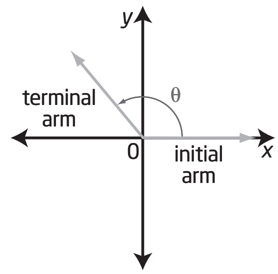
Chapter 2 Trigonometry

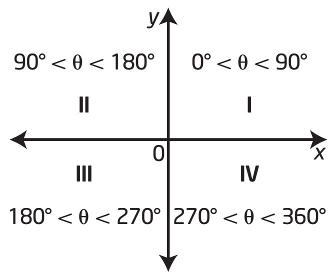
2.1 Angles in Standard Position

KEY IDEAS

- An angle θ is in standard position when
  - the vertex of the angle is located at the origin (0, 0) on a Cartesian plane
  - the initial arm of the angle lies along the positive *x*-axis

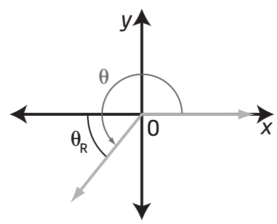


- Measure the angle θ in a counterclockwise direction from the initial arm to the terminal arm. The quadrants of the Cartesian plane are labelled with Roman numerals, also in a counterclockwise direction beginning from the positive *x*-axis.



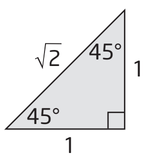
- Angles in standard position have a corresponding acute angle called the reference angle, θ_R. The reference angle is the acute angle formed between the terminal arm and the *x*-axis.

Quadrant III

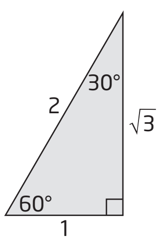


- There are two special right triangles for which you can determine the exact values of the primary trigonometric ratios.

Hint: The smallest angle is always opposite the shortest side.



$\sin 45° = \frac{1}{\sqrt{2}}$ $\cos 45° = \frac{1}{\sqrt{2}}$ $\tan 45° = 1$



$\sin 30° = \frac{1}{2}$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\tan 30° = \frac{1}{\sqrt{3}}$

$\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$ $\tan 60° = \sqrt{3}$

Working Example 1: Sketch an Angle in Standard Position, $0° \leq \theta < 360°$

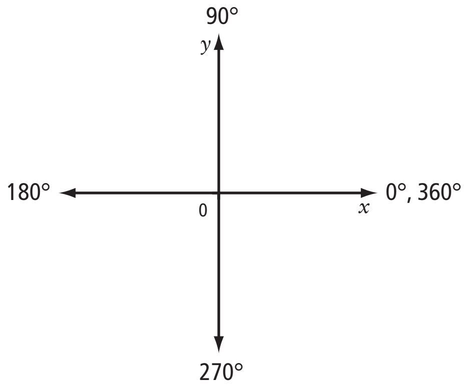
Sketch each angle in standard position. State the quadrant in which the terminal arm lies.

a) 80° **b)** 120° **c)** 266°

Solution

Consider the angle values for the axes: 0°, 90°, 180°, 270°, and 360°. Determine which region of the graph will contain the terminal arm of the angle.

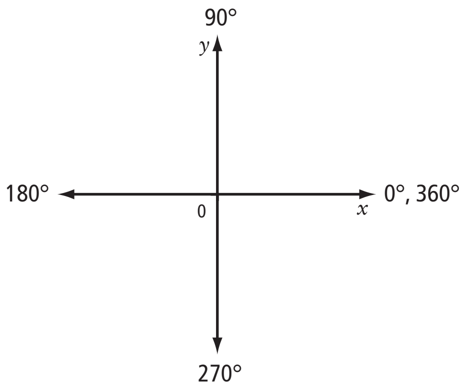
a) For $\theta = 80°$, the angle lies between ________ (angle 1) and ________ (angle 2).



Which axis is the terminal arm of $\theta = 80°$ nearest to?

The terminal arm lies in quadrant ________.

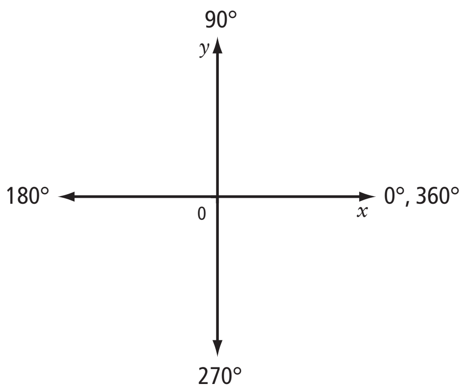
b) For $\theta = 120°$, the angle lies between ________ (angle 1) and ________ (angle 2).



Mentally subdivide each quadrant into halves (45° each) or thirds (30° each) to help you draw a reasonably accurate sketch without a protractor.

The terminal arm lies in quadrant ________.

c) For $\theta = 266°$, the angle lies between ________ (angle 1) and ________ (angle 2).



Show the rotation of the terminal arm using a curved arrow pointing counterclockwise.

The terminal arm lies in quadrant ________.

For more examples, see pages 79–80 of *Pre-Calculus 11*.

Working Example 2: Determine a Reference Angle

Determine the reference angle, θ_R, for each angle θ. Sketch θ in standard position and label the reference angle θ_R.

a) $\theta = 315°$

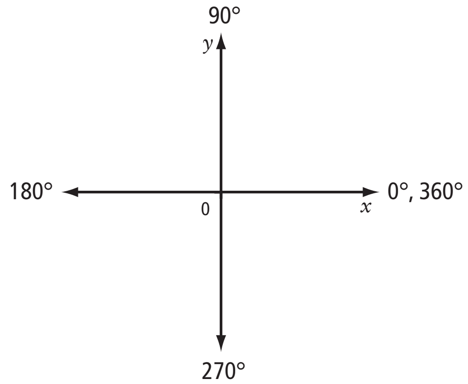
b) $\theta = 201°$

Solution

Sketch the angle θ in standard position.
Next, draw a vertical line from the terminal arm to the nearest part of the *x*-axis to make the reference triangle.
Label the acute angle between the *x*-axis and the terminal arm as θ_R.

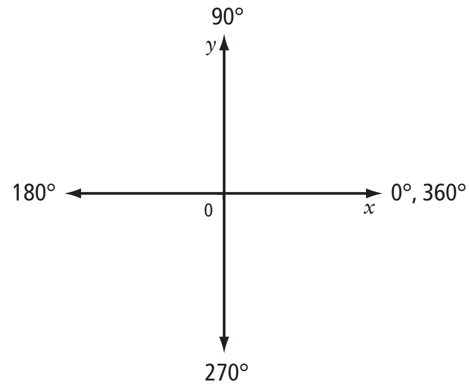
In quadrants I and IV, the reference angle will be formed with the positive *x*-axis (0° or 360°).
In quadrants II and III, the reference angle will be formed with the negative *x*-axis (180°).

a)



The terminal arm is in quadrant ________, so subtract θ from 360° to get θ_R. When $\theta = 315°$, $\theta_R =$ ________.

b)



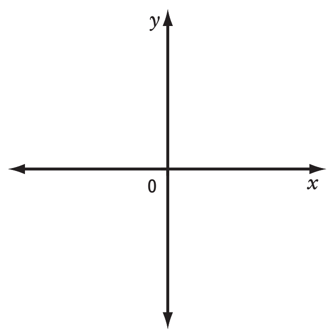
The terminal arm is in quadrant ________, so subtract 180° from θ to get θ_R. When $\theta = 201°$, $\theta_R =$ ________.

Working Example 3: Determine the Angle in Standard Position

Determine the measure of all angles in standard position, $0° < \theta < 360°$, that have a reference angle of 35°.

Solution

Sketch the angle $\theta = 35°$ in standard position.



Reflect the angle $\theta = 35°$ (quadrant ________)

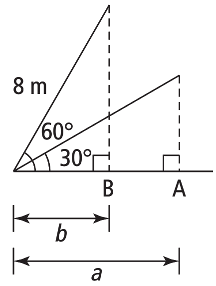
- in the *y*-axis (quadrant ________, $180° - \theta_R$)
- in the *x*-axis (quadrant ________, $360° - \theta_R$)
- in both the *x*-axis and the *y*-axis (quadrant ________, $180° + \theta_R$)

The angles are ________ (quadrant II), ________ (quadrant III), and ________ (quadrant IV).

The example on page 81 of *Pre-Calculus 11* asks the same question in a different way.

Working Example 4: Find an Exact Distance

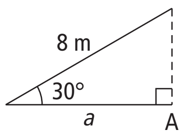
An 8-m boom is used to move a bundle of piping from point A to point B. Determine the exact horizontal displacement of the end of the boom when the operator raises it from 30° to 60°.



Solution

Use your knowledge of the exact trigonometric ratios in 30°-60°-90° triangles to calculate the exact distances *a* and *b*.

For distance *a*:



$$\cos 30° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\frac{\sqrt{3}}{2} = \frac{a}{8}$$

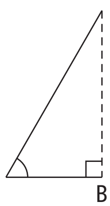
$$2a = 8\sqrt{3}$$

$$a = 4\sqrt{3}$$

Where did the value $\frac{\sqrt{3}}{2}$ come from?

For distance *b*:

Label the diagram with the given information.



Write the appropriate primary trigonometric ratio to solve for *b*.

Now solve for *b*, using exact values.

The total horizontal displacement is $a - b$ or exactly ________ m. (If you use your calculator to evaluate this exact value, you will find an approximate value of 2.93 m).

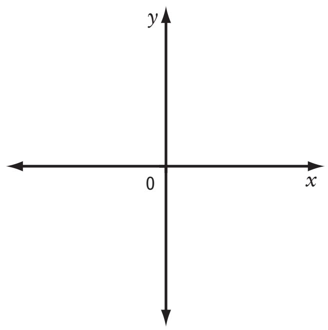
A different situation using exact values can be found on page 82 of *Pre-Calculus 11*.

Check Your Understanding

Practise

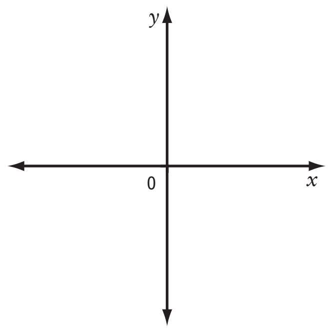
1. Sketch each angle in standard position. State the quadrant in which the terminal arm lies.

a) 100°



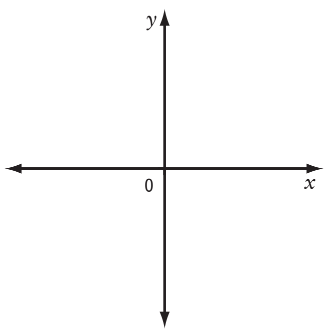
The terminal arm lies in quadrant ________.

b) 295°



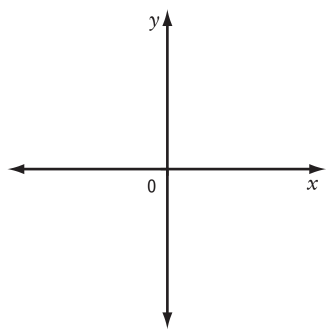
The terminal arm lies in quadrant ________.

c) 25°



The terminal arm lies in quadrant ________.

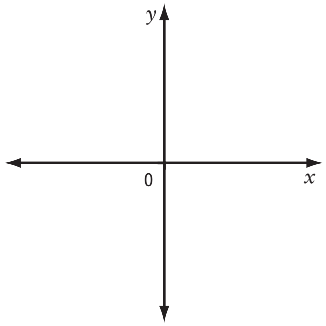
d) 230°



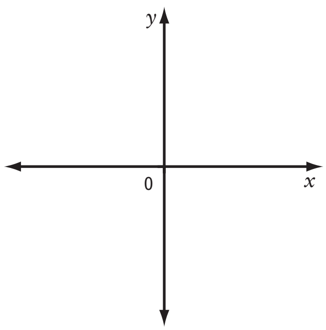
The terminal arm lies in quadrant ________.

2. Determine the reference angle, θ_R, for each angle θ. Sketch θ in standard position and label the reference angle θ_R.

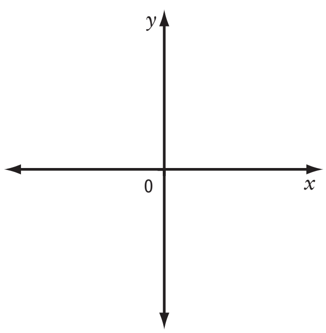
a) $\theta = 355°$ $\theta_R =$ ________



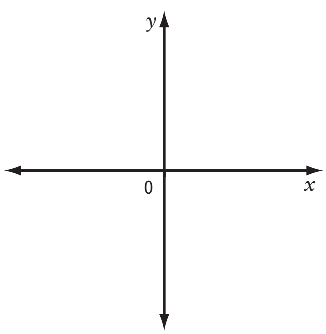
b) $\theta = 135°$ $\theta_R =$ ________



c) $\theta = 260°$ $\theta_R =$ ________

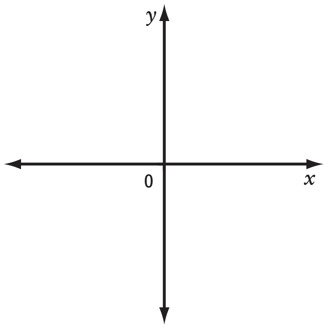


d) $\theta = 70°$ $\theta_R =$ ________



3. Determine the measures of the three other angles in standard position, $0° < \theta < 360°$, that have the given reference angle.

a) 70°

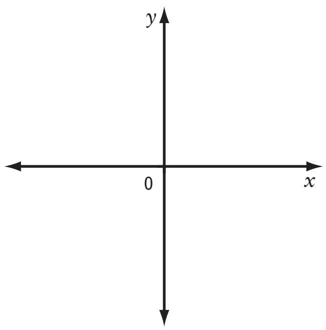


Reflect the angle $\theta = 70°$ (quadrant ________)

- in the y-axis (quadrant ________, $180° - \theta_R$)
- in the x-axis (quadrant ________, $360° - \theta_R$)
- in both the x-axis and the y-axis (quadrant ________, $180° + \theta_R$)

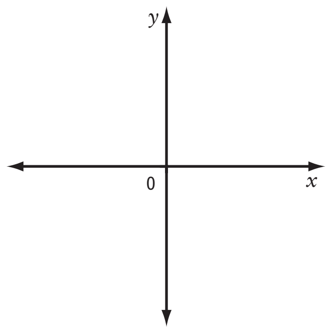
The angles are ________ (quadrant II), ________ (quadrant III), and ________ (quadrant IV).

b) 40°



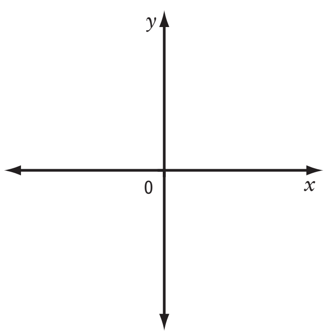
The angles are ________________.

c) 50°



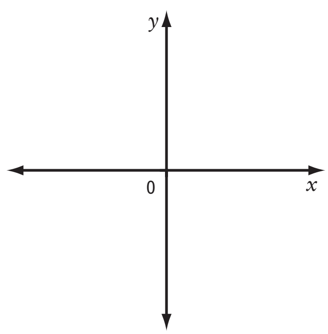
The angles are ________________.

d) 89°



The angles are ________________.

e) 27°

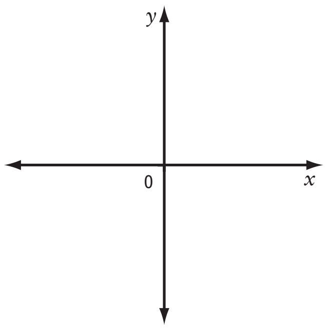


The angles are ________________.

4. Determine the measure of each angle θ in standard position, $0° \leq \theta < 360°$, given its reference angle and the quadrant in which the terminal arm lies.

Hint: This is like the previous question, but working backward.

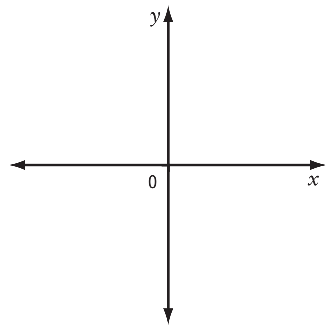
a) $\theta_R = 40°$, in quadrant II



The angle in standard position is

________.

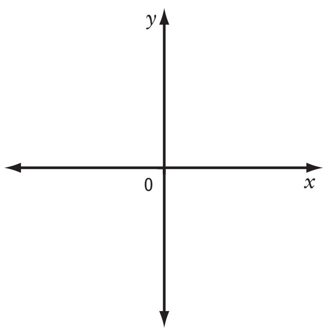
b) $\theta_R = 37°$, in quadrant IV



The angle in standard position is

________.

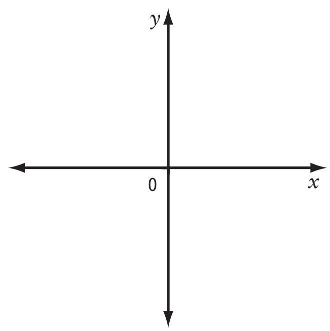
c) $\theta_R = 80°$, in quadrant III



The angle in standard position is

________.

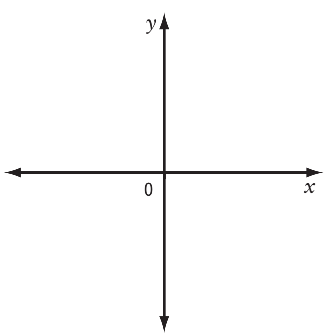
d) $\theta_R = 45°$, in quadrant I



The angle in standard position is

________.

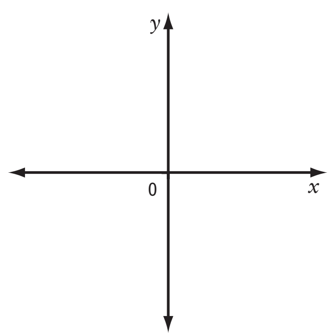
e) $\theta_R = 10°$, in quadrant II



The angle in standard position is

________.

f) $\theta_R = 85°$, in quadrant IV



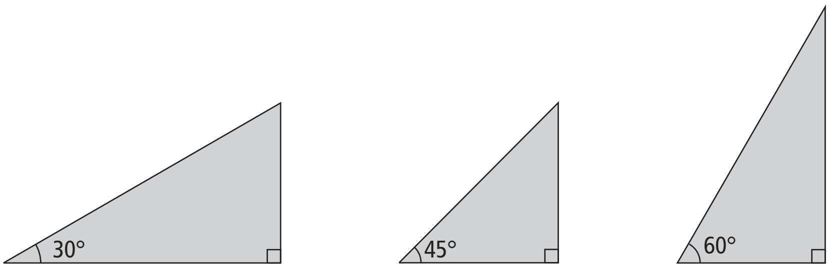
The angle in standard position is

________.

This question should help you complete #7 on page 83 of *Pre-Calculus 11*.

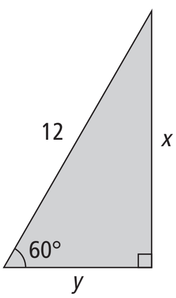
5. Label the sides of the right triangles shown with their exact lengths. The shortest side of each triangle should be 1 unit. Complete the table with the exact values of sine, cosine, and tangent for each angle.

> The phrases "exact value" and "exact length" are clues that you should be thinking in terms of special triangles and square roots, rather than using your calculator.



	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$
sin θ			
cos θ			
tan θ			

6. Find the exact values of the missing side lengths.



Write the primary trigonometric ratio you would use to solve for x.

Now solve for x, using exact values:

To find y, you could use the values y and 12 with the trigonometric ratio ________.

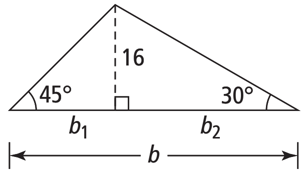
Or, you could use the values of x and y with the trigonometric ratio ________.

Or, you could use the Pythagorean Theorem.

Solve for y:

Apply

7. Find the exact area of the triangle shown.



Formula for area of a triangle: $A = \frac{1}{2}bh$

From the diagram, $h =$ ________.

Calculate b. One strategy is to divide the big triangle into two parts, each consisting of a right triangle. Which primary trigonometric ratio (sine, cosine, tangent) is used?

In the 45°-45°-90° triangle, solve for b_1:

In the 30°-60°-90° triangle, solve for b_2:

The total base of the triangle is $b = b_1 + b_2$ or ________.

Now, find the area.

$A = \frac{1}{2}bh$

$A =$

The exact area of the triangle is ________ units.

8. Find the exact area of an equilateral triangle with a height of 9 units.

Diagram:

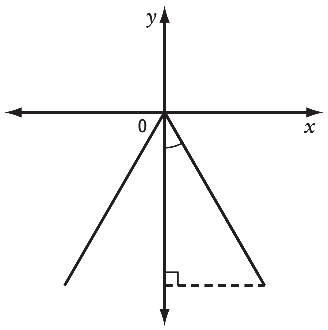
What information is known? What is unknown? Which primary trigonometric ratio will you use?

The total base of the triangle is:

The area of the triangle is:

9. A grandfather clock has a pendulum that is 1.40 m long and swings ±30° from centre. What are the minimum inner dimensions of the cabinet inside the clock that will accommodate the pendulum? Give exact values.

Label the diagram to model the information.



Let *a* represent the horizontal distance the pendulum travels from the centre to the end of its swing. Add this information to your diagram.

Write the primary trigonometric ratio you would use to solve for *a*:

Solve for *a*:

The total horizontal distance the pendulum travels is ________ m.

The pendulum is ________ m long, so the vertical dimension must be at least ________ m.

The cabinet must be at least ________ m wide and ________ m high.

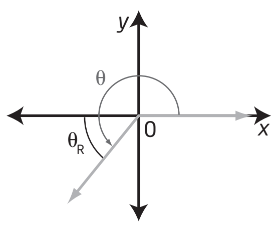
This question is similar to Example 4 on page 82 of *Pre-Calculus 11*.

10. An 8-m boom is used to move a bundle of piping from point A to point B. Determine the exact vertical displacement of the end of the boom when the operator raises it from 30° to 60°.

Hint: Use two triangles.

Connect

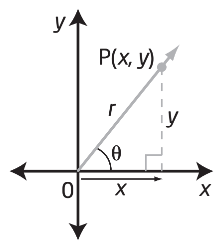
11. Complete the table for angles in standard position, $0° < \theta < 360°$.

Quadrant	Value of Angle θ	How to Calculate θ_R	Sketch
I	$0° < \theta < 90°$		
II	______ $< \theta <$ ______		
	______ $< \theta <$ ______		
	______ $< \theta <$ ______	$\theta_R = 360° - \theta$	

KEY IDEAS

- Recall that you measure an angle θ in a counterclockwise direction from the initial arm (on the positive x-axis) to the terminal arm. The quadrants of the Cartesian plane are labelled with Roman numerals, also in a counterclockwise direction.
- If the terminal arm lies on an axis, the angle is called a quadrantal angle (it separates the quadrants).
  For example, 0°, 90°, 180°, 270°, and 360°.
- If point P(x, y) lies on the terminal arm of an angle θ in standard position, the primary trigonometric ratios are

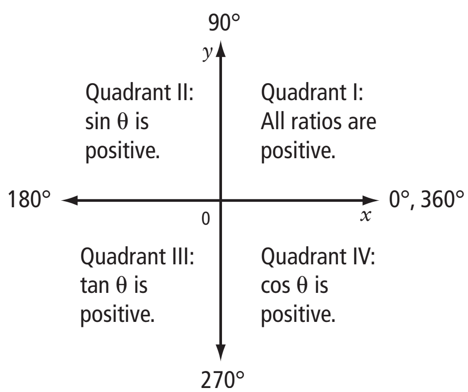
  $\sin \theta = \frac{y}{r}$ $\qquad \cos \theta = \frac{x}{r}$ $\qquad \tan \theta = \frac{y}{x}$

  where the distance from point P(x, y) to the origin (0, 0) is represented by the variable r. Then, by the Pythagorean Theorem,

  $$r = \sqrt{x^2 + y^2}$$

- Since r is a distance, it is always a positive number.
- sin θ is positive in quadrants I and II.
  sin θ = 0 on the x-axis and ±1 on the y-axis.
- cos θ is positive in quadrants I and IV.
  cos θ = ±1 on the x-axis and 0 on the y-axis.
- tan θ is positive in quadrants I and III.
  tan θ = 0 on the x-axis and is undefined on the y-axis.

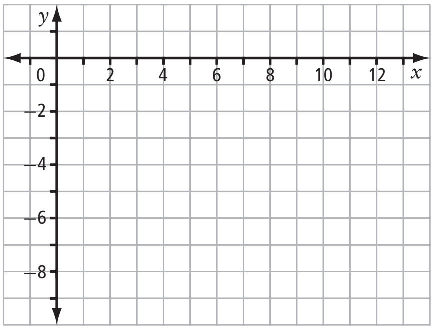


Working Example 1: Write Trigonometric Ratios for Angles in Any Quadrant

The point P(12, −9) lies on the terminal arm of an angle θ in standard position. Determine the exact trigonometric ratios for sin θ, cos θ, and tan θ.

Solution

Sketch the point P(12, −9) on the Cartesian plane.
Draw a line segment from the origin to P to represent the terminal arm of the angle.
Label the angle θ and the reference angle θ_R.



To draw the right triangle containing θ_R, make a vertical line from P to the nearest part of the *x*-axis.

From the coordinates of point P, you know that $x =$ ________ and $y =$ ________.

Calculate *r* using the Pythagorean Theorem.

$r = \sqrt{x^2 + y^2}$

Substitute the values of *x*, *y*, and *r* into the formulas to calculate the three ratios.

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

The wording "exact trigonometric ratios" means to leave your answer as a fraction in lowest terms.

$\sin \theta =$ ________ $\cos \theta =$ ________ $\tan \theta =$ ________

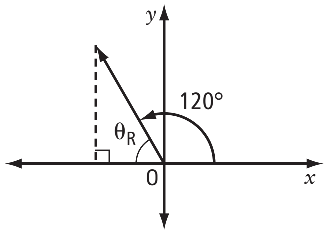
For another example, see page 91 of *Pre-Calculus 11*.

Working Example 2: Determine the Exact Value of Trigonometric Ratios

Determine the exact values of the sine, cosine, and tangent ratios for $\theta = 120°$.

Solution

Sketch the angle in standard position. Draw the reference angle, θ_R, and determine its measure.



The reference angle θ_R = ________.

The trigonometric ratios for the reference angle are

$\sin \theta_R =$

$\cos \theta_R =$

$\tan \theta_R =$

The reference angle corresponds to one of the special triangles you learned about in Section 2.1. Use exact ratios whenever possible.

In quadrant II, x is negative and y is positive (r is always positive). Determine the sign of each ratio.

$\sin \theta = \frac{y}{r}$ $\quad$ $\cos \theta = \frac{x}{r}$ $\quad$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{+}{+}$ $\quad$ $\cos \theta = \frac{-}{+}$ $\quad$ $\tan \theta = \frac{\square}{\square}$

$\sin \theta = +$ $\quad$ $\cos \theta = -$ $\quad$ $\tan \theta = \square$

Therefore, ________ is positive, and $\cos \theta$ and $\tan \theta$ are negative for angles in quadrant II.

$\sin 120° = +\sin 60°$ $\quad$ $\cos 120° = -\cos 60°$ $\quad$ $\tan 120° =$ ________

$\sin 120° =$ ________ $\quad$ $\cos 120° =$ ________ $\quad$ $\tan 120° =$ ________

Working Example 3: Determine Trigonometric Ratios

Suppose θ is an angle in standard position with terminal arm in quadrant IV and $\cos \theta = \frac{\sqrt{33}}{7}$. Determine the exact values of the other two trigonometric ratios.

Solution

Since $\cos \theta = \frac{x}{r}$, $x =$ ________ and $r =$ ________.

In quadrant IV, is x positive or negative?

Solve for y using the Pythagorean Theorem.

$x^2 + y^2 = r^2$

In quadrant IV, is y positive or negative? Select the positive or negative root accordingly.

The other two trigonometric ratios are $\sin \theta = \frac{y}{r} = \frac{\square}{\square}$ and $\tan \theta = \frac{y}{x} = \frac{\square}{\square}$.

Working Example 4: Solve for an Angle Given Its Sine, Cosine, or Tangent Value

Solve for θ.

a) $\tan \theta = -0.9004$, $0° \leq \theta < 360°$

b) $\cos \theta = -\frac{1}{\sqrt{2}}$, $0° \leq \theta < 360°$

Solution

a) Step 1: Determine which quadrants the solutions will be in by looking at the sign (+ or −) of the given ratio.

tan θ is positive in quadrants ________ and ________.

tan θ is negative in quadrants ________ and ________.

So, the solutions are in quadrants ________ and ________.

Step 2: Solve for the reference angle.

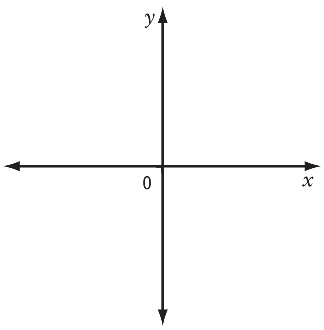
The reference angle is in quadrant I, where all three trigonometric ratios are positive.

$\tan \theta_R = +0.9004$

$\theta_R = \tan^{-1}(0.9004)$

$\theta_R \approx$ ________ to the nearest tenth of a degree

Step 3: Sketch the reference angle in the appropriate quadrants. Use the diagram to determine the measures of the two related angles in standard position.



Draw an angle in standard position in each quadrant where tan θ is negative.

The first angle is in quadrant ________.

$\theta = 180° - \theta_R$

θ = ________

The second angle is in quadrant ________.

$\theta = 360° - \theta_R$

θ = ________

The solutions to the equation $\tan \theta = -0.9004$, $0° \leq \theta < 360°$, are θ = ________ and θ = ________.

b) $\cos \theta = -\frac{1}{\sqrt{2}}$, $0° \leq \theta < 360°$

Step 1: Determine which quadrants the solutions will be in by looking at the sign (+ or −) of the given ratio.

cos θ is positive in quadrants ________ and ________.

cos θ is negative in quadrants ________ and ________.

So, the solutions are in quadrants ________ and ________.

Step 2: Solve for the reference angle.

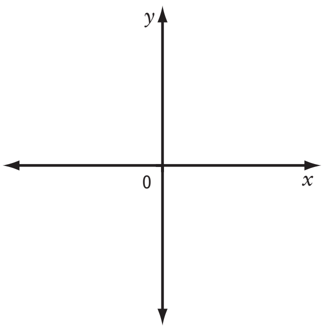
$\cos \theta_R = +\frac{1}{\sqrt{2}}$

$\theta_R = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\theta_R =$ ________

The values 1 and $\sqrt{2}$ correspond to the 45°-45°-90° special triangle. You can find this answer without using a calculator.

Step 3: Sketch the reference angle in the appropriate quadrants. Use the diagram to determine the measure of the related angle in standard position.



The first angle is in quadrant ________.

$\theta = 180° - \theta_R$

$\theta =$ ________

The second angle is in quadrant ________.

$\theta = 180° + \theta_R$

$\theta =$ ________

The solutions to the equation $\cos \theta = -\frac{1}{\sqrt{2}}$, $0° \leq \theta < 360°$, are $\theta =$ ________ and $\theta =$ ________.

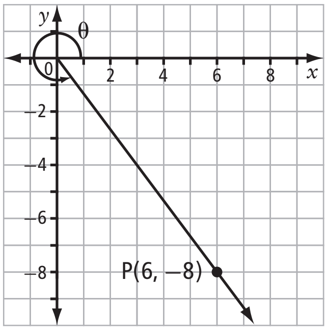
For more examples, see pages 94–95 of *Pre-Calculus 11*.

Check Your Understanding

Practise

1. The coordinates of a point P on the terminal arm of an angle θ are shown.

a) Determine the values of *x, y,* and *r*.



$x =$ ________

$y =$ ________

$r = \sqrt{x^2 + y^2}$

$r =$ ________

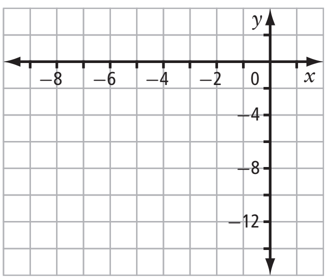
Leave your answers as fractions in lowest terms.

b) Write the exact trigonometric ratios sin θ, cos θ, and tan θ for the angle.

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\sin \theta =$ ________ $\cos \theta =$ ________ $\tan \theta =$ ________

2. Sketch an angle θ in standard position so that the terminal arm passes through P(−8, −15). Then, find the exact values of sin θ, cos θ, and tan θ for the angle.



$x =$ ________

$y =$ ________

$r = \sqrt{x^2 + y^2}$

$r =$ ________

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\sin \theta =$ ________ $\cos \theta =$ ________ $\tan \theta =$ ________

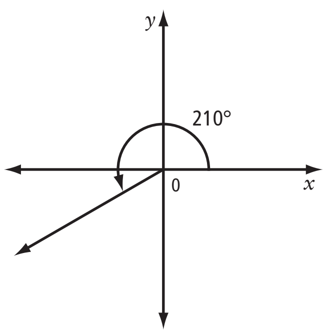
3. Determine the exact values of sin θ, cos θ, and tan θ if the terminal arm of an angle in standard position passes through the point P(1, 0).

x = ________ y = ________ r = ________

sin θ = ________ cos θ = ________ tan θ = ________

4. Determine the exact values of the sine, cosine, and tangent ratios for each angle.

a)



Draw in the special triangle that represents the reference angle.

The reference angle θ_R = ________.

The trigonometric ratios for the reference angle are

$\sin \theta_R =$ $\cos \theta_R =$ $\tan \theta_R =$

In quadrant ________, x and y are both negative (r is always positive). Determine the sign of each ratio.

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\sin \theta = \frac{-}{+}$ $\cos \theta = \frac{\square}{\square}$ $\tan \theta = \frac{\square}{\square}$

$\sin \theta = -$ $\cos \theta = \square$ $\tan \theta = \square$

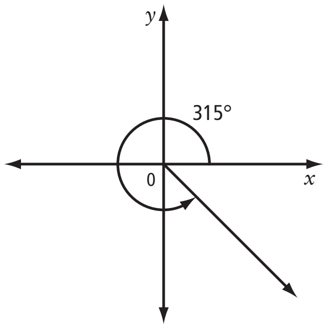
Therefore, ________ is positive, and ________ and ________ are negative for angles in quadrant III.

sin 210° = −sin 30° cos 210° = ________ tan 210° = ________

sin 210° = ________ cos 210° = ________ tan 210° = ________

Exact values for the trigonometric ratios in special right triangles can be found on page 79 of *Pre-Calculus 11*.

b)



The reference angle $\theta_R =$ ________.

The trigonometric ratios for the reference angle are

$\sin \theta_R =$ $\cos \theta_R =$ $\tan \theta_R =$

Determine the sign of each ratio in quadrant IV.

$\sin \theta = \frac{y}{r}$ $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$

$\sin \theta = -$ $\cos \theta = \square$ $\tan \theta = \square$

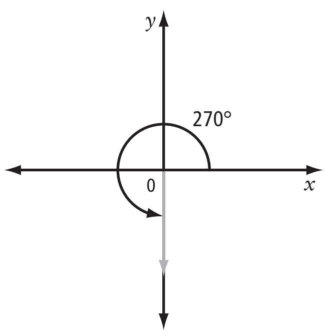
Therefore, ________ is positive, and ________ and ________ are negative for angles in quadrant IV.

$\sin 315° = -\sin \theta_R$ $\cos 315° =$ ________ $\tan 315° =$ ________

$\sin 315° = -\sin$ ________

$\sin 315° =$ ________ $\cos 315° =$ ________ $\tan 315° =$ ________

c)



$x = 0$

$y =$ ________

$r =$ ________

Choose any point P(0, *y*) on the terminal arm to calculate the three ratios.

For the quadrantal angle 270°, ________ = 0 and ________ is negative

(*r* is always ________________).

$\sin 270° = \frac{y}{r}$ $\cos 270° = \frac{x}{r}$ $\tan 270° =$ ________

$\sin 270° =$ ________ $\cos 270° =$ ________ $\tan 270° =$ ________

5. Suppose θ is an angle in standard position with terminal arm in quadrant III and $\tan \theta = \frac{6}{5}$. Determine the exact values of the other two primary trigonometric ratios.

In quadrant III, x is ________ and y is ________ (r is always positive).

Since $\tan \theta = \frac{y}{x}$, the known information is $x =$ ________ and $y =$ ________.

Solve for the unknown value using the Pythagorean Theorem.

$r = \sqrt{x^2 + y^2}$

The other two trigonometric ratios are $\sin \theta =$ ________ and $\cos \theta =$ ________.

6. Suppose θ is an angle in standard position with terminal arm in quadrant II and $\cos \theta = -\frac{7}{12}$. Determine the exact values of the other two primary trigonometric ratios.

In quadrant II, x is ________ and y is ________ (r is always ________________).

Since $\cos \theta = \frac{\square}{\square}$, the known information is ________ and ________.

Solve for the unknown value using the Pythagorean Theorem.

The other two trigonometric ratios are $\sin \theta =$ ________ and $\tan \theta =$ ________.

7. Suppose θ is an angle in standard position with terminal arm in quadrant IV and $\cos \theta = \frac{40}{41}$. Determine the exact values of the other two primary trigonometric ratios.

In quadrant IV, x is ________ and y is ________ (r is always ________________).

Since $\cos \theta = \frac{\square}{\square}$, the known information is ________ and ________.

Solve for the unknown value using the Pythagorean Theorem.

The other two trigonometric ratios are $\sin \theta =$ ________ and $\tan \theta =$ ________.

These questions should help you complete #8 on page 96 of *Pre-Calculus 11*.

Apply

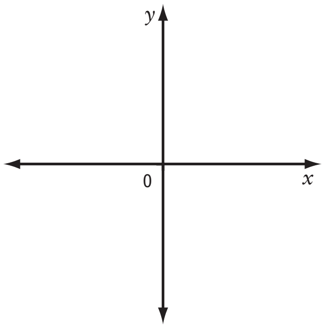
8. Solve for θ. Round your answer to the nearest degree.

a) $\cos \theta = 0.8829$, $0° \leq \theta < 360°$

$\cos \theta$ is positive in quadrants ________ and ________.

The reference angle is $\theta_R = \cos^{-1}(0.8829)$ or ________.

Sketch the reference angle in the appropriate quadrants.



To calculate the values of the two angles, recall the following:

- In quadrant II, $\theta = 180° - \theta_R$.
- In quadrant III, $\theta = 180° + \theta_R$.
- In quadrant IV, $\theta = 360° - \theta_R$.

The solutions to the equation $\cos \theta = 0.8829$, $0° \leq \theta < 360°$, are $\theta =$ ________ and

$\theta =$ ________.

b) $\tan \theta = -1.9626$, $0° \leq \theta < 360°$

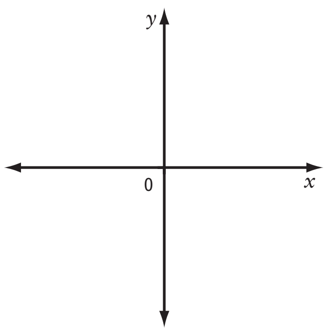
$\tan \theta$ is negative in quadrants ________ and ________.

The reference angle is in quadrant I, where sine, cosine, and tangent are all positive.

The reference angle is $\theta_R = \tan^{-1}(1.9626)$

or ________.

Sketch the reference angle in the appropriate quadrants. Then, calculate the values of the two angles.



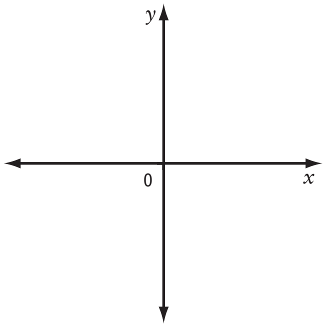
The solutions to the equation $\tan \theta = -1.9626$, $0° \leq \theta < 360°$, are $\theta =$ ________

and $\theta =$ ________.

Question 9 on page 97 of *Pre-Calculus 11* is similar but involves special triangles.

9. Point P(−4, −5) is on the terminal arm of an angle θ in standard position.

a) Sketch the angle.



From the coordinates of the point P, you know that

$x =$ ________ and $y =$ ________.

Calculate tan θ.

$\tan \theta = \frac{y}{x}$

$\tan \theta =$

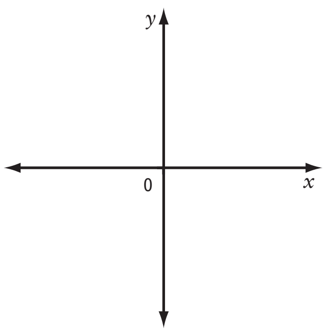
b) What is the measure of the reference angle, to the nearest degree?

The reference angle is $\theta_R = \tan^{-1}($________$)$ or ________.

c) What is the measure of θ, to the nearest degree?

10. Point P(12, –3) is on the terminal arm of an angle θ in standard position.

a) Sketch the angle.



From the coordinates of the point P, you know that

$x =$ ________ and $y =$ ________.

$\tan \theta = \frac{y}{x}$

$\tan \theta =$

b) What is the measure of the reference angle, to the nearest degree?

The reference angle is $\theta_R = \tan^{-1}($________$)$ or ________.

c) What is the measure of θ, to the nearest degree?

Question 27 on page 99 of *Pre-Calculus 11* asks you to explain the process used above.

Connect

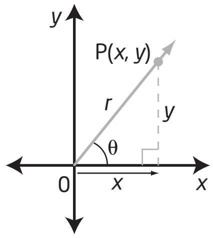
11. Complete the table, using the following symbols where appropriate.

+ (positive) − (negative) 0 1 −1 undefined

Trigonometric Ratio	$\sin \theta = \frac{y}{r}$	$\cos \theta = \frac{x}{r}$	$\tan \theta = \frac{y}{x}$
0°	0		
Quadrant I	+		
90°	1		undefined
Quadrant II			
180°			
Quadrant III			
270°			
Quadrant IV			
360°	0		

Chapter 2 Skills Organizer A

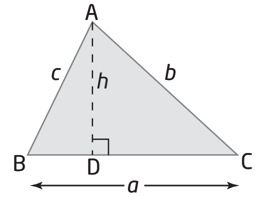
Complete the table for each point P on the terminal arm of an angle θ in standard position.

	Quadrant	Sketch	sin θ	cos θ	tan θ
P(x, y)	I				
P(0, 1)	N/A		$\sin\theta = 1$		
P($-x$, y)			$\sin\theta = \frac{y}{r}$, $\sin\theta > 0$	$\cos\theta = \frac{-x}{r}$, $\cos\theta < 0$	$\tan\theta = \frac{y}{-x}$, $\tan\theta < 0$
P(−1, 0)	N/A				
P($-x$, $-y$)					
P(0, −1)			$\sin\theta = -1$		
P(x, $-y$)					
P(1, 0)					

2.3 The Sine Law

KEY IDEAS

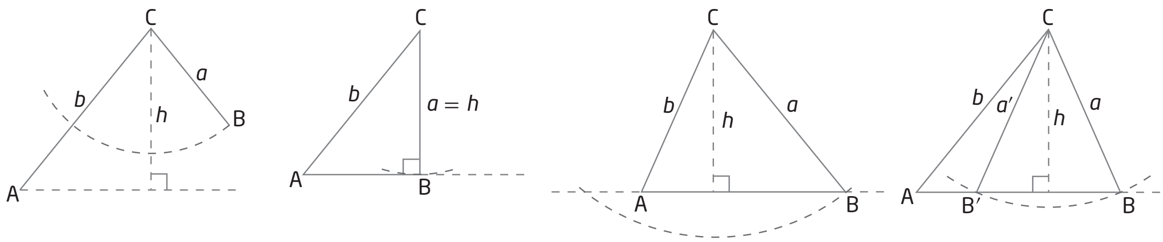
- A triangle that does not contain a right angle is an oblique triangle. You can use the sine law to solve problems involving oblique triangles.



- To find a side length using the sine law, you must know the measure of the angle opposite that side, plus one other angle–opposite side pair (ASA).

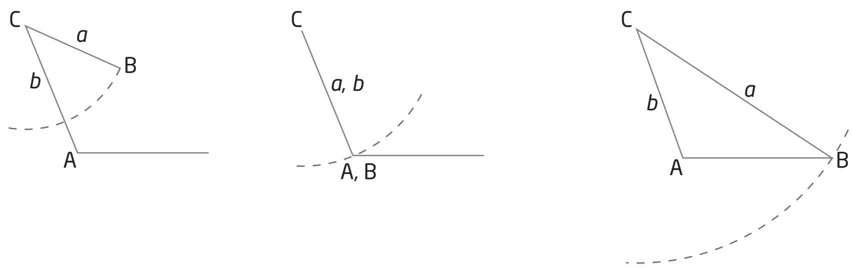
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- To find an angle using the sine law, you must know the length of the side opposite that angle, plus one other angle–opposite side pair (SSA). Note that there are two solutions for any given $\sin \theta$, $0° \leq \theta < 180°$. One solution is an acute angle (in quadrant I), and the other solution is the obtuse angle (in quadrant II) with the same θ_R. This is known as the ambiguous case of the sine law.
- For an acute ∠A and $a < b$, calculate the altitude $h = b \sin A$ to determine if there are 0, 1, or 2 triangles.



No triangles when…	One triangle when…		Two triangles when…
$a < h$ $a < b \sin A$	$a = h$ $a = b \sin A$ $\angle B = 90°$	$a \geq b$ ∠B is acute	$h < a < b$ $b \sin A < a < b$

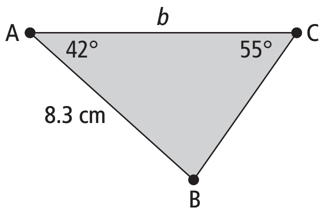
- For an obtuse ∠A, compare the lengths of a and b to determine if there are 0 or 1 triangles.



No triangles when…	One triangle when…
$a < b$ or $a = b$	$a > b$ ∠B is acute

Working Example 1: Determine an Unknown Side Length

Determine the length of side b, to the nearest millimetre.



Solution

This is an oblique triangle, so you can use the sine law. To solve for side b, you need to know its opposite angle, ∠B.

The angles in a triangle sum to ________°, so

$\angle B = 180° - (42° + 55°)$

$\angle B =$ ________

You also know one other angle–opposite side pair: ∠C = ________ and side c = ________ cm. According to the sine law,

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin \square} = \frac{\mathbf{8.3}}{\sin \mathbf{55°}}$$

$$b = \sin \square \left(\frac{8.3}{\sin 55°}\right)$$

$b =$ ________…

Side b is ________ cm.

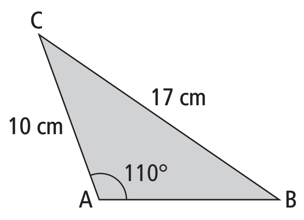
Rearrange the formula before you enter the numbers into your calculator, and complete the calculations all in one step.

Is your calculator in "degree" mode?

"To the nearest millimetre" means round your answer to the nearest 0.1 cm.

Working Example 2: Determine an Unknown Angle

Determine the measure of ∠B, to the nearest degree.



Solution

Use the sine law.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \square°}{\mathbf{17}} = \frac{\sin B}{\mathbf{10}}$$

$$10\left(\frac{\sin \square°}{17}\right) = \sin B$$

$$\sin^{-1}\left(\frac{10 \sin \square°}{17}\right) = \angle B$$

$\angle B =$ ________

Since ∠A is obtuse and $a > b$, there is one triangle with an acute ∠B.

Do the calculations all in one step on your calculator. Do not round off until the final answer.

Working Example 3: Determine the Number of Triangles

Determine the number of triangles that will satisfy the following conditions.

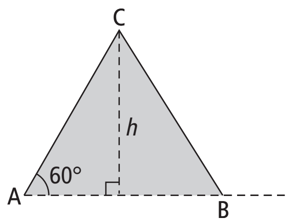
a) In △ABC, ∠A = 60°, a = 14 units, and b = 15 units.

b) In △JKL, ∠J = 42°, j = 6.9 cm, and k = 10.3 cm.

Solution

a) Sketch a possible diagram.

Hint: Place the known angle at the lower left corner of your diagram and the angle to be calculated at the lower right. Then, your altitude, h, will always be a vertical line.



In the diagram, label sides a (opposite ∠A) and b (opposite ∠B) with their lengths.

Since ∠A is acute and $a < b$, calculate the altitude.

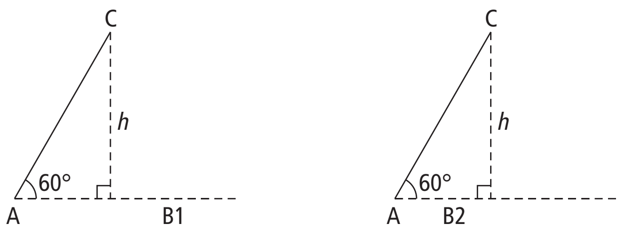
Make your sketch as close to scale as possible, especially in the measure of ∠A.

$h = b \sin A$

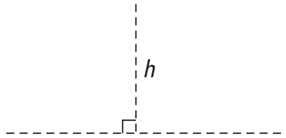
$h =$

In order from smallest to largest, ________ (h) < ________ (a) < ________ (b).

Therefore, two triangles exist that satisfy the given conditions: one with acute ∠B and one with obtuse ∠B. Sketch the two possible triangles.



b) Sketch a possible diagram. Place the known ∠J at the lower left corner of your diagram.



Known information:

∠J = ________°, j = 6.9 cm, k = ________ cm

∠J is acute and $j < k$, so calculate the altitude.

$h = k \sin J$

$h =$

The altitude is the length of the hypotenuse multiplied by the sine of the known angle.

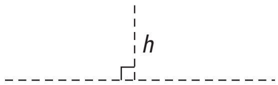
Since $h = j$, there is/are ______________ triangle(s). ∠K is a right angle.
(*zero, one,* or *two*)

Working Example 4: Use the Sine Law in an Ambiguous Case

In △ABC, $a = 3$, $b = 6$, and ∠A = 70°. Determine the measure of ∠B, to the nearest degree.

Solution

Sketch a possible diagram. Place the known ∠A at the lower left corner of your diagram.



Known information:

∠A = ________°, a = ________, b = ________

∠A is acute and $a < b$, so calculate the altitude.

$h = b \sin A$

$h =$

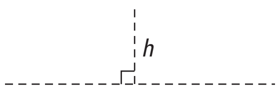
Since $a < h$, there is/are ________________ triangle(s). You cannot calculate angle ∠B.
(*zero, one,* or *two*)

Working Example 5: Solve a Triangle

In △ABC, $a = 4.8$ cm, $b = 6.4$ cm, and ∠A = 18°. Solve the triangle. Round angles to the nearest degree and sides to the nearest tenth of a centimetre.

Solution

Sketch a possible diagram. Place the known ∠A at the lower left corner of your diagram.



Known information:

∠A = ________°, a = ________ cm, b = ________ cm

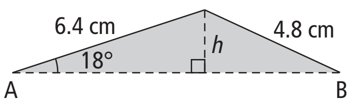
∠A is acute and $a < b$, so calculate the altitude.

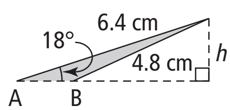
$h = b \sin A$

$h =$

Since $h < a < b$, there is/are ________________ triangle(s).
(*zero, one,* or *two*)

Solving a triangle means to find the measures of all angles and all sides. Since there are two triangles that match the situation given, you will complete two full sets of calculations.





Triangle 1: ∠B is acute

Calculate ∠B using the sine law:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \mathbf{18°}}{\mathbf{4.8}} = \frac{\sin B}{\mathbf{6.4}}$$

$$6.4\left(\frac{\sin 18°}{4.8}\right) = \sin B$$

$$\angle B = \sin^{-1}\left(\frac{6.4 \sin 18°}{4.8}\right)$$

∠B = ________°

Find ∠C:

∠C = 180° − (18° + 24°)

∠C = ________°

Calculate side c using the sine law:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{\mathbf{4.8}}{\sin \mathbf{18°}} = \frac{c}{\sin \mathbf{138°}}$$

$$c = \sin 138°\left(\frac{4.8}{\sin 18°}\right)$$

c = ________

Triangle 2: ∠B is obtuse

To calculate ∠B, find the angle in quadrant II with $\theta_R = 24°$:

∠B = 180° − 24°

∠B = ________°

Find ∠C:

Calculate side c using the sine law:

c = ________

The two possible triangles are as follows:

- acute △ABC: ∠A = ________°, ∠B = ________°, ∠C = ________°, a = ________ cm, b = ________ cm, and c = ________ cm
- obtuse △ABC: ∠A = ________°, ∠B = ________°, ∠C = ________°, a = ________ cm, b = ________ cm, and c = ________ cm

See a similar example on page 106–107 of *Pre-Calculus 11*.

Check Your Understanding

Practise

1. Solve for the unknown side (to one decimal place) or angle (to the nearest degree) in each.

a) $\frac{a}{\sin 48°} = \frac{20}{\sin 75°}$

$$a = \sin 48° \left(\frac{20}{\sin 75°}\right)$$

$$a =$$

b) $\frac{x}{\sin 125°} = \frac{15}{\sin 30°}$

c) $\frac{\sin B}{10} = \frac{\sin 45°}{16}$

$$\sin B = 10\left(\frac{\sin 45°}{16}\right)$$

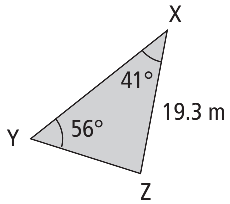
$$\angle B = \sin^{-1}\left(\boxed{}\right)$$

$$\angle B =$$

d) $\frac{\sin \theta}{5} = \frac{\sin 110°}{25}$

2. Determine the length of side x, to one decimal place, in each $\triangle XYZ$.

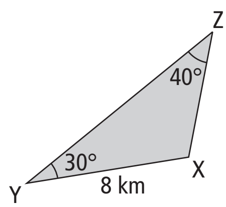
a)



$$\frac{x}{\sin X} = \frac{y}{\sin Y}$$

The length of side x is ________ m.

b)

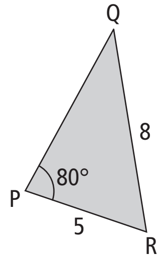


Find ∠X. Then, set up the sine law with pairs of angles and their opposite sides, and then solve.

The length of side x is ________ km.

3. Determine the measure of $\angle$Q, to the nearest degree, in each $\triangle$PQR.

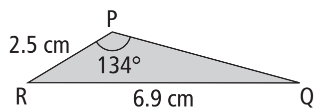
a)



$$\frac{\sin Q}{q} = \frac{\sin P}{p}$$

$\angle$Q measure ________°.

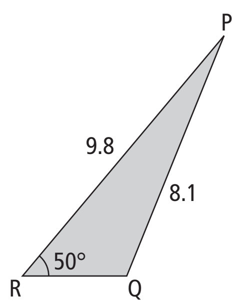
b)



$\angle$Q measure ________°.

Set up the sine law with pairs of angles and their opposite sides, and then solve.

c)

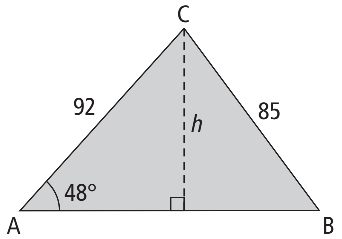


Your calculator gives you the reference angle (in quadrant I) by default. Calculate the actual obtuse angle (in quadrant II) at vertex Q.

$\angle$Q measure ________°.

4. Calculate the altitude, h, for each triangle, to two decimal places.

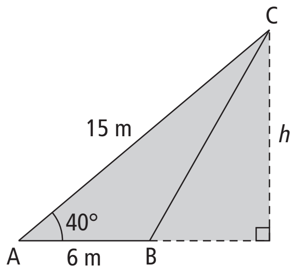
a)



$h = b \sin A$

$h =$

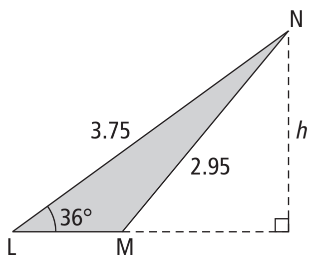
b)



$h =$ ________ $\sin A$

$h =$

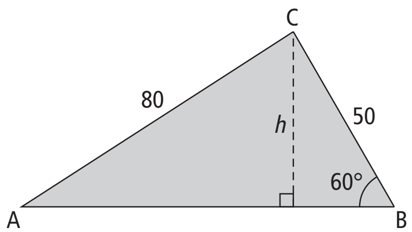
c)



$h =$ ________ $\sin$ ________

$h =$

d)



$h =$

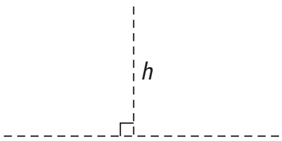
$h =$

Caution! For this triangle, the known angle is ∠B, not ∠A.

5. Determine how many triangles satisfy the following conditions.

a) $\angle A = 65°$, $a = 4.0$ cm, and $b = 4.4$ cm

Sketch a possible diagram. Place the known ∠A at the lower left corner of your diagram.



$h = b \sin A$

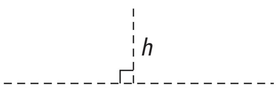
$h =$

∠A is acute and $a < b$, so calculate the altitude.

Since $a = h$, there is/are ________________ triangle(s).
(*zero, one,* or *two*)

∠B is a(n) ________________ angle.
(*acute*, *obtuse*, or *right*)

b) $\angle A = 20°$, $a = 6$, and $b = 25$



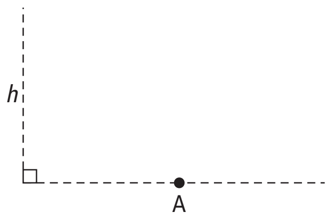
$h = b \sin A$

$h =$

Compare the length of side *a* to the altitude.

Since $a < h$, there is/are ________________ triangle(s).
(*zero, one,* or *two*)

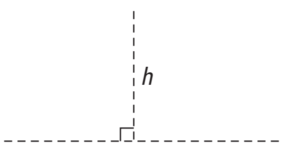
c) $\angle A = 116°$, $a = 10$ cm, and $b = 10$ cm



∠A is obtuse, so compare the lengths of *a* and *b*.

Since *a* ________ *b*, there is/are ________ triangle(s).
(< or > or =) (*zero, one,* or *two*)

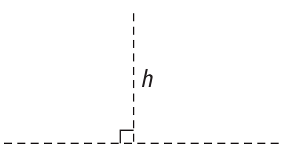
d) $\angle A = 65°$, $a = 15$, and $b = 16$



When ∠A is acute, compare the lengths of *a*, *b*, and *h*.

Since ________ < ________ < ________, there is/are ________________ triangle(s).
(*zero, one,* or *two*)

e) $\angle A = 60°$, $a = 80$, and $b = 75$

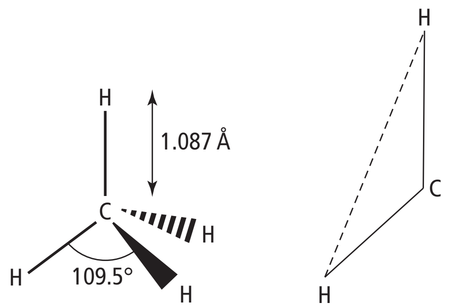


Since $b \leq$ ________, there is/are ________________ triangle(s).
(*zero, one,* or *two*)

This question should help you complete #7 on page 108 of *Pre-Calculus 11*.

Apply

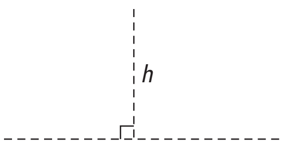
6. Methane (CH_4) is the major component of natural gas. It can be found in underground gas reservoirs, oil wells, coal mines, marshland, agricultural sites, sewage sludge, and landfills. The molecule is highly symmetrical, with each C-H bond 1.087 Å (angstroms) long, and each H-C-H angle 109.5°. Calculate the distance, in angstroms, between H atoms.



Fill in the triangle with the known information. Which form of the sine law is best when solving for a distance?

This question should help you complete #15 on page 110 of *Pre-Calculus 11*.

7. Determine the unknown side and angles in △ABC, where ∠A = 35°, a = 120, and b = 100. If two solutions are possible, give both.



Compare the lengths of *a*, *b*, and *h*.

Since b ≤ ________, there is/are ______________ triangle(s).
(*zero, one,* or *two*)

Solve for ∠B: $\dfrac{\sin B}{b} = \dfrac{\sin A}{a}$

Solve for *c*: $\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

Solve for ∠C: ∠A + ∠B + ∠C = 180°

In △ABC, ∠A = ________°,

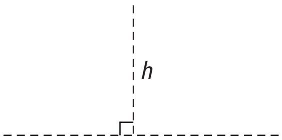
∠B = ________°, ∠C = ________°,

a = ________, *b* = ________,

and *c* = ________.

8. Determine the unknown side and angles in △ABC, where ∠A = 41°, a = 12.3 cm, and b = 15.6 cm. If two solutions are possible, give both.

Compare the lengths of *a*, *b*, and *h*.



Since ________ < ________ < ________, there is/are ______________ triangle(s).
(*zero, one,* or *two*)

Triangle 1: ∠B is acute

Calculate $\angle B_1$ using the sine law:

$$\frac{\sin A}{a} = \frac{\sin B_1}{b}$$

$\angle B_1$ = ________°

Find ∠C:

$\angle A + \angle B_1 + \angle C = 180°$

∠C = ________°

Calculate side c using the sine law:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

c = ________

Triangle 2: ∠B is obtuse

To calculate $\angle B_2$, find the angle in quadrant II with $\theta_R = \angle B_1$:
$\angle B_2 = 180° - \angle B_1$

$\angle B_2$ = ________°

Find ∠C:

∠C = ________°

Calculate side c using the sine law:

c = ________

The two possible triangles are as follows:

- acute △ABC: ∠A = ________°, ∠B = ________°, ∠C = ________°, a = ________ cm, b = ________ cm, and c = ________ cm
- obtuse △ABC: ∠A = ________°, ∠B = ________°, ∠C = ________°, a = ________ cm, b = ________ cm, and c = ________ cm

For more practice, try #8 on page 109 of *Pre-Calculus 11*.

Connect

9. Solving a triangle means finding the measure of all unknown sides and angles. Create a flowchart or other graphic organizer describing how to use the sine law to solve triangles in the following situations. Include formulas and diagrams.

a) You are given two angles and one side (ASA).

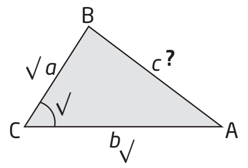
b) You are given two sides and an angle opposite one of those sides (SSA). Be sure to include information on the ambiguous case.

📖 The Key Ideas on page 107 of *Pre-Calculus 11* may give you some helpful ideas.

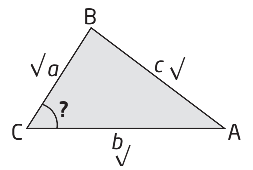
2.4 The Cosine Law

KEY IDEAS

- A triangle that does not contain a right angle is called an oblique triangle. You can also use the cosine law to solve problems involving oblique triangles.
- To find a side length using the cosine law, you must know the measure of the angle opposite that side, plus the lengths of the other two sides (SAS). If the unknown side in △ABC is c, the cosine law is written as
  $c^2 = a^2 + b^2 - 2ab \cos C$



- To find an angle using the cosine law, you must know the length of all three sides in the triangle (SSS). If you are solving for the angle at vertex C in △ABC, the cosine law is written as
  $c^2 = a^2 + b^2 - 2ab \cos C$



- You can choose to rearrange the formula before or after substituting the known values.

$$c^2 - a^2 - b^2 = -2ab \cos C$$

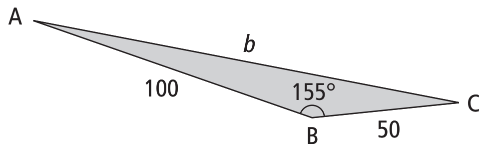
$$\frac{c^2 - a^2 - b^2}{-2ab} = \cos C$$

$$\cos^{-1}\left(\frac{c^2 - a^2 - b^2}{-2ab}\right) = \angle C$$

- There is no ambiguous case for the cosine law because acute angles (in quadrant I) have positive values of cos θ and obtuse angles (in quadrant II) have negative values of cos θ.
- You can use cosine law in combination with the sine law to solve oblique triangles.

Working Example 1: Determine an Unknown Side Length

Determine the length of side b, to one decimal place.



Solution

This is an oblique triangle. You know two sides and the angle at the vertex that joins them (SAS), so use the cosine law.

First, write the cosine law in terms of the unknown side, b. The known sides are a and c. The angle opposite the unknown side is ∠B.

$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = \square^2 + \square^2 - 2(\square)(\square) \cos (\square°)$ Substitute the known values.

$b^2 = \square + \square - \square \cos 155°$ Simplify.

$b^2 =$ ________ Determine a value for b^2. Do not round off yet.

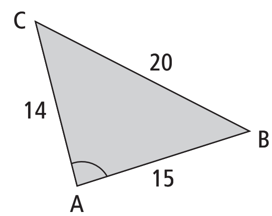
$b \approx$ ________ Take the square root.

Why do you use only the positive root of b^2?

See the example on pages 116–117 of *Pre-Calculus 11*, which is a word problem.

Working Example 2: Determine an Unknown Angle

Determine the measure of ∠A, to the nearest degree.



Solution

This is an oblique triangle. You know three sides and want to find an angle (SSS), so use the cosine law.

First, write the cosine law in terms of the unknown ∠A.
The side opposite the known angle, a, is isolated on the left side of the equation.
The two known sides are b and c, and they appear on the same side of the equation as the unknown angle.

$a^2 = b^2 + c^2 - 2ab \cos A$

$\square^2 = \square^2 + \square^2 - 2(\square)(\square) \cos A$ Substitute the known values.

$\square = \square + \square - \square \cos A$ Simplify.

$\square = -\square \cos A$ Isolate the term containing cos A. Subtract b^2 and c^2 from both sides.

$\cos A =$ ________ Divide both sides by the coefficient of cos A. Do not round off yet.

$\angle A \approx$ ________ Use the $\cos^{-1}$ function to find the angle.

Working Example 3: Choose Your Method

Determine the appropriate method to solve for the requested information.

a) In △ABC, ∠A = 60°, a = 15 units, and b = 14 units. Find the measure of ∠B.

b) In △JKL, ∠L = 46°, j = 6.9 cm, and k = 10.3 cm. Find the length of side l.

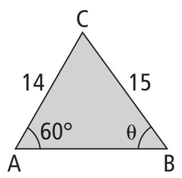
c) In △LMN, ∠L = 75°, ∠M = 28°, and m = 100. Find the length of side l.

d) In △XYZ, ∠X = 90°, x = 5 cm, and z = 3 cm. Find the measure of ∠Z.

Solution

a) What kind of triangle is △ABC? ______________
(*oblique* or *right*)

Sketch a diagram and label the known information.



You are given information that deals with two pairs of angles and their opposite sides: ∠A and a, ∠B and b. So, use the sine law. Alternatively, you are given side a, side b, and ∠A, or SSA. Therefore, use the sine law.

b) What kind of triangle is △JKL? ______________
(*oblique* or *right*)

Sketch a diagram and label the known information.

You are given two sides (j and k) and the angle at the vertex that joins these sides, ∠L. So, use the cosine law to find the length of the third side.

Alternatively, you are given side j, ∠L, and side k, or SAS. Therefore, use the cosine law.

c) What kind of triangle is △LMN? ______________
(*oblique* or *right*)

Sketch a diagram and label the known information.

You know: ______________________

You want to find: ______________________

Therefore, use the ______________.

Alternatively, you are given side m, ∠L, and ∠M, or ________.

So, use the ______________.

d) What kind of triangle is △XYZ? ________________
(*oblique* or *right*)

Sketch a diagram and label the known information.

You know the opposite side and the hypotenuse, and you want to find: ________________

Therefore, use the ________________.

Working Example 4: Solve a Triangle

In △ABC, $b = 14$ cm, $c = 15$ cm, and ∠A = 110°. Solve the triangle.

Solution

Sketch a diagram. Place the known ∠A at the lower left corner of your diagram.

What kind of triangle is △ABC? ________________
(*oblique* or *right*)

You are given two sides and the angle at the vertex that joins them (SAS). To find the length of the third side,

use the ________________.

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = \square^2 + \square^2 - 2(\square)(\square) \cos (\square°)$ Substitute the known values.

$a^2 = \square + \square - \square \cos (\square°)$ Simplify.

$a^2 =$ ________ Determine a value for a^2. Do not round off yet.

$a \approx$ ________ Take the square root. Round to one decimal place.

Now, you know all three sides (SSS), so you can use the ________________.
Write the cosine law in terms of the unknown ∠B.

You could also use the sine law, since two pairs of angles and their opposite sides (∠A and *a*, ∠B and *b*) are known.

$b^2 = a^2 + c^2 - 2ac \cos B$

$\square^2 = \square^2 + \square^2 - 2(\square)(\square) \cos B$ Substitute the known values.

$\square = \square + \square - \square \cos B$ Simplify.

$\square = -\square \cos B$ Isolate the term containing cos B. Subtract a^2 and c^2 from both sides.

$\cos B =$ ________ Divide both sides by the coefficient of cos B. Do not round off yet.

∠B ≈ ________ Use the $\cos^{-1}$ function to find the angle.

∠C = 180° − (∠A + ∠B)

∠C =

Therefore, ∠A = ________°, ∠B = ________°, ∠C = ________°, $a =$ ________ cm,

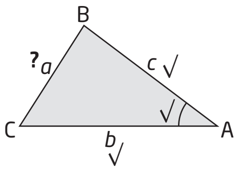
$b =$ ________ cm, and $c =$ ________ cm.

Check Your Understanding

Practise

1. Write the cosine law in terms of the appropriate variables to solve for the unknown side.

a) In △ABC, sides b and c and ∠A are known.

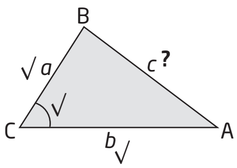


The side opposite the known angle is ________.

Use the form of the cosine law that starts and ends with that letter. The letters representing the other two sides go in the middle.

$$\square^2 = b^2 + c^2 - 2bc \cos \square$$

b) In △ABC, sides a and b and ∠C are known.

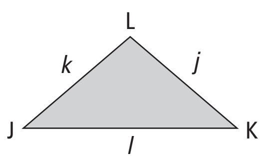


The side opposite the known angle is ________.

The form of the cosine law to use is

$$c^2 = \square^2 + \square^2 - 2(\square)(\square) \cos C$$

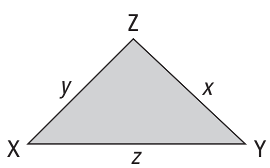
c) In △JKL, sides j and k and ∠L are known.



The form of the cosine law to use is

$l^2 =$ ______________

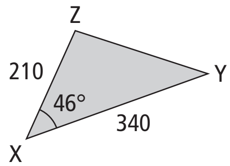
d) In △XYZ, sides x and z and ∠Y are known.



The form of the cosine law to use is

$y^2 =$ ______________

2. Determine the length of side x, to the nearest unit, in $\triangle XYZ$.



The form of the cosine law to use is

$x^2 =$

Substitute the known values into the formula and solve for x.

$x^2 = \square^2 + \square^2 - 2(\square)(\square)\cos(\square^\circ)$

3. In $\triangle ABC$, $a = 10$ cm, $b = 8$ cm, and $\angle C = 25°$. Determine the length of side c, to the nearest millimetre.

Sketch:

The form of the cosine law to use is

$c^2 =$

Substitute the known values into the formula and solve for c.

4. In $\triangle XYZ$, $x = 15$ cm, $z = 14$ cm, and $\angle Y = 60°$. Determine the length of side y, to the nearest millimetre.

Sketch:

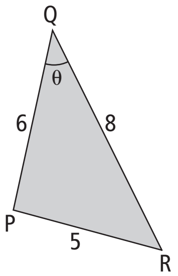
The form of the cosine law to use is

$y^2 =$

Substitute the known values into the formula and solve for y.

5. Determine the measure of ∠Q, to the nearest degree, in each triangle.

a)



$q^2 = p^2 + r^2 - 2pr \cos Q$

$\square^2 = \square^2 + \square^2 - 2(\square)(\square) \cos Q$

$\square = \square + \square - \square \cos Q$

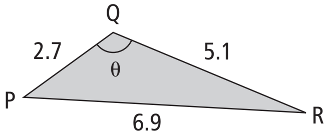
$\square = -\square \cos Q$

$\cos Q =$

$\angle Q \approx$

Do not round off until you find ∠Q.

b)



$q^2 = p^2 + r^2 - 2pr \cos Q$

The value of cos Q should be negative because ∠Q is an obtuse (quadrant II) angle.

∠Q measures ________°

c) In △PQR, $p = 30$, $q = 20$, and $r = 15$.

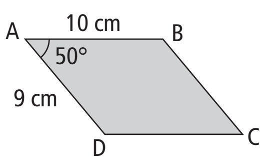
Sketch:

Set up the cosine law for this triangle.

∠Q measures ________°

Apply

6. In parallelogram ABCD, the measure of the acute angle is 50°. If the sides are 10 cm and 9 cm, what is the measure of the shortest diagonal, to one decimal place?

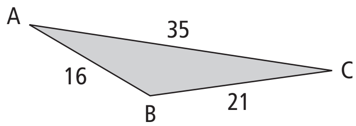


Draw in the diagonal to make a triangle. Label its length with a variable.

The form of the cosine law to solve for the diagonal is

The measure of the shortest diagonal is ________ cm.

7. Solve △ABC. Round angles to the nearest degree.



You know the measure of all three sides (SSS).

Therefore, use the ________________
(*sine law* or *cosine law*)

to find any angle. Decide which angle you will find first. ________

The form of the cosine law to solve for ________ is ________________.

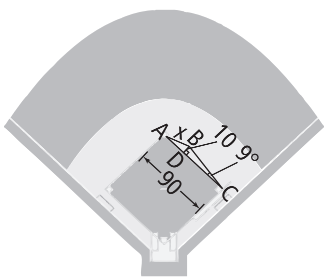
Next, solve for ________ using the sine law or cosine law.

Solve for the remaining angle using the fact that the sum of the angles in a triangle is 180°.

In △ABC, ∠A = ________°, ∠B = ________°, ∠C = ________°, a = ________,

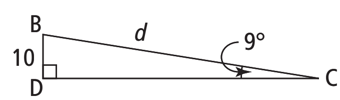
b = ________, and c = ________.

8. In a baseball diamond, the bases are 90 ft apart. The second baseman stands back from the base line, AC, to cover the territory between first base, C, and second base, A. If the second baseman is standing at point B, which is 10 ft from the baseline at an angle of 9° to first base, how far is the player from second base?



To use the cosine law to find distance x, you need the opposite angle, ∠C, and the lengths of the two sides AC and BC.

You know AC = ________ ft and ∠C = ________°. You need to calculate BC (call this distance d).



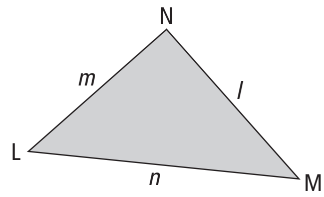
The form of the cosine law to solve for x is

Therefore, the distance from the player to second base is ________ ft.

Use a similar multiple-step approach in #10 to #23 on pages 121–123 of *Pre-Calculus 11*.

Connect

9. Write the form of the cosine law required to solve for each of the angles and sides of △LMN.



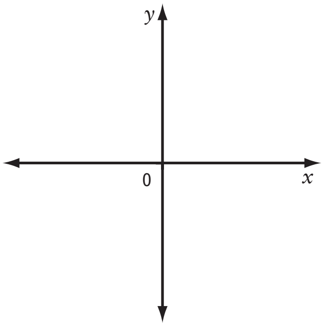
Given Information	Solve For	Formula
m, ∠L, n (SAS)		
	m	
	n	
l, m, n (SAS)	∠L	cos L =
	∠M	
	∠N	

Chapter 2 Review

2.1 Angles in Standard Position, pages 56–67

1. Sketch each angle in standard position. State which quadrant the angle terminates in and the measure of the reference angle.

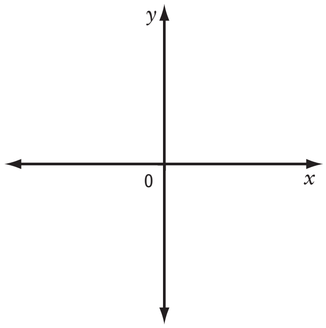
a) 35°



quadrant ________

θ_R = ________

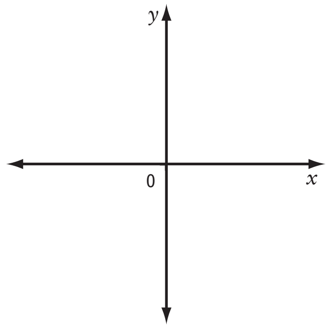
b) 165°



quadrant ________

θ_R = ________

c) 216°

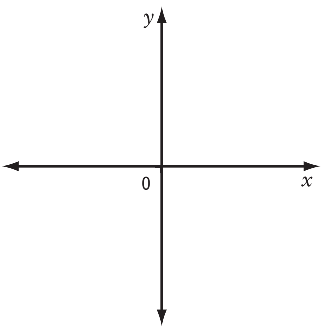


quadrant ________

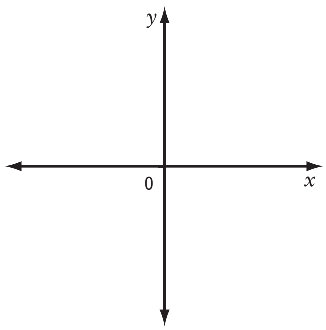
θ_R = ________

2. Determine the exact value of the following ratios without using technology.

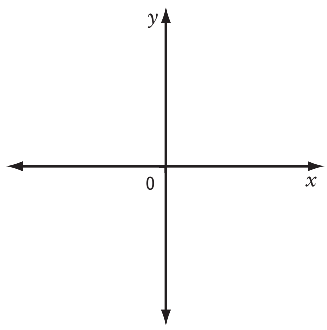
a) cos 180° = ________



b) tan 210° = ________

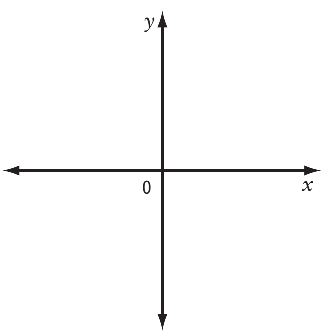


c) sin 315° = ________

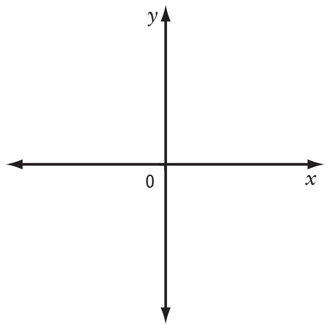


2.2 Trigonometric Ratios of Any Angle, pages 68–79

3. A point P(−4, 5) lies on the terminal arm of an angle θ in standard position. Determine the exact trigonometric ratios for sin θ, cos θ, and tan θ.

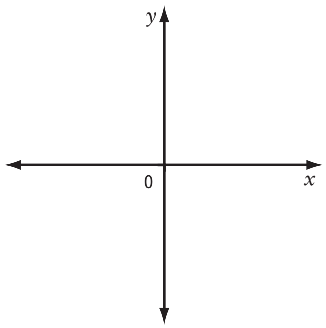


4. Suppose θ is an angle in standard position with terminal arm in quadrant II and $\sin \theta = \frac{15}{17}$. Determine the exact values of the other two primary trigonometric ratios.

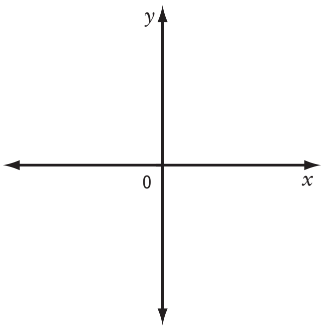


5. Solve for θ, $0° \le \theta < 360°$.

a) $\cos \theta = 0.5877$



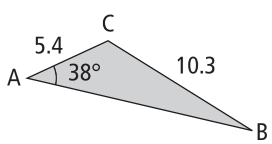
b) $\sin \theta = -\frac{\sqrt{3}}{2}$



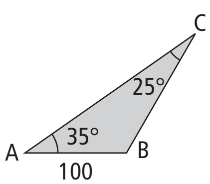
2.3 The Sine Law, pages 81–93

6. Find the indicated side or angle.

a) ∠B = ________

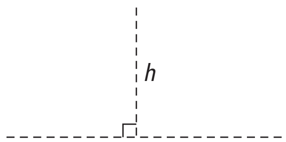


b) side b = ________

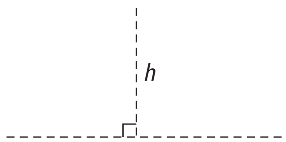


7. Determine how many $\triangle$ABCs satisfy the following conditions.

a) $\angle A = 69°$, $a = 10.1$ cm, and $b = 11.4$ cm



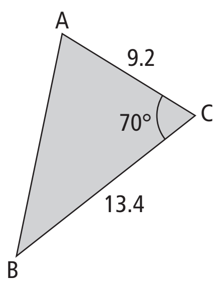
b) $\angle A = 28°$, $a = 4$, and $b = 6$



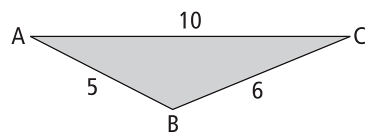
2.4 The Cosine Law, pages 94–102

8. Find the indicated side or angle.

a) side c = ________

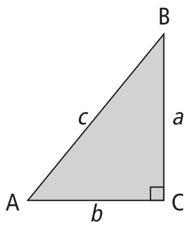


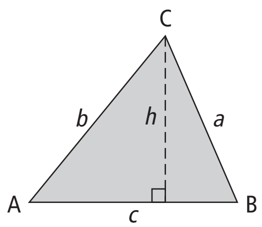
b) $\angle A$ = ________



Chapter 2 Skills Organizer B

Solving a triangle means finding the measure of all unknown sides and angles. Complete the chart with all relevant formulas to solve any triangle.

<table>
<tr><th>Triangle</th><th>Known Info</th><th>How to Calculate Side</th><th colspan="2">How to Calculate Angle</th></tr>
<tr><td rowspan="3">

right triangle</td><td rowspan="3">$\angle C = 90°$</td><td>$a^2 =$</td><td>$\sin A = \frac{a}{c}$</td><td>$\sin B = \frac{b}{c}$</td></tr>
<tr><td>$b^2 =$</td><td>$\cos A =$</td><td>$\cos B =$</td></tr>
<tr><td>$c^2 = a^2 + b^2$</td><td>$\tan A =$</td><td>$\tan B =$</td></tr>
<tr><td rowspan="5">

oblique triangle</td><td rowspan="3">SSS
or
SAS</td><td>$a^2 = b^2 + c^2 - 2bc \cos A$</td><td colspan="2">$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$</td></tr>
<tr><td>$b^2 =$</td><td colspan="2">$\cos B =$</td></tr>
<tr><td>$c^2 =$</td><td colspan="2">$\cos C =$</td></tr>
<tr><td>ASA
or
SSA*</td><td>$\frac{a}{\sin A} =$ _______ = _______</td><td colspan="2">$\frac{\sin A}{a} =$ _______ = _______</td></tr>
<tr><td colspan="4">* Notes about the ambiguous case:

	Acute Angle	Obtuse Angle
0 triangles		
1 triangle		
2 triangles		
</td></tr>
</table>