Chapter 5 Radical Expressions and Equations

5.1 Working With Radicals

KEY IDEAS

Definitions		
Term	**Description**	**Examples**
Mixed radical	• the product of a monomial and a radical	index, radical sign $-8\sqrt[3]{45}$ coefficient, radicand
Entire radical	• radical with a coefficient of 1 or −1	$\sqrt{30}$, $-\sqrt[3]{100}$, $\sqrt{98y^3}$
Like radicals	• radicals with the same index and the same radicand	$9\sqrt{2}$ and $-5\sqrt{2}$ $\frac{1}{4}\sqrt[3]{7x^2}$ and $-\sqrt[3]{7x^2}$
Radical in simplest form	a radical term where the • radicand does not contain a fraction • radicand does not contain a factor that can be removed • denominator does not contain a radical	$\sqrt{20} = \sqrt{4(5)}$ $= \sqrt{4}(\sqrt{5})$ $= 2\sqrt{5}$
Working With Radicals		
	Strategies	**Examples**
Compare and order radicals	• Convert mixed radicals to entire radicals. If the radicals have the same index, compare the radicands.	• Order $5\sqrt[3]{4}$, $\sqrt[3]{153}$, $\sqrt[3]{46}$ $5\sqrt[3]{4} = \sqrt[3]{5^3(4)}$ $= \sqrt[3]{125(4)}$ $= \sqrt[3]{500}$ So, $\sqrt[3]{46}$, $\sqrt[3]{153}$, $5\sqrt[3]{4}$ are in ascending order.
	• Compare the coefficients of like radicals.	• $5\sqrt[3]{4}$, $7\sqrt[3]{4}$, $11\sqrt[3]{4}$ are in ascending order
	• Compare the indices of radicals with equal radicands. When the radicands are equal, the higher the indices, the smaller the number.	• $\sqrt[4]{16}$, $\sqrt[3]{16}$, $\sqrt[2]{16}$ are in ascending order
Adding and subtracting radicals	• Combine the coefficients of like radicals: $q\sqrt[n]{x} \pm r\sqrt[n]{x} = (q \pm r)\sqrt[n]{x}$, where n is a natural number q, r, and x are real numbers • If n is even, then $x \geq 0$.	• $6\sqrt{2} + 4\sqrt{2} = 10\sqrt{2}$ • $6\sqrt{2} - 4\sqrt{2} = 2\sqrt{2}$

Restrictions on variables in radicand	• Identify the values of the variables that make the radical a real number. For radicals to be real numbers, – denominators cannot be equal to zero – if the index is even, the radicand must be positive – if the index is odd, the radicand may be any real number	• $\frac{8}{\sqrt{0}}$ is undefined • $\sqrt[4]{xy}$, $x \geq 0$, $y \geq 0$ • $\sqrt[5]{xy}$, no restrictions • $\sqrt{-4}$ is undefined, but $\sqrt[3]{-8} = -2$

Working Example 1: Convert Mixed Radicals to Entire Radicals

Express each **mixed** radical in **entire** radical form. State the restrictions on the variables.

a) $5\sqrt{3}$ **b)** $2b\sqrt{b}$ **c)** $-4x\sqrt[3]{7x^2}$

Solution

a) $5\sqrt{3}$

$5 = \sqrt{\square}$ Write the coefficient, 5, as a square root.

$5\sqrt{3} = (\sqrt{5^2})(\sqrt{3})$ Multiply the radicands of the square roots.

$= \sqrt{\square(3)}$

$= \sqrt{\square}$

The entire radical form of $5\sqrt{3}$ is ________.

b) $2b\sqrt{b}$

Is there any restriction on the value of *b*? Explain.

$2b = \sqrt{\square}$ Write the coefficient, $2b$, as a square root.

$2b\sqrt{b} = (\sqrt{4b^2})(\sqrt{b})$ Multiply the radicands.

$= \sqrt{\square(b)}$

$= \sqrt{\square}$

The entire radical form of $2b\sqrt{b}$ is $\sqrt{\square}$. Since the index is ________________,
(*odd* or *even*)

then b ________ 0.

c) $-4x\sqrt[3]{7x^2}$

$-4x = \sqrt[3]{\square}$ Write the coefficient, $-4x$, as a cube root.

$-4x\sqrt[3]{7x^2} = (\sqrt[3]{-64x^3})(\sqrt[3]{7x^2})$ Multiply the radicands.

$= \sqrt[3]{\square}$

The entire radical form of $-4x\sqrt[3]{7x^2}$ is $\sqrt[3]{\square}$. Since the index of the radical is

________________, the value of the variable x can be ________________.
(*odd* or *even*)

See page 274 of *Pre-Calculus 11* for more examples.

Working Example 2: Express Entire Radicals as Mixed Radicals

Express each entire radical as a mixed radical in simplest form.

a) $\sqrt{192}$ **b)** $\sqrt[3]{y^7}$

Solution

a) $\sqrt{192}$

Method 1: Use the Greatest Perfect-Square Factor

The following perfect squares are factors of 192: 1, ________, ________, and ________.
Write $\sqrt{192}$ as a product using the greatest perfect-square factor.

$$\sqrt{192} = \sqrt{\boxed{}(3)}$$
$$= (\sqrt{64})(\sqrt{3})$$
$$= \underline{}\sqrt{3}$$

How might a table help you find the greatest perfect square?

Therefore, $\sqrt{192}$ written as a mixed radical is ________.

How can you verify the answer?

Method 2: Use Prime Factorization

Express 192 as a product of prime factors.

$$\sqrt{192} = \sqrt{(2)(2)(2)(2)(2)(2)(\boxed{})}$$

Since the index is 2, combine pairs of identical factors.

$$\sqrt{192} = \sqrt{(2^2)(2^2)(2^2)(\boxed{})}$$
$$= (2)(2)(2)\sqrt{\boxed{}}$$
$$= \underline{}\sqrt{3}$$

Therefore, $\sqrt{192}$ written as a mixed radical is ________.

b) $\sqrt[3]{y^7}$

Method 1: Use Prime Factorization

Express the radicand as a product of prime factors.

$$\sqrt[3]{y^7} = \sqrt[3]{(y)\boxed{}}$$

Since the index is ________, combine triplets (groups of three) of identical factors.

$$\sqrt[3]{y^7} = \sqrt[3]{(y)(\boxed{})(\boxed{})}$$
$$= (y)(y)\sqrt[3]{\boxed{}}$$
$$= y^2\sqrt[3]{\boxed{}}$$

Therefore, $\sqrt[3]{y^7}$ written as a mixed radical is ________.

Method 2: Use Powers

$\sqrt[3]{y^7} = y^{\frac{7}{3}}$

Write the rational exponent as the sum of a whole number and a fraction.

$y^{\frac{7}{3}} = y^{\frac{6}{3} + \frac{1}{3}}$

$= \left(y^{\frac{6}{3}}\right)\left(y^{\frac{1}{3}}\right)$

$= \underline{\qquad\qquad}\left(y^{\frac{1}{3}}\right)$

$= y^2\sqrt[3]{\boxed{\qquad\quad}}$

What whole number is $\frac{6}{3}$?

Use the rule of adding exponents to explain why $y^{\frac{6}{3} + \frac{1}{3}}$ is equal to $\left(y^{\frac{6}{3}}\right)\left(y^{\frac{1}{3}}\right)$.

Therefore, $\sqrt[3]{y^7}$ written as a mixed radical is ________. Since the index is ________________,
(*odd* or *even*)
the value of y can be ________________.

Compare these methods to those on page 275 of *Pre-Calculus 11*.

Working Example 3: Compare and Order Radicals

Without using technology, arrange the following real numbers in ascending order.

$\sqrt{149}$, 13, $4\sqrt{10}$, $2(42)^{\frac{1}{2}}$, $3\sqrt{19}$

Ascending is from least to greatest. What is the arrangement greatest to least called?

Solution

Express each number as an entire radical.

$\sqrt{149}$ is already written as an ________________.

$13 = \sqrt{\boxed{\qquad\quad}^2}$

$= \sqrt{\boxed{\qquad\quad}}$

$4\sqrt{10} = \sqrt{(4)^2(\boxed{\qquad\quad})}$

$= \sqrt{\boxed{\qquad\quad}}$

$2(42)^{\frac{1}{2}} = 2\sqrt{\boxed{\qquad\quad}}$

$= \sqrt{(2)^2(\boxed{\qquad\quad})}$

$= \sqrt{\boxed{\qquad\quad}}$

$3\sqrt{19} = \sqrt{\boxed{\qquad\quad}}$

$= \sqrt{\boxed{\qquad\quad}}$

All these radicals have the same indices. Compare the five radicands and order them from least to greatest: $\sqrt{149} < \sqrt{\boxed{\qquad\quad}} < \sqrt{\boxed{\qquad\quad}} < \sqrt{\boxed{\qquad\quad}} < \sqrt{171}$.

The real numbers written in ascending order are ____.

Compare these methods to those on page 276 of *Pre-Calculus 11*.

Working Example 4: Add and Subtract Radicals

Simplify the radicals and combine like terms.

a) $\sqrt{27} + 2\sqrt{12}$ **b)** $5\sqrt{8} - 3\sqrt{18} + \sqrt{3}$ **c)** $3\sqrt{32a} - 4\sqrt{162a}$, $a \geq 0$

Explain the restriction on the variable for part c).

Solution

a) $\sqrt{27} + 2\sqrt{12}$

Rewrite each radical as a mixed radical in simplest form.

$\sqrt{27} = \sqrt{(\boxed{})(3)}$ $\quad 2\sqrt{12} = 2\sqrt{(\boxed{})(3)}$

$= \_\_\_\_\_\_\sqrt{3}$ $\quad = \_\_\_\_\_\_\sqrt{3}$

$3\sqrt{3}$ and $4\sqrt{3}$ are like radicals because ____________________________________.
Therefore, add the coefficients.

$\sqrt{27} + 2\sqrt{12} = 3\sqrt{3} + 4\sqrt{3}$

$= \_\_\_\_\_\_$

b) $5\sqrt{8} - 3\sqrt{18} + \sqrt{3}$

Rewrite each radical as a mixed radical in simplest form.

$5\sqrt{8} =$ ______, $3\sqrt{18} =$ ______, $\sqrt{3}$ is already in simplest form

The like radicals are ______ and ______. Since $\sqrt{3}$ has a different __________,
it is not a like radical with the other two terms.

$5\sqrt{8} - 3\sqrt{18} + \sqrt{3} = \_\_\_\_\_\_\sqrt{2} - 9\sqrt{2} + \sqrt{3}$

$= \_\_\_\_\_\_$

c) $3\sqrt{32a} - 4\sqrt{162a}$

$3\sqrt{32a} = 3\sqrt{16(\boxed{})a}$ $\quad 4\sqrt{162a} = 4\sqrt{(\boxed{})2a}$

$= (3)(\_\_\_\_\_\_)\sqrt{2a}$ $\quad = (4)(\_\_\_\_\_\_)\sqrt{2a}$

$= \_\_\_\_\_\_$ $\quad = \_\_\_\_\_\_$

Since $12\sqrt{2a}$ and ______ have the same radicand, they are ______ radicals. Therefore,

add their __________.

$3\sqrt{32a} - 4\sqrt{162a} = \_\_\_\_\_\_ - \_\_\_\_\_\_$

$= \_\_\_\_\_\_$

Compare these methods to those on pages 276–277 of *Pre-Calculus 11*.

Check Your Understanding

Practise

1. Complete the table.

Mixed Radical Form	$\sqrt{125} = \sqrt{(25)(5)}$ $= \underline{\qquad\quad}\sqrt{5}$	$6\sqrt{7}$		$-5\sqrt{15}$		$3\sqrt[4]{6}$
Entire Radical Form	$\sqrt{125}$	$6\sqrt{7} = \sqrt{(6)^2(7)}$ $= \sqrt{\boxed{}}$	$\sqrt{63}$		$\sqrt[3]{-56}$	

2. Write each entire radical as a mixed radical in simplest form. Write each mixed radical as an entire radical. State any restrictions on the variables.

a) $\sqrt[3]{b^4y} = \sqrt[3]{b^3(\boxed{})}$

$= b\sqrt[3]{\boxed{}}$

When the index is odd, is there any restriction on the variables? Explain.

b) $2\sqrt{a} = \sqrt{2^2(a)}$

$= \sqrt{\boxed{}}$

When the index is even, what restriction is there on the variable?

c) $\sqrt[3]{-48x^3y}$

d) $\sqrt[4]{32a^5}$

e) $7y\sqrt{2z}$

f) $-3m^2\sqrt{5m}$

#1 and 2 use the same concepts as #1 and 4 on pages 278 and 279 of *Pre-Calculus 11*.

3. From the following list, identify like radicals. Show your method.

$\sqrt{63}$, $\sqrt[3]{250}$, $4\sqrt[3]{16}$, $\sqrt{7}$, $-\sqrt{27}$, $3\sqrt{75}$

Like radicals have the same indices and the same radicands.

4. State a radical expression that would form a like radical pair for each given expression.

a) $\sqrt{28} =$ ________ $\sqrt{7}$

$\sqrt{28}$ and ________ are like radicals.

b) $7\sqrt[3]{2}$ and ________ are like radicals.

c) $-4\sqrt{2m}$ and ________ are like radicals.

d) $\sqrt[4]{32a^5}$ and ________ are like radicals.

e) $-8x\sqrt{18}$ and ________ are like radicals.

f) $\sqrt{20xy^3}$ and ________ are like radicals.

What is the index and radicand of $\sqrt{28}$ when it is reduced to its simplest form?

For parts d), e), and f), what do you need to do before you can find a like radical?

The concepts used in #3 and 4 are also used in #5 on page 279 of *Pre-Calculus 11*.

5. Arrange each set of numbers in descending order without using technology.

a) $-\sqrt{60}, -2\sqrt{17}, -5\sqrt{3}, -8$

b) $\sqrt{300}, \frac{3}{10}\sqrt{3500}, 18, 9\sqrt{\frac{15}{4}}$

c) $5, 3\sqrt[4]{8}, \sqrt[4]{615}, 4\sqrt[4]{\frac{9}{4}}$

6. Simplify each expression.

a) $\sqrt{7} - \sqrt{28} + 3\sqrt{63}$

b) $3\sqrt{175} - 6\sqrt{32} + \sqrt{98}$

c) $\sqrt[3]{9} + \sqrt{9} - 1$

d) $\sqrt[4]{48} - \frac{2}{3}\sqrt[4]{243}$

#6 is similar to #8 and 9 on page 279 of *Pre-Calculus 11*.

7. Simplify each expression. State any restrictions on the variable.

a) $8\sqrt{m} - \sqrt{m} + 6\sqrt{m} =$ ________$\sqrt{m}$, $m \geq$ ________

b) $5\sqrt{3x^3} - 3\sqrt{12x^3} = 5$ ________ $\sqrt{3\boxed{}}$ $- 3$ ________ $\sqrt{\boxed{}}$

$=$ ________, $x \geq 0$

c) $\sqrt{32a^2b^3} - ab\sqrt{98b}$

d) $\frac{\sqrt{64y^3}}{2} - \sqrt{9y^3} + \frac{1}{5}\sqrt{25y^3}$

#7 is similar to #10 on page 279 of *Pre-Calculus 11*.

Apply

Unless otherwise stated, express all answers as radicals in simplest form.

8. The flow-rate equation of a nozzle of a hose is $r = 6d^2P$, where r is the flow rate in gallons per minute, d is the diameter of the nozzle in inches, and P is the pressure of the nozzle in pounds per square inch. What is the diameter of a hose with nozzle pressure of 3 lb/in.^{2} and flow rate of 162 gal/min?

The quantities in the formula I know are ________ and ________.

How do you rewrite the formula to isolate d?

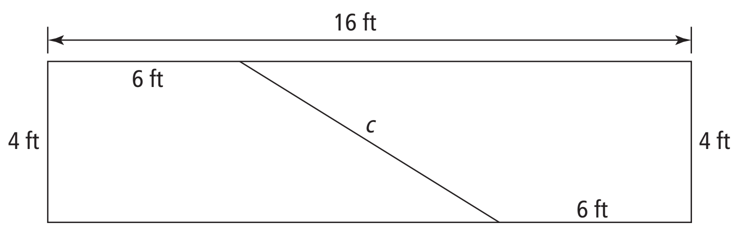
9. The formula $t = \sqrt{\frac{2d}{g}}$ can be used to find the time, t, in seconds, it takes for an object to fall from a height or distance, d, in metres, using the force of gravity, g (9.8 m/s^2). A ball is dropped from the top of a building 122.5 m in height. How long does it take for the ball to reach the ground (ignoring other forces, such as air pressure)?

What values in the formula do you know?

10. Tyson has a square dog run with a perimeter of 32 ft. He wants to split the dog run diagonally. What is the length of the diagonal section of fencing that Tyson will need to install? Show your reasoning.

Consider drawing a diagram when solving word problems.

11. Jodie has a rectangular piece of wood with a width of 4 ft and length of 16 ft. She is cutting it to create two large tables of the exact same size. She cuts the length diagonally to create two trapezoids. The smaller side of each trapezoid that results from the cut is 6 ft. What is the length of the diagonal cut, *c*? Show your reasoning.



12. Tyrus is flying a kite. He lets out 155 ft of string, ties the string to a tent peg, and pounds the peg into the ground. He then measures the distance to the point directly below the kite. He finds this distance to be 85 ft. Assuming that the kite string is taut, how high is the kite? Show your reasoning.

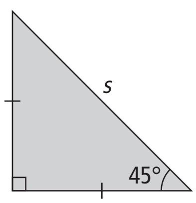
Connect

13. Describe the error in the following simplification. Then, show the correct simplification.

$$\sqrt{16b} + \sqrt{4b} = \sqrt{20b}$$
$$= 2\sqrt{5b}$$

14. What radical expression could be added to $-3\sqrt{45} + 2\sqrt{12} + 3\sqrt{27} - 3\sqrt{20}$ to obtain a sum of $\sqrt{500} + \sqrt{300}$?

15. Use the relationship between the sides of a 45°-45°-90° triangle to show that the area of an isosceles right triangle can be found using the formula $A = \frac{s^2}{4}$, where s is the length of the hypotenuse and A is the area in square units.



5.2 Multiplying and Dividing Radical Expressions

KEY IDEAS

Definitions		
Term	**Description**	**Examples**
Rationalize the denominator	• write an equivalent expression in which the denominator is a rational number	$\frac{3}{\sqrt{2}} = \frac{3(\sqrt{2})}{\sqrt{2}(\sqrt{2})}$ $= \frac{3\sqrt{2}}{2}$
Conjugates	• two binomial factors whose product is the difference of two squares • To find the conjugate of a binomial, reverse the sign of the second term in the binomial, creating the difference of two squares. The result is a rational number.	binomial: $4 - \sqrt{6}$ conjugate: $4 + \sqrt{6}$ Check: $(4 - \sqrt{6})(4 + \sqrt{6})$ $= 16 + 4\sqrt{6} - 4\sqrt{6} - 6$ $= 10$
Working With Radicals		
Action	**Strategies**	**Examples**
Rationalize the denominator when it contains a square root binomial	• The three steps to simplify an expression where the binomial in the denominator contains a radical are 1) find the conjugate of the denominator 2) multiply the numerator and denominator by the conjugate 3) simplify	• Simplify $\frac{3}{4 - \sqrt{6}}$. The conjugate of the denominator is $4 + \sqrt{6}$. $\frac{3}{4 - \sqrt{6}}\left(\frac{4 + \sqrt{6}}{4 + \sqrt{6}}\right)$ $= \frac{12 + 3\sqrt{6}}{16 + 4\sqrt{6} - 4\sqrt{6} - 6}$ $= \frac{12 + 3\sqrt{6}}{10}$
Multiply radicals with the same indices	• Multiply the coefficients and multiply the radicands: $(m\sqrt[k]{a})(n\sqrt[k]{b}) = mn\sqrt[k]{ab}$, where k is a natural number m, n, a, and b are real numbers • If the index, k, is even, then $a \geq 0$ and $b \geq 0$. • When multiplying radicals with more than one term, use the distributive property and simplify.	• $(6\sqrt[4]{8})(3\sqrt[4]{5})$ $= (6)(3)\sqrt[4]{(8)(5)}$ $= 18\sqrt[4]{40}$ • $(6\sqrt[4]{8})(3\sqrt[4]{5 + x})$, $x \geq -5$ $= (6)(3)(\sqrt[4]{8(5 + x)})$ $= 18\sqrt[4]{40 + 8x}$

Divide two radicals with the same indices	• Divide the coefficients and divide the radicands: $\frac{m\sqrt[k]{a}}{n\sqrt[k]{b}} = \frac{m}{n}\sqrt[k]{\frac{a}{b}}$, where k is a natural number m, n, a, and b are real numbers $n \neq 0$, $b \neq 0$ • If k is even, $a \geq 0$, $b > 0$.	$\frac{14\sqrt{6}}{2\sqrt{2}} = \left(\frac{14}{2}\right)\left(\frac{\sqrt{6}}{2}\right)$ $= 7\sqrt{3}$

Working Example 1: Multiply Radicals

Simplify.

a) $(3\sqrt{5})(\sqrt{10})$

b) $(4\sqrt{14x})(2\sqrt{7x^3})$, $x \geq 0$

c) $\sqrt{3}(-5\sqrt{10} + \sqrt{6})$

d) $(5\sqrt{2x} + \sqrt{5})(-4\sqrt{2x} + \sqrt{5x})$, $x \geq 0$

e) $\left(7x\sqrt[3]{8xy^2}\right)\left(3\sqrt[3]{8x^2y^2}\right)$

Solution

a) The indices are the same, so the two can be multiplied.

$$\begin{aligned}(3\sqrt{5})(\sqrt{10}) &= (\underline{\hspace{3em}})(\underline{\hspace{3em}})\sqrt{(\boxed{})(\boxed{})} \\ &= \underline{\hspace{3em}}\sqrt{50} \\ &= (3)(5)\sqrt{\boxed{}} \\ &= 15\sqrt{2}\end{aligned}$$

Multiply the coefficients and multiply the radicands.

b)

$$\begin{aligned}(4\sqrt{14x})(2\sqrt{7x^3}) &= (\underline{\hspace{3em}})(\underline{\hspace{3em}})\sqrt{(\boxed{})(\boxed{})} \\ &= 8\sqrt{98x^4} \\ &= 8(\underline{\hspace{3em}})(\underline{\hspace{3em}})\sqrt{2} \\ &= \underline{\hspace{3em}}\sqrt{2}\end{aligned}$$

c)

$$\begin{aligned}\sqrt{3}(-5\sqrt{10} + \sqrt{6}) &= \sqrt{3}(\underline{\hspace{3em}}) + \sqrt{3}(\underline{\hspace{3em}}) \\ &= -5\sqrt{(\boxed{})(\boxed{})} + \sqrt{(\boxed{})(\boxed{})} \\ &= -5\sqrt{30} + \sqrt{\boxed{}} \\ &= -5\sqrt{30} + \underline{\hspace{3em}}\sqrt{2}\end{aligned}$$

Use the distributive property.

Multiply the coefficients and the radicands.

d) $(5\sqrt{2x} + \sqrt{5})(-4\sqrt{2x} + \sqrt{5x})$

$= \underline{\qquad\qquad}(-4\sqrt{2x} + \sqrt{5x}) + \sqrt{5}\left(\boxed{}\right)$ Use the distributive property.

$= 5(-4)\sqrt{(2x)(2x)} + 5\sqrt{(2x)(5x)} + \left(-4\sqrt{(\boxed{})(\boxed{})}\right) + \sqrt{(\boxed{})(\boxed{})}$

$= -20(\underline{\qquad\qquad}) + 5\,\underline{\qquad\qquad}\sqrt{10} - 4\sqrt{10x} + 5\sqrt{\boxed{}}$

$= -40x + 5x\sqrt{10} - 4\sqrt{10x} + 5\sqrt{x},\ x \geq 0$

e) $\left(7x\sqrt[3]{8xy^2}\right)\left(3\sqrt[3]{8x^2y^2}\right) = (\underline{\qquad\qquad})(\underline{\qquad\qquad})\sqrt[3]{(\boxed{})(\boxed{})}$

$= 21x\sqrt[3]{\boxed{}}$ Group the perfect cube terms.

$= 21x\sqrt[3]{(4)^3x^3y^3(\boxed{})}$ Simplify.

$= \underline{\qquad\qquad}\sqrt[3]{y}$

Working Example 2: Divide Radicals

Simplify. Rationalize denominators where needed.

a) $\dfrac{\sqrt{15xy}}{\sqrt{5x}},\ x > 0,\ y \geq 0$

b) $\dfrac{8\sqrt{5}}{2\sqrt{3}}$

c) $\dfrac{7}{5\sqrt{3} - \sqrt{2}}$

d) $\sqrt[3]{\dfrac{8k}{3}}$

Why are there restrictions on the variables in part a)?

Why are there no restrictions on *k* in part d)?

Solution

a) $\dfrac{\sqrt{15xy}}{\sqrt{5x}} = \sqrt{\dfrac{15xy}{5x}}$ Divide the dividends.

$= \sqrt{\boxed{}\,y}$, where $x > 0,\ y \geq 0$

b) $\dfrac{8\sqrt{5}}{2\sqrt{3}} = \dfrac{\boxed{}}{\boxed{}}\sqrt{\dfrac{\boxed{}}{\boxed{}}}$ Divide the coefficients. Divide the dividends.

$= 4\sqrt{\dfrac{\boxed{}}{\boxed{}}}$

$= 4\dfrac{\sqrt{5}}{\sqrt{3}}\left(\dfrac{\sqrt{3}}{\sqrt{3}}\right)$ Rationalize the denominator.

$= \dfrac{4\sqrt{\boxed{}}}{3}$ Simplify.

c) $\dfrac{7}{5\sqrt{3} - \sqrt{2}}$

Multiply the top and bottom by the conjugate of the denominator. What is the conjugate of $5\sqrt{3} - \sqrt{2}$? ________________

$$\frac{7}{5\sqrt{3} - \sqrt{2}} = \frac{7\boxed{}}{(5\sqrt{3} - \sqrt{2})\boxed{}}$$

$$= \frac{35\sqrt{3} + 7\sqrt{2}}{\boxed{}}$$

$$= \frac{35\sqrt{3} + 7\sqrt{2}}{\boxed{}}$$

$$= \frac{\boxed{}}{\boxed{}}$$

If the denominator was $\sqrt{3}$, you would rationalize the denominator by multiplying top and bottom by $\sqrt{3}$. In this case, why can't you multiply the numerator and denominator by $5\sqrt{3} - \sqrt{2}$? Try it.

d) $\sqrt[3]{\dfrac{8k}{3}} = \dfrac{\sqrt[3]{8k}}{\sqrt[3]{3}}$ Separate the radical into the quotient of two radicals.

$= \dfrac{\boxed{}\sqrt[3]{k}}{\sqrt[3]{3}}$ Simplify.

$= \left(\dfrac{2\sqrt[3]{k}}{\sqrt[3]{3}}\right)\left(\dfrac{\boxed{}}{\boxed{}}\right)$

$= \dfrac{2\sqrt[3]{9k}}{\boxed{}}$

What would happen if you simply multiplied the numerator and denominator by $\sqrt[3]{3}$? Remember that you are multiplying by a value that allows you to get rid of the radical in the denominator.

Compare these methods to those on pages 287–288 of *Pre-Calculus 11*.

Check Your Understanding

Practise

1. Multiply. Express as mixed radicals in simplest form.

   a) $(6\sqrt{3})(5\sqrt{2})$

   The product of the coefficients is ________. The product of the radicands is ________.

   $(6\sqrt{3})(5\sqrt{2}) = \_\_\_\_\_\_\_\_ \sqrt{\boxed{}}$

   Always check to see if you can simplify further. Can this term be simplified?

   b) $(4\sqrt{18a^2})(\sqrt{3a^2})$

   c) $(-5\sqrt{28x})(\sqrt{7x^3})$, where $x \geq 0$

   d) $\left(\sqrt[3]{81y^4}\right)\left(\frac{1}{3}\sqrt[3]{9y^3}\right)$

#1 uses the same concepts as #1 on page 289 of *Pre-Calculus 11*.

2. Multiply using the distributive property. Simplify.

   a) $2\sqrt{5}(\sqrt{6} + 2)$

   $= 2\sqrt{(\boxed{})(\boxed{})} + \_\_\_\_\_\_\_\_\sqrt{5}$

   $= 2\sqrt{\boxed{}} + \_\_\_\_\_\_\_\_\sqrt{5}$

   b) $\sqrt{3}(-5\sqrt{10} + \sqrt{6})$

   c) $\sqrt{14x}(3\sqrt{10} - \sqrt{2x})$, $x \geq 0$

   d) $\sqrt{21a}(5 - \sqrt{7a} + 2\sqrt{3})$, $a \geq 0$

#2 uses the same concepts as #2 on page 289 of *Pre-Calculus 11*.

3. Expand, using the distributive property. Simplify.

a) $(5 - 4\sqrt{3})(-2 + \sqrt{3})$

$= 5(-2 + \sqrt{3}) - 4\sqrt{3}(-2 + \sqrt{3})$

$= \_\_\_\_\_ + \_\_\_\_\_\sqrt{\square} + \_\_\_\_\_\sqrt{\square} - \_\_\_\_\_\sqrt{\square}$

$= \_\_\_\_\_ + \_\_\_\_\_\sqrt{\square}$

b) $(-2 - 3\sqrt{6})^2 = (-2 - 3\sqrt{6})(-2 - 3\sqrt{6})$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

c) $(7 - 3\sqrt{5})(7 + 3\sqrt{5})$

d) $(\sqrt{7} - 4)(3\sqrt{3} + \sqrt{7} + 2)$

#3 uses the same concepts as #4 on page 290 of *Pre-Calculus 11*.

4. Expand and simplify. State any restrictions on the values for the variables.

a) $(-3\sqrt{3k} + 4)(\sqrt{3k} - 5)$

$= \_\_\_\_\_\sqrt{\square} + \_\_\_\_\_\sqrt{\square} + \_\_\_\_\_\sqrt{\square} - \_\_\_\_\_$

$= \_\_\_\_\_ + \_\_\_\_\_\sqrt{\square} - \_\_\_\_\_$, where $k \geq \_\_\_\_\_$

b) $(\sqrt{2} - 3\sqrt{5m})^2 = (\sqrt{2} - 3\sqrt{5m})(\sqrt{2} - 3\sqrt{5m})$

$= \_\_\_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_$, where m _____ 0

c) $(5\sqrt{2x} + \sqrt{5})(-4\sqrt{2x} + \sqrt{5x})$

d) $\left(8\sqrt[3]{4y^2} - y\right)\left(\sqrt[3]{2y} + 5y\right)$

Since the index is 3, y can be __________.

#4 uses the same concepts as #5 on page 290 of *Pre-Calculus 11*.

5. Divide. Rationalize the denominators, if necessary. Express each radical in simplest form.

a) $\dfrac{\sqrt{117}}{\sqrt{13}}$

b) $\dfrac{-5\sqrt{80}}{\sqrt{5}} = \_\_\_\_\_\_\_\_ \sqrt{\dfrac{\square}{\square}}$

$= \_\_\_\_\_\_\_\_ \sqrt{\square}$

$= \_\_\_\_\_\_\_\_$

c) $\dfrac{3\sqrt{28}}{4\sqrt{4}}$

d) $\dfrac{-3\sqrt{3a}}{4\sqrt{8a}}$, where $a > 0$

e) $\dfrac{\sqrt{15xy}}{\sqrt{10xy^3}}$, where $x > 0$ and $y > 0$

f) $\dfrac{3 - 3\sqrt{3a}}{4\sqrt{8a}}$, where $a > 0$

#5 uses the same concepts as #6 and 8 on page 290 of *Pre-Calculus 11*.

6. Complete the table.

Binomial	Conjugate	Product of the Two Binomials
$4 + \sqrt{5}$		
$-5 - 3\sqrt{3}$		
$7\sqrt{5} + 4\sqrt{2}$		
$2\sqrt{z} - \sqrt{3}$, $z \geq 0$		

#6 uses the same concepts as #9 on page 290 of *Pre-Calculus 11*.

7. Rationalize each denominator. Simplify.

a) $\dfrac{4}{\sqrt{2}-7} = \left(\dfrac{4}{\sqrt{2}-7}\right)\left(\dfrac{\sqrt{2}+7}{\sqrt{2}+7}\right)$

$= \dfrac{\boxed{\qquad}\sqrt{\boxed{\quad}} + \boxed{\qquad}}{\boxed{\qquad\qquad}}$

$= \dfrac{\boxed{\qquad\qquad}}{\boxed{\quad}}$

b) $\dfrac{3\sqrt{5}}{\sqrt{6}+4}$

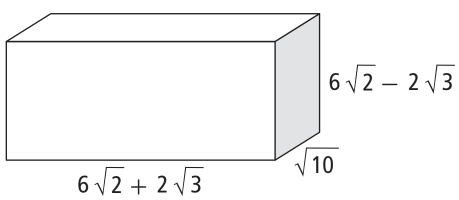
c) $\dfrac{\sqrt{5}+2\sqrt{2}}{4-5\sqrt{5}}$

d) $\dfrac{9}{4-\sqrt{x}}$, where $x \geq 0$ and $x \neq 16$

#7 uses the same concepts as #11 on page 290 of *Pre-Calculus 11*.

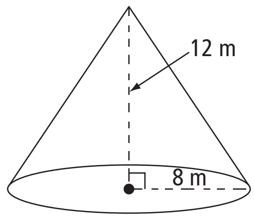
Apply

8. Express the volume of the right rectangular prism in simplest radical form.



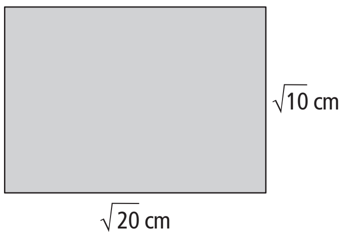
9. The lateral surface area of a cone can be found using the formula $LSA = \pi r\sqrt{r^2 + h^2}$, where r is the radius of the base and h is the height of the cone. Find the lateral surface area of the cone in the diagram. Write the answer in simplest radical form containing π.

Lateral surface area, *LSA*, is the entire surface area, excluding the area of the base.

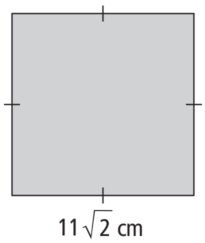


10. Find the area of the each of the following polygons. Express the answers in simplest radical form. Be sure to include units in your answers.

a)



b)



c) a right triangle with legs of $6\sqrt{3}$ cm and $4\sqrt{2}$ cm

11. Tineka simplified the following expression. Identify and explain any errors. Then, correct the errors and state the correct solution.

$$\begin{aligned}(3\sqrt{2} - \sqrt{5})^2 &= (3\sqrt{2})^2 - (\sqrt{5})^2 \\ &= 9(2) - 5 \\ &= 18 - 5 \\ &= 13\end{aligned}$$

12. Without the use of technology, arrange the following expressions in ascending order.

$\sqrt{7}(\sqrt{7} + 2)$, $(\sqrt{7} + 2)(\sqrt{7} - 2)$, $(2 - \sqrt{7})^2$, $(\sqrt{7} + 2)^2$

13. To find the radius of a right cylinder, you can use the formula $r = \sqrt{\frac{V}{\pi h}}$, where r is the radius, V is the volume in cubic units, and h is the height. A grain silo is a right cylinder. It has a height of 14 m and a volume of 224 m^3. Find the radius of the silo. Express the answer rounded to the nearest hundredth of a metre.

14. Anthony simplified the expression $\left(\frac{\sqrt{m} - 2}{6 - \sqrt{m}}\right)$ as shown. Identify any errors he made, including the restriction he has identified. Show the correct simplification and restriction(s) on the variable.

$$\left(\frac{\sqrt{m} - 2}{6 - \sqrt{m}}\right) = \left(\frac{\sqrt{m} - 2}{6 - \sqrt{m}}\right)\left(\frac{6 + \sqrt{m}}{6 + \sqrt{m}}\right)$$

$$= \frac{6\sqrt{m} - 12 + m}{6 - m}$$

$$= 6\sqrt{m} - 2\text{, where } m > 0$$

Connect

15. State three conditions that must be true for a radical expression to be in simplest form.

16. Is the following statement true or false? Explain.

$(\sqrt{-8})(\sqrt{-2}) = 4$

Chapter 5 Skills Organizer A

Complete the missing information in the chart for the topics in Section 5.1 and 5.2.

Entire Radical	Mixed Radical	Like Radical
An entire radical is… Example:	A mixed radical is … Example:	Like radicals are… Example:

Strategy for converting entire radical to mixed radical:	Strategy for converting mixed radical to entire radical:

Comparing and Ordering Radicals	When radicals have the same index, I…	Example:
	If they are radicals, I…	Example:
	If they have the same radicand, I…	Example:

Multiplying Radicals	Dividing Radicals
If radicals have the same indices, I can multiply them by… Example:	If radicals have the same indices, I can divide them by… Example:

Radicals in Simplest Form	Conjugate
A radical is in simplest form if: 1) 2) 3)	A conjugate is: I use a conjugate to… Example:

5.3 Radical Equations

KEY IDEAS

Solving a radical equation is similar to solving a linear or quadratic equation, as shown in the table.

Working With Radicals		
	Strategies	**Example**
Isolate one of the radical terms	• Perform the same mathematical operation on each side of the equation to isolate a radical term.	$\sqrt{4x} - 7 = 13,\ x > 0$ $\sqrt{4x} - 7 + 7 = 13 + 7$ $\sqrt{4x} = 20$
Eliminate a root	• For a square root, raise both sides of the equation to the exponent 2. • For a cube root, raise both sides of the equation to the exponent 3.	• $\sqrt{4x} = 20$ $(\sqrt{4x})^2 = (20)^2$ $4x = 400$ $x = 100$ • $\sqrt[3]{4x} = 20$ $(\sqrt[3]{4x})^3 = (20)^3$ $4x = 8000$ $x = 2000$
Check the answer or root	• Substitute the calculated value into the original equation to check.	Substitute $x = 100$. Left Side: $\sqrt{4x} - 7$ $= \sqrt{4(\mathbf{100})} - 7$ $= \sqrt{400} - 7$ $= 20 - 7$ $= 13$ Right Side: 13
Extraneous root	• Check for a solution, or root, that does not satisfy the restrictions on the variable or does not make sense in the context of the problem.	• For $\sqrt{x}$, a root of $x = -3$ does not meet the restrictions because x must be greater than or equal to zero. • For an area problem, $A = -4$ does not make sense because you cannot have a negative area.

Working Example 1: Solve an Equation With One Radical Term

a) Solve $\sqrt{y-1} + 7 = 13$.

b) State the restrictions on y so the radical is a real number.

c) Verify the solution.

If this equation was $x + 7 = 13$, how would you solve it?

Solution

a) Isolate the radical term.

$$\sqrt{y-1} + 7 = 13$$

$$\sqrt{y-1} + 7 - \underline{\qquad\qquad} = 13 - \underline{\qquad\qquad}$$

$$\sqrt{y-1} = \underline{\qquad\qquad}$$

Raise both sides of the equation to the exponent 2.

$$\left(\sqrt{y-1}\right)^2 = 6^2$$

$$\underline{\qquad\qquad} = \underline{\qquad\qquad}$$

Solve for y.

$$y - 1 + \underline{\qquad\qquad} = 36 + \underline{\qquad\qquad}$$

$$y = \underline{\qquad\qquad}$$

b) The radicand, $y - 1$, must be greater than or equal to zero for the radical to be a real number. So, for $y - 1 \geq 0$, $y \geq$ ________.

c) The solution is 37, which satisfies the restriction. To check this solution, substitute 37 into the original equation.

Left Side	Right Side
$\sqrt{y-1} + 7$	13
$= \sqrt{\mathbf{37} - 1} + 7$	
$= \sqrt{36} + 7$	
$= 6 + 7$	
$= 13$	

Left Side = Right Side, so the solution $y = 37$ is correct.

Working Example 2: Radical Equation With an Extraneous Root

a) Solve the equation $m = \sqrt{2 - m}$.

b) Check the solution for extraneous roots.

Remember that an extraneous root is one that does not fit the restrictions, or does not make sense in the context of the question.

c) State the restrictions on the variable so that the equation involves real numbers.

Solution

a) Raise both sides of the equation to the exponent 2.

$$m = \sqrt{2 - m}$$
$$m^2 = (\sqrt{2 - m})^2$$
$$m^2 = \_\_\_\_\_\_\_\_$$

Set the quadratic equation equal to 0 by subtracting the binomial from the right side.

$m^2 + \_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_ = 2 - m + m - \_\_\_\_\_\_\_\_$

$m^2 + \_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_ = 0$

Solve the quadratic equation by either factoring or using the quadratic formula.

Method 1: Use Factoring

$$m^2 + m - 2 = 0$$

$(\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_) = 0$

Use the zero product property by setting each factor equal to zero.

$m + 2 = 0$ $\qquad$ $m - 1 = 0$

$m = \_\_\_\_\_\_\_\_$ $\qquad$ $m = \_\_\_\_\_\_\_\_$

So, $m = \_\_\_\_\_\_\_\_$ and $\_\_\_\_\_\_\_\_$.

Method 2: Use the Quadratic Formula

$m^2 + m - 2 = 0$

What is the coefficient of a variable that does not have a number in front of it?

$a = \_\_\_\_\_\_\_\_$ $\qquad$ $b = \_\_\_\_\_\_\_\_$ $\qquad$ $c = \_\_\_\_\_\_\_\_$

Substitute into the quadratic formula, $m = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$m = \frac{-\boxed{} \pm \sqrt{\boxed{}^2 - 4(\boxed{})(-2)}}{2(\boxed{})}$$

When multiplying, do not forget about the negative sign that precedes 4 in the term "$-4ac$." What is $-4(-2)$?

$$= \frac{-1 \pm \sqrt{\boxed{}}}{2}$$

So, $m = \_\_\_\_\_\_\_\_$ and $\_\_\_\_\_\_\_\_$.

b) Check the two solutions by substituting into the original equation: $m = \sqrt{2 - m}$.

For $m = 1$:

Left Side	Right Side
m	$\sqrt{2 - m}$
1	$= \sqrt{2 - \mathbf{1}}$
1	= ________

Left Side ________ Right Side
(= or ≠)

For $m = -2$:

Left Side	Right Side
m	$\sqrt{2 - m}$
−2	$= \sqrt{2 - (\mathbf{-2})}$
−2	= ________

Left Side ________ Right Side
(= or ≠)

Therefore, the solution is $m =$ ________. The value $m = -2$ is ____________________.

c) The radicand $2 - m$ must be positive for the radical equation to have a real value.

$$2 - m \geq 0$$
$$2 - m + m \geq 0 + m$$
$$2 \geq m$$

Written another way, m ________ 2. So, the value of m must be less than or equal to 2 for the value of the radical equation to be a real number. The solution $m = 1$ satisfies this condition.

Working Example 3: Solve an Equation With Two Radicals

Solve the radical equation $\sqrt{2x + 5} = 2\sqrt{2x} + 1$, $x \geq 0$. Check your solution.

Solution

$\sqrt{2x + 5} = 2\sqrt{2x} + 1$

Square both sides.

$$(\sqrt{2x + 5})^2 = (2\sqrt{2x} + 1)^2$$
$$\_\_\_\_\_\_\_\_ = (\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_)$$
$$2x + 5 = 4(2x) + \_\_\_\_\_\_\_\_ + 1$$

When squaring a binomial, a trinomial is formed.

$$2x + 5 = 8x + 4\sqrt{2x} + 1$$
$$2x + 5 - 8x - 1 = 8x - 8x + 4\sqrt{2x} + 1 - 1$$

Isolate the remaining radical.

$$-6x + 4 = 4\sqrt{2x}$$
$$(-6x + 4)^2 = (4\sqrt{2x})^2$$

Square both sides.

$$(\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_) = (\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_)$$
$$36x^2 - \_\_\_\_\_\_\_\_ + 16 = 32x$$

Simplify.

Set the quadratic equal to zero.

$$36x^2 - 48x - \underline{\qquad\quad} + 16 = 32x - \underline{\qquad\quad}$$

$$36x^2 - 80x + 16 = 0$$

$\underline{\qquad\quad}(9x^2 - 20x + \underline{\qquad\quad}) = 0$ Remove common factor.

$4(9x - \underline{\qquad\quad})(x - \underline{\qquad\quad}) = 0$ Factor.

Using the zero property, set each factor equal to zero and solve.

$9x - 2 = 0$	$x - 2 = 0$
$9x = \underline{\qquad\quad}$	$x = \underline{\qquad\quad}$
$x = \underline{\qquad\quad}$	

Check $x = \frac{2}{9}$ and $x = 2$ by substituting into the original equation: $\sqrt{2x+5} = 2\sqrt{2x} + 1$.

For $x = \frac{2}{9}$:

Left Side	Right Side
$\sqrt{2x+5}$	$2\sqrt{2x} + 1$
$= \sqrt{2\left(\mathbf{\frac{2}{9}}\right) + 5}$	$= \sqrt[2]{2\left(\mathbf{\frac{2}{9}}\right)} + 1$
$= \underline{\qquad\quad}$	$= \sqrt[2]{\frac{4}{9}} + 1$
$= \underline{\qquad\quad}$	$= \underline{\qquad\quad}$
$= \frac{7}{3}$	$= \underline{\qquad\quad}$

Left Side $\underline{\qquad\quad}$ Right Side
(= or ≠)

For $x = 2$:

Left Side	Right Side
$\sqrt{2x+5}$	$2\sqrt{2x} + 1$
$= \sqrt{2(\mathbf{2}) + 5}$	$= \sqrt{2(\mathbf{2})} + 1$
$= \underline{\qquad\quad}$	$= \underline{\qquad\quad}$
$= \underline{\qquad\quad}$	$= \underline{\qquad\quad}$
$= \underline{\qquad\quad}$	$= \underline{\qquad\quad}$

Left Side $\underline{\qquad\quad}$ Right Side
(= or ≠)

Therefore, $x = \frac{2}{9}$ is a solution, but $x = 2$ is an extraneous root.

See pages 297–298 of *Pre-Calculus 11* for similar examples.

Check Your Understanding

Practise

1. Square each expression.

a) $4x - 5$

$(4x - 5)^2 = (\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_)$

$= \_\_\_\_\_\_\_\_$

b) $\sqrt{7y}$, $y \geq 0$

c) $\sqrt{2x - 3}$, $x \geq \frac{3}{2}$

d) $6\sqrt{9m}$, $m \geq 0$

$(6\sqrt{9m})^2 = (6\sqrt{9m})(6\sqrt{9m})$

$= \_\_\_\_\_\_\_\_ (9m)$

$= \_\_\_\_\_\_\_\_$

e) $3\sqrt{n} - 8$, $n \geq 0$

#1 uses the same concepts as #1 on page 300 of *Pre-Calculus 11*.

2. Mitchell obtained $x = 6$ and $x = 2$ as the solutions to the radical equation $3 = x + \sqrt{2x - 3}$. Do you agree with Mitchell's solutions? Show your reasoning.

For $x =$ ________:

Left Side	Right Side

Left Side ________ Right Side
(= or ≠)

For $x =$ ________:

Left Side	Right Side

Left Side ________ Right Side
(= or ≠)

Therefore, ____.

#2 uses the same concepts as #5 on page 300 of *Pre-Calculus 11*.

3. Solve. State any restrictions and check for extraneous roots.

a) $\sqrt{x - 2} = 9$

Square both sides.

Solve for x.

Verify.

b) $10 = \frac{\sqrt{m}}{10}$

c) $-8 + \sqrt{5a - 5} = -3$

d) $p = \sqrt{2 - p}$

e) $-n + \sqrt{6n + 19} = 2$

f) $\sqrt{7c - 54} - c = -6$

#3 uses the same concepts as #3, 4, and 6 on page 300 of *Pre-Calculus 11*.

4. Solve. State any restriction on variables.

a) $\sqrt{9x^2 + 4} = 3x + 2$

b) $\sqrt{2x^2 - 9} = 3 - x$

c) $\sqrt{x}\left(\sqrt{x - 7}\right) = 12$

#4 uses the same concepts as #7 and 8 on page 301 of *Pre-Calculus 11*.

5. Solve the following equation with two radicals. Check for extraneous roots.

a) $\sqrt{3n} = \sqrt{4n - 1}$

Square both sides.

Solve for n.

Verify.

b) $\sqrt{\dfrac{x}{10}} = \sqrt{3x - 58}$

c) $\sqrt{2y + 3} = 1 + \sqrt{y + 1}$

d) $\sqrt{c + 7} + \sqrt{2c - 3} = 4$

#5 uses the same concepts as #9 and 10 on page 301 of *Pre-Calculus 11*.

Apply

6. Examine the following steps that Su Ling used to solve the equation $4 + \sqrt{-3m + 10} = m$. Is her work correct? If not, solve correctly. Show your reasoning.

$$4 + \sqrt{-3m + 10} = m$$

$$\sqrt{-3m + 10} = m - 4$$

$$\sqrt{-3m + 10}^2 = (m - 4)^2$$

$$-3m + 10 = m^2 - 16$$

$$0 = m^2 + 3m - 26$$

Substitute a, b, and c into the quadratic formula and solve.

$a = 1$, $b = 3$, $c = -26$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-\mathbf{3} \pm \sqrt{\mathbf{3}^2 - 4(\mathbf{1})(\mathbf{-26})}}{2(\mathbf{1})}$$

$$m = \frac{-3 \pm \sqrt{113}}{2}$$

#6 uses the same concepts as #12 on page 301 of *Pre-Calculus 11*.

7. The kinetic energy, E_k, in joules, of a moving object can be expressed as $E_k = \frac{1}{2}mv^2$, where m is the mass, in kilograms, and v is the speed at which the body is moving, in metres per second.

a) If the kinetic energy of a car with a mass of 1000 kg is 189 000 J, determine the car's speed to the nearest hundredth of a metre per second.

b) State the speed of the object to the nearest kilometre per hour.

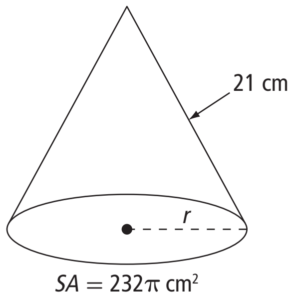
8. The speed, V, in feet per second of outflow of a liquid from an orifice is given by the formula $V = 8\sqrt{h}$, where h is the height, in feet, of the liquid above the opening. How high, to the nearest hundredth of a foot, is a liquid above an orifice if the velocity of outflow is 50 ft/s?

9. Strings of certain musical instruments are under tension. When they are plucked or struck, the speed of the wave of the string can be calculated using the formula $V = \sqrt{\frac{FL}{M}}$, where

- V represents the speed of a wave on a string in metres per second
- F represents the force of the tension in newtons
- M represents the mass per unit length in kilograms
- L represents the unit length in metres

If a wave travels through a string with a mass of 0.2 kg at a speed of 9 m/s, it is stretched by 10.6 N. Use the formula to find the length of the string to the nearest thousandth of a metre.

10. The surface area of a cone (SA) with slant height (h) and radius of base (r) can be found using the formula $SA = \pi r^2 + \pi rh$. Find the radius of the base of the cone shown here.



Connect

11. Why does $a^2 - b^2 \neq (a - b)^2$?

a) Explain algebraically.

b) Explain by substituting values for a and b.

c) Are there any values for a and b for which the two sides of the equation are equal? Explain.

12. Sandra was absent for the lesson on solving radical equations. Your teacher has asked you to help her catch up. Explain, step by step, the procedure for solving the equation $\sqrt{4n + 8} - 3 = n$.

13. a) Show that there is no solution to the equation $\sqrt{2a + 9} - \sqrt{a - 4} = 0$.

b) State an example of another radical equation that has no solution. Show why it has no solution.

Chapter 5 Review

5.1 Working With Radicals, pages 188–198

1. Convert each entire radical to a mixed radical in simplest form. State any restrictions on the variable(s).

a) $\sqrt{288}$

b) $\sqrt{128c^2}$

c) $\sqrt{24a^4b^3}$

d) $\sqrt[3]{250x^3y^5}$

2. Convert each mixed radical to an entire radical. State any restriction on the variable(s).

a) $4\sqrt{6} = \sqrt{\left(\boxed{}\right)^2(6)}$

$= \sqrt{\boxed{}}$

b) $-5m\sqrt{7}$

c) $3y\sqrt[3]{2y^2}$

d) $-2x\sqrt[4]{6xy^3}$

3. Simplify. State any restrictions on the values for the variables.

a) $3\sqrt{6} - 4\sqrt{6} =$ ________ $\sqrt{6}$

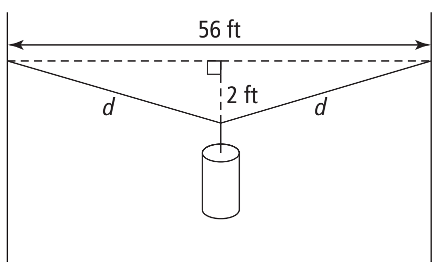
b) $-\sqrt{45} + 2\sqrt{5} - \sqrt{20}$

c) $-3\sqrt{18} + 3\sqrt{8x} - \sqrt{32x^3}$

d) $2\sqrt[3]{6x^2y} - \sqrt[3]{48x^2y}$

4. Put the following values in ascending order: $3\sqrt{30}$, $\sqrt{250}$, 16, $4\sqrt{15}$.

5. A wire is pulled taut between two posts. A weight is placed in the middle of the wire, which pulls the wire down at its centre by 2 ft. How long is the wire after the weight is place on it? Write the answer in simplest radical form.



5.2 Multiplying and Dividing Radical Expressions, pages 199–209

6. Multiply. Express each product in simplest form. State any restrictions on the values for the variables.

a) $(\sqrt{6})(\sqrt{14})$

b) $(\sqrt{3x^2})(2\sqrt{3x^4})$

$= \_\_\_\_\_\_\_\_ \sqrt{(\boxed{})(\boxed{})}$

$= \_\_\_\_\_\_\_\_, x \_\_\_\_\_\_\_\_ 0$

c) $(-10y\sqrt{5})(4\sqrt{50})$

d) $(5 - 4\sqrt{3})(3 + 3\sqrt{3})$

e) $(\sqrt{2} - 3\sqrt{5r})^2$

f) $(3 - \sqrt{2x})(3 + \sqrt{2x})$

7. Rationalize each denominator. State any restrictions on the values for the variable(s).

a) $\frac{4}{\sqrt{5}} = \left(\frac{4}{\sqrt{5}}\right)\left(\frac{\boxed{}}{\boxed{}}\right)$

$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$

b) $\frac{-\sqrt{2}}{8\sqrt{3}}$

c) $\frac{3}{\sqrt{5} + 4}$

d) $\frac{3 + 4\sqrt{3}}{\sqrt{2} + 2\sqrt{5}}$

e) $\frac{\sqrt{15xy}}{\sqrt{10xy^3}}$

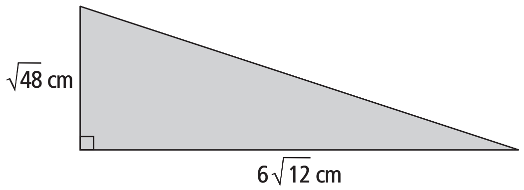
f) $\frac{3n^2 + \sqrt{2n^2}}{\sqrt{10n}}$

8. Write the conjugate of each expression.

a) $\sqrt{3k} - 5$

b) $-3\sqrt{2} - 4\sqrt{7}$

9. For the given right triangle, express the following in simplest radical form.



a) the perimeter

b) the area

5.3 Radical Equations, pages 211–222

10. Solve each radical equation. State any restrictions on the values for the variable(s).

a) $-8 + \sqrt{5a - 5} = -3$, $a \geq$ ________

$\sqrt{5a - 5} =$ ________

$(\sqrt{5a - 5})^2 = ($________$)^2$

________ $=$ ________

$5a =$ ________

$a =$ ________

b) $\sqrt{2n - 88} = \sqrt{\frac{n}{6}}$

c) $b - 6 = \sqrt{18 - 3b}$

d) $\sqrt{x + 4} - \sqrt{x - 4} = 2$

11. Two adjacent sides of a parallelogram have the measures $\sqrt{14n - 45}$ cm and $2n$ cm. Determine the actual lengths of the two sides if the perimeter of the parallelogram is 54 cm.

12. The Japanese game called Chu Shogi uses a square board. The board is covered with smaller squares that are alternating black and white. Each of these squares is 3 cm by 3 cm. If the diagonal of the square playing board is $\sqrt{2592}$ cm, how many small squares are on the board?

Chapter 5 Skills Organizer B

Complete the organizer for the concepts in Section 5.3, Radical Equations.