Quadratic Equations

Opener

***Pre-Calculus 11*, pages 204–205**

Suggested Timing
30–45 min

Blackline Masters
BLM 4–2 Chapter 4 Prerequisite Skills
BLM U2–1 Unit 2 Project Checklist

Key Terms

quadratic equation	extraneous root
root(s) of an equation	quadratic formula
zero(s) of a function	discriminant

What's Ahead

In this chapter, students explore the concept of resolving quadratic equations graphically and algebraically. They are introduced to methods using both pencil and paper, and technology. Students explore graphical representations of quadratics in section 4.1, learning the relationship between x-intercepts and the roots of a quadratic equation. In section 4.2, students begin to work with quadratics algebraically by factoring. section 4.3 extends the algebraic treatment of quadratics, solving quadratic equation by completing the square. In section 4.4, students explore the development of the quadratic formula and how it is applied to find the roots of quadratic equations.

By the end of the chapter, students should have a deep understanding of what it means to solve a quadratic equation. They should understand that they are accomplishing the same thing when they find the x-intercepts of the function graphically or solve for the roots algebraically. They should also understand when it is best to factor, complete the square, or use the quadratic formula.

Planning Notes

Begin Chapter 4 by discussing with students that any situation involving a maximum or minimum will use quadratics. Explain that the graph of a quadratic is a parabola, and draw two parabolas on the board: one that opens upward and one that opens downward. Discuss the relationship between independent and dependent variable as it relates to each parabola. Emphasize the maximum and minimum point of a parabola and their association with the vertex of the graph.

Based on the attributes of a quadratic that you have discussed, ask students,

- How are parabolic shapes, or quadratic functions, related to the objects in the opener? How are these objects similar? What characteristics do they share?
- Where else have you seen quadratic equations in everyday life? What types of situations might be represented by a quadratic equation?
- Can you think of examples that might fit each of the diagrams of parabolas? For example, the height of a basketball in relation to the time the ball is in the air can be represented by a parabola that opens downwards.

You might consider breaking the class into groups and asking each group to investigate functions of parabolas as reflectors in flashlights, headlights, satellite dishes, and so on. How might this topic be related to the functions of tipis and sweat lodges, which are used as healing and ceremonial places by First Nations peoples? In particular, consider the heat generated by a central fire pit.

Draw students' attention to the Did You Know? on page 204. Tell them that, while Apollonius was the first to study parabolas in depth, the history of quadratic equations and functions date back much earlier, to the Babylonian, Chinese, and East Indian cultures. Consider asking students to research the ancient origins and applications of quadratic equations.

In addition to the Key Terms listed, you might have students revisit the definition of the following terms:

- roots of a function
- x-intercepts of a function
- real numbers

Unit Project

At the beginning of the chapter, go over the Unit 2 Project Wrap-Up. Students should be aware of what the project is and where it is going. Go over the Project Corners as a class. Students should have a section in their Foldable to take notes and generate ideas they can use on their project. The information in the Project Corners will provide opportunities for good class discussions, as well as giving students ideas for working on their project.

Chapter Summary

Students should record notes and a number of examples in their graphic organizer. At the end of each section, they should summarize the material in their own words. After completing the practice questions, they will also have a better understanding of the areas with which they have difficulty. They can make notes about these problem areas in their chapter summary. They will then be able to revisit these notes to remind themselves of areas they need to improve on and focus on for the unit test. They should also revisit the unit project after each section in the chapter. The skills learned in each section will be useful when completing the unit project.

Meeting Student Needs

- Consider having students complete the questions on **BLM 4–2 Chapter 4 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- Post the student learning outcomes in the class to prepare students for the chapter.
- Have students write the Key Terms in their notebooks. As they work through the chapter, have them develop their own definitions, including graphs and figures.
- After sketching the diagrams of parabolas on the board, go outside and throw stones or lumps of snow up in the air. Ask student to describe the path of the projectiles. Have them try to see the parabolic shape.
- Hand out to students **BLM U2–1 Unit 2 Project Checklist**, which provides a list of all the requirements for the Unit 2 Project.

Gifted

Challenge students to use a graphing calculator and their previous knowledge of quadratic functions to collect data that model the path of projectiles that might be used to create video games. Such projectiles include basketballs, footballs, and golf balls. When considering student responses, look for data that include distances that approximate the real situation. This will help students to make the connection between the elements of a quadratic that give the characteristics of height and distance in real-world situations.

Enrichment

Encourage students to explore how a series of quadratics could model the multiple bounces of a projectile, such as a golf ball bouncing down the fairway. When considering student responses, look for a series of parabolas that open downward and whose maximum heights are declining as a bouncing ball naturally would.

Career Link

You could talk to students about what a career in Robotics Engineering entails. Ask students what they already know about this field. Ask students how quadratics might be used in robotics. Engineers model real-life objects and actions using mathematics. They compute the solutions to many equations to find the answers, and the quadratic equation is among those that are most useful.

Web Link

For additional careers involving quadratic equations, teachers go to www.mhrprecalc11.ca and follow the links.

Graphical Solutions of Quadratic Equations

4.1

***Pre-Calculus 11*, pages 206–217**

Suggested Timing
90–135 min

Materials
- grid paper
- graphing calculator
- spreadsheet program
- ruler

Blackline Masters
Master 2 Centimetre Grid Paper
BLM 4–3 Chapter 4 Warm-Up
BLM 4–4 Section 4.1 Extra Practice
TM 4–1 How to Do Page 208 Example 1 Using Microsoft® *Excel*
TM 4–2 How to Do Page 208 Example 1 Using TI-Nspire™

Mathematical Processes
- ✓ Communication (C)
- ✓ Connections (CN)
- ☐ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcomes
RF4 Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including:
- vertex
- domain and range
- direction of opening
- axis of symmetry
- x- and y-intercepts

and to solve problems.
RF5 Solve problems that involve quadratic equations.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–5, 9, 17, 18
Typical	#1–3, 5, 6, two of 8–10, 12, 17–19
Extension/Enrichment	#4, 6, 12, 13, 16

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce prerequisite skills needed for this section.

You could open the class by asking students to consider the water fountains in the school or even go and study them. Ask,
- Why do you think the fountains are designed as they are? What do you think determines the shape of the bowl? What would happen, say, if the bowl was smaller/shorter (the water would flow out of them when the handle was on fully).
- What do you notice about the shape of the water flow? What does altering the force of the water with the handle do to the shape of the water flow? You might even have students experiment by measuring the height of the water and the point it strikes the bowl in relation to how far they turn the handle.

Students can work in pairs to discuss the design of water fountains.
- What are some of the most innovative designs they have seen?
- Can they suggest design ideas of their own? What will they have to consider in their designs?

The class can then come together to discuss their ideas.

Investigate Solving Quadratic Equations by Graphing

The intent of this investigation is to get students started with some simple algebraic manipulations. The provided equation simplifies considerably, making it easier for students to relate its characteristics to the x-intercepts. Students should figure out that $y = 0$ at the point where the graph crosses the x-axis (x-intercepts).

While students are asked to use their graph to find where the lights should be placed, you could also ask students to find the maximum point of the jet of water.

In #2, students develop a family of curves where a changes, but b and c are the same for each equation (12 and 0 respectively). To heighten students' understanding of the relationship of the graph to its equation, you may also consider helping students make the connection between the coefficient of the x^2 term and the changing shape of the parabolic arc.

For #2b), as the value of x increases, the height of the graph (its maximum) increases; also, while one of the x-intercepts stays at 0, the other increases. Try to lead students to make a connection between the factors and the graph (foreshadowing later sections). You should also reiterate that x-intercepts are also called the *roots of an equation*.

The intent of the Reflect and Respond is to get students to see that different quadratic equations can have the same x-intercepts, even though their graph might have different shapes.

Meeting Student Needs

- Provide students with **Master 2 Centimetre Grid Paper**.
- Draw students' attention to the margin definition of a quadratic equation on page 208, which makes reference to the term *standard form*. Discuss with them what it means for a quadratic equation to be in vertex and standard form. You may even extend this discussion to revisit how each form is written and how to convert between the two.
- Students may need to be reminded that to expand $(x - 1)^2$, they must multiply $(x - 1)$ by itself. They must also remember to distribute the -6 to each term inside the brackets after $(x - 1)^2$ has been expanded.
- If students are having difficulty, consider working through #2 as a class.
- When discussing zeros of a function, it may help some students to see examples of parabolic graphs that intersect the x-axis.
- Consider having students work in small groups to complete the Reflect and Respond. The class could then reconvene to discuss.

Common Errors

- Some students may forget the middle term when expanding $(x - 1)^2$. That is, $(x - 1)^2 \neq x^2 - 1$.

R_x Remind students to write out the equivalent expression and then apply the distributive property for each term: $(x - 1)^2 = (x - 1)(x - 1)$.

- Some students may forget to multiply -6 by each term inside the bracket when distributing -6 inside the brackets.

R_x Remind students to multiply each term inside the brackets by -6:
$-6(x^2 - 2x + 1) = -6(x^2) - 6(-2x) - 6(1)$

Web Link

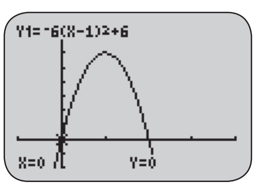
For more information about fountains, go to www.mhrprecalc11.ca and follow the links.

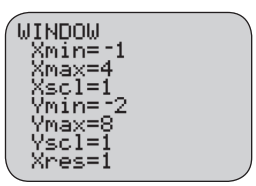
Answers

Investigate Solving Quadratic Equations by Graphing

1. a) Example: A suggested table of values, graph, and window settings from a TI-83 Plus/84 Plus:

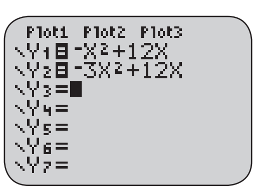
x	y
0	0
0.5	4.5
1	6
1.5	4.5
2	0

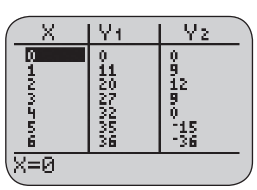


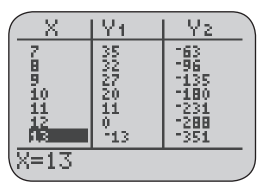


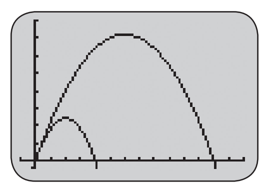
b) The zeros of the graph are at 0 m and 2 m. These represent when the height of the water is zero: at the jet and when the stream hits the pool. So, the light should be placed 2 m from the jet.

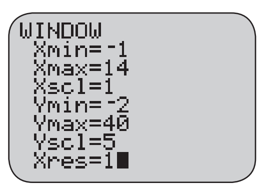
2. a) Example: A suggested table of values, graph, and window settings from a TI-83 Plus/84 Plus.











b) Both of these graphs have a larger second x-intercept, showing that the water stream travels farther. They also have a higher maximum value, indicating that the stream travels higher.

c) The x-intercepts are called zeros because these are the points where the function equals zero. In this case, the function models the height of the water, so the zeros represent the points where the height of the water is 0 m.

3. a) You could aim the jet at a sharper angle to make the stream land closer to the jet or at a more gentle angle to make it land farther away.

b) Yes. If two streams were aimed at different angles, the two would travel up to different heights, but could both land at the same spot.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• You may wish to have students work in pairs. • You may wish to have students complete #1 and 2 independently and then complete #3 as a class. • Encourage students to sketch a diagram of possible scenarios that could represent the water's path. Have them compare their diagrams to other's in their class. • If there is insufficient space in the classroom for a water investigation, consider doing a demonstration. Have students suggest how they could test #3a) in a simulation.

Link the Ideas

Consider using the definition of a quadratic as an opportunity to talk about the value of a. Ask students why a can't equal zero? If a did equal 0, what would the graph of the function look like? What type of function would this be?

You could also discuss maximum and minimum points and the direction the parabola opens. Ask students what happens to the shape of the graph when $a < 0$? What happens to the shape of the graph when $a > 0$?

Students should work in pairs to complete the examples. This will give them an opportunity to brainstorm and compare if they have difficulties or questions.

Example 1

For Method 1, ask students

- From the table, how do you know there is only one solution?
- What if there was a decimal root?

The disadvantage of a table is that if there are decimal solutions, you do not automatically get the exact solution. You should emphasize the importance of doing a check. When doing a check, remind students to use brackets when substituting the x-values and to use order of operations to determine if the solution is correct.

Method 2 provides students with the opportunity to use a spreadsheet. This is a good way to link and connect to daily life situations involving collecting and analyzing data, and making conclusions based on the graphs of the data.

For the Your Turn question, consider helping advanced students recognize that the Example and the Your Turn questions are similar in that they are both perfect square trinomials, so they both have two equal roots. You may have to help some students recognize Example 1 as a perfect square because of the -1 coefficient.

Example 2

Finding the x-intercepts is like finding the zeros of the function, as seen in the previous example. In Method 1, it is important to use positive and negative values for x, to see how the function behaves in quadrants I and II. In Method 2, the spreadsheet graph provides an opportunity for you to talk about the y-intercept and the maximum point. Before students do the Your Turn question, ask them what the difference is between Example 2 and Example 1. Why does Example 2 have two roots, when Example 1 only had one? The quadratic in Example 1 is a perfect square while Example 2 is not.

Example 3

Consider using this example as an opportunity to talk with advanced students about imaginary roots. You should only introduce this concept at this point, going into it in greater detail in sections 4.3 and 4.4.

Example 4

Before working through this example, you may want to discuss with students some of the characteristics of suspension bridges and the shape of the cables described in the example. When suspension bridges are being built, the support cables sag in the shape of a catenary curve before they are tied to the deck below. Over time, the cables take on a parabolic curve as additional connecting cables are tied to connect the main suspension cables with the bridge deck below.

Ask students why they need to find the x-intercepts. What does the x-intercept mean in the context of this question? The x-intercept represents the location when the height from the top of the tower to the cable is zero. The two x-intercepts represent the distance each tower is from the end of the bridge. So, to determine the distance between the two towers, students must determine the difference between the two x-intercepts.

Key Ideas

Since a quadratic has a degree of 2, there will be zero or two roots.

- If there are zero roots, the quadratic does not cross the x-axis.
- If there are two roots, the roots may be equal or non-equal.

Meeting Student Needs

- Provide students with **Master 2 Centimetre Grid Paper**.
- Discuss the relationship between zeros of the function and roots of the equation. How are they similar? How are they different? Help students create a relationship between the zeros in the Investigation and the roots in the first section of Link the Ideas.
- Have a class discussion about the different methods used in the examples for solving a quadratic equation. Demonstrate the use of a table of values, a spreadsheet, and graphing calculator for Example 1.
- Consider creating posters listing the steps involved for each of the three methods described in the examples. Discuss why one method might be preferred over another in a particular situation, but ask which method students prefer. Tell students that the method they use is a personal choice.
- Ensure that students understand that there may be zero, one, or two real roots for a quadratic equation. These roots may or may not be rational, but it is more likely that they will have rational roots.
- Consider working through the calculator example as a class. Graphing calculators can speed up both instruction, and the learning process.
- You may wish to provide students with **TM 4–1 How to Do Page 208 Example 1 Using Microsoft® *Excel*** and **TM 4–2 How to Do Page 208 Example 1 Using TI-Nspire™**.

Web Link

For more information about using technologies, including Microsoft® *Excel*, for graphing quadratic equations, go to www.mhrprecalc11.ca and follow the links.

Answers

Example 1: Your Turn

The root is 3.

Example 2: Your Turn

Increasing or decreasing the sweatshirt price by \$10 will result in no revenue.

Example 3: Your Turn

The graph does not intersect the x-axis, so there are no real roots.

Example 4: Your Turn

The horizontal distance between the two towers is 138.6 m to the nearest tenth of a metre.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to try all three methods and select the two with which they are most comfortable. • Some students may benefit from partnering to discuss the technology applications. • You may need to help student recall the steps required in using the technology for the problem. • Ensure students understand the meaning of *root*. It may be necessary to sketch out several examples of parabolas touching the x-axis at its vertex, and using these examples to revisit the meaning of root. • Suggest that students show a verification of their answer.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to revisit the term *zero* and have students sketch a parabola with two zeros to ensure that they understand this term. • Ensure students make the link between a root and "no revenue." • Encourage students to try all three methods and select the two with which they are most comfortable. • Some students may benefit from partnering to discuss the technology. • You may need to reactivate student's knowledge of the steps required in using the technology for the problem. • Have students verbally describe to a partner the difference in appearance of a parabola with zero or one root, versus one with two roots. • Ensure students are able to use substitution to verify their conclusion.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students verbally describe the difference between the appearance of a parabola with one, two, or no zeros before proceeding. • Ensure students understand the connection between no real roots and no x-intercept. You may wish to sketch several unlabelled examples of quadratics that have one, two, or no real roots, and then ask students to describe the roots of each. Provide them with a choice in how they want to describe the differences (i.e., orally, using a calculator, using a paper and pencil sketch, etc.).
Example 4 Have students do the Your Turn related to Example 4.	• Discuss with students why not all roots are integers. • Help students recall the process of locating roots using technology. • Have students explain to a partner how many roots the quadratic has and how they know. Encourage them to solve (i.e., explain) it in more than one way. • Suggest that students identify the variables in the quadratic and explain what each represents. This should help to clarify what students should be looking for.

Check Your Understanding

Practise

For #3, you may need to help students remember to set the equation to zero in parts d) and e). Students have not seen a question like these in the lesson. They can use inverse operations to set this equation to zero and then solve in a manner similar to those found within the lesson.

For #4, students can estimate the roots to the nearest tenth. You can show them how to find the roots using their graphing calculator.

Apply

For #5, ask students when the height of the function is zero. They need to recognize that this happens before the ball is kicked and when the ball first hits the ground. This is why there are two x-intercepts.

For #6, students may need help finding the equation. One way to do this is to say that there are two unknown numbers, x and y. Since the sum of these numbers is 9, $x + y = 9$. Therefore, the two numbers are x and $9 - x$. So the equation becomes $x(9 - x) = 20$. Students can apply the distributive property and make the equation equal to zero.

For #11, you could ask students what the graph means within the context of this question. Ask, "What does it mean when the curve is above and below the x-axis?" When the curve is above the x-axis, the diver is above the surface of the water.

In #12, some students may be interested to know that the Borden Bridge referred to in the question is no longer in use. In 1985, it was replaced with a modern four lane bridge.

Extend

For #13a), students should understand one real root actually means that the function has two equal roots, and that a quadratic with two equal roots is a perfect square trinomial. Students have to find the value of k that makes this quadratic a perfect square. Conversely, in part b), for a quadratic to have two distinct roots, it cannot be a perfect square. For part c), students need to understand that a quadratic has no real roots if its graph does not intersect with the x-axis. This happens when the graph opens up and its vertex is above the x-axis, or if it opens down and its vertex is below the x-axis. Since a in this equation is positive, the quadratic opens up. For the quadratic to have no real roots, the vertex must be above the x-axis. The value of k, then, must be one that places the quadratic above the x-axis.

Create Connections

Question #16 is an excellent question to help students use the visual representation of a quadratic.

For #17, you will need to help students recall the definition of *axis of symmetry*. The axis of symmetry is an imaginary line that cuts the quadratic into two symmetrical halves. The distance the x-intercepts are from the axis of symmetry is equal on both sides.

For #18, *vertex* is a term that you might have to revisit with students. The vertex is the point at which a quadratic changes direction. It is the halfway point of the graph, so the graph is symmetrical on both sides of the vertex.

Meeting Student Needs

- Provide students with **Master 2 Centimetre Grid Paper**.
- Provide **BLM 4–4 Section 4.1 Extra Practice** to students who would benefit from more practice.
- Give students the opportunity to solve equations using their own methods. You may discuss the different methods with them and then let them decide which method works best for them. • Encourage students to create graphs, using both pencil and paper and a graphing calculator. This encourages hands-on activity, accommodating concrete and kinesthetic learners.
- If you notice that students are struggling with any of the concepts or methods of solving, consider discussing some of the questions as a class.
- Students could work through the first four questions individually, and then work through the rest in pairs or small groups.
- A strong knowledge of zeros or roots is needed for #13. Ensure that all students analyse the equation to discover the value of k for each situation.
- Have students work in small groups to work on one question from 16 to 18. Have them present to the class their solution and the method they chose to solve the problem.
- Consider having students research the Canadian connection to sports that involve quadratic equations, such as hockey, lacrosse, and basketball. They may also research traditional Aboriginal sports and games that involve quadratic equations.

Gifted

- Tell students that anti-collision technology slows a vehicle automatically when a potential collision is detected. Ask students to consider what variables would have to be considered for this technology to work. Ask them to explore the relationship between distance, deceleration, and time during the deceleration process, and to create a mathematical function to model the deceleration process. Deceleration is negative acceleration. $D = \frac{1}{2}at^2$ is a commonly used formula that relates distance travelled, acceleration, and time.

Enrichment

- Ask students to create an equation that models a basketball free throw. Ask them to explain what the two solutions represent in terms of the path of the basketball. When considering student responses, look for the kind of thinking that has been expressed in the student resource. The two roots represent the point at which the ball leaves the player's hand and a point behind the backstop. You may want to ask them what would make the second root irrelevant (the ball would hit the basket or backboard, or would be intercepted by another player).

Web Link

For more information about the motion of kickoffs and the HSBC Celebration of Light, go to www.mhrprecalc11.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–5, and 9. Students who have no problems with these questions can go on to the remaining questions.	• Questions 1-4 all require students to find the roots of a quadratic given the graph or by graphing. • Encourage students to sketch a diagram for #9 before solving.
Assessment *as* Learning	
Create Connections Have all students complete #17 and 18.	• Some students may benefit from sketching the information provided in #17. • Encourage students to draw a diagram of the information in #18 in order to determine the symmetry around the axis of symmetry. • Encourage students to go over the terminology and revisit the diagrams they may have drawn for Chapter 3. Have them identify the important characteristics of a quadratic with their partner.

Factoring Quadratic Equations

4.2

***Pre-Calculus 11*, pages 218–233**

Suggested Timing

135–180 min

Materials

- grid paper
- graphing calculator or computer with graphing software
- algebra tiles
- blank laminated cards
- ruler

Blackline Masters

Master 2 Centimetre Grid Paper
BLM 4–3 Chapter 4 Warm-Up
BLM 4–5 Section 4.2 Extra Practice

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ☐ Technology (T)
- ✓ Visualization (V)

Specific Outcomes

RF1 Factor polynomial expressions of the form:
- $ax^2 + bx + c, a \neq 0$
- $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$
- $a(f(x))^2 + b(f(x)) + c, a \neq 0$
- $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$

where *a*, *b* and *c* are rational numbers.

RF5 Solve problems that involve quadratic equations.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2a), c), 3a), 4a)–c), 5a), b), 7a), c), d), 9a), b), d), 11, 30, 32
Typical	#1, 2a), c), 3a), b), 4a)–c), 5, 6a), b), 7a), b), d), 9b), c), f), 10c), 13, 16, 20, 29, 31, 32
Extension/Enrichment	#5, 6, 7c), e), f), 8c)–e), 9b), c), f), 10e), 17, 23, 29–32

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce prerequisite skills needed for this section. Ask students,

- When you throw a ball, does it go in a straight line or a parabolic curve? How do you know?
- What would happen if a ball, when thrown, did not follow a parabolic curve? What impact would this have on sports?
- What other factors might affect the path of a projectile, such as a ball? How does knowledge of this movement relate to a job, such as a game designer/programmer? Can you think of other jobs for which this knowledge might be important?

Investigate Solving Quadratic Equations by Factoring

For #1d), students need to recognize that the context of this question relates to dimensions and areas. If they are having difficulty with this question, ask them if a dimension can be negative. Since it cannot be, what implications does this have for the roots they determined?

When working on step 2, ensure that students have fully factored the expression. Some students may not recognize that $x^2 - 81$ is a difference of squares. Students may have difficulty starting step 2d). Make sure they understand that *r* and *s* are variables representing the two *x*-intercepts. Encourage students to assign numerical values to *r* and *s* from the context of this investigation. They can then relate the factored expression to the values of the intercepts and then substitute the variables *r* and *s*. You may wish to remind students to write their equation in standard form.

Meeting Student Needs

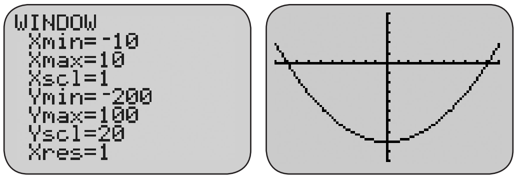
- Provide students with **Master 2 Centimetre Grid Paper**.
- Discuss the student learning outcomes presented in this section.
- For #1, some students might need help creating the area equation. Help them recognize why the length is $2x$.
- Some students might benefit from using grid paper to sketch various volleyball courts that have area of 162 m^2. They can work through this until they find one with the dimension where length is twice the width. This method presents a visual representation of the Investigation.

Answers

Investigate Solving Quadratic Equations by Factoring

1. a) $x(2x) = 162$, so in standard form $2x^2 - 162 = 0$.

b) A graph with these window settings shows there are two x-intercepts. They are -9 and 9.



c) The roots are -9 and 9. They are the x-values that make the equation equal to 0.

d) The root -9 is not acceptable because the dimension of a volleyball court cannot be negative.

2. a) $2x^2 - 162 = 2(x^2 - 81)$
$= 2(x + 9)(x - 9)$

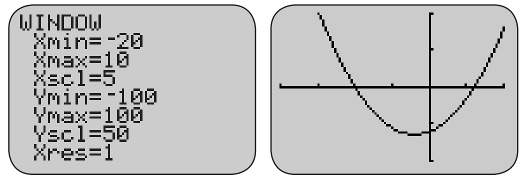
b) It is exactly the same.

c) The factored expression is equal to 0 when $x = -9$ or $x = 9$. Therefore, the x-intercepts are the same as the roots of the equation.

d) If r is an intercept, then $(x - r)$ is a factor. If s is an intercept, then $(x - s)$ is a factor. You can then multiply the two factors and set the result to equal zero. You will not be able to ascertain the exact equation unless you have more information.

3. a) $x(x + 4) = 60$, so in standard form $x^2 + 4x - 60 = 0$

b) A graph and the window settings are shown. There are two x-intercepts: -10 and 6.



4. a)
$$\begin{aligned}(x - r)(x - s) &= 0\\ (x - (-10))(x - (6)) &= 0\\ (x + 10)(x - 6) &= 0\end{aligned}$$

b) The graph is the same.

5. If $(x - r)$ is a factor, then the value of x that makes $(x - r) = 0$ is an x-intercept. That x-value is a zero and a root of the equation.

6. $x^2 - 5x - 6$ factors to $(x - 6)(x + 1)$. The solutions to $(x - 6)(x + 1) = 0$ are $x = 6$ and $x = -1$.

7. If 3 is a root, then $(x - 3)$ is a factor. If -5 is a root, then $(x + 5)$ is a factor. We can then multiply them and set the result equal to 0. So, $x^2 + 2x - 15 = 0$ is a possible equation. Others are possible, but they would have the same x-intercepts.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• You may wish to have students work through the Investigation in pairs. • Some students may benefit from revisiting the concept of multiplying binomials.

Link the Ideas

You might want to work through the Link Your Ideas section as a class. You may then have students work together as partners or small groups to work through the examples. It is important that students are comfortable with the different factoring strategies, and know when to use them.

When discussing how to factor a trinomial of the form $ax^2 + bx + c$, help students recognize that when rewriting the middle term as a sum of two terms, it does not matter which term they write first. You could illustrate this by having students work through the example, $3x^2 + 8x + 4$, first writing $8x$ as $(6x + 2x)$ and then writing it $(2x + 6x)$.

Example 1

For #1b), students often struggle when dividing by a fraction. Take extra time presenting this step.

Example 2

Students will need several examples of these types of questions because these are difficult concepts to grasp. For part a), you might show students an alternative way of factoring, in which they expand and group like terms, and then factor.

$$\begin{aligned}&12(x + 2)^2 + 24(x + 2) + 9\\ &= 12(x^2 + 4x + 4) + 24x + 48 + 9\\ &= 12x^2 + 48x + 48 + 24x + 57\\ &= 12x^2 + 72x + 105\\ &= 3(4x^2 + 24x + 35)\\ &= 3(2x + 5)(2x + 7)\end{aligned}$$

Regardless of the method used, remind students that it is important to use brackets for all substitutions.

Example 3

You can use this example and the paragraphs before it as an opportunity to talk about what it means to solve a quadratic equation. Solving a quadratic equation algebraically means to find the equation's roots, which are the values for the variable that make the function equal to zero. Solving a quadratic equation graphically means finding the x-intercepts, which are the points at which the graph crosses the x-axis. You should make the connection between algebraic solutions and graphical solutions.

Students must understand that once they have identified possible roots of the equation, it is important to do a check. Each root should be checked in the original question. The check ensures your answer is correct and that one of the roots is not an extraneous root. Extraneous roots are defined on page 236. Remind students to use brackets for all substitutions, especially when negative numbers are involved.

Example 4

The quadratic equation is often given to students when they are solving word problems. These types of questions can become monotonous and students will automatically factor and find the roots of the equation. Consider word problems as an opportunity to discuss the relationship between the independent and dependant variables. Stress to students that they must understand *why* they are finding the roots of the equation. What do the roots mean in the context of the problem? For example, in Example 4, students are finding the roots of the equation because they want to determine the horizontal distance a dog jumps to land in a body of water, meaning the height is zero.

Key Ideas

Have students summarize some of the strategies they use when factoring a quadratic equation. Two examples follow:

- When factoring, always look for a common factor to remove first.
- When factoring, and the middle term is broken into the sum of two terms, if the variable in the second term is negative, always factor out a negative to make the variable positive. For example, for $(2r^2 - 5r) + (-6r + 15)$, you would factor -3 to make the variable positive: $r(2r - 5) + (-3)(2r - 5)$.

Meeting Student Needs

- Provide students with **Master 2 Centimetre Grid Paper**.
- Draw students' attention to the zero product property described on page 223 of the student resource. You might help students develop the idea of the zero product property by presenting examples, such as
  - if $(3)(?) = 0$, then $?$ must equal zero
  - if $(3)(x) = 0$, then x must equal zero
  - if $(x)(y) = 0$, then x or/and y must equal zero
  - if $(x + 3)(x - 5) = 0$, then $(x + 3) = 0$ or/and $(x - 5) = 0$
- Consider quickly revisiting the concepts around factoring polynomials with students, such as
  - common factor
  - grouping
  - difference of perfect squares
  - perfect trinomial squares
  - $1x^2$ trinomials, for which factors use a product of c and a sum of b
  - ax^2 trinomials, for which you could introduce the idea of a "magic number," $(a)(c)$ (i.e., factors use a product of $(a)(c)$ and a sum of b)
- Allow students to use algebra tiles to factor the polynomial. Some students may need time to refresh this skill in order to be successful with the rest of this section.
- You might consider using colour-coded "substitution cards" for Example 2. Prepare the following:
  - two cards of equal size from a sheet of Bristol board. Write r^2 on one and $(x +2)^2$ on the other.
  - two cards of equal size from a different color of Bristol board. Write r on one and $(x + 2)$ on the other.

  Then, demonstrate part a) on the whiteboard, using the Bristol board cards where appropriate.
- You might consider preparing blank, laminated cards with magnets on the back. The original question can be written on the whiteboard, with blank, laminated cards in the variable parts. Then, use a dry-erase marker to write on each card, as required.

Common Errors

- Student often have difficulty squaring a negative number, calculating $(-2)^2 = 4$.

R_x Have students write out the intermediary step: $(-2)^2 = (-2)(-2) = 4$.

- Students forget when taking the square root of a number that there are two possible roots: the positive and negative value.

R_x Remind students about the rules for multiplying positive and negative values.

- When students factor by grouping they often make mistakes when one of the middle terms is negative. For example,
$3x^2 + x - 4$
$= 3x^2 + 4x - 3x - 4$
$= (3x^2 + 4x) - (3x - 4)$

R_x Get students to rewrite the middle term using addition. Remind them that the variable needs to be positive. For example,
$3x^2 + x - 4$
$= 3x^2 + 4x + (-3x) - 4$
$= (3x^2 + 4x) + (-3x - 4)$
$= x(3x + 4) + (-1)(3x + 4)$

Answers

Example 1: Your Turn

a) $3x^2 + 3x - 6 = 3(x - 1)(x + 2)$

b) $\frac{1}{2}x^2 - x - 4 = \frac{1}{2}(x + 2)(x - 4)$

c) $0.49j^2 - 36k^2 = (0.7j - 6k)(0.7j + 6k)$

Example 2: Your Turn

a) $-2(n + 4)(n - 4)$

b) $(2x - 2 - 0.5y)(2x - 6 + 0.5y)$

Example 3: Your Turn

a) 5

b) 4 and -4

c) 2 and $-\frac{4}{3}$

Example 4: Your Turn

3 ft

Example 5: Your Turn

The table is 5 ft by 9 ft.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work in pairs. • Each of the questions in Your Turn are quite different and require a different approach. If students are having difficulty, suggest they revisit the examples of the form with which they are having difficulty. Consider providing them with another similar question to solve. • If students find grouping difficult, you may wish to use the trial and error method by inspection or decomposition. This alternative approach may help clarify understanding. • Some students may need to clarify what it means to "factor," and what this means in the context of finding roots.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work in pairs. • Some students may need remediation in complex problems by starting them with simple forms of differences of squares and working up to the complex. A more scaffolded approach may assist students in transitioning to more complex questions and using patterning. • Remind students to clearly identify what they are substituting into the quadratic.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work in pairs. • Have students state the zero product property in their own words to ensure they understand it and can apply it. • You may wish to have students create examples of factored equations and determine the roots of the equation.
Example 4 Have students do the Your Turn related to Example 4.	• You may wish to have students work in pairs to work through the example and complete the Your Turn question.
Example 5 Have students do the Your Turn related to Example 5.	• You may wish to have students work in pairs to work through the example and to complete the Your Turn question.

Check Your Understanding

Practise

For #1, remind students to always remove a common factor first whenever possible.

For #4, suggest students rewrite each expression as a difference of squares first to make factoring easier. For example, $4y^2 - 9x^2 = (2y)^2 - (3x)^2$.

For #5, to simplify, students can expand, group like terms, and then factor, or replace each function with a single variable, then factor, and then substitute the original function. Encourage them to try both methods to see which they like better.

For #6b), students should first remove the common factor or the greatest common factor.

For #7, the temptation is to expand, group like terms, and then factor. Remind students that they need only to set each factor equal to zero.

For #9, remind students that they always want the variable of the highest exponent to be positive.

Apply

For #11, students need to write an equation that represents the area of the rectangle as a function of its dimensions. They can then substitute the given area into the equation and set the equation equal to zero to determine the value of x.

For #13, the flare will follow a parabolic path. To determine how long it takes the flare to return to the sea, students must find the x-intercepts. In finding the x-intercepts, students are finding how long it takes the flare to have a height of zero. The height will be zero when it is first launched and when it returns to the sea.

For #16, students are no longer finding the root of the equation because the height is not zero. This makes this question different from others in this practise set. Instead, they are required to determine the time the ball has been in the air when $h = 3$. Students have to read the question carefully in order to answer it correctly. Once students have substituted the height for 3, they must make the equation equal to zero, and then find the roots.

For #20, you may need to help students reactivate their knowledge of the Pythagorean Theorem.

Extend

For #28, students may struggle in determining the expression. They might need help setting this question up.

Create Connections

Students must work backward in #30, making it an excellent question. You could ask students to solve #9b) from start to finish. Then, ask them to explain how to find the equation starting with the roots. Students would merely follow the same process in reverse. Seeing a question starting with the quadratic equation and ending with the roots might help students see how to start with the roots and end with the quadratic equation.

For #31, ask students
- If the quadratic equation cannot be solved by factoring, does it mean there is no solution?
- If the quadratic equation cannot be factored, how might you find the roots?

Students might remember completing the square from the previous chapter or remember solving quadratic equations graphically as done in the beginning of this chapter.

Project Corner

You might take the opportunity to discuss the Unit 2 project described in the Unit 2 opener at the beginning of Chapter 3, as well as the Unit 2 Project Wrap-Up at the end of the chapter. Both will give students insight into the project.

The Project Corner in this section presents several points of information about avalanches and why avalanche control managers attempt to deliberately set off avalanches. It also introduces the “launchers” that are used to initiate the slide. These points should provide sufficient prompts for students to begin their research. They could research the conditions in which avalanches are likely to occur, where avalanche experts position themselves to stay out of danger, launcher and missile design, and the parabolic trajectory of projectiles.

Have students identify any bullets or facts from the list that may assist them in designing their project. Consider providing students with **BLM U2–1 Unit 2 Project Checklist**, which provides an overview of the project. Explain to students that they will build their project by completing the Unit 2 project questions and activities in Chapter 4, and include them in a presentation or a report. Have students store all the work for the Unit 2 project in a project portfolio.

Meeting Student Needs

- Provide students with **Master 2 Centimetre Grid Paper.**
- Provide **BLM 4–5 Section 4.2 Extra Practice** to students who would benefit from more practice.
- Place posters around the classroom, illustrating the different types of question students may encounter in the Check Your Understanding, and the factoring strategies they might use. Some posters could also contain key words with definitions.
- Have algebra tiles available for students, as well as blank laminated cards. Students may wish to work on the white board where they can physically substitute a variable for a function in certain questions, such as #6.
- Give students the opportunity to solve equations using their own methods. You may discuss the different methods with them, and then let them decide which method works best for them.
- If you notice that students are struggling with any of the concepts or methods of solving, consider discussing the additional questions in the practice section as a class.

Gifted

- Ask students to create challenging factoring questions and include the correct solutions. When considering student responses, ask students to examine what defines a "challenging factoring question," and look for the accuracy of the solutions.

Enrichment

- Have students examine the path of golf shots, and attempt to create quadratics that mimic the path of golf shots, using different clubs. For example, a wedge creates a high, short shot because it has a loft of 46° or more. A 5-iron has a loft of about 27°, which creates a medium-height, medium-length shot. And a driver produces a long, lower shot because it has a loft of 14° or less. Alternative situations that students might examine include basketball shots or even the path of a paper ball thrown to a wastepaper basket from various distances. Students familiar with *APS* on smart phones can use these games as a guide. When considering student responses, look for equations that produce reasonable distances of height and flight.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 2 a), c), 3a), 4a)–c), 5a), b), 7a), c), d), 9a), b), d), 11. Students who have no problems with these questions can go on to the remaining questions.	• You may wish to have students work in pairs. • Encourage students to try to factor the questions in #1-4 using more than one method. • Remind students of the importance of identifying variables when using substitution for #6. • For students having difficulty linking roots with intercepts, have them sketch a diagram of the solution they obtain for #7. This may assist in linking the concepts together.
Assessment *as* Learning	
Create Connections Have all students complete 30, 32.	• Question #30 provides students an opportunity to show that they can write roots in factored form and use the factors to determine the original quadratic. For parts c) and d), encourage students not to use fractions in their final answer. Visual learners may benefit from plotting the roots on the x-axis and obtaining a visual of where the trajectory would be for the problem. • Have students complete #32 and share and compare their two numbers with another student.

Solving Quadratic Equations by Completing the Square

4.3

***Pre-Calculus 11*, pages 234–243**

Suggested Timing

90–135 min

Materials

- grid paper
- graphing calculator or computer with graphing software
- ruler

Blackline Masters

Master 2 Centimetre Grid Paper
BLM 4–3 Chapter 4 Warm-Up
BLM 4–6 Section 4.3 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF4 Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including:
- vertex
- domain and range
- direction of opening
- axis of symmetry
- x- and y-intercepts

and to solve problems.

RF5 Solve problems that involve quadratic equations.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 3d), 5a), c), f), 6a), c), d), 7a), c), e), 8, 9, 20
Typical	#1c), e), 5a), c), f), 6a), c), d), 7a)–d), 10–12, 20
Extension/Enrichment	#3e), f), 5b), d), e), 6e), f), 7f), 11, 13–15, 20, 21

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce prerequisite skills needed for this section. You could ask students to locate Roger's Pass on a map. Discuss why it might be important to keep a pass like this open. Measure off 15 m and see exactly how deep the snow gets here. Ask students if there is anywhere in their region that gets that much snow.

Before beginning the lesson, discuss the learning outcomes with students and link the outcomes to the material in the previous chapter. There, students used completing the square to write a quadratic function in the vertex-graphing form.

Web Link

For more information about Roger's Pass and graphing projectiles, go to www.mhrprecalc11.ca and follow the links.

Investigate Solving Quadratic Equations by Completing the Square

Completing the square is a method students learned in Chapter 3. Completing the square is useful for quadratics that cannot be factored.

For #2, students should start looking at selecting appropriate methods for solving quadratics equations, depending on the situation and the form of the equation. This question highlights the need to go beyond graphical methods when considering solutions.

For #5, students need to recognize they are determining the value of x when $h = 0.75$. Once they have substituted $h = 0.75$ in the original question, they can set the equation equal to zero and find the roots.

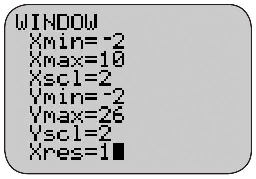
For #6, students should be judging whether their solutions are reasonable. Stress that they should always consider their solution in the context of the problem, and eliminate any solution that may be mathematically correct, but not reasonable for the scenario.

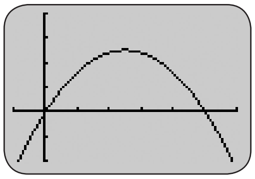
Meeting Student Needs

- Provide students with **Master 2 Centimetre Grid Paper**.
- For #3, the emphasis is on using completing the square to solve a quadratic. Students should already be proficient at this method from the previous chapter, but revisiting the steps might help some students.

Answers

Investigate Solving Quadratic Equations by Completing the Square

1. 



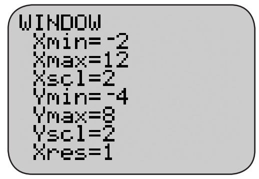
2. The x-intercepts are approximately −0.02 and 10. These could represent very large or important numbers and, therefore, might have to be more exact.

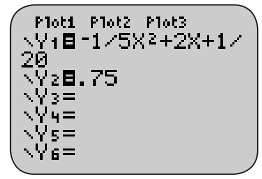
3. a) $f(x) = -\frac{1}{5}(x - 5)^2 + \frac{101}{20}$

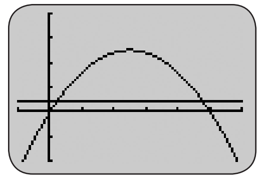
b) $x = 5 + \frac{\sqrt{101}}{2}$ and $x = 5 - \frac{\sqrt{101}}{2}$

4. The roots are $5 + \frac{\sqrt{101}}{2}$ and $5 - \frac{\sqrt{101}}{2}$. These represent the starting point and landing point of the projectile. The first is very near 0 and the last being about 10 units away from the starting point.

5. They could find out when the height of the shell is 750 m, or 0.75 km, by graphing the howitzer's trajectory and the line $y = 0.75$, and finding the intersection points.







The horizontal distances are approximately 0.36 km and 9.64 km. These distances are the x-values of the intersection points and show when the shell is at 0.75 km high.

6. Example: I would choose the farther distance because the closer one would set off an avalanche very close to the firing.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• You may wish to complete this Investigate as a class, but then allow individuals or pairs of students to draw their own conclusions for #6. • Students may benefit from quick going over graphing technology and setting appropriate windows. • Some students may require reactivation of the processes used in Chapter 3 for completing the square. • Ensure that students understand what they have found by having them share their response to #6. As this question supports the unit project, sharing and discussing responses would benefit all students.

Link the Ideas

Students should work in pairs to complete the examples. This will give them an opportunity to brainstorm and compare if they have difficulties or questions. You could also suggest that they discuss any terms that are new to them and help each other come to a fuller understanding of them.

Example 1

For the Your Turn question, you may want to help students reactivate their knowledge of the area of a circle. You could also consider having students check their answers by measuring a two dollar coin.

Example 2

In this example students follow the steps to complete the square. You could use this example as an opportunity to discuss the difference between exact roots and approximate roots. You may also want to have a discussion with students about why the ± symbol only appears on the right side of the equation, when we take the square root of both sides.

Example 3

Ensure that students understand why the plus/minus symbol is used in the sixth step of the solution. The concept of square roots and solving radical equations will be further developed in Chapter 5.

Example 4

Ask students to explain how the roots of the equation will help them find the horizontal distance. The height is zero before the kick and when the ball first hits the ground. As a result, there are two roots of the equation. The difference between the roots represents the distance the ball travels.

Key Ideas

Have students write down the Key Ideas in their own words and refer to them when required.

Meeting Student Needs

- Provide students with **Master 2 Centimetre Grid Paper**.
- Often students will do well when first learning a concept if a pattern is provided. You may wish to build posters for the wall, and templates that students can use for $1x^2$ and ax^2 quadratics. For example, you could provide something like the following for finding the third term of a perfect square:

$$x^2 + 10x - 21 = 0$$

$$x^2 + 10x + \underline{\qquad} = 21 + \underline{\qquad}$$

$$(x + 5)^2 = \underline{\qquad\qquad}$$

- If students are having difficulty with completing the square, have them work in pairs or small groups. You might work with these groups, providing mini-lessons for each type of question and giving immediate feedback while students work through the questions.

Answers

Example 1: Your Turn

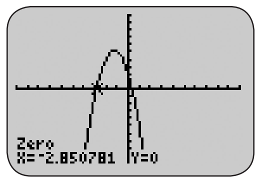
$w = 0.6$ cm

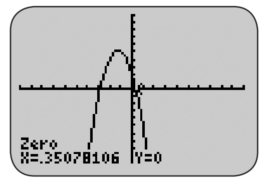
Example 2: Your Turn

$p = -1.9$ and 5.9 to the nearest tenth.

Example 3: Your Turn

The roots are $\frac{-5 + \sqrt{41}}{4}$ and $\frac{-5 - \sqrt{41}}{4}$. The roots, to the nearest hundredth, are -2.85 and 0.35. Using technology they are verified by finding the zeros of the related function.





Example 4: Your Turn

The ball travels approximately $54.6 - 17.4$, or 37.2 m.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Students may benefit from working with a partner to discuss the process used. • Remind students that they must consider both the negative and positive root on the right side and to consider if either of these is a extraneous root. • You may want to help students recall the formula for the surface area of a circle.
Example 2 Have students do the Your Turn related to Example 2.	• Students may benefit from working with a partner to discuss the process used. • Remind students that they are looking for two answers, which are not necessarily unique. • Before starting, have students verbalize to a partner the process they will use for solving. • Remind students to read the questions carefully and note the level of precision required in their answer. • Ensure students remember the sign for their answer. • Ask students how they could verify their answer using technology. They should draw a link between the roots they got and those on the screen. They should be identical.
Example 3 Have students do the Your Turn related to Example 3.	• Students may benefit from working with a partner to discuss the process used. They may have more difficulty with the process considering the value of the square root. Remind students that they are looking for two answers (not necessarily unique). • Have them verbalize their process to a partner or to you before starting. • Remind students to read the questions carefully and note the level of precision required in their answer. • Ensure students remember the sign for their answer.
Example 4 Have students do the Your Turn related to Example 4.	• You may wish students to work in pairs. The process modeled in the example needs to be supported for understanding. • Encourage students to think about the answer they determine and whether it represents the full motion of the ball. Unless something stops the ball, it will continue to bounce. The equation only deals with when the ball first strikes the ground.

Check Your Understanding

Practise

For #2, students should complete the square and isolate the square binomial.

For #5, students need to remember that there are two solutions when taking the square root: one positive and one negative.

Apply

For #11, students may need help setting up the formula.

For #12, again, students may struggle finding the equation and may require assistance.

Extend

Question #13 is an excellent question that requires students to work backwards. Ask students to solve #5a) from start to finish and to then explain how they would find the equation by starting with the roots. Students simply follow the same process in reverse. Seeing a question starting with the quadratic equation and ending with the roots might help students see how to start with the roots and end with the quadratic equation.

For #14, it might help students to complete the square for a normal question side-by-side with a question with a second variable. Following the steps for a question with only one variable might help students solve the question with the additional variable, k.

Project Corner

Section 4.3 Project Corner addresses points relating to the actual targeting and firing of a missile. Students could be encouraged to research both the "avalauncher" and the howitzer, draw the trajectory of each. By writing or obtaining a quadratic equation for each of the trajectories, the students can also compare vertices and the characteristics of the graphs.

Have students store all the work for the Unit 2 project in a project portfolio. You may wish to provide students with **BLM U2–1 Unit 2 Project Checklist**, which includes a list of all the requirements for Chapter 3 and Chapter 4. Students can use the checklist to check their progress or to prepare their final submission.

Meeting Student Needs

- Provide students with **Master 2 Centimetre Grid Paper**.
- Have students work in pairs for #1 to 5. The partners should take turns explaining the answer.
- Provide **BLM 4–6 Section 4.3 Extra Practice** to students who would benefit from more practice.
- Give students the opportunity to solve equations using their own methods. You may discuss the different methods with them, and then let them decide which method works best for them.
- If you notice that students are struggling with any of the concepts or methods of solving, consider discussing the additional questions in the practice section as a class.
- Consider bringing a flying disc to the classroom (or gymnasium) when discussing #9. One student can throw the disc to a partner. The rest of the class can estimate the height of the disc and then measure the distance between the thrower and catcher (or have one person throw the disc without it being caught, and then measure where the disc lands). Students could then write the equation representing the flight of the disc to see how it compares to the given equation. They should start with the equation $y = a(x - p)^2 + q$, where (p, q) is the disc's maximum height, and (x, y) is the distance between the thrower and where the disc ends up. Students could then determine the value of a, and then convert the equation into standard form.
- For #12, you might encourage students to draw a diagram to help them set up this question.

Gifted

- Ask students to focus on #13 in the extend section. Have them write a description of the process of writing an equation when given the roots. Then ask them to comment on the role of the general form of a quadratic and its implications for using the completing the square method. When considering student responses, look for answers that support the thinking found in the solutions to the questions in the student resource. The responses should also make the connection between the general form and processes, such as completing the square, which call for other forms to come into play.

Enrichment

- Ask students to explain what the value of finding exact roots of quadratics would have been to mathematicians prior to the invention of the calculator. Look for responses that note the value of root answers as opposed to numerical forms. For example, root answers can be cancelled in some circumstances, but must be rounded if needed for actual distance measurements.

Web Link

For more information about the 2009 Canadian Ultimate Championship and avalanche control at Lake Louise, go to www.mhrprecalc11.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 3d), 5a), c), f), 6a), c), d), 7a), c), e), 8, and 9. Students who have no problems with these questions can go on to the remaining questions.	• You may wish to encourage students to work with a partner, so you can determine where students are with respect to completing the square. • Question #1 assesses whether the student understands what a perfect square is. Coach them through the first part and provide examples of squares that have been completed (e.g., $(x + 2)^2$). • Questions #2–7 require students to complete the squares of various forms of quadratics. Helping student step through the modelled solutions for each form might assist them in moving forward. • Encourage students to verify their solutions with their calculators. Remind them to watch for the units for the solutions • Ask students to graph the equation they create in part #8b) and the provided equation in #9. Then, ask them to graph the equation once they have completed the square. Ask, "How do the results compare to your solutions?"
Assessment *as* Learning	
Create Connections Have all students complete #20.	• Question #20 provides students with an opportunity to explain their learning through solving a quadratic equation using different methods • You may wish to ask students to identify what still causes them concern, based on their lessons.

4.4 The Quadratic Formula

***Pre-Calculus 11*, pages 244–257**

Suggested Timing
90–135 min

Materials
- graphing calculator

Blackline Masters
BLM 4–3 Chapter 4 Warm-Up
BLM 4–7 Section 4.4 Extra Practice

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☐ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

Specific Outcomes

RF1 Factor polynomial expressions of the form:
- $ax^2 + bx + c, a \neq 0$
- $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$
- $a(f(x))^2 + b(f(x)) + c, a \neq 0$
- $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$

where a, b and c are rational numbers.

RF4 Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including:
- vertex
- domain and range
- direction of opening
- axis of symmetry
- x- and y-intercepts

and to solve problems.

RF5 Solve problems that involve quadratic equations.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 3a), c), e), 4a), b), 6, 7a), d), 11, 21–24
Typical	#2, 3a), c), e), 4a), b), 5b), c), d), 6, 7a), b), d), 8, 9, 13, 15, 21–24
Extension/Enrichment	#2, 3d), f), 4e), f), 5b), c), e), 6, 11, 13, 17–19, 21–24

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce prerequisite skills needed for this section.

Investigate the Quadratic Formula

For #1, the emphasis is on students explaining what is happening in the proof. They should understand how the proof is developed, but they are not developing the proof on their own.

For #3, you may want to direct students so that they understand that the quadratic formula cannot be used on a quadratic that does not cross the x-axis.

For #4, you may want to discuss with students the positive and negative aspects of the quadratic formula. If you are adding or subtracting a number, two possible solutions will occur. If you are adding or subtracting zero, two equal solutions will occur. Students should be reminded that you cannot take the square root of a negative number, in which case there is no real solution.

Meeting Student Needs

- Consider using #3 and 4 as an opportunity to talk with advanced students about imaginary roots.
- Students may wish to solve the equation in a different way. Encourage them to try. Ask them what other ways there might be to solve this equation. You could have them work in pairs or small groups to complete this task.

Answers

Investigate the Quadratic Formula

1. Example: Isolate the terms that contain the variable, x. Add the square of half the coefficient of the x-term to both sides. Express the side containing variables as a square; and simplify the other side of the equals sign. Take the square root of both sides and then isolate x.

2.
$$\begin{aligned} ax^2 + bx + c &= 0 \\ x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 \\ x^2 + \frac{b}{a}x &= -\frac{c}{a} \\ x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 &= -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \\ \left(x + \frac{b}{2a}\right)^2 &= -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \\ \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ x + \frac{b}{2a} &= \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \\ x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

3. a) No, it must be converted into the form $ax^2 + bx + c = 0$, $a \neq 0$.
b) Example: The quadratic formula is useful when the equation is not easily factored, and a graph would either be difficult or not accurate enough.
c) Example: If the quadratic equation is not given in the form $ax^2 + bx + c = 0$, it may be easier to use a strategy other than the quadratic formula.

4. Two. The quadratic formula is degree 2. The $\pm$ will yield one answer when the square root is added and one answer when the square root is subtracted.

5. If $b^2 - 4ac = 0$, then we are adding or subtracting 0, which results in only one root.

6. Yes, if $b^2 - 4ac < 0$ because we can't take the square root of a negative number.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• You may wish to complete this investigation as a class, comparing each of the steps of the specific and general example as you go through it. • Some students may benefit from trying to use the quadratic formula to solve a trinomial that could have been easily factored, and one for which students may already know the roots. Doing so should help students when answering the first question in #3. • If students are having difficulty with #4 and 5, encourage them to revisit some of the examples and graphs in section 4.1 • You may wish to prompt students by asking them under what conditions the square root of a number is not possible. This may assist them in generalizing a solution for non-real roots.

Link the Ideas

Consider engaging students in a discussion regarding the discriminant and how it is linked to the nature of the roots. Since a quadratic is a polynomial of degree 2, there will always be zero or two solutions:

- two equal roots if the discriminant is zero.
- two unequal roots if the discriminant is a positive number.
- no solution if the discriminant is less than zero because you can't take the square root of a negative number.

Students should work in pairs to complete the examples. This will give them an opportunity to brainstorm and compare if they have difficulties or questions.

Example 1

Help students see the connection between the nature of the roots and the number of x-intercepts.

Avoid saying that there is no solution, but rather say that there is no *real* solution. Students will benefit from knowing that there is no solution in the real number system, but that there is a solution in the imaginary number system.

You should also avoid saying that there is one solution because this is not the case with quadratics. When there is only one root, there are really two equal solutions. Saying that there are "two equal solutions" will help students become familiar with this concept.

Example 2

Ask students why the equation must be in standard form and why the equation must equal zero. You can also do examples where the variable term, the variable squared term, or the constant are missing. In these cases a, b, or c would be zero.

Students have not seen how to use the quadratic formula when fractions are involved. So, for question b) of the Your Turn, consider either showing students how to use the quadratic formula with fractions or show them how to eliminate the fractions by multiplying each term, including the zero, by the greatest common factor.

Example 3

Encourage students to remove common factors. Alternatively, whenever possible, they can divide each term by the greatest common factor. However, when doing so, students must remember to also divide the zero on the right-hand side of the equation by the greatest common factor.

Discuss with students how they can eliminate the decimals in the Your Turn question. Ask,

- If you multiply each term by 10, will this eliminate the decimals?
- What number would you have to multiply each term by to eliminate the decimals?

When employing this strategy, caution students to remember to multiply the right-hand side of the equation by the same number they multiplied the left-hand side of the equation.

Example 4

Some students may have difficulty understanding why $2x$ is added to the sides in the area formula, rather than just x. Remind them that the picture is placed in the centre of the mat. Discuss what this means in terms of the measurements (there are two equal widths, one on each side of the painting). You might also consider asking how this problem could be solved by placing the picture in the corner of the mat and then calculating how much space to leave around the edges in order to centre the painting. In that case, you would work with only x, and then divide this distance by two to determine how much space to leave on each side.

Meeting Student Needs

- You might tell students that one way of remembering the quadratic formula is to sing the following to the tune of *Pop Goes the Weasel*:
  x equals negative *b*
  plus or minus square root
  b squared minus four *ac*
  all over two *a*.
- Suggest that students try to create a mnemonic device of their own to remember this formula.
- Suggest students solve all three equations in Example 1 both graphically and using the quadratic formula. Again, doing this will help students see the correlation between the discriminant and the nature of the roots.
- In Example 2, have the students highlight the discriminant in the process of solving. This will help them make the connection between the discriminant and the nature of the roots.
- In Example 3a) iii), the solution does not move the constant to the right side of the equation, which is the method of solving with which students will be most accustomed. If students are having difficulty, consider presenting the example in the format with which they are more familiar, as follows:

$$
\begin{aligned}
6x^2 - 14x + 8 &= 0\\
6x^2 - 14x &= -8\\
x^2 - \frac{7}{3}x &= -\frac{4}{3}\\
x^2 - \frac{7}{3}x + \frac{49}{36} &= -\frac{4}{3} + \frac{49}{36}\\
x^2 - \frac{7}{3}x + \frac{49}{36} &= -\frac{48}{36} + \frac{49}{36}\\
\left(x - \frac{7}{6}\right)^2 &= \frac{1}{36}\\
x - \frac{7}{6} &= \pm\sqrt{\frac{1}{36}}\\
x &= \frac{7}{6} \pm \frac{1}{6}
\end{aligned}
$$

- Suggest that students create a recipe card on which to write the quadratic formula, a quadratic equation in standard form, the discriminant, and the three scenarios that the discriminant can produce. They could colour code the information on the card. However, encourage students to memorize the formula, using the "song version" of the formula or a mnemonic device of their own. It is much quicker for them to work with the quadratic formula if they have it memorized, rather than referencing a formula sheet.
- If some students are struggling with any of the concepts or methods of solving, consider discussing the additional questions in the practice section topic as a class. You may want to provide practice questions for students to finish solving. For example:

$$x = \frac{-4 \pm \sqrt{36 - 4(1)(9)}}{6} \qquad x = \frac{8 \pm \sqrt{48 - 4(1)(9)}}{6}$$

$$x = \frac{-8 \pm \sqrt{24 - 4(1)(9)}}{4}$$

Common Errors

- Consider the following example: $5v^2 - 7v - 1 = 0$; $a = 5$, $b = -7$, and $c = -1$. When either or both b and c are negative, students can sometimes make errors when inserting the values into the quadratic formula and making the calculations. There are several different errors student might make:
  - when b is negative, students sometimes forget to change the sign for $-b$, or forget to change the sign for b^2.
  - when c is negative, some students forget to change the sign of the term $-4ac$.

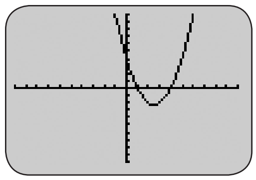
Rx Help students avoid these mistakes by emphasizing the importance of using parentheses for all substitutions. For example,

$$v = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(-1)}}{2(5)}$$

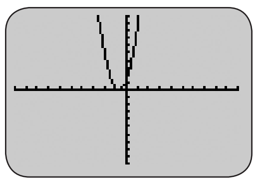
Answers

Example 1: Your Turn

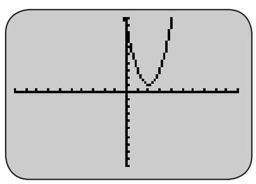
a) $(-5)^2 - 4(1)(4) = 9$; two distinct, real roots



b) $(4)^2 - 4(3)\left(\frac{4}{3}\right) = 0$; one real root or two equal real roots



c) $(-8)^2 - 4(2)(9) = -8$; no real roots



Example 2: Your Turn

a) $\frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2(3)}$, $x = \frac{1}{3}$ and $x = -2$

b) $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\left(\frac{1}{2}\right)\left(-\frac{5}{2}\right)}}{2\left(\frac{1}{2}\right)}$, $x = 1 + \sqrt{6}$ and $x = 1 - \sqrt{6}$, or approximately 3.45 and -1.45

Example 3: Your Turn

Example: Because the question has decimals, it's probably not easily factored. So, graph with technology or use the quadratic formula.

$\frac{-(-3.7) \pm \sqrt{(-3.7)^2 - 4(0.57)(-2.5)}}{2(0.57)}$, so x is approximately 7.11 and -0.62.

Example 4: Your Turn

The equation is $x^2 - 51x + 378 = 0$, so 9 cm and 42 cm. You can't reduce either side by 42 cm, so the width of the removed strips is 9 cm.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• It may benefit some students to reactivate their knowledge of the order of operations. • Coach students through the formula for the discriminant and solve the first question with them. Have students verbalize what values they will substitute into the formula.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work in pairs. • Have students identify the values for *a*, *b*, and *c*. Coach students to determine the nature of the roots by the discriminant. • Ensure students understand the difference between an *exact* and an *approximate* root. • Some students may have difficulty recognizing that the ± results in two solutions for the quadratic formula. To help them, suggest that they work through the equations separately, showing both calculations.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students either work with a partner, or complete the question on their own and then compare with a partner. • Encourage students to solve the quadratic in more than one way. Explain that they may have a favourite approach, but they should be confident in more than one approach.
Example 4 Have students do the Your Turn related to Example 4.	• You may wish to have student work with a partner. • Some students may have difficulty generating the area formula. Suggest they draw a diagram of the situation. Alternatively, they could mimic the problem by cutting 2 inches from the bottom and side of an 8.5 × 11 in. piece of paper. Then they could centre this reduced piece of paper on a full sheet. Then, label the paper according to the problem.

Check Your Understanding

Practise

Remind students to simplify the equations by eliminating all fractions and decimals and equating the equation to zero. Placing the equation in descending powers of the variable will also help keep things organized. Students must remember to use brackets for all substitutions involving the quadratic formula.

Apply

For #8, ask student whether either the area or perimeter is affected by the fact that the horse corral uses the barn for one side.

For #9, students might need to recall how to find the percent of a number.

For #11, you could ask students why they are finding the roots of the equation to determine how wide the arch is at the base. Again, connecting roots of an equation to x-intercepts of the corresponding graph, students are trying to find the point at which $h = 0$. The height of the arch will be zero in two points: the start and the end of the arch.

For #12, ask students how they can tell that it is an open-topped box from the diagram. Students could use the dimensions of the cardboard and the area of the base of the box to find the side length. Encourage them to find the side length using an alternative method. For example, since the total area is 360 (i.e., 30 by 12) and the area of the base is 208, all the remaining pieces have an area of 152. The top and bottom pieces (the box and the missing corners) have an area of $30x$. The two sides of the box have an area of $12x - 2x^2$. The equation then becomes $30x + 12x - 2x^2 + 12x - 2x^2 + 30x = 152$. You can then solve for x by grouping like terms, equating the equation to zero, and then factoring.

For #13 students are not initially finding the roots of the equation. They must substitute the given values for d, equate the equation to zero, and then solve. You could ask students why the parabola of the given equation opens up and not down. The parabola opens down because as the car's speed increases, the stopping distance increases as well. Similarly, as the car's speed decreases, the stopping distance decreases. The parabola opens up because of this relationship. Ask students, in terms of the relationship between speed and stopping distance, what would it mean if the parabola opened downward?

For #15, students might struggle to find the equation. They might not see that there is only one variable, and that variable represents the number of \$15 decreases in price. Discuss the concept of revenue with students. Revenue, or the money brought in, is found by multiplying the number of ski jackets sold by the price per ski jacket. Once students have found the equation and have solved for the roots, they may think the answer is the decrease in price. You may need to help them recognize that the variable represents the number of times the price is decreased by \$15. So, if, for example, $x = 5$, the price was not reduced by \$5, the price was reduced by $5 \times \$15$, or \$75.

Extend

For #17, you could discuss with students how to verify roots. To verify the solution of -8, students would substitute -8 for x in the equation, and then solve for b. Once students have found b, ask them how they can find the other root.

For #18, you might want to help students recall the formula for surface area of a cylinder: $SA = 2\pi r^2 + 2\pi rh$.

For #19, you could ask students what they need to find the area of the right triangles. They know that the area of the three triangles are equal. They also know that one triangle has a height and base of x, and a second triangle has a height of 6 and base of $(6 - x)$. So, $x^2 = (6(6 - x))$, or $x^2 + 6x - 36 = 0$.

Create Connections

You could tie #22 to the idea of symmetry about the vertex. Since parabolas are symmetrical about the vertex, the halfway point between the two x-intercepts would equal the vertex. Once students have the vertex, they can easily determine the axis of symmetry.

Project Corner

The Project Corner in section 4.4 provides information about contour maps. As students will have discovered, the placement of the missiles is important when initiating a controlled avalanche. Contour maps will assist students with elevations. The knowledge students have gained about the use of the quadratic formula will help them solve any quadratic equation that they may develop.

Consider providing students with **BLM U2–1 Unit 2 Project Checklist**, which includes a list of all the requirements for Chapter 4. Students can use the checklist to check their progress or prepare their final submission.

Web Link

For more information about reading contour maps and elevations, go to www.mhrprecalc11.ca and follow the links.

Meeting Student Needs

- Provide **BLM 4–7 Section 4.4 Extra Practice** to students who would benefit from more practice.
- Give students the opportunity to solve equations using their own methods. You may discuss the different methods with them, and then let them decide which method they feel works best.
- Suggest that students colour-code their substitutions to emphasize the placement of the coefficients. They could use one colour for a, one for b, and another for c.
- Students should refer to the notes they created on recipe cards earlier. Each student should add other ideas/concepts to their cards as they work through the questions. These will serve both as a reference and as study notes.
- For question #7, students could prepare small debates as to which method is better for specific questions. Two students could present their cases to the class and then the class could vote for the method they support.

Enrichment

- Ask students to explain why the value of the discriminant can be used to indicate the number and type of solutions. Why does it mean that there is one solution and the solution is real when the discriminant is 0? When considering student responses, look for an understanding of the fact that there is no real solution to the square root of a negative number, and that if the discriminant is 0, there is only one point where the line touches the x-axis. Both of these aspects of student solutions point to an understanding of the impact of the square root and squared components.

Gifted

- Challenge students to explain the origin of the quadratic formula and explain its uses, especially in technology. This is a wonderful opportunity for students to see the historical development of a problem solving strategy. Look for accuracy as well as a sense of the various approaches to quadratics and their role in problem solving.

Web Link

For information about the history of the quadratic formula, go to www.mhrprecalc11.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do : #1, 3a), c), e), 4a), b), 6, 7a), d), 11. Students who have no problems with these questions can go on to the remaining questions.	• Question #1 provides students with an opportunity to determine the nature of the solutions for a quadratic, and to verify their conclusion by graphing. Students may wish to have a summary table beside them, identifying under what conditions the number of roots occur. Encourage students to graph in more than one way. • Questions #3 and 4 require students to use the quadratic formula to solve for the roots, but the answers are expressed differently. Discuss with students the difference between an exact root and an approximate root. • Question #7 provides an opportunity for students to demonstrate their learning to solve a quadratic by using their own approach. • Coach students through #8 to determine the expressions to multiply. • Encourage students to draw a graph of #11 to visualize what the problem is asking for.
Assessment *as* Learning	
Create Connections Have all students complete #21, 22, 23, 24	• Students may need coaching through the simplification of the roots presented. In particular, they may need help seeing two answers. • Have students recall the meaning of the axis of symmetry. Encourage them to sketch a graph of the quadratic to assist in determining the axis of symmetry. • Question #23 provides students an opportunity to explain their thinking and to identify a preferred method for solving. Allow students to identify two preferred methods. • Question #24 allows students to design a graphic organizer, which will demonstrate their learning as well as serve as a summary page when studying for their final exam.

4 Chapter 4 Review

***Pre-Calculus 11*, pages 258–260**

Suggested Timing
90–135 min

Materials
- grid paper
- graphing calculator or computer with graphing software

Blackline Masters
Master 2 Centimetre Grid Paper
BLM 4–4 Section 4.1 Extra Practice
BLM 4–5 Section 4.2 Extra Practice
BLM 4–6 Section 4.3 Extra Practice
BLM 4–7 Section 4.4 Extra Practice
BLM 4–8 Chapter 4 Review #22

Planning Notes

Have students who are not confident with their skills and understanding discuss strategies with you or a classmate. Encourage them to refer to their summary notes, worked examples, and previously completed questions in the related sections of the student resource.

Have students make a list of questions with which they need
- no help
- a little help
- a lot of help

They can use these lists to help them prepare for the Practice Test.

Meeting Student Needs

- Students who require more practice on a particular topic may refer to **BLM 4–4 Section 4.1 Extra Practice**, **BLM 4–5 Section 4.2 Extra Practice**, **BLM 4–6 Section 4.3 Extra Practice**, or **BLM 4–7 Section 4.4 Extra Practice**.
- Provide students with **Master 2 Centimetre Grid Paper**.
- Ensure students have access to algebra tiles, as well as their prepared notes.
- Have students refer to the list of Student Learning Outcomes to determine which areas need the most support before the assessment.
- Break students into groups. Each group could prepare a poster illustrating the key concepts and ideas found in the chapter. The posters could be placed in the classroom for students to access while working on the Review questions.

Enrichment

Ask students to explore the intersection of two parabolas. For example, suppose a scientist was trying to test if an object falling from space could be intercepted by an object fired from Earth. Since both are travelling in parabolic form, it should be possible to determine the interception of the two functions. Have students test this hypothesis. The intersection of two parabolas, based on two objects in flight, is the point where x and y are solutions to both equations. Students may solve simultaneous equations or use technology to find such points.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 4 Review The Chapter 4 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against answers in the student resource.	• Have students revisit any section that they are having difficulty with prior to working on the chapter test.

Chapter 4 Practice Test

4

***Pre-Calculus 11*, pages 261–262**

Suggested Timing
45–60 min

Materials
- grid paper

Blackline Masters
BLM 4–9 Chapter 4 Test

Planning Notes

Have students start the practice test by writing the question numbers in their notebook. Have them indicate for which questions they need no help, a little help, and a lot of help. Have students first complete the questions they know they can do, followed by the questions they know something about. Finally, suggest to students that they do their best on the remaining questions.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1–4, 7, 8, 11.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1	4.1	Example 2	✓ describe the relationship between the roots of a quadratic equation, the zeros of the corresponding quadratic function, and the *x*-intercepts of the graph of the quadratic function
#2	4.2	Example 1	✓ factor a variety of quadratic expressions
#3	4.2	Example 1	✓ solve quadratic equations by factoring
#4, 8	4.4	Example 2	✓ solve quadratic equations, using the quadratic formula
#5	4.2	Example 4	✓ solve problems that involve quadratic equations
#6	4.1	Example 1 Example 3	✓ solve quadratic equations by graphing the corresponding quadratic function
#7	4.3	Example 2	✓ solve quadratic equations, using the process of completing the square
#9	4.4	Example 1	✓ use the discriminant to determine the nature of the roots of a quadratic equation
#10	4.4	Example 4	✓ select and use an appropriate method for solving a quadratic equation
#11	4.4	Example 4	✓ select and justify a method for solving a quadratic equation
#12, 13, 14	4.4	Example 4	✓ model and solve a quadratic equation

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 4 Self-Assessment Have students use their responses on the practice test and work they completed earlier in the chapter to identify skills or concepts they may need to reinforce.	• Have students review their concept map and compare it to the outcomes • Before the chapter test, coach students in areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 4 Test After students complete the practice test, you may wish to use **BLM 4–9 Chapter 4 Test** as a summative assessment.	• Have students use their concept map, their I Can statements, and the results of their Review questions to determine areas with which they require extra work before beginning the chapter test.

UNIT 2

Unit 2 Project Wrap-Up

***Pre-Calculus 11*, page 263**

Suggested Timing

60–90 min

Blackline Masters

Master 1 Project Rubric
BLM U2–1 Unit 2 Project Checklist
BLM U2–2 Unit 2 Project Rubric – Option 1
BLM U2–3 Unit 2 Project Rubric – Option 2

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

RF4 Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including:
- vertex
- domain and range
- direction of opening
- axis of symmetry
- x- and y-intercepts

and to solve problems.

RF5 Solve problems that involve quadratic equations.

Planning Notes

The Unit 2 Project Wrap-Up can be completed at the end of section 4.4. Make sure students are aware of the project information provided on page 139, and the Unit 2 Project Wrap-Up on page 263. Take time to revisit the Project Corners in Chapter 3 and 4. Students need to consider examples of quadratic functions in real-world contexts. They can discuss in general where quadratic equations are used and when a quadratic function is considered a good model for a given situation.

If students have not yet chosen an application of quadratic functions, they should do so at this time. You might have students work in pairs or small groups to research possible applications of quadratic functions. Encourage them to choose an application that is of personal interest. They can choose either of the two options below.

Option 1: Quadratic Functions in Everyday Life

Have students search the Internet for two images or video clips, one related to objects in motion and one related to fixed objects. These images or clips should demonstrate a quadratic relationship. Note that the examples that students choose may not actually represent quadratic functions, but they should be close enough to warrant investigation. If they are having difficulty selecting a topic to research, it might help them to revisit the Project Corners in the chapters, and the situations modelled throughout the investigations, worked examples and problems.

After they have made their choices, have students model each image or clip with a quadratic function. Students might superimpose a set of axes and examine the relationship shown by the curve. Or, they might find a set of data that suggests a quadratic relationship, graph the data, and use the graph to analyse the relationship. Ambitious students may wish to collect their own data for an application of quadratic functions, such as projectile motion. Next, have students determine the accuracy of the model. For each image or clip, have them research the situation involved to determine what evidence there is for a quadratic relationship.

Throughout their project work, students may wish to talk about their ideas with each other. Peer coaching may allow students to improve their skills.

Option 2: Avalanche Control

In Chapter 4, students are introduced to the idea of avalanche safety. Students could go over the Project Corners throughout the chapter, and share their own ideas and experiences with avalanche safety and avalanche control.

Have students research where the best locations are for avalanche cannons. They can then do further research into why and how avalanches are intentionally triggered. Students are to determine three quadratic functions to model the trajectory of an avalanche control projectile and create a graph of each. Each graph should illustrate the specific coordinates of where the projectile will land. Students could refer to problems, examples, and investigations that relate to the equation for the trajectory of a projectile. Students will then complete a one-page report summarizing their findings.

Have students use **BLM U2–1 Unit 2 Project Checklist** to make sure that all parts of their project have been completed. As a class, brainstorm different ways students can do their presentations. You may wish to limit the time each student is allowed to present.

Students do best if they know exactly how they will be evaluated. One way to increase student motivation is to work with the class to create a rubric for the project. You may wish to use the **Master 1 Project Rubric** template and review the general holistic points within the 1–5 scoring levels. Discuss with students how they might achieve each of these levels in the Unit 2 project.

Ask questions such as the following:
- What are the big ideas in the unit?
- Which of the big ideas are involved in the project?
- What part of the project could you complete or get partially correct to indicate that you have a basic understanding of what was learned in the two chapters? (Should you get a pass mark if you can show…)
- What would be on a level 1 project? What might you start on correctly? What could be considered a significant start?
- What would be expected for a level 5 project? What should it include? Try to help students realize that a level 5 project may have a minor error or omission that does not affect the final result.
- Knowing the expectations of levels 1, 3, and 5 projects, what would I expect for a level 4? Help students to understand that this is still an honours level and therefore the work should be reflective of this. However, even an honours project may have a minor error or omission. Discuss the difference between a major conceptual error and a minor calculation or omission. Understanding this point will help clarify for students the expectations and differences between a pass and above average result and may encourage some students to work toward the highest level. Repeat the process for level 2.

BLM U2–2 Unit 2 Project Rubric–Option 1 and **BLM U2–3 Unit 2 Project Rubric–Option 2** models completed rubric for this project. Note that these provide one idea for completing a rubric. Your class rubric may have more detail. Use what your students have developed or this example to ensure that students understand the criteria for an acceptable level, as well as what would warrant either an unacceptable or an honours grading.

Assessment	Supporting Learning
Assessment *of* Learning	
Unit 2 Project This unit project gives students an opportunity to apply and demonstrate their knowledge of the following: • quadratic equations in both the form $y = a(x - p)^2 + q$ and $y = ax^2 + bx + c$ • analyzing quadratic equations to determine, both graphically and algebraically, their vertex, domain and range, direction of opening, axis of symmetry, and x- and y-intercepts • solving problems that involve quadratic equation Work with students to develop assessment criteria for this project. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing students work on the Unit 2 project.	• You may wish to have students use **BLM U2–1 Unit 2 Project Checklist**, which provides a list of the required components for the Unit 2 project. • Revisiting the Project Corner boxes at the end of each section of Chapters 3 and 4 will assist students in developing appropriate data presentations. • Make sure students recognize what is expected for the minimum requirements for an acceptable project as well as the difference between levels 5 and 4. • Clarify the expectations and the scoring with students using **Master 1 Project Rubric** or the rubric you developed as a class. It is recommended to go over the scoring rubric at the beginning of the project, as well as intermittently throughout the project to refresh student understanding of the project assessment.

UNIT 2

Cumulative Review and Test

***Pre-Calculus 11*, pages 264–267**

Suggested Timing
60–90 min

Materials
• grid paper

Blackline Masters
Master 2 Centimetre Grid Paper
BLM U2–4 Unit 2 Test

Planning Notes

Have students work independently to complete the review and then compare their solutions with a classmate's. Alternatively, you may wish to assign the cumulative review to reinforce the concepts, skills, and processes learned so far; if students encounter difficulties, provide an opportunity for them to discuss strategies with other students. Encourage them to refer to their notes, and then to the specific section in the student resource. Once they have determined a suitable strategy, have students add it to their notes. Consider having students make a list of the questions that they found difficult. They can then use the list to help them prepare for the unit test.

Meeting Student Needs

- Provide students with **Master 2 Centimetre Grid Paper.**

Assessment	Supporting Learning
Assessment *for* Learning	
Cumulative Review, Chapters 3 and 4 The cumulative review provides an opportunity for students to assess themselves by completing selected questions pertaining to each chapter and checking their answers against the answers in the back of the student resource.	• Have students revisit their notes from each chapter to identify items that they had problems with, and do the questions related to those items. Have students do at least one question that tests skills from each chapter. • Have students revisit any chapter section with which they are having difficulty.
Assessment *of* Learning	
Unit 2 Test After students complete the cumulative review, you may wish to use the unit test on pages 266 and 267 as a summative assessment.	• Consider allowing students to use their graphic organizers. • You may wish to have students complete **BLM U2–4 Unit 2 Test**, which provides a sample unit test. You may wish to use it as written or adapt it to meet the needs of your students.