they label key points on the graph to facilitate comparisons.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 3 Have students do the Your Turn related to Example 3.	• Remind students that once a quadratic function is converted to vertex form, they can obtain important characteristics of its graph. • Review what the values *a*, *p*, and *q* represent and their effect on a quadratic function. • Students may benefit from comparing their completed square with that of a classmate.
Example 4 Have students do the Your Turn related to Example 4.	• Highlight the importance of identifying the variables in a contextual problem. • Encourage students to record the key words in the problem (e.g., price, session fee, \$1 increase, fewer sessions, maximum revenue). • Coach students through determining the factors of the quadratic function. • Encourage students to compare their expansion of the quadratic function and their answer to the process of completing the square with a classmate.

Check Your Understanding

Practise

These questions focus almost exclusively on the process of completing the square and using this process as a strategy for determining the characteristics of a graph of a quadratic function. Some students may record only the last line of the solution, namely, the quadratic function in vertex form, and show very few or no steps in the process. Encourage students to organize their work and show the solution step by step. Consider allowing students to work in pairs for some problems. Alternatively, they might work individually on one or two questions before comparing solutions with a classmate.

Consider using the following prompts to help students learn the process of completing the square:

- What is the relationship between the value of b in the standard form and the value of p in the vertex form?
- How are the values of b and p connected to the value that is used to complete the square?
- Why is it necessary to both add and subtract a value when completing the square?
- How does a leading coefficient (i.e., $a \neq 1$) affect the process of completing the square?
- How does a leading coefficient that is negative (i.e., $a < 0$) affect the process?

There are many variations on the process of completing the square. If some students prefer to use a variation that makes more sense to them, encourage them to do so and explain why the process works. They might consider one of the following variations (using the example in the Key Ideas):

Variation 1

$$\begin{aligned} y &= 5x^2 - 30x + 7 \\ y &= 5(x^2 - 6x) + 7 \\ y &= 5(x^2 - 6x + 9 - 9) + 7 \\ y &= 5(x^2 - 6x + 9) - 5(9) + 7 \\ y &= 5(x - 3)^2 - 45 + 7 \\ y &= 5(x - 3)^2 - 38 \end{aligned}$$

Variation 2

$$\begin{aligned} y &= 5x^2 - 30x + 7 \\ y &= 5(x^2 - 6x) + 7 \\ y &= 5(x^2 - 6x + 9) - 45 + 7 \\ y &= 5(x - 3)^2 - 38 \end{aligned}$$

Variation 3

$$\begin{aligned} y &= 5x^2 - 30x + 7 \\ y - 7 &= 5x^2 - 30x \\ y - 7 &= 5(x^2 - 6x) \\ y - 7 + 5(9) &= 5(x^2 - 6x + 9) \\ y - 7 + 45 &= 5(x - 3)^2 \\ y + 38 &= 5(x - 3)^2 \\ y &= 5(x - 3)^2 - 38 \end{aligned}$$

Encourage students to use the method that they prefer, as long as they understand the method and can explain how it works.

Apply

These questions allow students to work with the concepts of quadratic functions in a less guided way. As they work, encourage students to consider if there is more than one strategy to solve each problem. Also, have them consider how they might verify their work.

The intent for #10 and 11 is for students to use the process of completing the square. Some students may suggest other strategies such as using technology or using $x = -\frac{b}{2a}$ to find the x-coordinate of the vertex and then substituting to find y. Let them know that while these other strategies work, it is important that they understand how to use the process of completing the square to determine characteristics of the graph of the function without graphing.

Encourage students who find #12 challenging to look carefully at each step and check values and signs.

For #13 to 15 and 17, students determine a maximum or minimum value in a situation for which the quadratic function is given in standard form. Students need to recognize that the problem can be solved by completing the square, which is the intent for these problems. Some students may suggest using technology to create a graph and determine the solution. Point out to them that it is important to be able to solve problems algebraically and graphically.

For #18 to 24, students need to generate quadratic functions to model a variety of real-life situations. You might point out that the quadratic functions involved for these problems arise out of two quantities being multiplied. Encourage students to do the following:

- Identify variables clearly, including the target quantity to be maximized or minimized.
- If appropriate, sketch a diagram and label it using variables derived from the situation.
- Write equations that relate the variables in the situation.
- Combine the equations to express the target quantity as a function of only one other variable.
- Use completing the square to determine the vertex of the quadratic function.
- Use the vertex to determine the solution.

Consider asking students to verify the solution for each problem.

For #18, you might point out that *Digging Roots* is an award winning First Nations band.

Extend

For #25, students work with completing the square on a quadratic function that has three fractional coefficients. Encourage them to show each step in the solution.

Many students may find #28 challenging. Consider having only stronger students attempt this problem. Advise them to sketch and label their own diagram. Have students write equations for how the quantities are related and use the equations to generate a quadratic function. Some students may not understand the key to the solution. Once they recognize that the width is connected to the radius of the circle, students can consider the area of the semicircle and rectangle separately to generate a model function.

For #27, students make connections between their learning in this section and their learning in section 3.2 about the meaning of the characteristics of a graph of a quadratic function. You may wish to have students work in pairs before developing an individual response.

Create Connections

For #29, students re-examine and build their understanding of quadratic functions in standard form and vertex form. Consider having students explain their thinking in a class discussion after they have had an opportunity to do so with a partner.

Project Corner

The Project Corner box provides students with information about situations involving motion that might be modelled using quadratic functions. Encourage students to brainstorm some possibilities. They may find the related Web Link at the end of this section interesting.

Meeting Student Needs

- Encourage students to sketch a diagram to model problems.
- Allow students to use algebra tiles as they work on learning the process of completing the square.
- Have students create their own question containing errors similar to #12. Have them exchange their question with a classmate and try to identify and correct any errors.
- Solving #17 in section 3.2 should help students complete #21 and 22.
- Have students refer to their list of learning outcomes for the chapter and assess their progress. Tell them to highlight in yellow each outcome addressed in this section. Once they have completed the Check Your Understanding questions, have them use a different colour to highlight the outcomes that they have no problem with. Have students identify the outcomes that remain highlighted in yellow and require more work. Provide any needed coaching.
- Provide **BLM 3–6 Section 3.3 Extra Practice** to students who would benefit from more practice.

ELL

- Some students may not be familiar with the following terms: *overstock*, *gymnast*, *trampoline*, *archery club*, *arrow*, *parabolic microphone*, *sound waves*, *nature audio-recording*, *sports broadcasting*, *concert promoter*, *holding pen*, and *market research*. Use a combination of descriptions, visuals, and examples to assist in student understanding.
- For #28, use the diagram to clarify the meaning of a *Norman window*.

Enrichment

- Challenge students to create two or three new questions that involve completing the square and provide the solutions.

Gifted

- Ask students to consider the function $y = -|-x|^2$. Ask students to predict the general characteristics of the graph of the quadratic function. (Students should show that the graph has the general shape of $y = x^2$ but opens downward. This is because the absolute value (similar to the power of 2) cancels the impact of the $-x$, and the first negative sign results in opening downward.

Common Errors

- Students may make arithmetic and sign errors when completing the square.

R_x Encourage students to organize their work clearly and neatly and to show each step. Have them ask themselves why a value needs to be both added and subtracted. Some students may find it helpful to refer back to using algebra tiles to complete the square.

Web Link

For information about applications of quadratic functions involving motion, such as volcanoes, torque, and astronomy, go to www.mhrprecalc11.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1a), b), 2a), b), 4a), b), 5a), d), 7a) to c), 8a), b), 9, and 12 to 14. Students who have no problems with these questions can go on to the remaining questions.	• Ensure that students have access to algebra tiles—particularly for #1 and 2—since some students may prefer to model the process before solving. • Stress the importance of being able to use more than one method to solve problems. Encourage students to refer to the multiple methods shown in the worked examples and have them attempt several different methods to verify their work for #4a) and b), and for #5a) and d). • Some students may benefit from being coached through the process of completing the square for #8a). Then, assign them to complete #8b) and c) on their own and use their work to check for understanding. • Ensure that students have recorded the process for completing the square in their notes. Ask them if it matters in what order they carry out the process of completing the square. Clarify any misunderstandings. • Have students work in pairs to compare their explanations and corrections for #12. • Encourage students to sketch a diagram for #14 and 15. They may wish to use technology to create a graph initially. Allow them to do so but then ask them to solve the problem algebraically. Emphasize the importance of being able to solve problems algebraically and graphically. Have students recall how they know that a point is the maximum value or minimum value.
Assessment *as* Learning	
Create Connections Have all students complete #29 and 30.	• For #29, students show their understanding of quadratic functions in standard form and vertex form. They may benefit from modelling the function using algebra tiles to discover that the square is already complete. Have them compare their written response with their partner's and revise as needed. • For #30, encourage students to work through the solution and then compare it with the given solution.

3 Chapter 3 Review

***Pre-Calculus 11*, pages 198–200**

Suggested Timing
60–90 min

Materials
- grid paper
- ruler
- graphing calculator
- graphing software (optional)
- algebra tiles

Blackline Masters
Master 2 Centimetre Grid Paper
Master 3 0.5 Centimetre Grid Paper
Master 4 Algebra Tiles (Positive Tiles) (optional)
Master 5 Algebra Tiles (Negative Tiles) (optional)
BLM 3–4 Section 3.1 Extra Practice
BLM 3–5 Section 3.2 Extra Practice
BLM 3–6 Section 3.3 Extra Practice

Planning Notes

Have students who are not confident identify strategies with you or a classmate. Encourage them to refer to their summary notes, worked examples, and previously completed questions in the related sections of the student resource.

Have students make a list of questions that they need no help with, a little help with, and a lot of help with. They can use this list to help them prepare for the practice test.

Meeting Student Needs

- Make algebra tiles available for students to use.
- Students who require more practice on a particular topic may refer to **BLM 3–4 Section 3.1 Extra Practice**, **BLM 3–5 Section 3.2 Extra Practice**, and **BLM 3–6 Section 3.3 Extra Practice**.
- Before students begin the questions, invite them to review their summary of the Key Ideas for each section. Invite students to clarify any misunderstandings. Alternatively, invite students who have prepared presentations about the Key Ideas to make their presentation to the class. Encourage the audience to ask questions of the presenters.
- Individualize the chapter review. Have students choose three questions from each section to begin. Correct the questions and analyse errors. Encourage students to request assistance for the questions they are unable to complete successfully. Students can then choose additional practice questions based on their results.
- If it has not been done already, post all of the learning outcomes. Invite students to ask questions about any outcomes that they do not understand.

ELL
- Encourage students to refer to their vocabulary dictionaries as they work on the questions.
- The language in #6 about solar energy collectors may be challenging to some students. Have another student use the photograph to explain the terms used. Encourage students to sketch their own diagram of the situation.
- The language in #12 may be challenging to some students who are not familiar with the sport of soccer. Have another student who has knowledge of soccer explain the terms used.

Enrichment
- Encourage students to develop a chapter study card using only one side of a 3 in. by 5 in. card. This card should show all the necessary information in order to achieve superior results on a summative assessment. (The study cards should include some of the more difficult aspects of quadratic functions.)

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 3 Review The Chapter 3 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against answers in the student resource.	• Have students review the statements on the self-assessment blackline master to check for processes and skills that they are still unsure of. Have them use this feedback to help select questions. • Have students revisit any section that they are having difficulty with prior to working on the chapter test.

Chapter 3 Practice Test

3

***Pre-Calculus 11*, pages 201–203**

Suggested Timing
45–60 min

Materials
- grid paper
- ruler
- graphing calculator
- graphing software (optional)
- algebra tiles

Blackline Masters
Master 2 Centimetre Grid Paper
Master 3 0.5 Centimetre Grid Paper
Master 4 Algebra Tiles (Positive Tiles) (optional)
Master 5 Algebra Tiles (Negative Tiles) (optional)
BLM 3–7 Chapter 3 Test

Planning Notes

Have students start the practice test by writing the question numbers in their notebook. Have them indicate which questions they need no help with, a little help with, and a lot of help with. Have students first complete the questions they know they can do, followed by the questions they know something about. Finally, suggest to students that they do their best on the remaining questions. Ensure that students get coaching for questions that they indicated they need help with.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #2–8, 10–12.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1,	3.1	Link the Ideas	✓ identify quadratic functions
#2, 5, 8b)	3.1	Example 2	✓ write a quadratic function given a graph
#3	3.1	Example 1	✓ identify the range of a quadratic function
#4, 7	3.3	Link the Ideas Example 1	✓ convert a quadratic function from standard to vertex form by completing the square
#6	3.1	Link the Ideas Example 3	✓ determine whether a quadratic function has zero, one, or two *x*-intercepts using the values of *a* and *q*
#8a)	3.2	Example 1	✓ identify features of a graph, such as the vertex, and equation of symmetry
#9, 10	3.1	Example 1	✓ analyse and graph quadratic functions using transformations
#11	3.3	Link the Ideas Example 2	✓ correct errors in an example of completing the square
#12, 13	3.2	Example 2	✓ solve a problem by analysing a quadratic function
#14, 16	3.3	Example 4	✓ write a function to model a given situation ✓ solve problems by analysing functions
#15	3.3	Example 4	✓ write a function to model a given situation

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 3 Self-Assessment Have students use their responses on the practice test and work they completed earlier in the chapter to identify skills or concepts they may need to reinforce.	• Students may wish to review the summary notes they completed during the chapter before they begin the practice test. • Make algebra tiles available to students to help them solve problems. • Before the chapter test, coach students in areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 3 Test After students complete the practice test, you may wish to use **BLM 3–7 Chapter 3 Test** as a summative assessment.	• Coach students through any areas in which they are still experiencing difficulty. • Some students may benefit from redoing selected questions related to their area(s) of weakness. Choose questions from the student resource or the extra practice blackline masters.