Systems of Equations

***Pre-Calculus 11*, pages 422–423**

Suggested Timing
30–45 min

Blackline Masters
BLM 8–2 Chapter 8 Prerequisite Skills
BLM U4–1 Unit 4 Project Checklist

Key Terms
system of linear-quadratic equations
system of quadratic-quadratic equations

What's Ahead

In section 8.1, students are introduced to linear-quadratic and quadratic-quadratic systems of equations. They are shown how these systems can represent real-world situations and how the solutions to these systems can be determined graphically by finding points of intersection. In section 8.2, students extend their knowledge of linear-quadratic and quadratic-quadratic systems of equations, learning how to solve them algebraically by both substitution and elimination.

Planning Notes

Begin this chapter by helping students reactivate their knowledge of systems of equations. Ask:

- What is a linear equation? Give an example. What does the graph of a linear equation look like?
- What is a quadratic equation? Give an example. What does the graph of a quadratic equation look like?
- What is a linear system of equations? Give an example.
- How do you find the solutions to a linear system of equations?
- What are the possible number of solutions to a linear system of equations?

Then, tell students that in this chapter they are going to explore linear-quadratic and quadratic-quadratic systems of equations. They will solve these systems graphically and algebraically. Ask:

- What do you think the graph of a linear-quadratic system of equations would look like?
- What do you think a graph of a quadratic-quadratic system would look like?

Up to this point, students are only familiar with quadratic equations of the form $y = ax^2 + bx + c$, so they may not raise questions about other conic sections, such as circles, ellipses, and hyperbolas. Should students raise this issue, you may wish to tell them that the quadratic equations dealt with at this level correspond to quadratic functions only. Other quadratic equations, such as those for circles, ellipses, and hyperbolas, will be addressed in future math courses.

Unit Project

In this project, students choose an object that they think could be enhanced by the use of nanotechnology. The object will have linear and parabolic design lines. In Chapter 8, students create a futuristic design of their object and determine equations that control its shape. In Chapter 9, they complete a cost analysis for part of the construction of their object.

Throughout the chapter, Project Corner boxes provide information related to the unit project. These features are not mandatory, but they are recommended because they provide some background for the final unit project assignment. If you are going to develop a project rubric with the class, you may want to start now. See pages 341–342 in this Teacher's Resource for information on working with students to develop a class rubric.

Chapter Summary

Students should be aware of the benefits of keeping a summary of what they are learning in the chapter. Discuss methods for keeping a summary, such as a

- Foldable
- mind map
- concept map
- spider map
- Frayer model
- KWL chart

Meeting Student Needs

- Consider having students complete the questions on **BLM 8–2 Chapter 8 Prerequisite Skills** to activate prerequisite skills for this chapter.
- Hand out **BLM U4–1 Unit 4 Project Checklist**, which provides a list of all the requirements for the Unit 4 project.

- Encourage students to research nanotechnology on the Internet. You may wish to show a few pictures of objects created with nanotechnology.
- Student athletes in the classroom may wish to share the use of technology and mathematics in the development of their area of skill. The Internet has many video clips illustrating the correct technique for various track and field events. Some of these videos include the mathematics involved in these events. Consider showing a couple of these clips or suggesting that students research them and share them with the class.

Enrichment

- Encourage students to imagine that they have been shrunk to the size of the period at the end of this sentence. What could they do at that size that they could not do at their present size? Suggest that they consider the opportunities that would be open to them in electronics or medicine. Such is the world of nanotechnology. Ask students to consider the mathematics of miniaturization as they work through the chapter.

Gifted

- Ask students to view the image on page 423 of the student resource, which shows Dr. Ian Foulds holding a microrobot that he and his students developed. Based on the images, ask students to estimate the dimensions of this "microbot."

Career Link

Suggest that students who are interested in what university researchers do find out more about this career online. Students may also research nanotechnology and Ian Foulds (or other Canadian researchers inside and outside the field of nanotechnology). In what areas of study would students like to do research? Are there emerging fields of knowledge with which they would like to be involved? Are there any universities, Canadian or otherwise, that are particularly prominent in these areas?

Web Link

To learn more about nanotechnology and how systems of equations are used in other careers, go to www.mhrprecalc11.ca and follow the links.

Solving Systems of Equations Graphically

8.1

***Pre-Calculus 11*, pages 424–439**

Suggested Timing

120–150 min

Materials

- grid paper
- graphing calculator
- computer with graphing software

Blackline Masters

Master 3 0.5 Centimetre Grid Paper
BLM 8–3 Chapter 8 Warm-Up
BLM 8–4 Section 8.1 Extra Practice
TM 8–1 How to Do Page 428 Example 2 Using TI-83/84
TM 8–2 How to Do Page 428 Example 2 Using TI-Nspire™

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF6 Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–6, 8, 9, 11, 19, 20
Typical	#1–11, one of 12–14, 20
Extension/Enrichment	#12, 15–18, 20

Planning Notes

Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce prerequisite skills needed for this section. Before beginning work on the chapter, provide the student learning outcomes either on the whiteboard or in a handout. Consider having available two or three illustrations of systems of equations, including both linear-quadratic and quadratic-quadratic systems. Discuss the significance of the point(s) of intersection on the graphs.

You may need to reactivate students' knowledge of the following five topics.

1. Linear Equations

Provide students with an example of a linear equation, such as $3x - y - 10 = 0$. Ask:

- What is the equation's degree?
- What variables does it contain?
- What would the graph look like?
- What are the x-intercepts? y-intercepts?
- What is the slope-intercept form for the equation? ($y = mx + b$, so for the above example $y = 3x - 10$)
- How do you write an equation of a line given a point and the slope? (Given slope (m) and point (x_1, y_1), the point-slope formula is $y - y_1 = m(x - x_1)$.)
- How do you write an equation given two points? (Given the points (x_1, y_1) and (x_2, y_2), the formula is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.)

2. Quadratic Equations: Parabolas

Ask:

- What is the vertex form of the equation of a parabola? (The form is $y = a(x - p)^2 + q$.)
- What does a represent in the vertex form?
- What is the vertex?
- What is the axis of symmetry?
- Given the equation $y = -3(x - 4)^2 + 5$, what is the vertex? Which way does the parabola open? What is the axis of symmetry?
- Given the standard form of a quadratic equation, $y = ax^2 + bx + c$, how would you determine the vertex form? Be sure that students are familiar with how to complete the square, and then either work through the following example with students, or have them work through it on their own:
  $y = 2x^2 - 12x - 11$
  $y = 2(x^2 - 6x + 9) - 11 - 18$
  $y = 2(x - 3)^2 - 29$

3. Solving Quadratic Equations

Ask:

- Give an example of a quadratic equation.
- How many solutions can a quadratic equation have?
- How can you solve a quadratic equation?

Example:
Solve by factoring.
$0 = x^2 - 5x + 6$
$0 = (x - 3)(x - 2)$
$x - 3 = 0$ or $x - 2 = 0$
The solutions are 3 and 2.

Solve using the quadratic formula.
$0 = 2x^2 + 24x + 66$
$0 = 2(x^2 + 12x + 33)$

Apply the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(33)}}{2(1)}$$

$$x = \frac{-12 \pm \sqrt{12}}{2}$$

$$x = -6 \pm \sqrt{3}$$

$$x \approx -4.268 \text{ or } -7.732$$

4. Solving Linear Systems of Equations
Provide the following example:
Consider the system
$x + 2y - 10 = 0$
$5x - 2y - 26 = 0$

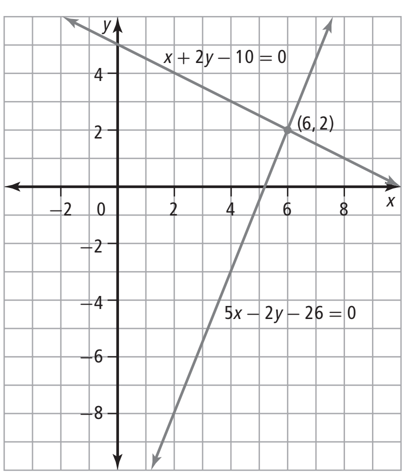
Then, ask students:
- What type of system of equations is this? (linear)
- How many solutions are possible to a linear system of equations?
- What are the methods for solving a linear system of equations?
- How do you verify the solution to a system of linear equations?

Solve the system graphically, and then algebraically using both substitution and elimination.

Solve graphically.
Write equations in slope-intercept form, and then graph.

$$y = \frac{-x}{2} + 5$$

$$y = \frac{5x}{2} - 13$$



The solution is (6, 2), where the lines intersect.

Solve algebraically by substitution.
Solve for x in equation 1:
$x = -2y + 10$

Substitute $-2y + 10$ for x in equation 2, and solve for y.

$$\begin{aligned} 5(-2y + 10) - 2y - 26 &= 0 \\ y &= 2 \end{aligned}$$

Substitute $y = 2$ into either equation and solve for x.

$$\begin{aligned} x + 2(2) - 10 &= 0 \\ x &= 6 \end{aligned}$$

The solution is (6, 2).

Solve algebraically by elimination.

$$\begin{aligned} (x + 2y - 10 &= 0) \\ 5x - 2y - 26 &= 0 \\ \hline 6x - 36 &= 0 \\ x &= 6 \end{aligned}$$

Substitute $x = 6$ into either equation and solve for y.

$$\begin{aligned} 6 + 2y - 10 &= 0 \\ y &= 2 \end{aligned}$$

The solution is (6, 2).

Remind students that they should verify their solutions by substituting $x = 6$ and $y = 2$ into each equation in the system. A true statement should result for both.

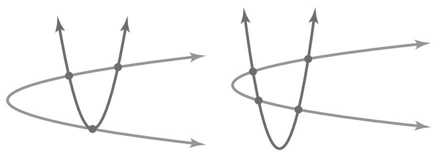
5. If $a = b$ and $a = c$, then $b = c$.
Ask students, "If $y = \frac{-x}{2} + 5$ and $y = \frac{5x}{2} - 13$, what single equation can be formed involving only the variable x? How could you use this to solve a system of equations?"

Investigate Solving Systems of Equations Graphically

Consider having students work through the Investigate with a partner or in groups of three. Before they begin, have a class discussion. Ask:
- In a linear system in which the lines intersect, what is the point where the lines cross called? What is the significance of this intersection point?
- How can the demand for an item affect its cost?
- Aside from demand, what influences the cost of an item?
- What determines a change in revenue?
- Sketch a graph with a maximum value. Sketch a graph with a minimum value. What do you call these types of graphs?

For #4b), ask students to draw their sketches such that they can be seen and shared by the whole class.

For #5b), some students may draw quadratic-quadratic systems with three or four solutions. For example,



3 intersection points   4 intersection points

In these cases, be prepared to discuss that the parabolas that students will be working with are functions, so the parabolas will open up or down, and not to the right or left.

Meeting Student Needs

- Have a brief class discussion about marginal cost and marginal revenue. Discuss what factor(s) may affect the total revenue a company would expect when selling a certain product. Refer to items of interest to students, such as personal music devices, CDs, DVDs, etc.
- Consider giving each student a copy of the graph. Each student would then be able to draw lines and write notes directly on the handout. These graphs could then be placed in students' binders for future reference.
- Emphasize that students should not get caught up in the nuances of the economic terms and running a business. The process of solving a system of equations is the primary outcome of the investigation.

ELL

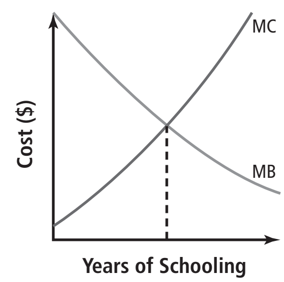
- Ensure that students understand the language of business that is used in the Investigate, such as *maximum profit* (and *maximized profit*), *revenue*, *marginal*, *production*, *business factors*, etc.
- Suggest students add the following terms to their vocabulary dictionary: *linear-quadratic*, *quadratic-quadratic*, *intersection*, etc. Encourage them to include any term they encounter with which they are not familiar. Suggest that they include a verbal description, diagram, and/or example for each term.

Enrichment

- The demand curve illustrates the decreased demand for a product as the price of the product increases: the higher the price, the less the demand. Ask students what might shift the demand curve of, for example, a computer. As the price rises for a product, the manufacturers produce more, so supply increases. Ask students to draw the supply and demand curves, and to explain the importance of the point at which they intersect.

Gifted

- Marginal benefits are the additional benefits gained from producing one more of an item. Marginal cost is the additional cost of producing one more item. The graph below shows the marginal cost of additional years of education, and marginal benefits in terms of one's career advancement.



Ask students to explain the importance of the intersection point of these two curves.

Web Link

To learn more about solving systems of linear-quadratic and quadratic-quadratic systems of equations, go to www.mhrprecalc11.ca and follow the links.

Answers

Investigate Solving Systems of Equations Graphically

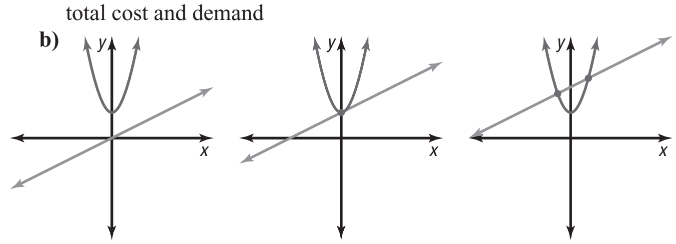
1. The price is $6.00 when 600 are produced.

2. Yes, the intersection is at the minimum value on the curve.

3. Demand will decrease. The demand line has a negative slope, so as you increase quantity you move down the demand function line.

4. a) system 1: marginal cost and marginal revenue; system 2: average total cost and demand

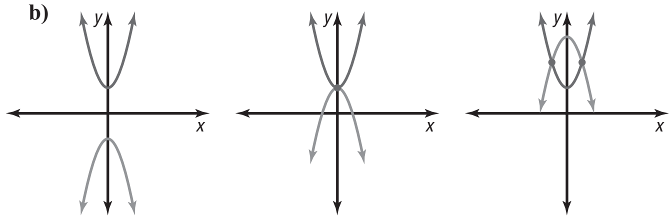
b)



Answers

5. a) marginal cost and average total cost

b)



6. Find the x-values and y-values of the point(s) of intersection.

7. It is the only pair of values that solves both the marginal revenue equation and the marginal cost equation.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Refer students to the graph provided and ask them to orally identify a linear-quadratic and a quadratic-quadratic system. • Have students identify the different curves and how each of them is related to the axes. • Review locating the points of intersection on the graph.

Link the Ideas

Discuss the following with the class:

- What does the graph of a linear-quadratic system look like? How many solutions can it have? Sketch each possibility.
- What does the graph of a quadratic-quadratic system look like? How many solutions can it have? Sketch each possibility. (Even though there are no questions in this lesson in which this occurs, you may want to guide students to realize that it is possible that the two quadratic equations are equivalent, so there could be an infinite number of solutions.)

Encourage students to place the answers and examples in their Foldable or graphic organizer, or with their summary notes.

Example 1

Some students may not be familiar with the movements made by the diver and the board during a springboard dive. It may be helpful for them to see a video or simulation illustrating this motion. (See the Web Link on the following page.)

Discuss the Example with students. Ask:

- What does the green line represent? the blue line?
- How are the x-axis and y-axis labelled? Describe the relationship these units suggest. (Height is dependent on time.)

Rather than following the examples in order, an alternative might be to discuss Example 5 after Example 1, since they both involve bodies in motion.

Example 2

If you did not review slope-intercept form of the equation of a line and the vertex form for the equation of a parabola as outlined in the Planning Notes for this section, do so now. Also, revisit the concept of completing the square with students.

Students may need assistance entering linear or quadratic equations into their graphing calculator or graphing software. (See the Web Link on the next page.) Ask:

- How would you graph the system without the aid of technology?
- What non-technology methods can you use to graph the line? (table of values, point-slope form of the equation of a line, or slope-intercept form of the equation of a line)

You might have students explore how technology can be used to support their answers. You may wish to provide students with **TM 8–1 How to Do Page 428 Example 2 Using TI-83/84** or **TM 8–2 How to Do Page 428 Example 2 Using TI-Nspire™**.

Example 3

Ask:

- How would you write each equation in vertex form?
- Sketch the graphs using the vertex form. Use your graphing calculator or graphing software to graph the quadratic equations. Compare these graphs to your sketches. How are they the same? different?
- How do you know that there is only one intersection point?

Example 4

Discuss this concept of vertical curves with students or have them research it. (See the Web Link at the end of this page.) Students will encounter this topic again in #10 in the Check Your Understanding section. Some student may need to recall tangent lines. Ask:

- What would the road surface be like without vertical curves?
- Where have you used the term *tangent* in your past mathematics work?
- How does tangent relate to trigonometry?
- What does it mean when a line is tangent to a curve? to a circle?
- What does it mean for each grade line to be tangent to the curve?
- Describe a real-life scenario in which there can be negative intersection points.

Some students may enjoy drawing the parabolic curve as illustrated in the Did You Know? feature.

Example 5

Go through this example with students or have them work through it in pairs. It may be beneficial for some students to view a video or simulation of the stunt. (See the Web Link at the end of this page.) Ensure that students understand the strategy for finding the value of a in the vertex form of a parabola, given the vertex and a point on the parabolas. This is a skill students will need in later questions. Students also need to recognize that the independent variable is time, not horizontal distance.

Key Ideas

Ask students to rewrite the Key Ideas in their own words with examples, and place their work in their summary notes.

Meeting Student Needs

- Invite students to create sketches of quadratic-quadratic systems where both parabolas open upward, or both open downward.
- Students should study the graphs given in Example 1. Ask students to describe a different situation for each graph.
- Emphasis in this section should be placed on solving the systems graphically. Revisit the various methods students can use to graph systems. Invite students to make suggestions as to which of these methods can be used for linear relations and which can be used for quadratic relations.
- Revisit the formulas $y = mx + b$, $y = a(x - p)^2 + q$, and the concepts of x-intercepts and y-intercepts.
- Discuss the meaning of the point of intersection for each example. Some students may wish to discuss the meaning for each example, while others may wish to write out their descriptions.
- Students may wish to research other Cirque du Soleil acts and discuss the use of systems for the various stunts performed. Ask:
  - What type of system would be involved in a particular stunt?
  - What is the mathematics involved for each stunt?
- Provide students with an Exit Slip at the end of class and ask them to summarize, in five points or less, the Key Ideas learned in this lesson. These ideas may differ from the Key Ideas presented in the student resource.

ELL

- Ensure that students understand the terminology used in the examples, such as *springboard*, *Trans-Canada Highway*, *surveyors*, *grade*, *seesaw*, *corresponding function*, etc.
- Suggest students add the following terms to their vocabulary dictionary: *tangent*, *vertical curves*, *parabolic*, etc. Encourage them to include any term they encounter with which they are not familiar. Suggest that they include a verbal description, diagram, and/or example for each term.

Common Errors

- Students sometimes forget to verify the solution in both equations.

R_x Consider having a discussion with students about the nature of a system of equations versus a single equation. What makes the two different? Make sure they understand that the solution to a system satisfies all equations in the system, not just one.

- Some students forget to consider and determine multiple points of intersection, if they exist, stopping after finding one first point.

R_x Ask students, "How do you determine from your graph if there is more than one point of intersection?"

Web Link

To learn more about entering equations into a graphing calculator, information on motion involving a springboard, vertical curves, and seesaws, go to www.mhrprecalc11.ca and follow the links.

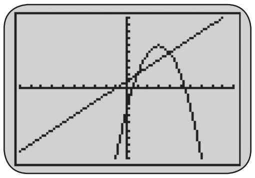
Answers

Example 1: Your Turn

a) System A

b) The diver on the 3-m board will always be in the air longer and at a higher height than the diver on the 1-m board, so the two graphs will never intersect.

Example 2: Your Turn

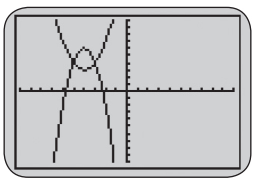


(1, 2) and (4, 5)

Verify (1, 2):
Right side = 0
Left side: (1) − (2) + 1 = 0
Right side = 0
Left side: $(1)^2 - 6(1) + (2) + 3 = 0$

Verify (4, 5):
Right side = 0
Left side: (4) − 5 + 1 = 0
Right side = 0
Left side: $(4)^2 - 6(4) + (5) + 3 = 0$

Example 3: Your Turn



(−3, 4) and (−5, 4)

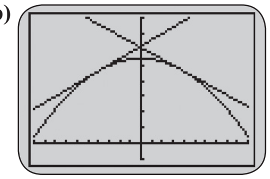
Verify (−3, 4):
Right side = −26
Left side: $2(-3)^2 + 16(-3) + (4) = -26$
Right side = −19
Left side: $(-3)^2 + 8(-3) - (4) = -19$

Verify (−5, 4)
Right side = −26
Left side: $2(-5)^2 + 16(-5) + (4) = -26$
Right side = −19
Left side: $(-5)^2 + 8(-5) - (4) = -19$

Example 4: Your Turn

a) $y = -0.002x^2 + 5.4$
$y = 0.08x + 6.2$
and
$y = -0.002x^2 + 5.4$
$y = -0.075x + 6.103\ 125$

b)

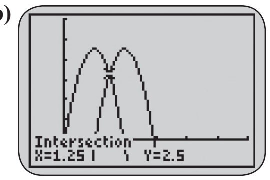


c) (−20.00, 4.60) and (18.75, 4.70)

Example 5: Your Turn

a) The system of equations is as follows:
$y = -8(x - 0.75)^2 + 4.5$
$y = -8(x - 1.75)^2 + 4.5$

b)



The solution is (1.25, 2.5).

c) Example: The solution means that the performers are at the same height, 3.28 m, at the same time, 1.46 s after the first performer is launched into the air. This will be 0.46 s after the second performer starts the stunt. It is at this point that the performers give each other a high five.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Prompt students to identify the similarities between the example and the problem in the Your Turn. • Ask students to explain how the height of the diving board in the example could be related to the different heights of the divers in the Your Turn. • Ask students to verbalize what each axis represents. • Ask students to verbalize what is happening to each diver independently, before linking the graphs to the intersection point.
Example 2 Have students do the Your Turn related to Example 2.	• Ask students to verbalize what each axis represents. • Allow students to use the trace or intersection function on their graphing calculators to locate the intersection point(s). Then, ask them to orally describe what the point(s) represents. • Show students how to access the table of values for equations they enter into their calculator. Some students may find the table of values to be an easier way to identify or verify the intersection point(s).
Example 3 Have students do the Your Turn related to Example 3.	• Assist students in identifying which variable to isolate before entering the equation in the *y* editor. • Ask students to verbalize what each axis represents. • Allow students to use the trace or intersection function on their calculators to locate the intersection point, and to then orally describe what the point represents. • Show students how to access the table of values for equations entered on their calculator. Some students may find this an easier way to identify or verify the intersection point(s).

Assessment	Supporting Learning
Assessment *for* Learning	
Example 4 Have students do the Your Turn related to Example 4.	• Students may have difficulty writing equations for word problems, and may avoid word problems as a result. Either provide the problem as a handout or suggest that students write out the problem. Then, suggest that students use the study skill of underlining key parts of the information given. They will then see that the equations are given and need only be written together as a system. • In using the process of graphing, review with students all the ways they can locate the intersection points. Listing them on the board and leaving them posted will assist students throughout this section and the next. • Revisit with students the meaning of a *point of tangency*. Show them how to locate this point on a table of values. • Discuss with students how they can identify a linear-quadratic and a quadratic-quadratic from the equations alone.
Example 5 Have students do the Your Turn related to Example 5.	• Ask students to verbalize what each axis will represent. • Prompt students to make a reasonable sketch before attempting to solve the problem. This visual will assist them in understanding what the intersection point represents. • You may need to reactivate students' understanding of the parameters in the general equation, $h = a(t - p)^2 + q$. Discuss the effects of $+a$ and $-a$.

Check Your Understanding

Practise

Students unfamiliar with motocross jumps and stunts may benefit from viewing a video on the sport before attempting #1. (See the Web Link at the end of this section.)

If students are having difficulty with #1, have them verbalize what each axis represents. Then, ask:

- Are the launch heights the same or different?
- Are the launch heights above ground level or at ground level?
- Are the speeds different or the same?
- Are the launch angles the same or different?

In #2, students often confuse $-x^2$ with $(-x)^2$. Discuss this confusion to ensure students are clear about the effect of this error. (See Common Errors at the end of this section.)

The equation for each graph is given in #3, so students only need to write the equations as a system, and then read the solution from the graph.

In #2 and 4, students often use the subtraction key when entering negative values in their graphing calculator, rather than using the negative key. Discuss with students the different functions of these keys and work through an example to see the effects. Perhaps have students work in pairs. Have one of the partners enter the first equation in #2 using the subtraction key, and the second partner using the negative key. Then have them share and discuss their results. They could then work through the first equation in #4b) together, again using both the subtraction and negative keys, predicting the outcome before pressing Enter.

When entering the equations in the graphing calculator in #5, students must designate which variable is x, and which is y. This is usually determined alphabetically, but not so in parts a), and c) to e). Ask:

- Does it matter which variable is designated as x (the independent variable) when entering the equation into the y editor on your graphing calculator?
- How do you determine which variable is x and which is y when these variables are not used in an equation?

Apply

For #6 and 7, students could work with a partner and then share their findings with the class.

When discussing #8, ask

- How did you find your equations?
- How did you show or verify that there was no solution? one solution? two solutions?

If students are having difficulty with #9, ask:

- What does the x-axis represent? What is the scale for each grid unit on the x-axis?
- What does the y-axis represent? What is the scale for each grid unit on the y-axis?
- How are the terms *revenue* and *profit* related in the context of this question?

In order to understand #10, students must be familiar with vertical curves. If students have not discussed and worked through Example 4, they should do so before attempting this question. To assist them with how to proceed with the question, ask:

- What system would be written and solved to find the coordinates at the beginning of the curve?
- What system would be written and solved to find the coordinates at the end of the curve?

If students are having difficulty with #11, discuss the variables and what they represent. Ask:

- Which variable in the formula represents x?
- Which variable in the formula represents y?
- What does t represent?
- Is it possible to have a value for t that is less than zero for the first car?
- What values of t for the second car do not make sense?

To assist students as they work through #12, ask:

- Is it possible that there is no intersection of arcs in this question? If so, sketch the streams of water showing no points of intersection.
- Is it possible that there is one intersection point of the arcs? If so, sketch the streams of water showing one intersection point.
- Is it possible that there are two intersection points of the arcs? If so, sketch the streams of water showing two intersection points.

There is not a worked example of the type of problem presented in #13. Some students may need help to build the equations.

For #14, some students may benefit from seeing a video or simulation of a 540° jump. (See the Web Link at the end of this section.)

Students may be more successful with #15 if they graph the situation. Ask:

- On which axis would you place the horizontal distance?
- On which axis would you place the height?

Then suggest that students sketch and label a graph to illustrate the jumps of the frog and the grasshopper. Ask:

- What is the vertex for the arc of the frog's first jump?
- What is the vertex for the arc of the grasshopper's first jump?
- How can you use these vertices to write a system of equations?
- What must happen for the frog to catch the grasshopper?

Extend

If students want to find more information on the Delian problem referred to in #16, suggest that they do some research on the Internet. (See the Web Links on the next page.)

For #18, ask

- What must be true of a solution point(s) for a system of three equations?
- Can the point(s) satisfy only two of the equations?

Create Connections

Students could place their response to #19 in their journals with their summary notes.

After making their predictions in #20, suggest that students enter the equations in a graphing calculator to check their predictions. Ask:

- Which did you predict correctly?
- What did you consider when making your predictions?
- For any that you predicted incorrectly, what did you neglect to consider, leading to your error?

Project Corner

This Project Corner provides students with examples of some areas in which nanotechnology is being used. Several everyday examples and uses are listed, which should prompt student thinking for their own area of interest, which they will develop later in the chapter. You may wish to discuss the examples as a class and make an additional list on the board for students to use as a reference.

Meeting Student Needs

- Provide **BLM 8–4 Section 8.1 Extra Practice** to students who would benefit from more practice.
- Provide students with an opportunity to use graphing calculators throughout this section.
- For #2, some students may prefer to write out an explanation of how to verify a solution, instead of actually completing the mathematical calculation.
- For #6 and 7, students should share their sketches with others. Is there more than one correct response?
- For #13, have students work together and review how to write the equations.
- Ask students to compare the sketch for #15 prior to making the calculations.

ELL

- Ensure students are familiar with the concepts in the problems, including *motocross*, *pricing scheme*, *roller coasters*, *accelerates* (*acceleration*), *duplicating*, *plague*, etc.

Common Errors

- Students often confuse $-x^2$ with $(-x)^2$.

R_x Evaluate $-x^2$ and $(-x)^2$ by replacing x with 5. Ask students if the results are the same, and challenge them to explain why or why not. Then, replace x with -4 and ask students to compare and explain the results. Ask, "For what values of n is $-x^n = (-x)^n$ true?

Web Link

To learn more about motocross, snowboarding jumps, and the duplicating cube (Delian) problem, go to www.mhrprecalc11.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1 to 6, 8, 9, and 11. Students who have no problems with these questions can go on to the remaining questions.	• To assist with #2, remind students that the system is given to them already. Ask how they can identify the points of intersection. Have them read off one solution for you orally before asking how they can verify it. If a list of the different methods for verifying points has not been posted in the class for reference, consider posting one now. Have students identify the methods they know to verify whether the point is correct. (Enter equations in the calculator and verify the intersection point is the same, table of values, trace, intersect function, and read off the values). • Clarifying #2 and ensuring that students understand the process will assist them with #3 to 5. • Students who are having difficulty with #6 may best be referred to the Link the Ideas section. • For #8 and 9, you may need to review how to write an equation in slope-intercept form: $y = mx + b$. It may also benefit students to make a sketch first. If they are not using technology, provide students with **Master 3 0.5 Centimetre Grid Paper** and encourage them to graph a sketch that crosses at integer values. Doing so will make it easier to read the initial intersection points.
Assessment *as* Learning	
Create Connections Have all students complete #19 and 20.	• Some students may benefit from working with a partner so that they can compare and discuss responses. This may be especially helpful for #19. • Encourage students to describe what each equation represents in #20. This will help them to visualize what type of intersection is possible. It may also assist some students to sketch a diagram without the use of technology. Encourage students to discuss their responses as a class.

8.2 Solving Systems of Equations Algebraically

***Pre-Calculus 11*, pages 440–456**

Suggested Timing
100–120 min

Materials
- grid paper
- graphing calculator
- computer with graphing software

Blackline Masters
BLM 8–3 Chapter 8 Warm-Up
BLM 8–5 Section 8.2 Extra Practice

Mathematical Processes
- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcomes
RF6 Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	# 1, 3a), b), d), 4a)–d), 6, 22, 24
Typical	#1, 2, 3a), c), e), 4, 6, 8, one of 10–13, 14, 16 or 17, 22, 24
Extension/Enrichment	#6, 12, 15, 20–22, 24

Planning Notes

Before beginning section 8.2, you may want to revisit some of the concepts that students will need to be successful in the rest of this chapter:

- the quadratic formula (See Planning Notes for section 8.1.)
- the substitution and elimination methods for solving systems of linear equations (See Planning Notes for section 8.1.)
- factoring (See Planning Notes for the Chapter 6 Opener.)
- the meaning of *roots* (For example, in step 7 of the Investigate, students need to understand that *root* means *solution*.)

Investigate Solving Systems of Equations Algebraically

For step 2 of the Investigate, ask:

- If you were to use the elimination method to solve this system, which variable would you eliminate? Could you eliminate x^2? x? Explain your reasoning. In step 6, in which students go through steps 1 to 5 again to solve a second system, elicit from students that when using the elimination method, only y can be eliminated for this second system. Ask, "Why can you not eliminate x?"
- Try both the elimination and substitution methods of solving the system. What do you notice about the resulting equation? (The results for both methods are the same. Emphasize to students that they do not need to use both methods to solve a system.)

Meeting Student Needs

- Display posters in the room reminding students of the steps involved in solving a system of equations by substitution or elimination. Include examples on the poster. Alternatively, you could create these posters as a class.
- Have students work in partners or small groups to discuss the similarities and differences in solving systems of equations using substitution and elimination.

ELL

- Ensure that students understand the concepts used in the opener, including *ancient civilization*, *geometrically*, *popularized*, *symbolism*, and *notation*.
- Suggest students add the following terms to their vocabulary dictionary: *algebraic*, *elimination*, *substitution*, *initial system*, etc. Encourage them to include any term they encounter with which they are not familiar. Suggest that they include a verbal description, diagram, and/or example for each term.

Common Errors

- When using the elimination method, students often make errors when subtracting and when using the distributive property.

R_x Present students with the following example and ask them to identify where the error has been made:

Example 1:

$$\begin{aligned} x^2 + 12x + y + 36 &= 0 \quad (1) \\ 3x + 2y - 5 &= 0 \quad (2) \end{aligned}$$

Solution:

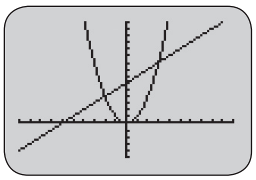
$$\begin{array}{l} 2x^2 + 24x + 2y + 36 = 0 \\ \quad\;\; -(3x + 2y - \;\; 5 = 0) \\ \hline 2x^2 + 21x \qquad\;\; + 31 = 0 \end{array}$$

(In the first line of the solution, the constant, 36, has not been multiplied by 2. In the final line of the solution, subtracting the negative value, −5, has not been done correctly.)

Answers

Investigate Solving Systems of Equations Algebraically

1.



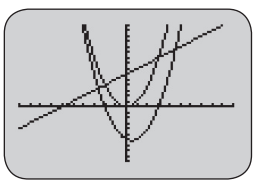
Calculate intersection points: (−2, 4) and (3, 9).

2. a) Example: Substitute $(x + 6)$ from the first equation into y in the second equation.

b) $x^2 - x - 6 = 0$

3. The x-values represent the x-coordinates of the points of intersection.

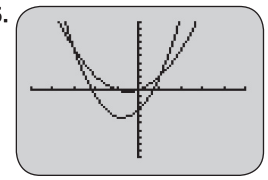
4.



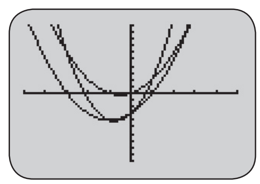
The line would intersect at the x-intercepts. The x-intercepts are the solutions to the quadratic equation in #2b).

5. The x-intercepts, or roots, are the x-values of any point of intersection.

6.



Calculate the intersection points: (−3, 6) and (1, 2).
Substitute $(2x^2 + 3x - 3)$ from the first equation for y in the second equation. The equation becomes $x^2 + 2x - 3 = 0$.
The x-values represent the x-coordinates of the point of intersection.
The line would intersect at the x-intercepts. The x-intercepts are the solutions to the quadratic equation in #2b).



The x-intercepts are the x-values of any point of intersection.

7. The equation created using substitution or elimination represents the points of intersection of the two original curves and involves only one variable, x. The roots of this equation are the x-values that satisfy both of the original equations. The roots are the x-coordinates of the points of intersection of the original curves. To find the y-value for the original equations, find the coordinates on the original graphs that correspond to the x-intercepts of the equation created by substitution or elimination.

8. Example: I would solve the system in terms of one of the variables, and then substitute that value into one of the original equations to solve for the second variable.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Revisit with students the methods used in solving systems of linear equations. • Ask students to verbalize the substitution and elimination methods and how they will decide which to use in a system. If they need a specific example to work from, provide them with the system $2x - y = 8$ and $-2x - 3y = 5$. Be aware that some students will prefer to use one method over the other. Even though this is not incorrect, encourage students to use both methods so they gain an understanding of each. Minimally, they should be able to identify which method is being used in a solution, and be able to follow the example through, identifying any errors.

Link the Ideas

Suggest students rewrite the second statement in their own words and place it in their Foldable or graphic organizer, or with their summary notes.

Example 1

Discuss the example with students. Ask:

- Can you eliminate the x-terms in this system? Why or why not?
- Can you eliminate the x^2-term in this system? Why or why not?
- When using the substitution method, why is it desirable to use a variable with a coefficient of 1?

Ensure that students understand that they have to complete the solution by solving for y. Students often solve the quadratic equation and then forget to substitute this value(s) into one of the original equations to solve for the other variable.

Consider having students work through the Your Turn in pairs. One partner could use the substitution method, while the other uses elimination.

Example 2

Ask:

- Can you eliminate the S-term? Why or why not?
- Can the system be solved using another method, such as substitution or graphing? If so, find the solution using this method.

Note that some students may suggest writing one equation instead of a system. This can be done by letting L represent the larger number and $(46 - 2L)$ represent the smaller number. The equation is then $(46 - 2L)^2 - 3L = 93$. Solving this quadratic equation yields two solutions: 29.75 and 17. The 29.75 is eliminated because the problem states that the numbers are integers. Substituting 17 into $46 - 2L$ yields 12, so the two numbers are 12 and 17.

Example 3

Ask:

- If you were entering this system into your graphing calculator, which variable represents x?
- Sketch the situation or enter the system into your graphing calculator. How many solutions will there be?
- Are any of the solutions not practical or realistic in relation to the problem?
- Is it possible for time, t, to be a negative quantity?
- Are there any factors that are not considered in the system of equations that could affect the height of the crate?

Example 4

If factoring has not been reviewed, it may be beneficial to do so now. In particular, factoring by grouping should be reviewed. (See the Planning Notes for the Chapter 6 Opener.)

Ask:

- Is one method, substitution or elimination, preferable over the other? Why or why not?
- Is it possible to solve the system by eliminating the x-variable? the x^2-variable? Why or why not?

Example 5

You may have to explain a basketball “alley-oop” to some students. Show a video or simulation. (See the Web Link on the following page.)

Ask:

- Which variable, d or h, represents x?
- Rewrite the system using the variables x and y, and enter the system into a graphing calculator or graphing software. How does the solution found on the graph compare to the solution given in the example?

Key Ideas

Ask students to rewrite the Key Ideas in their own words with examples, and place their work in their summary notes.

Meeting Student Needs

- Have students discuss the examples in small groups. Invite them to share their findings on how to determine which variable to eliminate in each method.
- Some students may wish to work through Example 2 without the assistance of the textbook. If so, provide them with the question on a handout.
- Once students have worked through the examples, each student could write a summary of the Key Ideas for the section. You could assign this task as an Exit Slip activity, which is to be turned in at the end of class for assessment.
- Create two posters: one communicating the Key Ideas for solving using elimination, and the other presenting the Key Ideas for solving using substitution.

The following quadratic-quadratic system of equations has no solution. You may wish to demonstrate this type of example for students or have them work through the solution on their own.

$y = x^2 + 4x + 5$
$y = -x^2 + 2x - 3$

ELL

- Ensure that students understand the vocabulary in the examples, such as *conundrum*, *worthiness*, *cargo plane*, *crate*, *parachute*, etc.

Enrichment

- A black hole absorbs light that passes close enough to its enormous gravitational field. Light passing near a black hole, perhaps from a distant star, will have its path changed. Black holes have been located by studying the effect of their gravitational field on the path of light. Have students investigate how mathematics, particularly systems of equations, has been used to discover invisible objects, such as black holes.

Gifted

- A cyclotron accelerates subatomic particles to nearly the speed of light. High speed collisions are studied for the products of the collisions. Ask students to speculate on how solving systems of equations that model particle motion could be used to predict the coordinates of collisions and the paths of post-collision particles.

Web Link

To learn more about an "alley-oop" in basketball, go to www.mhrprecalc11.ca and follow the links.

Answers

Example 1: Your Turn

Isolate y in the first equation:

$y = -3x - 9$. Substitute for y in second equation and solve:

$$\begin{aligned} 4x^2 - x + (-3x - 9) &= -9 \\ 4x^2 - x - 3x &= 0 \\ 4x^2 - 4x &= 0 \\ 4x(x - 1) &= 0 \end{aligned}$$

$x = 0$ and $x = 1$

The solutions are $(0, -9)$ and $(1, -12)$.

Example 2: Your Turn

a) Let x represent first number.
Let y represent second number.
$2x + 14 = y$
$y + 1 = x^2$

b) $$\begin{aligned} (2x + 14) + 1 &= x^2 \\ 0 &= x^2 - 2x - 15 \\ 0 &= (x - 5)(x + 3) \end{aligned}$$

$x = 5$ and $x = -3$

The numbers are 5 and 24, or -3 and 8.

Example 3: Your Turn

a) about 9.45 s

b) 462 m

c) $462 = -4.9(9.45)^2 + 900$
$462 = 462.4$
$462 = -4(9.45) + 500$
$462 = 462.2$

Example 4: Your Turn

Using substitution

$y = 6x^2 - x + 1$

$$\begin{aligned} 4x^2 - 4x - (6x^2 - x + 1) &= -6 \\ 4x^2 - 4x - 6x^2 + x - 1 &= -6 \\ 0 &= 2x^2 + 3x - 5 \\ 0 &= (2x + 5)(x - 1) \end{aligned}$$

$x = -\frac{5}{2}$ or $x = 1$

The solutions are $\left(\frac{-5}{2}, 41\right)$ and $(1, 6)$.

Using elimination

$6x^2 - x - y = -1$
$4x^2 - 4x - y = -6$

Use subtraction.

$$\begin{aligned} 2x^2 + 3x &= 5 \\ 0 &= 2x^2 + 3x - 5 \\ 0 &= (2x + 5)(x - 1) \end{aligned}$$

$x = -\frac{5}{2}$ or $x = 1$

The solutions are $\left(\frac{-5}{2}, 41\right)$ and $(1, 6)$.

Example 5: Your Turn

a) $(18.96, 1.96)$ and $(22.15, -4.75)$

b) The catch is made at approximately 18.96 m from where the ball was struck, and at a height of 1.96 m. The solution 22.15 is dismissed because a height of -4.75 does not make sense in this context. The equations in the problem refer only to distance and height, and not time. We assume that Ruth has timed her leap such that her glove and the ball are at the same spot at the same time and that Ruth makes the catch.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Prompt students to identify how to determine which variable to isolate or eliminate when solving. • It may be necessary to remind students that a solution must be an ordered pair and not simply a solution for one variable. Prompt students to verbalize how they will find the missing coordinate for the intersection point(s).

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Remind students to select appropriate variables when formulating their own equations. • It may assist some students in determining their equations if they identify which is the smallest value and assign a variable to this value first. From there, the rest of the equation can be built. • Revisit the quadratic formula with students who are having difficulty. Provide a quadratic on the board, and ask them to identify the values for the parameters of the quadratic formula. Then ask students to calculate the values. Some students may also benefit from being shown how to calculate this using technology.
Example 3 Have students do the Your Turn related to Example 3.	• Assist students in identifying what the axes represent. • Ask students to verbalize what the system will be, given the equations. • Ask students to identify the method they feel most comfortable with when solving the system. They may choose any method at this point, but encourage them to be proficient in at least two. • Prompt students to verify their solution in at least two ways.
Example 4 Have students do the Your Turn related to Example 4.	• Ask students to verbalize how they can decide which method of solving would be better to use for the example. • Ask students to identify the method they feel most comfortable with when solving the system. They may choose any method at this point, but encourage them to be proficient in at least two. • Prompt students to verify their solution in at least two ways.
Example 5 Have students do the Your Turn related to Example 5.	• Ask students to verbalize which variable should be isolated in the substitution method. • Prompt students to identify which variable should be eliminated if using the elimination method. • Ask students how they will verify that the solution is correct. They have several possible approaches. It would be helpful for them to refer to the summary chart that they have constructed throughout the chapter individually or as a class.

Check Your Understanding

Practise

For #1, students may need clarification on whether the ordered pair is (k, p) or (p, k).

In #2, students may need clarification on whether the ordered pair is (w, z) or (z, w). Also, be prepared to discuss what the graphs look like. Some students may recognize that the given quadratics are not parabolas, but others may not.

After students have attempted #3, ask:

- Why is substitution the preferred method to solve these systems?
- How did you decide what variable to isolate for substituting? (Elicit that one could substitute for the other variable, but that involves more work.)
- Could elimination be used? Why or why not?

For #4, ask students:

- Why is elimination the preferred method to solve these systems?
- How did you decide what variable to eliminate?
- Could substitution be used? Why or why not?

Apply

Consider using #6 as a discussion question or a journal entry question. Ask:

- Which person's explanation, Alex's or Kaela's, is more understandable to you?
- Can you find another explanation for why this system has no solution?

For #9, students may make the common error of forgetting to divide the product of the base and height by 2 when finding the area of a triangle. Ask:

- What are the expressions for the base and height of the triangle?
- What is the area formula of a triangle?

For #10, ask:

- Can two integers having a difference of -30 both be positive? Do they both have to be negative, or can one be positive and the other negative?
- Is there more than one pair of integers that form the solution?
- Could you solve the resulting quadratic equation by factoring?

In #13, students often confuse $-4.9t^2$ with $(-4.9t)^2$. To assist students who make this error, ask:

- What is the order of operations?
- When evaluating $-4t^2$, which operation is performed first?
- Evaluate $-4t^2$ and $(-2t)^2$ for $t = 3$. Are the values the same? Explain.
- In #13b), is the answer obtained a reasonable height? If not, how must you interpret the value to obtain the answer?

In #14, students need to find the vertex and the value of a for the vertex form for the equation of a parabola. Ask students:

- What is the vertex of each parabola? How did you find the vertices?
- How do you find the value for a for a parabola?
- From the sketch of the graph of the system, how many points of intersection are there?

Warn students about the negative sign in front of the first equation in #16. Suggest that they either change the fraction to a decimal, or the decimal to a fraction.

In #17, students should be aware that there may be a slight difference due to rounding, depending on which formula they substitute the value of x into.

Extend

For #18, suggest that students sketch or enter into their graphing calculator the three equations. Ask:

- Is there one point of intersection for all three parabolas?
- How many systems of equations are needed to find the three vertices? What are they?
- What is the approximate shape of the sail? What formula would you use to find an approximation of the area?

For #19, ask students:

- How are the slopes of two perpendicular lines related?
- How do you find the distance between two points on a segment that is non-vertical or non-horizontal?

For #20, suggest that students rewrite the second equation in the system, solving for y.

Create Connections

Suggest that students use #22 for a journal entry or a summary of the methods studied. Students could write the ideas contained in the Key Ideas for sections 8.1 and 8.2 in their own words and place this writing where they keep their notes, examples, and other summaries.

For #23, ask students:

- Is there enough information given to sketch the graphs of the two parabolas? If so, sketch them.
- Can you determine the number of intersection points? How many are there?
- What do you need to find in order to write the equation of each parabola?

You may wish to suggest that students program the quadratic formula into their calculator.

For #24, discuss with students what it means when the discriminant is negative. Ask:

- What is the quadratic equation that results when the two $y =$ portions of the system are equated?
- When using the quadratic formula on this equation, what is the value of the discriminant?
- What is the meaning of a negative discriminant?

Project Corner

This Project Corner introduces students to carbon nanotubes. These tubes are 100 times stronger than steel and only $\frac{1}{16}$ of the mass. These nanotubes are used to reinforce plastics and materials, such as those used to construct common sports equipment. This Project Corner may prompt students' thinking for future possibilities in a design of their own. You may wish to continue to build on the list of items students created in Section 8.1. Have students brainstorm further ideas and uses to help them narrow down an area of interest. You may wish to refer students' to the photo at the top of page 421 in the student resource. This photo shows nanospheres arranged in layers.

Meeting Student Needs

- Provide **BLM 8–5 Section 8.2 Extra Practice** to students who would benefit from more practice.
- For #1 and 2, some students may prefer to write out the method used to verify a solution, rather than completing the verification.
- For #3 or 4, ask students to choose one question and to then solve it using both methods. Once complete, they could write an explanation as to which method worked better for that particular system of equations.
- Break the class into small groups. Create a set of cards containing the numbers 6 to 17, representing the questions in the Apply section. Randomly choose five of the cards and assign those questions to one group. Replace the cards and repeat this procedure for each group. Assign one student in each group to be the reporter for the group. When the groups have

completed their questions, each reporter can lead a verbal discussion about the questions the group completed. What were the challenges? Did the group use one method more than the other? Why?

ELL

- Ensure that students understand the vocabulary used in the problems, such as *mechanical energy*, *kinetic energy*, *initial* (as applied to height, velocity, etc.), *potential energy*, *horizontal*, *stuntman*, *skis*, *free-fall*, *volcano*, *lava fragments*, *approximated*, *summit*, *avalanche*, *industrial design engineer*, *disadvantages*, etc.
- Suggest students add the following terms to their vocabulary dictionary: *perimeter*, *circumference*, *perpendicular*, etc. Encourage them to include any term they encounter with which they are not familiar. Suggest that they include a verbal description, diagram, and/or example for each term.

Enrichment

- Ask students to explain the similarities and differences between solving systems of equations algebraically and solving systems graphically. Ask them to explain how changing the scale of problems, such as those at the microscopic level, might change the need for accuracy.

Gifted

- Ask students to picture a movie set where a projectile is launched and lands on top of a moving vehicle. This occurrence must be modelled mathematically in order to set up the stunt. Challenge students to create simultaneous equations whose intersection is the point of collision.

Common Errors

- Some students will not know how to find the common denominator in #20.

R_x Have them verbalize which method they think would be easiest to use to solve the system. Encourage students to explain their thinking.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 3a), b), 4a) to d), and 6. Students who have no problems with these questions can go on to the remaining questions.	• Students having difficulty with #1 can refer to the list of methods that can be used to verify solutions (i.e., graphing, substitution, evaluation, a table of values, and elimination). They should also refer to Example 1. • To assist with #3 and 4, coach students to identify which variable to isolate. Remind them of the importance of writing similar terms for the system in the same order. • Ask students who are having difficulty with #6 to verbalize how they know when a system has no solution. What should they look for? You may wish to revisit with them how to recognize when a linear system has no solution. Then, return their attention to question.
Assessment *as* Learning	
Create Connections Have all students complete #22, and 24.	• Some students may benefit from working in pairs so they can compare and discuss their responses, especially for #22 and 24. They should refer to the common list of how to solve systems of equations graphically and algebraically, which was developed throughout sections 8.1 and 8.2.

Chapter 8 Review

8

***Pre-Calculus 11*, pages 457–458**

Suggested Timing
90—120 min

Materials
- grid paper
- graphing calculator
- computer with graphing software

Blackline Masters
Master 3 0.5 Centimetre Grid Paper
BLM 8–4 Section 8.1 Extra Practice
BLM 8–5 Section 8.2 Extra Practice

Planning Notes

Have students who are not confident with the concepts in the chapter discuss strategies with you or a classmate. Encourage them to refer to their summary notes, worked examples, and previously completed questions in the related sections of the student resource. Have students make a list of questions that they need no help with, a little help with, and a lot of help with. They can use this list to help them prepare for the practice test.

Meeting Student Needs

- Students who require more practice on a particular topic may refer to **BLM 8–4 Section 8.1 Extra Practice** or **BLM 8–5 Section 8.2 Extra Practice**.
- Students should be encouraged to determine their own level of understanding by working through the student learning outcomes. They can then complete questions of their own choosing in the chapter review based on their own assessment.
- You may wish to provide students with **Master 3 0.5 Centimetre Grid Paper**.

ELL

- Ensure that students understand the vocabulary used, such as *biopharmaceutical*, *cell cultures*, *nutrient-rich medium*, etc.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 8 Review The Chapter 8 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against answers in the student resource.	• If students are unsure which questions they should try, they should be able to complete #1–3, 5–9, 10a), b), and 12. • Have students revisit any section that they are having difficulty with prior to working on the chapter test. • Students may find it helpful to refer to the summative list of methods for solving and verifying systems of equations that they compiled in the chapter sections. • Encourage students to make sure that they can demonstrate graphing and one algebraic method proficiently.

8 Chapter 8 Practice Test

***Pre-Calculus 11*, pages 459–460**

Suggested Timing

60—90 min

Materials

- grid paper
- graphing calculator
- computer with graphing software

Blackline Masters

Master 3 0.5 Centimetre Grid Paper

BLM 8–6 Chapter 8 Test

Planning Notes

Students should work individually through the practice test. Some students may need to complete every step to find the solutions, while others may intuitively know the answers. Allow for the differences in learning styles.

Prior to students starting the test, you should ensure that they are comfortable with all vocabulary in the questions so that their focus is on the question. Discuss terms that may be unfamiliar, such as *choreographer*, *jete jumps in canon*, *computer animators*, *double-jump mechanic*, *trajectory*, etc. Have students look over all questions and bring to your attention any word/concept with which they are unfamiliar.

Suggest that students start the practice test by writing the question numbers in their notebook. They can then indicate which questions they need no help with, a little help with, and a lot of help with. They can begin by completing the questions they know they can do, followed by the questions they know something about. Finally, suggest to students that they do their best on the remaining questions.

Once students have completed the test, they may mark their own work to identify areas that will need more review prior to formal assessment. You might also consider grouping students according to areas of need and allowing groups time to discuss any difficulties.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1, 2, 4, 5, 8, 10, and 12.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1	8.1	Example 2	✓ determine the solution of a system of linear-quadratic equations graphically ✓ explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations
#2	8.1	Link the Ideas Example 2	✓ understand why a system of linear-quadratic or quadratic-quadratic equations may have zero, one, two, or an infinite number of solutions ✓ explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations
#3	8.1	Example 3	✓ explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations
#4	8.1 8.2	Example 3 Example 4	✓ determine and verify the solution of a system of quadratic-quadratic equations graphically, with technology ✓ determine and verify the solution of a system of quadratic-quadratic equations algebraically
#5, 7b)	8.2	Example 1	✓ determine and verify the solution of a system of linear-quadratic equations algebraically
#6, 7a)	8.2	Example 4	✓ determine and verify the solution of a system of quadratic-quadratic equations algebraically
#8, 11	8.1	Example 3	✓ relate a system of quadratic-quadratic equations to the context of a problem ✓ determine the solution of a system of quadratic-quadratic equations graphically, with technology ✓ explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations
#9	8.2	Example 2 Example 3	✓ relate a system of linear-quadratic equations to the context of a problem ✓ model a situation, using a system of linear-quadratic or quadratic-quadratic equations ✓ model a situation, using a system of linear-quadratic or quadratic-quadratic equations
#10	8.1 8.2	Example 5 Example 4	✓ model a situation, using a system of linear-quadratic or quadratic-quadratic equations ✓ determine and verify the solution of a system of quadratic-quadratic equations algebraically
#12	8.2	Example 3	✓ relate a system of linear-quadratic equations to the context of a problem ✓ solve a problem that involves a system of linear-quadratic equations

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 8 Self-Assessment Have students use their responses on the practice test and work they completed earlier in the chapter to identify skills or concepts they may need to reinforce.	• Students may wish to review the summative lists they completed in the chapter sections before they begin the practice test. These lists can also be used to identify any areas of weakness. • Before students attempt the chapter test, coach them in areas with which they are having difficulties.
Assessment *of* Learning	
Chapter 8 Test After students complete the practice test, you may wish to use **BLM 8–6 Chapter 8 Test** as a summative assessment.	• For students uncertain which questions to complete, a minimal set could include #1 to 9.

CHAPTER 8

Unit 4 Project

***Pre-Calculus 11*, page 461**

Suggested Timing

90—120 min

Materials

- grid paper
- graphing calculator
- computer with graphing software

Blackline Masters

BLM U4–1 Unit 4 Project Checklist

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

RF6 Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.

Planning Notes

Begin by having a class discussion about nanotechnology. Ask students:

- What have you learned through your research on nanotechnology?
- What areas have you found that will be affected by this new technology?

You could also have student pairs or small groups discuss this topic further.

Then, help students more clearly define their individual projects. Ask students to consider the following:

- What application of nanotechnology are you interested in or would like to work with? (Examples: electronics, energy, health care, environment, sports, transportation, clothing, architecture, household appliances, gadgets)
- Have you decided on an object you would like to work with? Does your object have linear and parabolic design lines? If no, students will have to choose another object. If yes, suggest they begin by sketching the object with these design lines present.
- How do you plan to change the object you have chosen?
- Sketch a new design for your chosen object. Does your design include intersection(s) of parabolic and linear design lines? If no, redesign your object.
- If you make changes in your object as you are designing it, keep a record of all changes you make, so an evolution of the object can be seen.
- For one or more sections of your design, can you write a system of linear-quadratic or quadratic-quadratic equations that could model the shape?
- Can you find the intersection point(s) of the systems of equations you wrote?
- What is the importance or significance of the point(s) of intersection?

Assessment	Supporting Learning
Assessment *for* Learning	
Unit 4 Project This unit project gives students an opportunity to explore a technology, nanotechnology, that will have a profound impact on our future. They explore how the mathematics of linear and quadratic equations and inequalities is related to this burgeoning field.	• You may wish to have students use the part of **BLM U4–1 Unit 4 Project Checklist** that provides a list of the required components for the Chapter 8 part of the Unit 4 project..