Linear and Quadratic Inequalities

Opener

***Pre-Calculus 11*, pages 462–463**

Suggested Timing
10–15 min

Materials
• computer with Internet access

Blackline Masters
BLM 9–2 Chapter 9 Prerequisite Skills
BLM U4–1 Unit 4 Project Checklist

Key Terms
solution region boundary test point

What's Ahead

In Chapter 9, students solve linear and quadratic inequalities. Students have had experience solving linear inequalities in one variable in grade 9. In this chapter, students solve quadratic inequalities in one variable, as well as linear and quadratic inequalities in two variables. To solve quadratic inequalities in one variable, students explore a number of strategies to help them determine a personal strategy. In solving quadratic inequalities in two variables, students address two types of problems: in one case, students reduce the inequality to the one-variable case; in the other, they solve quadratic inequalities graphically.

Planning Notes

The chapter opener refers to inequalities in several contexts. Have students brainstorm to recall what they learned about solving inequalities in grade 9. If the class has Internet access, have students follow the Web Link in the student resource to find out about some of the inequalities named after mathematicians. Suggest that they research Sir Isaac Newton, G.H. Hardy, J.E. Littlewood, G. Pólya, and D. Mitrinovic. Students may not think of Newton as a mathematician and may not be familiar with the others. Some quick Internet research could lead to a short class discussion about the contributions these individuals made to mathematics.

As students progress through the chapter, have them record the Key Terms and develop their own definitions. They can also refer to the definitions in the student resource. Have students explain their understanding of the Key Terms.

As a class, highlight the careers related to inequalities mentioned in the opener and throughout the chapter, and have students explain how inequalities are used in each career.

Unit Project

You might take the opportunity to discuss the Unit 4 project described in the Unit 4 opener. In this chapter, a Project Corner box provides information related to the unit project. This feature is not mandatory, but it is recommended because it provides some background for the final report for the Unit 4 project assignment. If you are going to develop a project rubric with the class, you may want to start now. See pages 341–342 in this Teacher's Resource for information on working with students to develop a class rubric.

Chapter Summary

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. If they have used Foldables™ before, you may wish to have them select a style they found useful to keep their notes in for Chapter 9. Discuss other methods of summarizing information. For example, many students may have used different types of graphic organizers, such as a mind map, concept map, spider map, Frayer model, and KWL chart. Discuss which one(s) might be useful in this chapter.

In section 9.1 of the student resource, #19 gives students the outline for a mind map to summarize the first section. Encourage students to develop similar mind maps for sections 9.2 and 9.3 in order to produce a chapter summary.

Alternatively, students could use a set of index cards to build a chapter summary. They could write a concept or method from the chapter on the front of each card, and construct an example to illustrate the concept on the back. Students could also include a number of cards with questions or problems, and exchange them with classmates to help prepare for exams and other assessments.

Encourage students to use a summary method of their choice. Allowing personal choice in this way will increase students' ownership in their work. It may also encourage some students to experiment with different summary techniques.

Give students time to develop the summary method they have chosen. Ask them to include some method of keeping track of what they need to work on. Explain the advantage of doing this.

Meeting Student Needs

- Consider having students complete the questions on **BLM 9–2 Chapter 9 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- You may wish to post the student learning outcomes for the entire chapter in the classroom, colour-coding the outcomes by section in the chapter. Ensure that students understand the outcomes as written, and be prepared to rewrite some outcomes into language they understand. Provide students with their own copy. They can then refer to the outcomes as they work through the chapter. This will help them to self-assess their progress and to identify areas of weakness.
- Consider having students create short-answer questions involving the Key Terms and then exchange them with a partner.
- Hand out to students **BLM U4–1 Unit 4 Project Checklist**, which provides a list of all the requirements for the Unit 4 project.

ELL

- Encourage students to create their own vocabulary dictionary for the Key Terms using written descriptions, examples, and diagrams.

Enrichment

- Explain to students that the laws of supply and demand in economics help to determine the price of goods. Yet, an item in a particular area might have a range of prices for a variety of reasons. Challenge students to create a scenario that might produce a range of prices for the same item. Ask them how this situation might be represented using inequalities.

Gifted

- Challenge students to consider and comment on how a chemical engineer who is attempting to reduce the emissions from a pulp mill might have to use inequalities in his or her analysis. For example, the bleaching process involves strong and dangerous chemicals such as sodium hydroxide. Ask students:
  - What might occur if too little of the chemical is used?
  - What might be the environmental considerations of too much?
  - How might this decision produce an inequality?

Career Link

You may wish to have students who are interested in learning more about chemical engineers research the career, including the training and qualifications required, and employment opportunities. Have students present their findings orally. Explain how this career connects to the chapter.

Web Link

For detailed biographies of mathematicians, including most of those mentioned in the chapter, go to www.mhrprecalc11.ca and follow the links.

Linear Inequalities in Two Variables

9.1

***Pre-Calculus 11*, pages 464–475**

Suggested Timing
60–120 min

Materials
- grid paper
- straight edge
- graphing calculator
- computer with graphing software

Blackline Masters
Master 2 Centimetre Grid Paper
Master 3 0.5 Centimetre Grid Paper
BLM 9–3 Chapter 9 Warm-Up
BLM 9–4 Section 9.1 Extra Practice
TM 9–1 How to Do Page 470 Example 4 Using TI-Nspire™
TM 9–2 How to Do Page 470 Example 4 Using TI-83/84

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
RF7 Solve problems that involve linear and quadratic inequalities in two variables.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 3, 4, 6–11, 19, 20
Typical	#2, 3, 5–9, 11–14, 19–21
Extension/Enrichment	#8, 9, 14, 16–21

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce prerequisite skills needed for this section. If you have posted the outcomes, refer to the outcomes for this section.

To be successful in this section, students need to be adept at graphing linear equations, with and without technology. You may want to begin with a pre-test to determine how well students remember the concepts and skills necessary to graph such lines. This will make clear whether a revisit of graphing linear relations is desirable.

Problems that can be expressed as linear inequalities in two variables are frequently referred to as *mixing problems*, since the task is to find the permissible amounts of two different quantities. The solution set that students shade as they solve problems in this section is a representation of all the possible ways to mix two quantities.

Investigate Linear Inequalities

The Investigate in this section is an example of a simple mixing problem. It is possible for students to list all the possible solutions to the problem. By graphing the linear function that forms the boundary to the solution set, students should see that finding this boundary is a more efficient way to solve the problem, as opposed to listing all the solutions. This activity leads to the generalization of a method for solving any linear equation in two variables.

As students work on the investigation, you might wish to ask:
- What method did you use to generate all of the combinations?
- How do you know you have all possible combinations?
- What do the x-intercepts and y-intercepts represent in this situation?

Draw their attention to the question in the margin of the student resource: "Is it convenient to find all possible combinations this way?" Ask students:
- How can you be sure that the equation you wrote is correct?
- Will all students write the same equation? Why or why not?
- How is the linear inequality in step 5 related to the linear equation in step 2?

In step 8, the intention is for students to recognize that the boundary to the set of all possible combinations is a straight line. They should be able to write the equation of that line.

As students answer the Reflect and Respond, you might ask:
- How would the conditions need to change to make it impossible to spend the entire gift card?
- Could this situation happen in real life?

As you discuss step 11 in Reflect and Respond, draw out from the class that in a situation such as this one, the solution set is a set of discrete points, best represented by marking individual points on the Cartesian plane. Contrast this to a situation with real-valued solutions, where so many points are marked that the entire region of the plane is shaded. Tell students that shading is often used for convenience to show the solution set, even when the solution should be discrete points. This case is particularly true when technology is used to show the solution.

Meeting Student Needs

- Invite students to research the cost of two areas of interest to them. Provide them with a budget and ask them to determine how much of each choice they can afford per month. Students can create visual displays of their areas of interest, including current costs. Ideas can range from extracurricular activities to cell phones to entertainment.
- Students could brainstorm situations within the school in which decisions involving two or more variables exist.
- Encourage students to explore situations in which the entire $15 would not be spent. Ask whether these situations are covered in the investigation.
- For step 3, discuss whether students can draw a line through the points. Ask them what this line would represent and whether a line is possible in reality.
- Discuss the current cost of downloading music. Encourage students to talk about the importance of purchasing the music as opposed to downloading pirated copies. Research and discuss the laws against pirating music.

ELL

- Ensure that students add the following term to their vocabulary dictionary: *inequality*. Encourage them to include a verbal description, diagram, and/or example.

Common Errors

- Students may mix up the quantities to be shown on each axis.

R_x Refer to step 2, in which variables are assigned for students.

- Students may mix up the direction of inequalities.

R_x Refer students to the one-variable case to help them recall prior skills. For example, have them state the meaning of or give solutions to $x < 7$.

- Students may forget the difference between $<$ and $\leq$, or $>$ and $\geq$.

R_x Remind students that the line under the inequality represents an equal sign.

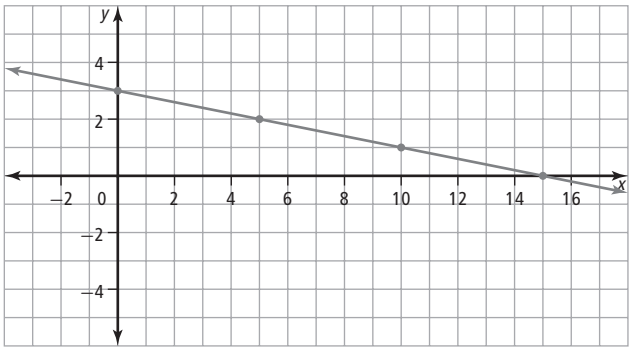
Answers

Investigate Linear Inequalities

1. 0 songs and 3 albums, 5 songs and 2 albums, 10 songs and 1 album, 15 songs and 0 albums

2. $x + 5y = 15$

3.



4. 1 to 15 songs and 0 albums, 0 to 10 songs and 1 album, 0 to 5 songs and 2 albums, 0 songs and 3 albums

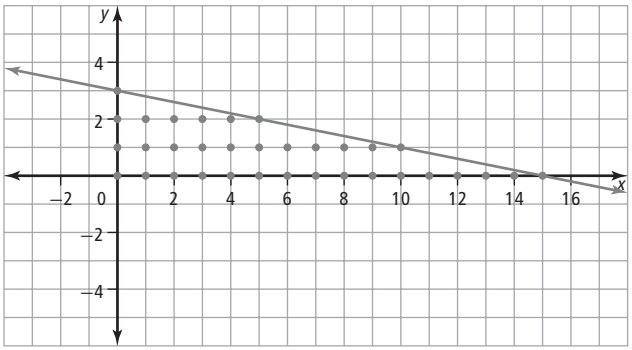
5. $y \leq 3 - \frac{1}{5}x$

6. Example: 5 songs and 2 albums:

$2 \leq 3 - \frac{1}{5}(5)$

$2 \leq 2$

8.



9. The line of the inequality indicates the values of x and y that would result in the spending of the entire value of the gift card.

10. Example: The inequality is partially represented on the graph, except that the graph includes only natural number values due to the context of the question.

11. If x and y could be any real number, the graph would show all values below and to the left of the line of the inequality.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• This Reflect and Respond should draw students' attention to the fact that they can buy only whole songs and complete albums. It is important to discuss step 11 with the class, since it applies to several real-world scenarios.

Link the Ideas

You may need to revisit with students some strategies for graphing linear relations. For example, they could write the equation of the line in the form $y = mx + b$ to identify the slope and y-intercept, and then sketch the graph.

To give students a concrete example of the new terms, ask them to identify each term in relation to their work in the Investigate. For example, have them look back and identify the boundary in the graph from the Investigate. Alternatively, you could have them generate an example for the new term, perhaps checking with a classmate to see if the other student agrees that the example fits the definition.

Students should understand that the solution to an inequality is usually an infinite number of points. This fact was also true with the inequalities they solved in grade 9. One change with the inequalities in grade 11 is that the solution set is an infinite number of points in the plane, rather than an infinite number of values on the real number line.

Example 1

Note that the methods used to graph the boundary should be familiar to students from their work on graphing linear equations in grade 10. Regardless of the method used to draw the boundary, students need to identify that the boundary must be drawn as a solid line.

You may want to ask students to generate other methods for determining the boundary. For example, they may generate a table of values and use the points in the table to draw the line. Students should understand that they can determine the boundary using methods that they explored in grade 10.

As students consider the example, you may want to ask:
- Why are these test points chosen?
- What is a different test point that would work?
- Are there any test points that are not suitable? Explain why or why not.

After students have completed the Your Turn, have them compare the method they used to graph the boundary to the method used by a classmate. Ask students:
- Why must the boundary line be a solid line in this case?
- Are there test points that you would prefer over (1, 3)? Explain why or why not.

You might have students explore how technology can be used to support their answers. You may wish to provide students with **TM 9–1 How to Do Page 470 Example 4 Using TI-Nspire™** or **TM 9–2 How to Do Page 470 Example 4 Using TI-83/84**.

Example 2

Before beginning this example, invite students to recall how solving linear inequalities differs from solving linear equations. Students should remember that when both sides of an inequality are multiplied or divided by a negative value, the direction of the inequality is reversed.

Ensure students understand from this example that the test point used must not be on the boundary, because this gives no information about which region to shade. Students should also see that since points on the boundary do not satisfy the inequality, it is important that the boundary is drawn as a dashed line.

As you work through the example, have students answer the questions posed in green in the student resource to draw out these important discussion points.

Example 3

In this example, students work backward from a graph to generate the inequality that the graph represents. You may want to ask students:
- What are some other ways to determine the equation of the line that forms the boundary?
- What are some other forms that can be used to write the equation of a line?
- What information does the dashed boundary line give us about the nature of the inequality?
- Is there another test point you would prefer to use? Why or why not?

If you have a student response system with your interactive whiteboard, you could have students submit answers to the Your Turn electronically to determine the level of understanding in the class. If such a system is unavailable, you could distribute index cards, and when everyone has completed the question, have each student hold up the card with their response written on it.

Example 4

This example is linked to the Unit 4 project, in which students consider the design for an object, and the materials used to construct the object.

Due to the values that appear in the example, this is a natural place to use technology to assist with graphing the solution region. The use of technology does not change the fact that students need to graph a linear function and decide how to shade the solution region.

Since this situation presents a restricted solution set, you may want to discuss the following with the class:

- Do we need to show all four quadrants in our solution?
- Is it possible for one of the quantities to be zero? How do you know?
- Is it possible for both of the quantities to be zero? Why or why not?

As students answer the Your Turn, you may want to ask what technology they would choose to help them answer this question, and why.

Key Ideas

If students are making index cards to summarize the chapter, you may want to suggest that students use the Key Ideas to make those cards. Whether or not students are making index cards, you could ask students to

- rewrite each Key Idea in their own words
- give an example to illustrate each Key Idea
- choose one or more of the examples in the student resource and show how each Key Idea relates to that example

Meeting Student Needs

- You may wish to discuss graphing solutions on a number line. What is the significance of the open circle? the closed circle? How do these relate to graphing in the Cartesian plane?
- Discuss what happens if the solution involves multiplying or dividing by a negative value.
- Post sketches of the four scenarios, clearly illustrating the significance of *equal to* in the inequality sign when drawing the boundary.
- Encourage students to use (0, 0) as a test point, since it is often the easiest point to mentally substitute into the inequality. However, remind them that (0, 0) is not an appropriate test point if it lies on the boundary.
- For Example 3, students may need to practise writing equations of linear functions given the slope and y-intercept.
- Remind students that a dashed line indicates that the points on the boundary are not part of the solution, and a solid line indicates that points on the boundary are part of the solution.
- You may wish to post the following information:
  - For the graph of $y > ax + b$ or $y \geq ax + b$, shade above the boundary.
  - For the graph of $y < ax + b$ or $y \leq ax + b$, shade below the boundary.

ELL

- Ensure that students add the following terms to their vocabulary dictionary: *infinite*, *boundary*, *region*, *intercepts*, and *slope*. Encourage them to include a verbal description, diagram, and/or example for each term.
- If students are not familiar with the word *mosaic* in the Your Turn for Example 4, show them pictures of mosaics to give them a visual understanding.
- To assist students in understanding some of the terms in the introductory statement for Example 4, such as *tabletop*, *aluminum*, and *laminated safety glass*, point them out in the photo that accompanies the example.

Enrichment

- Challenge students to describe situations in real life that can be represented by an inequality.

Gifted

- Challenge students to create a series of linear inequalities with solution regions that are an interesting mathematical shape (for example, an octagon).

Common Errors

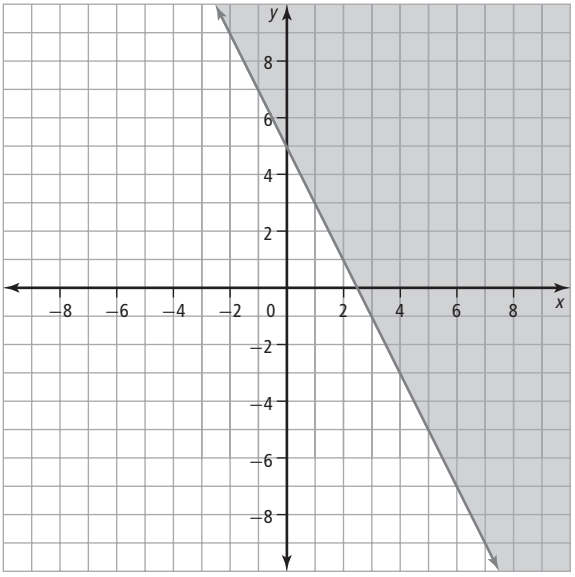
- Students may forget to change the direction of the inequality if they have multiplied or divided by a negative number.

R_x Suggest that students use the test point in the original inequality, as well as in the rearranged inequality. The same solution region should be indicated for shading by both substitutions. If not, an algebraic error has likely occurred.

Answers

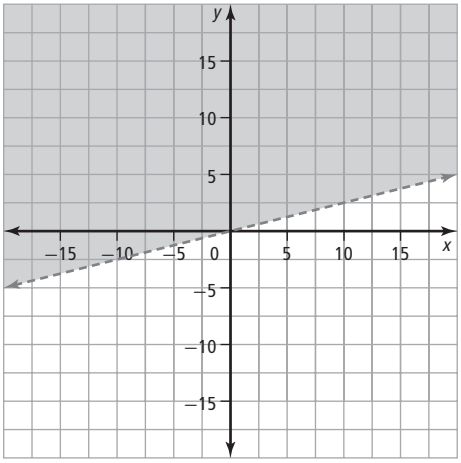
Example 1: Your Turn

a)



b) Yes, (1, 3) is part of the solution.

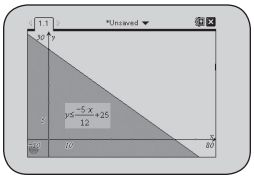
Example 2: Your Turn



Example 3: Your Turn

$y \geq -2x - 2$

Example 4: Your Turn



Every point in the first quadrant on or below this line is a solution.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Prompt students to write the inequality in terms of y. • It is important to ensure that students understand the idea that the solution region is the location of the solutions. Encourage them to choose test points on both sides of the boundary. Ensure they understand that the success or failure of a test means it is or is not a solution both graphically and algebraically.
Example 2 Have students do the Your Turn related to Example 2.	• Review with students when an inequality symbol reverses direction. • As a verification of their calculations, especially with students who confuse the meaning of the inequality signs, suggest that they check a point on each side of the boundary before shading. This will help prevent them from shading based on a computational error.
Example 3 Have students do the Your Turn related to Example 3.	• Coach students in determining the values of b and m for the equation $y = mx + b$. • To assist students who are having difficulty deciding which way to place the inequality, show them that the y-intercept can help them. If the area above the y-intercept is shaded, the inequality will be $>$ or $\geq$ (for an equation in the form $y = mx + b$), depending on whether the boundary is dashed or solid. If the area below the y-intercept is shaded, the inequality symbol will be $<$ or $\leq$ (for an equation in the form $y = mx + b$).
Example 4 Have students do the Your Turn related to Example 4.	• Prompt students to identify the key parts of the question. Ask them to verbalize which inequality sign might be used for terms such as *at least*, *at most*, and *a budget of*. This may assist them in setting up their inequality. A list of these and other word expressions could be posted on a wall in the classroom for reference.

Check Your Understanding

Practise

When students have completed #8, they could consult briefly with a partner to see if they agree on methods for graphing each inequality. After students have completed #9, they could consult with their partner again to discuss results and clarify which parameters influence the choice of technology for graphing inequalities.

Apply

In #11, have students compare answers to part d) concerning why Amaruq would want income from two sources. You could ask students to write a reason on an index card and hold the card up for you to view. You could then have the class sort the reasons into mathematical reasons (for example, the time it takes to sew the moccasins), and non-mathematical reasons (for example, the desire to diversify her income).

For #13, students are required to write an inequality, define variables, and then solve the inequality. You may want to poll the class to see how many students would put talk time on the x-axis, and how many would put it on the y-axis. Such a poll can be quickly accomplished if you have a student response system for your interactive whiteboard. This poll is meant to illustrate that in the context of mixing problems, either variable can be placed on either axis; in other words, there is no need to classify an independent and dependent variable.

Extend

The intention of #16 is, in part, for students to get a glimpse into section 9.3, in which they solve quadratic inequalities in two variables. A variety of answers is possible for this question. The emphasis should be on students' justification of their answers. Ideally, at least one student will think of a parabola as a boundary function. For part c), remind students to look back at the definition of a solution region in the student resource to check that their response is correct.

For #17, students need to determine the equations of four lines. This problem may be best completed in a group of two or four students, so each student can find the equation of one or two lines. Working in a group would also allow students to discuss the inequalities that they write to ensure that they are correct and, in particular, that the direction of each inequality is correct. You may want to encourage students to graph their inequalities to check that their graph matches the diagram given in the student resource.

Create Connections

Question 19 is designed to be accessible to all students, and it should help them clarify and quantify what they have learned so far in the chapter. Students should feel free to alter the format of the mind map in any way that they find more convenient or appropriate for their needs.

You may want to collect student responses to #20. An incentive for students in completing this question could be that the scenario they have created may appear on an assignment, quiz, or exam for this unit.

Meeting Student Needs

- If students do not have graphing paper, provide them with **Master 2 Centimetre Grid Paper** and/or **Master 3 0.5 Centimetre Grid Paper**.
- Students should be encouraged to use mental math for #1 and 2.
- In #13, students study the costs related to a cell phone. Students could be asked to research the options available from real service providers. They can use what they find out to determine which cell phone plan would be the best choice if they were looking for themselves.
- Provide **BLM 9–4 Section 9.1 Extra Practice** to students who would benefit from more practice.

ELL

- Ensure that students understand the terms *mentorship*, *marketing assistance*, *smart phone*, *nanomaterial*, and *nanotube*. Use a combination of verbal description, diagrams, and/or examples to help them.
- If students are not familiar with the term *moccasins* in #11, refer them to the picture that accompanies the question.
- For #18, you may wish to have English language learners work with another student to assist them with the language in the question.
- Ensure that students complete #19, which is a useful visual activity.

Common Errors

- For #3, some students may forget to divide each term by the coefficient of y.

R_x Remind students to make sure they understand what a question asks before attempting it. For this question, expressing y in terms of x means students need to isolate y and ensure that it has a coefficient of 1.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 3, 4, and 6 to 11. Students who have no problems with these questions can go on to the remaining questions.	• Prompt students to verbalize the meaning of the inequality signs in #1 before substituting values. Ensure they understand when the equal sign will and will not affect their decision about whether an ordered pair is a solution. • The basics of the section are covered in #3. Students should be very successful with this question before moving on. Encourage them to use parts a) and b) as guides for the remaining questions. Prompt students through parts a) and b), if needed, before they move on to parts c) and d). Success with #3 will assist them in completing #4, 7, and 8. • For students who are having difficulties, coach them to verbalize what happens when an inequality is divided by a negative. • Encourage students to identify the *y*-intercept and the slope for #9. Complete part a) with students, placing the values into slope–intercept form. Have students verbalize how they will determine which inequality symbol they should use. Have them pay close attention to whether the boundary is solid or dashed. Ask them how this might affect their decision. • Have students refer to Example 4 to assist them with #11. Remind them about choosing appropriate variables for their problem. Ask them to identify important words, such as "at least $250/week." Ask them how this value should be used to answer the question. • Although #14 is not assigned to Essential students, you may wish to direct them to it when they have completed their questions. Question 14 may assist them in formulating an appropriate costing scenario for their nanotechnology item when they work on their Unit 4 project.
Assessment *as* Learning	
Create Connections Have all students complete #19 and 20. Typical and Enrichment students can go on to #21 when completed.	• All students should be encouraged to complete the mind map in #19. Some students may wish to make the mind map into a Foldable format that they are familiar with. Encourage them to describe each of the four possible inequalities and corresponding examples in their own words. • For #20, it may assist students to brainstorm possible scenarios with a partner. Alternatively, you may wish to have the class generate some ideas and post them on the board. Students could then independently use one of these ideas as a springboard for their own scenario.

9.2 Quadratic Inequalities in One Variable

***Pre-Calculus 11*, pages 476–487**

Suggested Timing
120–180 min

Materials
- grid paper
- coloured pens, pencils, or markers
- graphing calculator
- computer with graphing software
- straight edge

Blackline Masters
Master 2 Centimetre Grid Paper
Master 3 0.5 Centimetre Grid Paper
BLM 9–3 Chapter 9 Warm-Up
BLM 9–5 Section 9.2 Extra Practice

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
RF8 Solve problems that involve quadratic inequalities in one variable.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 3–10, 13, 19, 20
Typical	#1, 3–9, 11–13, 18–20
Extension/Enrichment	#8, 9, 11, 12, 14–20

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce prerequisite skills needed for this section.

In this section, students solve quadratic inequalities using a wide variety of techniques. Regardless of the technique each student ultimately prefers, solving quadratics by factoring and by using the quadratic formula are a constant theme. A pre-test to ascertain how well students remember these skills is a good starting place for this section. Students need to be able to solve quadratics confidently in order to understand and use the strategies for solving quadratic inequalities.

Investigate Quadratic Inequalities

In this Investigate, students have a chance to see that their knowledge of solving linear inequalities can be leveraged to solve quadratic inequalities as well. The Investigate uses the graph of a quadratic, which students worked with in Unit 2, but focuses on describing characteristics of the parabola using only subsets of the domain.

As students work on the investigation, you might ask them why they start by obtaining the x-intercepts.

In step 1, after students answer parts b) and c), ask them what other ways there are to represent the sets, particularly if students use words rather than symbols.

In step 2, students are not given instructions on how to graph. You may want to have the class brainstorm some ways to graph; for example, they may complete the square and graph by hand, they may determine intercepts, or they may use technology. You may choose to have different groups use different methods of graphing, and then compare results.

In step 2d), the link to solving linear inequalities should become apparent to students, since they are asked to show their answer on a number line. To help them see the connection, you can ask them where they have used a similar strategy in the past.

As students answer the Reflect and Respond, you may want to use the think-pair-share process. Students write their own answers, and then discuss them with a partner to achieve consensus. When all pairs have decided on an answer, a number of them share their answers with the entire class to initiate a class discussion.

Meeting Student Needs

- Discuss the two situations, the roller coaster and the bicycle. Discuss the need for precision in the calculations. Which situation requires more precision? Why? Encourage students to understand the importance of all calculations and the impact that an incorrect calculation may have.

- You may wish to organize students into pairs to work on this section. Have students explore the information without using technology. The experience of calculating and graphing manually will promote understanding.
- For #1b) and c), inquire how students can determine the values of x that satisfy the inequality.
- While students are completing the investigation, you may wish to collect comments you overhear and post them when the Investigate is finished. Invite students to discuss the comments. Be sure to include responses regarding #4.

Common Errors

- Students may factor quadratics incorrectly.

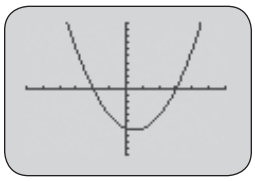
R_x Have students refer to Unit 2 and recall the strategies for factoring.

Answers

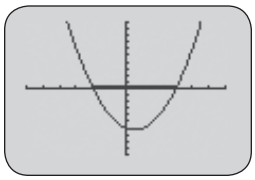
Investigate Quadratic Inequalities

1. a) $x = -1, x = 4$
b) $x < -1$ or $x > 4$
c) $-1 < x < 4$

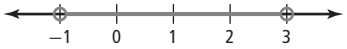
2. a)



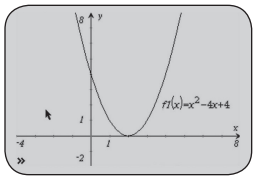
b) 2
c)



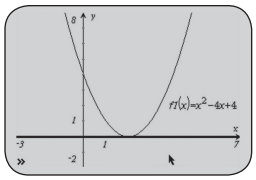
d) $-1 < x < 3$



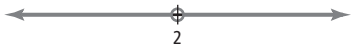
3. a)



b) 1
c)



d) $x < 2, x > 2$



4. a) Example: For step 2d), look at the graph to see where y is less than zero and determine the corresponding values of x. For step 3d), look at the graph to see where y is greater than zero and determine the corresponding values of x.
b) Example: Look for the zeros, because then it is easy to determine the domains for which the inequality is true.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• You may need to provide a sample graph for students to look at when reviewing and coaching them through these questions. Ask them to identify where values of x are negative and positive, and where values of y are negative and positive. Coach them by making a link between the two and then tying in to the x-intercepts.

Link the Ideas

The Link the Ideas refers to graphic and algebraic methods for solving quadratic inequalities. Students should look for similarities and differences among the methods they learn in this section, and especially look for a method that suits their own learning style and strengths. It is likely that students will create their own hybrid method by combining ideas from two or more of the methods presented. In this case, they should be able to articulate their method and why it works.

Example 1

Regardless of the method under consideration, you might ask students to explain why the symbols in the solution set are $\leq$ rather than $<$.

To help them connect this method to the Investigate, ask students why one section of the x-axis is highlighted in a different colour.

As students work through Method 2, ask:
- How is this concept similar to Method 1?
- How does this method differ from Method 1?

These questions should help them to see that they are using algebra to decide where the graph is above or below the x-axis, rather than looking at the graph to decide.

In section 9.1, students learned that some points are not suitable as test points. They are asked similar questions regarding Example 1 (see the green text in the margin of the student resource). Have students answer these questions for this example and for quadratic inequalities in general.

As students consider Method 3, ask them:
- Is factoring the quadratic important in this method?
- How is the use of number lines in Method 3 different than in Method 2?
- Why does Case 1 not lead to a solution for the inequality?
- How would the cases change if the inequality was $x^2 - 2x - 3 < 0$ or $x^2 - 2x - 3 > 0$?

After students have completed the Your Turn, you may want to poll students to see which methods they used most often. Have students give reasons for choosing or avoiding each method.

Example 2

Before students see the solution to the example, you may want to have them divide into three groups, one for each of the methods presented in Example 1. Invite each student to choose a group, rather than be assigned to one. Have each group work together to solve the inequality using the method they have chosen. When they are done, each group presents their solution to the class. Solutions from the group using roots and test points can be compared to the solutions in Example 2 of the student resource, but all groups should obtain the same solution.

Have each student complete the Your Turn using a different method than the one they used to solve the example.

Example 3

Students may need to recall the use of the quadratic formula from Chapter 4. You may want to ask what strategy they should use when they are unable to factor a quadratic. This discussion should guide them to the use of the formula.

As you work through the example, ask:
- How does the format of the x-intercepts change your use of the methods?
- How do you know where to place these exact values on the number line?

Ensure students answer the question in the student resource that asks whether this inequality can be solved using the methods of sign analysis and case analysis. Ensure students can justify their answers.

You may want students to complete the Your Turn working with a partner, so that they can discuss the correct use of the quadratic formula and perhaps have more success in completing the question.

Example 4

As students consider this example, ask:
- Why is technology used to solve this question?
- Would you prefer a different method? If so, explain why.

After students complete the question, ask:
- Could the solution to a problem like the one in Example 4 ever be something like $t > a$, where a is a real number?
- What would have to be true for such a solution?

Key Ideas

For the first Key Idea, have students consider what size the solution set for a quadratic inequality can be, and perhaps give examples to illustrate their answer.

If any students have created other methods of solving inequalities, they should add these methods to their notes as an additional Key Idea.

Meeting Student Needs

- In the classroom, provide visuals illustrating the various methods used to solve inequalities.
- For Example 1, divide students into three or six groups. Assign to each group one of the three methods and have them present their results to the class. If you have six groups, the other three groups could present the Your Turn answers, illustrating each different method. Allow students to use a variety of presentation methods.
- For Example 2, students could write a paragraph to explain the steps involved in each method, or explain verbally to a partner.

ELL

- Ensure that students add the following terms to their vocabulary dictionary: *graphically*, *algebraically*, *interval*, *satisfy* (in the context of a point that satisfies an inequality), and *roots*. Encourage them to include a verbal description, diagram, and/or example for each term.

- Ensure students understand that *between −1 and 3*, *inclusive* means the solution set includes −1 and 3.
- For Example 4, some students may not be familiar with baseball. You may wish to show them pictures and videos of the game and/or have another student explain the game. You can also refer them to the Web Link in the student resource.
- Check that students understand the names used for the strategies, such as *case analysis* and *sign analysis*, and that they make the connection between these names and the actual steps in the strategies. This understanding will be important when they are asked to use the strategies in the Check Your Understanding.

Enrichment

- Direct students to create an example of their own for each method of solving inequalities. They should be able to explain how each example is solved using each specific method.

Gifted

- Ask students to create a rationale for selecting a method to solve an inequality. Ask them to communicate what features of an inequality dictate the best method.

Common Errors

- Students may substitute values into the quadratic formula incorrectly.

R_x Suggest that students first rearrange the inequality so that zero is on one side.

Answers

Example 1: Your Turn

Example: The roots are $x = 2$ and $x = 8$. Test points give the solution as $\{x \mid 2 \leq x \leq 8, x \in \mathrm{R}\}$.

Example 2: Your Turn

Example: The roots are $x = -2$ and $x = 5$. Test points give the solution as $\{x \mid x < -2 \text{ or } x > 5, x \in \mathrm{R}\}$.

Example 3: Your Turn

Example: The roots are $x \approx -1.74$ and $x \approx 5.74$. Test points give the solution as $\{x \mid x < 2 - \sqrt{14} \text{ or } x > 2 + \sqrt{14}, x \in \mathrm{R}\}$.

Example 4: Your Turn

The ball is in flight between 0 s and approximately 3.56 s.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Have students verbalize which two methods they feel most comfortable with. You may suggest that they work in pairs or groups of three, matching them with students who choose a similar method. Encourage those students who complete the question quickly to try all three methods. • Each method provided in this example is important and results in each inequality being arranged slightly differently. Have students verbalize how each method is similar and different. This class discussion will benefit students at the end of the section when they complete Create Connections #19.
Example 2 Have students do the Your Turn related to Example 2.	• Again, have students verbalize which two methods they feel most comfortable with and work with students who chose a similar method. Then, have students who have no difficulty with their method try all four methods. • Some students may find it easier to combine the roots and test points method with sign analysis.
Example 3 Have students do the Your Turn related to Example 3.	• Prompt students to verbalize the inequality rewritten to zero. • Have students identify that they will need to solve using the quadratic formula. Encourage them to write down the values of the parameters *a*, *b*, and *c* for the quadratic formula. • Ask students why they will need an exact answer.
Example 4 Have students do the Your Turn related to Example 4.	• For students who are having difficulty, prompt them to use graphing technology to help them identify approximate *x*-intercepts and *y*-intercepts. Otherwise, encourage students to use the quadratic formula for exact values.

Check Your Understanding

Practise

Question 2 presents a case in which some inequalities have no solution and some have all real numbers as the solution. You may want to plan for a quick think-pair-share activity after this question to ensure that the whole class understands.

For #4 to 7, students are told the method to use in solving the inequalities. You may want to point out that the goal is for students to make comparisons among the methods to determine if there is a method that they prefer to use more often, or if some methods are more appropriate for certain types of questions. It is important that students understand each method before using one method exclusively.

For #9, students rearrange inequalities before solving. When students complete this question, encourage them to find a partner who rearranged one inequality in a different way. They should compare answers to confirm that the solution set is the same, though the quadratic that was solved may have been different.

Apply

Question #10d) relates to the non-linearity of these inequalities. Students' intuition is likely to tell them that twice the mass requires twice the thickness of ice to support it. You may want to ask them how the graph of the function shows that this is not true.

The context in #12 relates to the Unit 4 project. It also is connected conceptually to Example 4. When students answer part b) of this question and consider a reasonable domain for the situation, you may want to ask them to refer to Example 4.

Extend

The results of #2 are generalized in #14. Ensure students are aware of this connection so that they can try to work from specific to general as they complete #14. You may want to suggest that they generate a few more specific examples to help them recognize a pattern so that they can determine the general case. Also, you may want to help students with part c) by asking them how many values are in the interval $0 < x < 5$. Note that the choice of 0 and 5 is arbitrary, and that any interval will do.

Multiple answers are possible for each part of #15. Students should compare their answers to ascertain this. Alternatively, you could ask students to generate more than one solution to each inequality. To connect this question to Unit 2 and draw out the idea of vertical stretches and/or reflections, you could ask what all of the inequalities with the same solution have in common.

The graph in #17 is similar to that in the Chapter 9 Task for the Unit 4 project. It also follows up on the idea of #15, in that students see a different inequality that has the same solution as the one graphed. To prepare students for the Chapter 9 Task, discuss the meaning of points that are below the line but above the parabola, or above the line and below the parabola. This discussion will help students understand that each point has a relationship with each graph.

Create Connections

As students decide whether the solution in #20 is correct, you might suggest that they begin by solving the inequality without reference to Devan's work. Otherwise, they may make the same mistakes he did. Once students have completed their solution, they can review Devan's work for errors. Alternatively, you can recommend that they solve the inequality using a different method, so that they have more confidence in the correct solution before considering Devan's work.

Project Corner

In the Project Corner, one of the bullets states that costs decrease as nanotechnology improves. Have a discussion about items in technology that decrease in cost over time. Write the list on the board. Have students brainstorm why they believe the cost went down. Ask them to generate ideas about how they may derive an initial cost for the nanotechnology item they will use for the Chapter 9 Task. What do they believe will be the future price of their item? Encourage students to back up their initial costing with research.

Meeting Student Needs

- If students do not have graphing paper, provide them with **Master 2 Centimetre Grid Paper** and/or **Master 3 0.5 Centimetre Grid Paper**.
- For #1 to 3, it may be better to assess their understanding of the concepts orally.
- For #4, ask students what should be done to determine the solution. Then, suggest that students complete at least two of the four parts.
- Ask students to compare the processes of sign and case analysis in #5 and 6. Then, they may wish to complete just one question illustrating both processes.

- Prior to beginning #8, ask students to list all of the methods that could be used. Ask them to discuss with a partner which method they prefer and why.
- In #10, 11, and 12, three situations illustrate solving inequalities in everyday contexts. Be sure students read all three questions, even if they solve only one. Exposure to the content is important.
- For #11, students need to write an inequality using metres. You may wish to tell students that $1 \text{ ha} = 10\,000 \text{ m}^2$.
- For #15, students may wish to choose just one part and create a model to represent the inequality. For example, they may draw a graph, create an art project with string, or use another method of their choice.
- Provide **BLM 9–5 Section 9.2 Extra Practice** to students who would benefit from more practice.

ELL

- Ensure that students add *discriminant* to their vocabulary dictionary. Encourage them to include a verbal description, diagram, and/or example.
- In #10, students may not be familiar with ice fishing. You may wish to show them pictures and videos of the activity and/or have another student explain it.
- For #11 and 12, you may wish to have English language learners work with another student to assist them with the language in the questions. Have students refer to the Did You Know? accompanying #12 for more information about carbon fibre.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 3 to 10, and 13. Students who have no problems with these questions can go on to the remaining questions.	• You may wish to have students work with a partner to complete the assigned questions. • For students having difficulty with #1, prompt them through the question. Ask them to identify where the graph is less than zero and greater than zero. How could they use this information to help them answer the question? Then, assign #2. • For #4 to 7, have students refer back to Examples 1 and 2 for each different method. They should also use their response to Supporting Learning for Example 1 (i.e., their list of the methods' similarities and differences) to assist them in answering these questions. If they did not complete this question, it may be beneficial to the class to brainstorm and then post the similarities and differences between each of the methods, and how each inequality is used for each method. This will assist students in summarizing for #19. • In #8 and 9, students choose the method they feel most comfortable with. Encourage them to use at least two different methods, if possible. • For #10, students are given the inequality. Ask them what the link between 750 and 1500 is in the inequality. This connection should prompt them to create the second inequality.
Assessment *as* Learning	
Create Connections Have all students complete #19 and 20.	• All students should be encouraged to complete #19. Encourage them to generate their own examples of solving by each method, where possible. Alternatively, they could create sample questions and have a partner solve using the different methods. • Ensure that students justify their answer in #20a) to have a reference for study purposes. For students who are quicker at solving, encourage them to confirm the solution in more than one way.

9.3 Quadratic Inequalities in Two Variables

***Pre-Calculus 11*, pages 488–500**

Suggested Timing
60–120 min

Materials
- grid paper
- coloured pens, pencils, or markers
- graphing calculator
- computer with graphing software
- straight edge

Blackline Masters
Master 2 Centimetre Grid Paper
Master 3 0.5 Centimetre Grid Paper
BLM 9–3 Chapter 9 Warm-Up
BLM 9–6 Section 9.3 Extra Practice

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

Specific Outcomes
RF7 Solve problems that involve linear and quadratic inequalities in two variables.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–6, 9, 10, 16–18
Typical	#1–6, 9, one of 10–12, 16–18
Extension/Enrichment	#4, 5, 7, 13–18

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce prerequisite skills needed for this section.

Section 9.3 presents applications and extensions of the ideas in sections 9.1 and 9.2. Students solve quadratic inequalities in two variables in one of two ways. In the first case, they graph a quadratic equation, which separates the plane into two regions, one of which contains all the points that satisfy the inequality. This is analogous to the linear case in section 9.1, the only difference being the shape of the boundary. In the second case, students are given a quadratic inequality in two variables and specific information that enables them to transform it into an inequality in one variable. At that point, they use the strategies they mastered in section 9.2 to solve the quadratic. It is helpful for students to realize the links between the material in this section and the concepts and skills they acquired earlier in the chapter.

Investigate Quadratic Inequalities in Two Variables

You may wish to have students work in pairs for this Investigate. Each pair can discuss the instructions and their answers to the questions.

As students work on the investigation, you might ask:
- How do you know you are sketching the correct graph in step 1?
- Should the graph be drawn with a solid or dashed curve?
- How is this similar to what you learned about linear inequalities?
- How many test points do you think you need in general to guarantee you have the correct shading?
- How does this compare to sketching linear inequalities?

To address the Reflect and Respond, have students write their answers on three sticky notes, without identifying themselves on the notes. Then, designate three places in the classroom for students to post their answers when completed. When all students have posted their answers, allow a few minutes for students to read the responses. Students who wish to change their responses can do so on a new sticky note or in their own notes. The amount and depth of class discussion needed for this Investigate will become apparent from the responses on the sticky notes.

Meeting Student Needs

- To further the discussion in the opener, students may wish to investigate the strength of the design of the Volkswagen Beetle. It is a very sound structural application that includes openings for the doors. Encourage students to write a brief report about the vehicle.

- Discuss the student learning outcomes for this section, which you may have posted at the beginning of the chapter.
- Discuss any Key Terms, as needed, prior to completing the last section of the chapter.
- As a reminder, ask students the difference between a dashed boundary and a solid boundary on a graph.
- Students could work on the Investigate in pairs or small groups. Post the finished products in the classroom. Include captions and the equation.
- Students could research examples of graphs containing shaded regions using the Internet, magazines, or newspapers. Discuss their findings.

ELL

- Ensure that students have the term *parabola* in their vocabulary dictionary. Encourage them to include a verbal description, diagram, and/or example.

Common Errors

- Students may draw all graphs with solid curves.

R_x Refer them to the strategy they developed in section 9.1 to graph linear inequalities correctly.

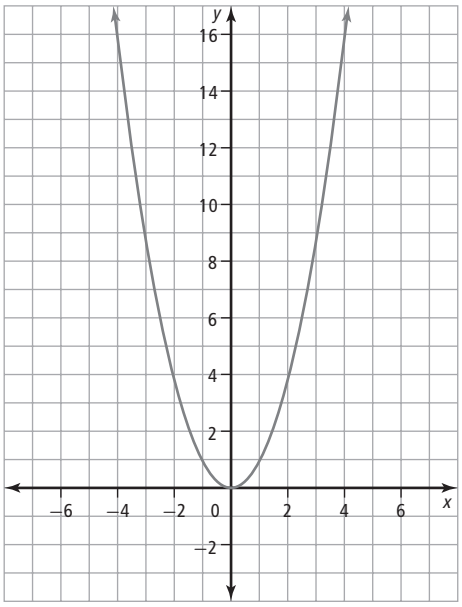
- Students may try to use the language *inside the parabola* or *outside the parabola* to describe the solution region.

R_x Remind students that a parabola is not a closed figure, and thus does not have an inside or outside. Rather, they should think of it as a graph that divides the plane into two parts, as a line does. As they did with lines, tell them to use the descriptors *above* and *below* to describe solution regions. If they need an illustration of the dangers of using the terms *inside* and *outside*, have them consider the inequalities $y \leq -x^2 + 1$ and $y \geq x^2 - 1$. In the first, the region below the parabola is shaded, and in the second, the region above is shaded, yet both regions could also incorrectly be described as inside the parabola.

Answers

Investigate Quadratic Inequalities in Two Variables

1.



2. a) Example:

x	y	Satisfies the Equation $y = x^2$?
3	9	$9 = 3^2$ Yes
2	4	$4 = 2^2$ Yes
4	16	$16 = 4^2$ Yes
5	25	$25 = 5^2$ Yes

b) Example: They satisfy the equation $y = x^2$.

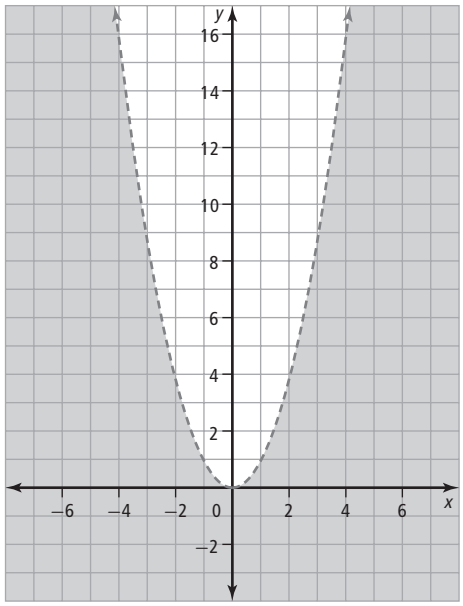
3. a) the region below the parabola

b) Example:

x	y	Satisfies the Inequality $y < x^2$?
3	3	$3 < 3^2$ Yes
2	1	$1 < 2^2$ Yes
−1	−1	$-1 < (-1)^2$ Yes
−3	1	$1 < (-3)^2$ Yes

4. Example: Yes, the test points satisfy the inequality.

5.



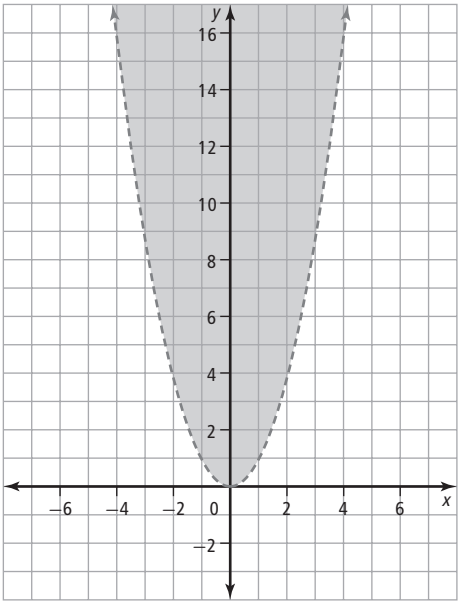
Answers

6. a) the region above the parabola
b) Example:

x	y	Satisfies the Inequality $y > x^2$?
−1	2	$2 > (-1)^2$ Yes
0	1	$1 > 0^2$ Yes
1	3	$3 > 1^2$ Yes
2	6	$6 > 2^2$ Yes

7. Yes

8.



9. Example: Every point in the shaded region is part of the solution set.

10. Example: If y is less than the function, then the solution set is the whole region below the function. If y is greater than the function, then the solution set is the whole region above the function.

11. The inequality contains one of the following signs: $\leq$ or $\geq$.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the Reflect and Respond questions. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• These questions are very similar to ones students have previously answered in this chapter and in Chapter 8. If not already done, you may wish to show them how the y-intercept can assist in determining what to shade as the solution region. Review with them when the boundary should be dashed or solid.

Link the Ideas

Students may need to review some strategies for graphing quadratic equations. For example, they may choose to sketch the graphs using transformations. It is important that students are comfortable sketching quadratic equations before they add the process of determining and shading the solution region. Also, students will more easily grasp the concepts if they see how similar the tasks in this section are to the tasks in the previous two sections.

Example 1

The quadratic inequality in this example is expressed in a form that should allow students to apply their knowledge of graphing quadratic equations. Students should be able to generate the graphs of quadratics in this section with and without technology.

As students consider the example, you may want to ask:

- Why were those particular test points used in Example 1?
- What is a different test point that would work?
- Do you have a favourite test point? What is it?
- Why is it your favourite?

- Can you use it every time? Why or why not?
- Are there any points that are not suitable as test points? Explain why or why not.

Example 2

You may want to divide the class into two or three groups based on how they choose to graph the quadratic inequality in this example. After they have graphed, you could have each group justify their method of choice. A group may decide that their method was not the most effective, so allow for that possibility as well. The groups should verify that all of the methods generated the same graph.

When students complete the Your Turn, have them use a different method for sketching the graph to ensure that they are comfortable with more than one method.

Example 3

In this example, students generate the equation of a quadratic from points on the parabola. You may want to have students work in pairs, which may help them to activate their prior knowledge. It may be a good idea to pause to discuss strategies for obtaining the equation before completing the example. To help students ensure their equation is reasonable, you might ask:

- How do you know there is no vertical or horizontal shift?
- Is it necessary to draw the parabola with its vertex at the origin? Explain why or why not.
- What would happen if you drew the parabola with its vertex somewhere else?
- How does the shape of the parabola compare to that of $y = x^2$?
- How do you know that the leading coefficient should be positive?
- Why is it necessary to restrict the domain in this case?

Students should complete the Your Turn with the same partner. Have them answer the above questions for the satellite dish scenario. When they are finished, have students work with another pair to compare and contrast the inequalities obtained for the microphone and the satellite dish. In particular, they should note that both situations require a restricted domain, but that they have different vertical stretches.

Example 4

To help students understand the restricted domain in this example, you might ask:

- How is the restriction to this domain different than that in Example 3?
- How does that difference appear in the algebraic statement of the domain?

If students use technology, you might ask what other way there might be to show the appropriate domain and range for the inequality.

When students have completed the Your Turn, ask:

- How do the domain and range for this situation compare to those in Example 4?
- Why is the domain different?

Key Ideas

Ask students to compare the Key Ideas for this section to those in sections 9.1 and 9.2. In particular, ask them which are the same as previous Key Ideas and which are extensions or generalizations. You may want students to create or find examples to show the similarities and differences between the Key Ideas in this section and those in the previous two sections.

Meeting Student Needs

- Place emphasis on solid versus dashed lines on the graph to ensure student understanding.
- Ask students to work through the various examples individually or in pairs. Then, ask them to write a short summary for you to examine. Add comments as needed and return the summaries to students for review purposes.
- Students could research the shape and dimensions of various satellite TV dishes. Have them try to write the equation that represents the parabolic curve of each dish. Ask them whether they think the shape of one dish is more advantageous than another, and have them write a sentence supporting their choice.
- Ask students to look through the investigation and examples to find where each of the Key Ideas is developed.
- You may wish to post the following information:
  - For the graph of $y > ax^2 + bx + c$ or $y \geq ax^2 + bx + c$, shade above the parabola.
  - For the graph of $y < ax^2 + bx + c$ or $y \leq ax^2 + bx + c$, shade below the parabola.

ELL

- For Examples 3 and 4, along with the accompanying Your Turn questions, have English language learners work with another student to assist them with the terms in the questions and solutions. Allow students to use pictures (including on the Internet) to help in their understanding of some of these terms.

Enrichment

- Challenge students to imagine a zip line whose form becomes a parabola as the traveller reaches the lowest point of the line. Have them produce a graph that represents the boundary of the inequality and whose solution region is a safe zone of travel. Ask students to speculate on what factors would change the equation of the boundary (for example, the weight of the traveller).

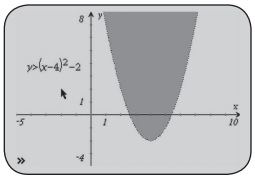
Gifted

- Ask students to find and describe an application for graphing quadratic inequalities in two variables. Refer them back to the parabolic microphone in Example 3.

Answers

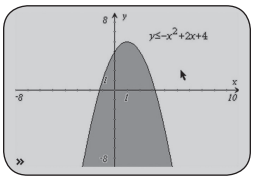
Example 1: Your Turn

a)



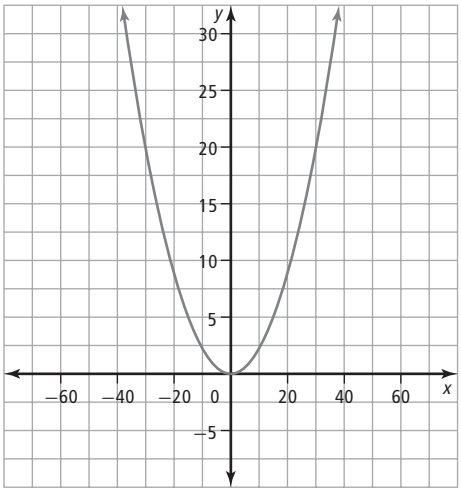
b) (2, 1) is not a solution to the inequality.

Example 2: Your Turn

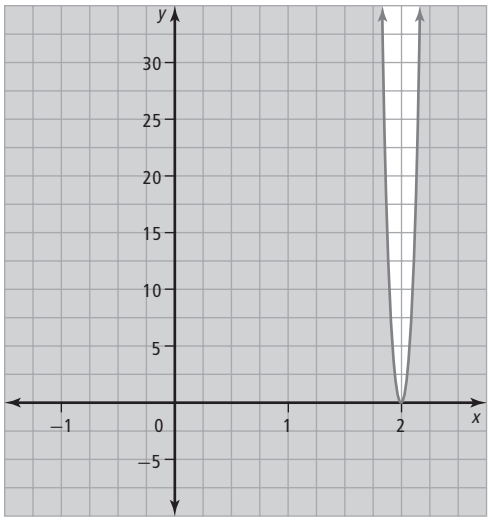


Example 3: Your Turn

$y \geq \frac{1}{45}x^2$, $\{x \mid -30 \leq x \leq 30, x \in \mathrm{R}\}$



Example 4: Your Turn



Example: You cannot have a negative value for the diameter of the rope or the mass. Therefore, the domain is $\{d \mid d \geq 0, d \in \mathrm{R}\}$ and the range is $\{M \mid M \geq 0, M \in \mathrm{R}\}$. One solution is (1, 1240). This means that a rope with a diameter of 1 in. would support a weight of 1240 lb.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to prompt students to answer this question using transformations. Ask students to verbalize what their vertex is. • Have students identify what methods they learned in section 9.2 that might assist in solving this question. What is their preferred method?
Example 2 Have students do the Your Turn related to Example 2.	• Students could use technology to work through the question. Ask them to identify two test points they could use. Have them try out their points and verbalize their conclusion.
Example 3 Have students do the Your Turn related to Example 3.	• Prompt students to begin by visualizing the problem and then drawing a diagram and labelling it with the given information. • Encourage students to use a test point from the solution region to verify the inequality symbol.
Example 4 Have students do the Your Turn related to Example 4.	• Students could use technology to work through the question. Ask them to identify and use two test points. • Ask them to write the domain and range and to verbalize their response. Note whether they mention that neither the diameter of the rope nor the mass can be a negative value. • Ensure they are able to verbalize their conclusion using a summary statement.

Check Your Understanding

Practise

For #3, remind students that they can use a test point to decide the direction of each inequality. Ask:

- How do you decide where to shade a graph?
- How can you adapt this strategy when you have a graph that is already shaded and you need to identify the inequality?

When students have completed #4 through 7, you may want to allow time for discussion, either as a class or in small groups. Students should consider the following questions:

- When is one method of graphing preferable to another?
- Do you have a favourite method? If so, why is it your favourite?
- Do all of the methods work in every situation?
- Are there pitfalls to watch for in some methods or some types of questions?
- How does using technology differ from the other methods? How much does this difference depend on the technology you are using?

Apply

For #10, ask students to consider how the values in the equation influence their strategy for solving the problem. Ask them to compare to #9 and explain how the numerical values in the questions affect the methods they choose.

In #11, students write an inequality to represent the water level of a river. In part c), they need to recognize that they are being asked to determine the domain for the inequality, similar to what they did in Example 3.

Extend

In #15, students encounter two inequalities on the same grid. While it may be somewhat intimidating initially for students to deal with two inequalities, you can suggest that they think of the question as two separate questions:

- How can you write an inequality that models the fact that the oil is below the upper parabola?
- How can you write an inequality that models the fact that the oil is above the lower parabola?

Create Connections

In #16, students see the type of question that is often posed to find a maximum. In this case, the maximum is not the goal; rather, students seek to guarantee that income stays above a specified level. To help students achieve a solution, ask them how they will define the independent variable in this situation. You might also direct their thinking:

- How can your choice of a variable account for the idea that each increase is $0.50?
- How can you express the new price of a candy-gram as a sum of the old price and the price increase?

Meeting Student Needs

- If students do not have graphing paper, provide them with **Master 2 Centimetre Grid Paper** and/or **Master 3 0.5 Centimetre Grid Paper.**
- Set up stations with various questions from the student resource on task cards. Have student pairs work on the task cards. You might wish to include the answer on the back of each card, allowing students to mark their own questions as they progress through the cards. Allow students to use any method they wish. They may wish to verbally explain the process they used to achieve their answer.
- Students could be asked to find a visual of a famous building or structure that was built in a parabolic shape (similar to the University Bridge in Saskatoon in #11). They could then superimpose a grid onto the visual of the structure to determine the approximate equation that represents the parabola. Examples are the Arc de Triomphe, the arch at the bottom of the Eiffel Tower, and the Gateway Arch in St. Louis.
- Provide **BLM 9–6 Section 9.3 Extra Practice** to students who would benefit from more practice.

ELL

- Ensure that students have the following terms in their vocabulary dictionary: *symmetry* and *curve*. Encourage them to include a verbal description, diagram, and/or example for each term.
- Ensure that students understand the terms *dam*, *exerts*, *arch*, *pier*, *water level*, *advertisements*, *revenue*, *student council*, and *candy-gram*. Use a combination of verbal description, diagrams, and/or examples to help them.
- For #10, some students may not be familiar with ski jumping. You may wish to show them pictures and videos of the sport and/or have another student explain it. You can also refer them to the picture in the student resource. Ensure they understand what the terms *jump* and *hip angle* refer to in this context.
- For #12, 15, and 17, you may wish to have English language learners work with another student to assist them with the language in the question.

Enrichment

- Ask students to create a drawing that shows the path sunlight takes when reflected to a focus by a parabolic mirror. Instruct them to draw lines from the focus to the ends of the mirror. Have students use these data to find inequalities whose intersection of regions shows the path the light takes. Encourage students to comment on the relationship between the convergence of the inequalities and the intensity of the light, both before and after the focal point.

Gifted

- Challenge students to consider the boundaries of inequalities. Specifically, ask them to speculate on how changing the equation of the boundary changes the solution region.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1 to 6, 9, and 10. Students who have no problems with these questions can go on to the remaining questions.	• Students should be successful with #1 to 3 before moving on to the other questions. Should students require a great deal of prompting, assign additional questions, similar in style, and then have students continue with the remaining Practise questions once they have gained understanding. • For #4 and 5, students are asked to sketch the diagram only. By using their previously learned knowledge of transformations and symmetry points, students should be able to complete the questions. As necessary, prompt learners to use technology and then to interpret the graphs. These questions should assist them with #6. • If students require further assistance with #6, coach them through the questions and then assign questions from #7 to confirm students' understanding. • Encourage students to sketch the parabola for #9 on grid paper and label the vertex. Prompt them to verbalize how they can use the vertex to derive the formula. Ask how they could determine a value for a in $y = a(x - p)^2 + q$. Should a be positive or negative? • Encourage students to use technology for #10. Ask them to verbalize an appropriate window setting, and write it on the board for the class. Ensure students recall the meaning of *range*.
Assessment *as* Learning	
Create Connections Have all students complete #16 to 18.	• You may wish to have students work in pairs to discuss their thinking. Also, working in pairs may assist those students who require some level of support for #18. • Students are given an opportunity to demonstrate their understanding in #18 by comparing Units 2 and 4. This comparison would serve as a valuable study tool for end-of-term exams.

Chapter 9 Review

9

***Pre-Calculus 11*, pages 501–503**

Suggested Timing
60–75 min

Materials
- grid paper
- straight edge
- coloured pens, pencils, or markers
- graphing calculator
- computer with graphing software

Blackline Masters
Master 2 Centimetre Grid Paper
Master 3 0.5 Centimetre Grid Paper
BLM 9–4 Section 9.1 Extra Practice
BLM 9–5 Section 9.2 Extra Practice
BLM 9–6 Section 9.3 Extra Practice

Planning Notes

Before students begin the review, suggest that they look over their chapter summary. Students should make sure that they are familiar with the terminology, processes, and concepts of the chapter. Have them identify any ideas that they find difficult, so that they can target those ideas as they work on the review.

Have students who are not confident discuss strategies with you or a classmate. Encourage them to refer to their summary notes, worked examples, and previously completed questions in the student resource.

Have students make a list of questions that they need no help, a little help, and a lot of help with. They can use this list to help them prepare for the practice test.

Meeting Student Needs

- If students do not have graphing paper, provide them with **Master 2 Centimetre Grid Paper** and/or **Master 3 0.5 Centimetre Grid Paper**.
- Ask students to review the list of student learning outcomes. For outcomes students feel proficient in, they need only complete a few questions in the review. For outcomes they do not feel proficient in, students should work through more questions.
- Have students review their summaries of the Key Ideas and clarify any misunderstandings.
- Students could produce a piece of artwork by graphing inequalities on the same sheet of graph paper. Students would need to provide the set of inequalities required for the artwork created. The areas could be shaded with coloured pencils or with string attached to pins found along the curves of the functions.
- Students who require more practice on a particular topic may refer to **BLM 9–4 Section 9.1 Extra Practice**, **BLM 9–5 Section 9.2 Extra Practice**, and **BLM 9–6 Section 9.3 Extra Practice**.

ELL

- Ensure that students have the following term in their vocabulary dictionary: *dimension* (in the context of measurement). Encourage them to include a verbal description, diagram, and/or example.
- Ensure that students understand the terms *budget*, *admission*, *commission*, *salesperson*, *laptop*, *decorative fountain*, *pathway*, *designer*, *storage shed*, *headlights*, *modify*, *cable*, *suspension bridge*, and *bridge deck*. Assist them by providing a combination of verbal description, visuals, and examples.
- For students who are not familiar with what wheat is, use a combination of visuals and description to assist their understanding. You might also refer them to the Did You Know? that accompanies #14.
- To help students with the language in #15, you may want to show them pictures and videos of roller coasters. Alternatively, have another student explain what a roller coaster is and its related vocabulary.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 9 Review The Chapter 9 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against answers in the student resource.	• You may wish to have students work in pairs. • Have students revisit any section that they are having difficulty with prior to working on the chapter test.

9 Chapter 9 Practice Test

***Pre-Calculus 11*, pages 504–505**

Suggested Timing
50–60 min

Materials
- grid paper
- straight edge
- coloured pens, pencils, or markers
- graphing calculator

Blackline Masters
Master 2 Centimetre Grid Paper
Master 3 0.5 Centimetre Grid Paper
BLM 9–7 Chapter 9 Test

Planning Notes

It may be beneficial for some students to complete the practice test as if it were an actual exam; that is, complete it individually without reference to the student resource or their notes. Also, they might complete the test in the amount of time that would be allotted for an actual exam.

Have students start the practice test by writing the question numbers in their notebook. Have them indicate which questions they need no help with, a little help with, and a lot of help with. Have students first complete the questions they know they can do, followed by the questions they know something about. Finally, suggest to students that they do their best on the remaining questions.

When they have completed the practice test, students should be able to identify areas that require more practice and/or study.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1–7, 9.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1	9.1	Example 1	✓ sketch, with or without technology, the graph of a linear inequality
#2	9.1	Example 3	✓ explain when a solid or a dashed line should be used in the solution to an inequality
#3	9.2	Example 1	✓ determine the solution of a quadratic inequality in one variable, using various strategies
#4	9.2	Example 1	✓ explain how to use test points to find the solution to an inequality
#5	9.2	Example 2	✓ determine the solution of a quadratic inequality in one variable, using various strategies
#6	9.1	Examples 1 and 2	✓ sketch, with or without technology, the graph of a linear inequality
#7	9.2	Example 3	✓ determine the solution of a quadratic inequality in one variable, using various strategies
#8	9.3	Example 2	✓ sketch, with or without technology, the graph of a quadratic inequality
#9	9.3	Example 4	✓ solve a problem that involves a quadratic inequality
#10	9.1	Example 4	✓ solve a problem that involves a linear or quadratic inequality
#11	9.1	Example 4	✓ solve a problem that involves a linear or quadratic inequality ✓ sketch, with or without technology, the graph of a linear inequality
#12	9.2	Example 4	✓ solve a problem that involves a quadratic inequality
#13	9.3	Example 3	✓ represent and solve a problem that involves a quadratic inequality in one variable and interpret the solution to a problem that involves a quadratic inequality in one variable

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 9 Self-Assessment Have students use their responses on the practice test and work they completed earlier in the chapter to identify skills or concepts they may need to reinforce.	• Before they begin the practice test, students may wish to review – the mind map they created in #19 of section 9.1 – their comparisons of the methods in #19 of section 9.2 – the chapter summary (index cards, Foldable, or graphic organizer) they created Students can use these items to identify any areas of weakness. • Before the chapter test, coach students in areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 9 Test After students complete the practice test, you may wish to use **BLM 9–7 Chapter 9 Test** as a summative assessment.	• Consider allowing students to use their chapter summary to complete the practice test.

CHAPTER 9

Unit 4 Project

***Pre-Calculus 11*, page 506**

Suggested Timing
60–105 min

Blackline Masters
BLM U4–1 Unit 4 Project Checklist

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☐ Technology (T)
- ☑ Visualization (V)

General Outcome
Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes
RF6 Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.
RF7 Solve problems that involve linear and quadratic inequalities in two variables.

Planning Notes

Content related to the Unit 4 project appears in several places in the chapter.

In section 9.1, Example 4 deals with the idea of using varying amounts of two different materials in designing an object, in this case a table. Check Your Understanding #14 revisits this example and asks students to consider the result of a change in the parameters. This activity parallels the choices students are asked to make when designing an object for their project.

In section 9.2, #12 asks students to consider how changes in the price of materials can affect the design and construction of an object. The graph in #17 is similar to that in the Chapter 9 Task, in which students consider the intersections of a quadratic with a linear graph.

The Project Corner at the end of section 9.2 refers to the costs of technology. While no task is set for students in the Project Corner, you may want to have a brief class discussion to draw out students' ideas and experiences. In particular, you could discuss how prices of new technology are often quite high when the technology is introduced but decrease as time passes. As well, choosing materials for non-financial reasons may be worthy of some debate. Students may want to brainstorm reasons that a more expensive material is sometimes chosen as part of a design.

In section 9.3, Example 3 refers to the use of new materials for a familiar object. The discussion from the Project Corner at the end of section 9.2 should help students see the relevance of this question to the project.

The Chapter 9 Task asks students to analyse the costs associated with new versus old technologies. You may wish to have students work in small groups to discuss the task and answer the questions before they work individually to write their own answers.

Meeting Student Needs

- Suggest that students outline their project using a web or other organizational tool of their choice. If students are presenting their projects, encourage them to consider all types of formats, including visual and oral.

Assessment	Supporting Learning
Assessment *for* Learning	
Unit 4 Project This unit project gives students an opportunity to demonstrate their knowledge of • systems of linear and quadratic equations • linear and quadratic inequalities	• You may wish to have students use the part of **BLM U4–1 Unit 4 Project Checklist** that provides a list of the required components for the Chapter 9 part of the Unit 4 project.

Unit 4 Project Wrap Up

UNIT 4

***Pre-Calculus 11*, page 507**

Suggested Timing
30 min

Blackline Masters
Master 1 Project Rubric
BLM U4–1 Unit 4 Project Checklist
BLM U4–2 Unit 4 Project Rubric

Mathematical Processes
- ☑ Communication (C)
- ☑ Connections (CN)
- ☑ Mental Math and Estimation (ME)
- ☑ Problem Solving (PS)
- ☑ Reasoning (R)
- ☑ Technology (T)
- ☑ Visualization (V)

General Outcome
Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes
RF6 Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.
RF7 Solve problems that involve linear and quadratic inequalities in two variables.

Planning Notes

The Project Wrap-Up for Unit 4 is the display of students' projects for the whole class. The format described in the student resource is a gallery walk. In a gallery walk, students display their project as though the classroom were an art gallery. After all of the work has been posted, students circulate through the room to see all of the projects created by the class. The gallery walk is not an assessment exercise—students do not pass judgment on their classmates' work. In addition, students do not talk about or explain their own work; so, they should understand that the display they set up has to speak for itself.

Ensure students are aware of the project information provided on page 421, the chapter tasks on pages 461 and 506, and the Unit 4 Project Wrap-Up on page 507. If students did not complete each part of the chapter task at the end of Chapters 8 and 9, have them do that now and then use the information to prepare their presentation.

Have students use **BLM U4–1 Unit 4 Project Checklist** to make sure that all parts of their project have been completed. You may wish to have students present their displays to the class. As a class, brainstorm different ways students can do their presentations. You may wish to limit the time each student is allowed to present.

Students do best if they know exactly how they will be evaluated. One way to increase student motivation is to work with the class to create a rubric for the project. You may wish to use the **Master 1 Project Rubric** template and review the general holistic points within the 1–5 scoring levels. Discuss with students how they might achieve each of these levels in the Unit 4 project.

Ask questions such as the following:
- What are the big ideas in the unit? (for example, understanding quadratic inequalities)
- Which of the big ideas are involved in the project?
- What part of the project could you complete or get partially correct to indicate that you have a basic understanding of what was learned in the two chapters? (Should you get a pass mark if you can show that you understand how to solve linear and quadratic equations but not inequalities?)
- What would be on a level 1 project? What might you start on correctly? What could be considered a significant start?
- What would be on a level 5 project? Help students realize that a level 5 project may have a minor error or omission that does not affect the final result.
- Knowing the expectations of levels 1, 3, and 5 projects, what would I expect for a level 4? Help students to understand that this is still an honours level and therefore the work should be reflective of this. However, even an honours project may have a minor error or omission. Discuss the difference between a major conceptual error and a minor miscalculation or omission. Understanding this point will help clarify for students the expectations and differences between a pass and above-average result, and may encourage some students to work toward the highest level. Repeat the process for level 2.

BLM U4–2 Unit 4 Project Rubric models a completed rubric for this project. Note that this is one idea for completing a rubric. Your class rubric may have more detail. Use what your students have developed or this example to ensure that students understand the criteria for an acceptable level, as well as what would warrant either an unacceptable or an honours grading.

Assessment	Supporting Learning
Assessment *of* Learning	
Unit 4 Project This unit project gives students an opportunity to apply and demonstrate their knowledge of the following: • systems of linear-quadratic equations in two variables • systems of quadratic-quadratic equations in two variables • linear inequalities in two variables • quadratic inequalities in two variables Work with students to develop assessment criteria for this project. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing students' work on the Unit 4 project.	• You may wish to have students use **BLM U4–1 Unit 4 Project Checklist**, which provides a list of the required components for the Unit 4 project. • Reviewing the Project Corner boxes in Chapters 8 and 9 will assist students in developing appropriate data for their presentations. • Make sure students recognize what minimum requirements are expected for an acceptable project, as well as the difference between levels 5 and 4. • Clarify the expectations and the scoring with students using **Master 1 Project Rubric** or the rubric you developed as a class. It is recommended to review the scoring rubric at the beginning of the project, as well as intermittently throughout the project to refresh student understanding of the project assessment.

UNIT 4

Cumulative Review and Test

***Pre-Calculus 11*, pages 508–512**

Suggested Timing
60–120 min

Materials
- grid paper
- straight edge
- coloured pens, pencils, or markers

Blackline Masters
Master 2 Centimetre Grid Paper
Master 3 0.5 Centimetre Grid Paper
BLM U4–3 Unit 4 Test

Planning Notes

Have students work independently to complete the review, and then compare their solutions with a classmate's. Alternatively, you may wish to assign the cumulative review to reinforce the concepts, skills, and processes learned so far; if students encounter difficulties, provide an opportunity for them to discuss strategies with other students. Encourage them to refer to their notes, and then to the specific section in the student resource. Once they have determined a suitable strategy, have students add it to their notes. Consider having students make a list of the questions that they found difficult. They can then use the list to help them prepare for the unit test.

Meeting Student Needs

- You may wish to provide students with **Master 2 Centimetre Grid Paper** and/or **Master 3 0.5 Centimetre Grid Paper**.
- Encourage students to draw and label diagrams, when appropriate.

ELL

- For terms that students are unfamiliar with, coach them, or have another student coach them, using a combination of visuals, descriptions, and examples.

Enrichment

- Have students develop new questions that are similar to the questions in the review, test, or chapters, or that are completely original. Students can then exchange questions and answer them for further practice.

Assessment	Supporting Learning
Assessment *for* Learning	
Cumulative Review, Chapters 8 and 9 The cumulative review provides an opportunity for students to assess themselves by completing selected questions pertaining to each chapter and checking their answers against the answers in the back of the student resource.	• Have students review their notes from each chapter to identify items that they had problems with, and to do the questions related to those items. Have students do at least one question that tests skills from each chapter. • Have students revisit any chapter section they are having difficulty with.
Assessment *of* Learning	
Unit 4 Test After students complete the cumulative review, you may wish to use the unit test on pages 510 to 512 as a summative assessment.	• Consider allowing students to use their graphic organizers. • You may wish to have students complete **BLM U4–3 Unit 4 Test**, which provides a sample unit test. You may wish to use it as written or adapt it to meet the needs of your students.