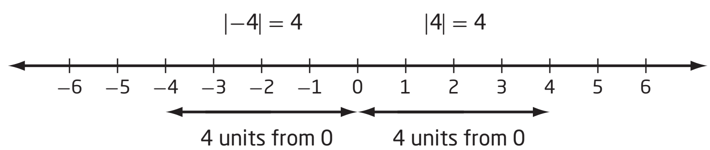
Chapter 7 Absolute Value and Reciprocal Functions

7.1 Absolute Value

KEY IDEAS

- Absolute value represents the distance from zero on a number line, regardless of the direction.



- Vertical bars around a number or expression represent its absolute value: $|-24|$, $|3 - 12|$.
- Absolute value is always zero or positive.
  - The absolute value of a positive number is positive: $|9| = 9$
  - The absolute value of a negative number is positive: $|-2| = 2$
  - The absolute value of zero is zero: $|0| = 0$

Working Example 1: Determine the Absolute Value of a Number

Evaluate.

a) $|25|$

b) $|-8|$

Solution

a) $|25| = 25$
By definition, $|a| = a$, for $a \geq$ ________.

b) $|-8| = -(-8)$
$= 8$
By definition, $|a| = -a$, for $a <$ ________.

See page 360 of *Pre-Calculus 11* for more examples.

Working Example 2: Compare and Order Absolute Values

Write the following real numbers in order from least to greatest.

$|-10.1|$, $\left|\frac{-21}{2}\right|$, -9.8, $|10|$, -10, $|-9.9|$, $\left|-9\frac{7}{8}\right|$, -9

Solution

Evaluate each number and express it in decimal form.

$|-10.1| =$ ________ $\left|\frac{-21}{2}\right| = 10.$________

$-9.8 = -9.8$ $|10| =$ ________

$-10 = -10$ $|-9.9| =$ ________

$\left|-9\frac{7}{8}\right| = 9.$________ $-9 =$ ________

Arrange the numbers in order: -10, ________, -9, 9.875, ________, 10, 10.1, ________

Show the original numbers in order: -10, -9.8, ________, $\left|-9\frac{7}{8}\right|$, ________, 10, ________, ________

This question is similar to Example 2 on page 361 of *Pre-Calculus 11*.

Working Example 3: Evaluate Absolute Value Expressions

Evaluate.

a) $|5 - 6|$ **b)** $|2| - |3(-4)|$ **c)** $|5(-2)^2 + 7(-3) - 15|$

Solution

a) $|5 - 6| = |$________$|$ Evaluate the expression inside the absolute value symbol.

$=$ ________ Evaluate.

Therefore, $|5 - 6| =$ ________.

b) $|2| - |3(-4)| = |2| - |$________$|$ Evaluate the expressions inside the absolute value symbol.

$=$ ________ $-$ ________ Evaluate the absolute values.

$=$ ________ Subtract.

Therefore, $|2| - |3(-4)| =$ ________________.

c) $|5(-2)^2 + 7(-3) - 15| = |5($________$) + 7(-3) - 15|$ Evaluate the power.

$= |$________$|$ Simplify the expression inside the absolute value symbol.

$=$ ________ Evaluate.

Therefore, $|5(-2)^2 + 7(-3) - 15| =$ ________.

This question is similar to Example 3 on page 361 of *Pre-Calculus 11*.

Check Your Understanding

Practise

1. Evaluate

a) $\left|\frac{-5}{6}\right|$

b) $\left|-1\frac{1}{4}\right|$

c) $|0.8|$

d) $|12|$

e) $|-1.2|$

f) $|0|$

2. Order the numbers from greatest to least.

$|-2.5|$, $\left|\frac{-15}{7}\right|$, -2, $-2\frac{6}{9}$, $|-2.09|$, $2\frac{3}{5}$

Determine each absolute value. Then, express each number in decimal form.

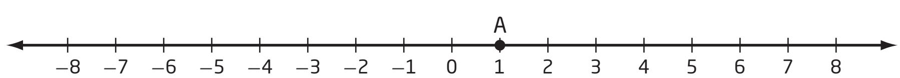
Order the decimal numbers from greatest to least.

____

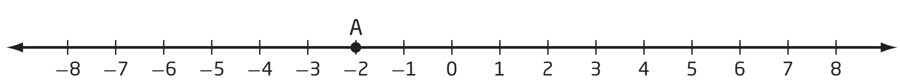
Show the original numbers in order from greatest to least.

____

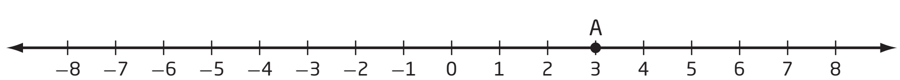
3. Given point A and the distance from A to B, show and label point B.

a) 

distance of 6 units

b) 

distance of 5 units

c) 

distance of 4 units

This question is similar to #5 on page 363 of *Pre-Calculus 11*.

4. Evaluate each expression.

a) $|1.2 - 1.5|$

$= |\_\_\_\_\_\_\_\_|$ Evaluate the expression inside the absolute value symbol.

$= \_\_\_\_\_\_\_\_$ Evaluate.

b) $|-11| - |-19|$

$= \_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_$

c) $|-11 - (-19)|$

$= |\_\_\_\_\_\_\_\_|$

$= \_\_\_\_\_\_\_\_$

d) $-4\left|\frac{1}{2} - \frac{3}{4}\right|$ Evaluate the expression inside the absolute value symbol.

$= -4\left|\frac{\square}{\square} - \frac{\square}{\square}\right|$ Find a common denominator.

$= -4\left|\frac{\square}{\square}\right|$ Subtract the fractions.

$= -4\left(\frac{\square}{\square}\right)$ Evaluate the absolute value.

$= \_\_\_\_\_\_\_\_$ Multiply.

e) $|1.2 - 1.5|^2$

$= |\_\_\_\_\_\_\_\_|^2$ Evaluate the expression inside the absolute value symbol.

$= (\_\_\_\_\_\_\_\_)^2$ Evaluate the absolute value.

$= \_\_\_\_\_\_\_\_$ Square the result.

f) $|-4 + 3^2| - |8 - (-10)| + |9 - 16| + |-10|$

$= \_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_$

$= \_\_\_\_\_\_\_\_$

This question is similar to #6 on page 363 of *Pre-Calculus 11*.

Apply

5. The surface of Great Slave Lake is 156 m above sea level.
At the deepest point, the bottom of the lake is 458 m below sea level.

a) Use absolute value symbols to write a statement to determine the depth of the lake.

b) What is the depth of the lake?

The depth of the lake is ______________.

6. The melting point of mercury is −39 °C. Its boiling point is 357 °C.

a) Use absolute value symbols to write a statement to determine the number of degrees between the boiling point and the melting point.

b) How many degrees is it?

The number of degrees is ______________.

7. Use the wind chill chart to answer the following questions.

Wind Speed (km/h)	Air Temperature (°C)		
	−15	−20	−25
15	−23	−29	−35
20	−24	−31	−37
25	−25	−32	−38
30	−26	−33	−39
35	−27	−33	−40
40	−27	−34	−41

a) The air temperature is −15 °C. The wind gusts from 15 km/h to 40 km/h. Use an absolute value expression to determine the change in wind chill temperature for the two wind speeds.

b) The air temperature is −25 °C. Use an absolute value expression to determine the change in wind speed that would change a wind chill reading of −35 °C to −40 °C.

8. A stock opened Monday's trading at $25.98/share. On Tuesday, the stock's value dropped by $2.31/share. On Wednesday, it dropped by $0.75/share, and on Thursday it rose by $1.15/share. On Friday the stock closed trading at $26.83/share.

a) Write a statement using absolute value symbols to show its change in value from Monday's opening to Friday's closing.

b) What was the change?

Connect

9. The formula for the distance between two points (x_1, y_1) and (x_2, y_2) on a Cartesian plane is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

How does the formula above compare to the following formula: $d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$?

Differences: ______________________________

Similarities: ______________________________

10. The number of strokes an expert golfer should take to play a hole is called "par" for the hole. The first table shows golf terms related to the number of strokes above or below par. The second table shows par values for the first nine holes for the Victoria golf course in Edmonton's River Valley.

Golf Term	Number of Strokes Above or Below Par
Double bogey	+2
Bogey	+1
Par	0
Birdie	−1
Eagle	−2

Par	Holes
4	1, 2, 6, 8, 9
3	3, 7
5	4, 5

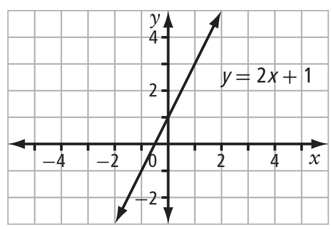
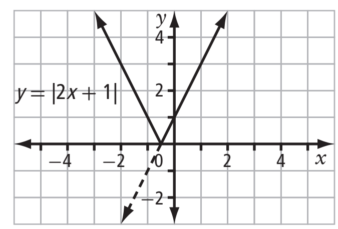
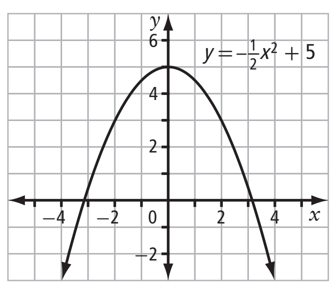
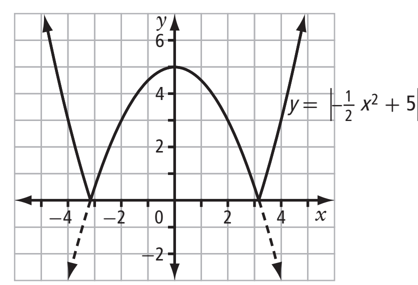
a) Write two problems that each use absolute value symbols and the information in the tables.

b) Solve each problem.

7.2 Absolute Value Functions

KEY IDEAS

- An absolute value function is a function that involves the absolute value of a variable. The graphs show some functions and their related absolute value functions.

Regular Function $y = f(x)$	**Related Absolute Value Function** $y = \lvert f(x) \rvert$
	
	

- To graph an absolute value function:
  - Make a table of values, and then graph the function.
  - Use $y = f(x)$ and $y = -f(x)$ to sketch the function. For $y = -f(x)$, all negative y-values are reflected in the x-axis.
- To analyse an absolute value function graphically:
  - First, graph the function.
  - Then, identify the characteristics of the graph, such as the x-intercept, y-intercept, minimum values, and domain and range.
- You can write any absolute value function, $y = \lvert f(x) \rvert$, as a piecewise function:

$$y = \begin{cases} f(x), \text{ if } f(x) \geq 0 \\ -f(x), \text{ if } f(x) < 0 \end{cases}$$

A piecewise function is made up of "pieces" of different functions. Each piece of the function has its own specific domain.

- The domain of an absolute function, $y = \lvert f(x) \rvert$, is the same as the domain of $y = f(x)$.
- The range of an absolute function, $y = \lvert f(x) \rvert$, depends on the range of $f(x)$. For an absolute linear or absolute quadratic function, the range will usually, but not always, be $\{y \mid y \geq 0, y \in \mathrm{R}\}$.
- An invariant point is a point that remains unchanged when a transformation is applied to it.

Working Example 1: Graph an Absolute Value Function of the Form $y = |ax + b|$

Consider the absolute value function $y = |3x - 1|$.

a) Determine the y-intercept and x-intercept.

b) Sketch the graph.

c) State the domain and range.

d) Express as a piecewise function.

Solution

a) The y-intercept occurs where the graph crosses the y-axis. The x-value at the y-intercept is ________.

Substitute, and solve for y.

$y = |3x - 1|$

$y = |3(\_\_\_\_\_\_\_\_) - 1|$

$y = |\_\_\_\_\_\_\_\_|$

$y = \_\_\_\_\_\_\_\_$

The y-intercept occurs at (0, ________).

The x-intercept occurs where the graph crosses the ____________________.

The y-value at the x-intercept is 0.

To determine the x-intercept, let $y =$ ________ and solve for x.

$y = |3x - 1|$

$\_\_\_\_\_\_\_\_ = 3x - 1$

$\_\_\_\_\_\_\_\_ = 3x$

$\_\_\_\_\_\_\_\_ = x$

Why was the absolute value symbol removed? What does $|0|$ equal?

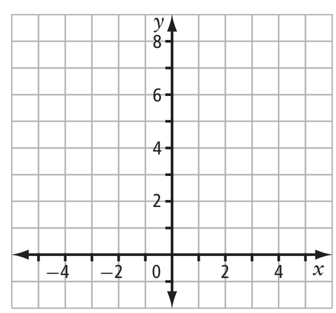
The x-intercept occurs at (________, 0).

b) Method 1: Sketch Using a Table of Values

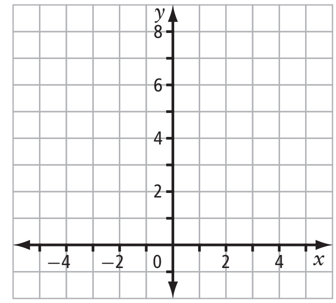
Complete the table:

x	$y = \|3x - 1\|$
0	
	0
2	
	8
−1	
−2	
−3	

Graph the function.



Method 2: Sketch Using the Graph of $y = 3x - 1$



Sketch the graph of $y = 3x - 1$, which is a line with a slope of ________ and a y-intercept of ________.

Use the x-intercept from part a): (________, 0).

Reflect all the points below the x-axis in the x-axis.
Use red to show the parts of the graph that represent $y = |3x - 1|$.

c) Any real number can be substituted for x into the function $y = |3x - 1|$.

Therefore, the domain is $\{x \mid x \in$ ________________$\}$.

For all values of x, $|3x - 1| \geq 0$, or $y \geq$ ________.

Therefore, the range is $\{y \mid y \geq$ ________________, $y \in$ ________$\}$.

d) The V-shaped graph of the absolute function $y = |3x - 1|$ is composed of two separate linear functions, each with its own domain.

When $x \geq \frac{1}{3}$, the graph of $y = |3x - 1|$ is the graph of $y =$ ________________.

When $x < \frac{1}{3}$, the graph of $y = |3x - 1|$ is the graph of $y = 3x - 1$ reflected in the ____-axis.

The equation of the reflected graph is $y = -(3x - 1)$ or $y = -3x$________________.

As a piecewise function, the function is $y = \begin{cases} 3x - 1, \text{ if } x \geq \text{\_\_\_\_\_\_\_\_} \\ -(\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}), \text{ if } x < \frac{1}{3} \end{cases}$

Working Example 2: Graph an Absolute Value Function of the Form $f(x) = |ax^2 + bx + c|$

Consider the absolute value function $y = |x^2 - 3x - 4|$.

a) Determine the y-intercept and x-intercept.

b) Graph the function.

c) Determine the domain and range.

d) Express as a piecewise function.

Solution

a) Determine the y-intercept by substituting $x =$ ________ in the function and solving for y.

$f(x) = |x^2 - 3x - 4|$

$f(x) = |($________$)^2 - 3($________$) - 4|$

$f(x) = |$________________________$|$

$f(x) =$ ________

The y-intercept occurs at (0, ________).

Determine the x-intercept by substituting $y =$ ________ into the function and solving for x.

$y = |x^2 - 3x - 4|$

________ $= |x^2 - 3x - 4|$

________ $= x^2 - 3x - 4$

________ $= (x - 4)($________$)$

$x - 4 = 0$ or ________ $= 0$

$x =$ ________ $x =$ ________

The x-intercepts occur at (________, 0) and (________, 0).

b) Use the graph of $y = f(x)$ to graph $y = |f(x)|$.

Complete the square to convert the function $y = x^2 - 3x - 4$ to vertex form, $y = a(x - h)^2 + k$.

$y = \left[x^2 - 3x + \left(\frac{3}{2}\right)^2\right] - 4 +$ ________

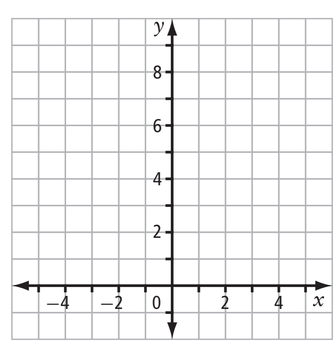
$y = (x$ ________________$)^2 -$ ________

Since $h =$ ________ and $k =$ ________, the coordinates of the vertex are (________, ________).

Since $a > 0$, the parabola opens ________________________.
(*upward* or *downward*)

Sketch the graph of $y = x^2 - 3x - 4$.

Reflect in the x-axis the part of the graph of $y = x^2 - 3x - 4$ that lies below the x-axis.



c) Any real number can be substituted for x into the function $y = |x^2 - 3x - 4|$.

The domain is $\{x \mid x$ ______________$\}$.

The range is all non-negative values of y,

$\{y \mid y \geq$ ________, $y \in$ ________$\}$.

d) The graph of $y = |x^2 - 3x - 4|$ consists of two separate quadratic functions. Use the x-intercepts to identify the specific domain for each function.

- When $x < -1$ and $x > 4$, the graph of $y = |x^2 - 3x - 4|$ is the graph of $y = x^2 - 3x - 4$;
  - opens ____________________ (*upward* or *downward*)
  - vertex at (______, −6.25)
  - y-intercept at (0, ________)
  - x-intercepts at (______, 0) and (______, 0)
- When $-1 \leq x \leq 4$, the graph of $y = |x^2 - 3x - 4|$ is the graph of $y = x^2 - 3x - 4$ reflected in the ______-axis. The equation of the reflected graph is $y = -(x^2 - 3x - 4)$ or $y =$ ________:
  - opens ____________________ (*upward* or *downward*)
  - vertex at (1.5, ______)
  - y-intercept at (0, ______)
  - x-intercepts at (______, 0) and (______, 0)

The function $y = |x^2 - 3x - 4|$ can be written as a piecewise function:

$$y = \begin{cases} x^2 - 3x - 4, \text{ if } x < \underline{\hspace{2cm}} \text{ or } x > \underline{\hspace{2cm}} \\ -(x^2 - 3x - 4), \text{ if } -1 \leq x \leq \underline{\hspace{2cm}} \end{cases}$$

This question is similar to Example 2 on pages 372–374 of *Pre-Calculus 11*.

Check Your Understanding

Practise

1. Given the values for $y = f(x)$, complete each table for $y = |f(x)|$.

a)

x	$y = f(x)$	$y = \|f(x)\|$
−4	0	
−2	−6	
0	−12	
2	−14	

b)

x	$y = f(x)$	$y = \|f(x)\|$
−2	7	
−1	4	
0	1	
1	−2	
2	−5	

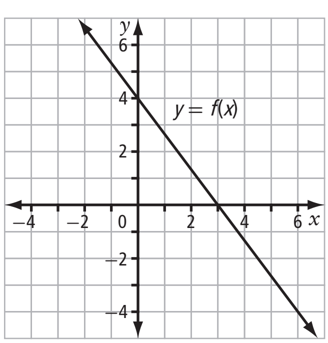
2. In the table, the first column is a point on the graph of $y = f(x)$. Complete the second column for the corresponding point on $y = |f(x)|$.

$f(x)$	$\|f(x)\|$
(−1, −10)	
(0, −6)	
(1, −2)	
(2, 2)	

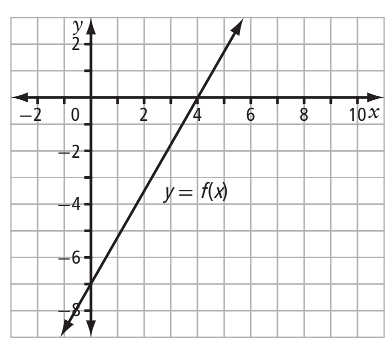
Questions 1 and 2 are similar to #1and 2 on page 375 of *Pre-Calculus 11*.

3. Given each graph of $y = f(x)$, sketch the graph of $y = |f(x)|$ on the same grid.

a)



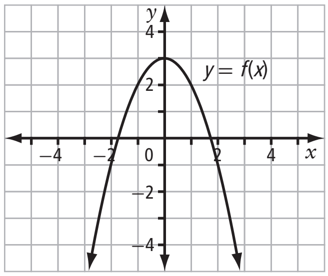
b)



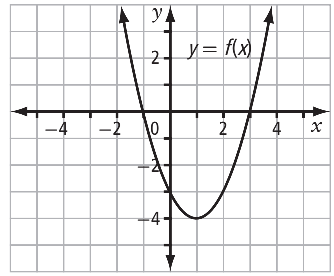
This question is similar to #5 on page 376 of *Pre-Calculus 11*.

4. Given each graph of $y = f(x)$, sketch the graph of $y = |f(x)|$ on the same grid.

a)



b)



This question is similar to #7 on page 376 of *Pre-Calculus 11*.

5. Sketch the graph of each absolute value function on the same grid.
Use different colours for each graph.
State the intercepts and the domain and range.

a) $y = |x + 2|$

x-intercept: (________, 0)

y-intercept: (0, ________)

domain: x ____________________

range: y ____________________

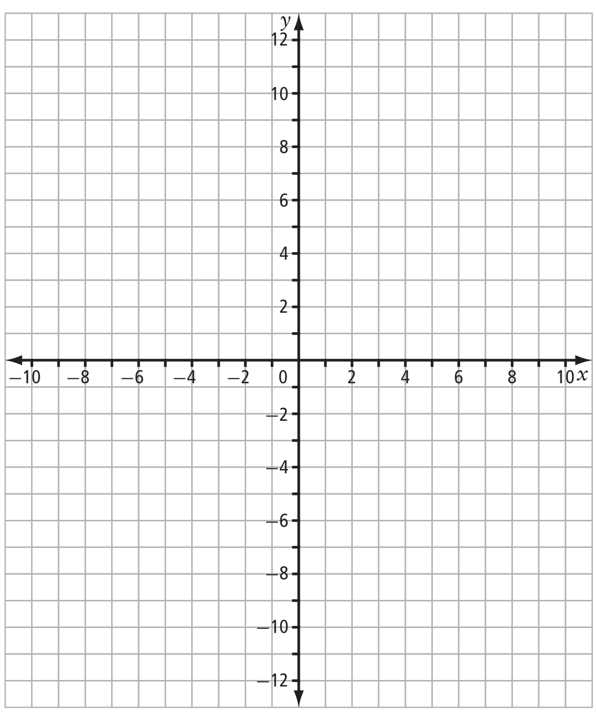
b) $y = |-(x - 1)^2 + 5|$

x-intercept(s): (________, 0)

y-intercept: (0, ________)

domain: x ____________________

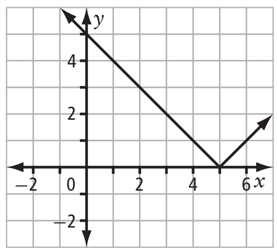
range: y ____________________



This question is similar to #3, 4, and 6 on pages 375 and 376 of *Pre-Calculus 11*.

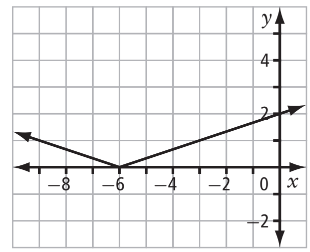
6. Write the piecewise function that represents each graph.

a)



$$y = \begin{cases} x - 5, \text{ if } x \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_} \\ -(x - 5), \text{ if } x \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_} \end{cases}$$

b)

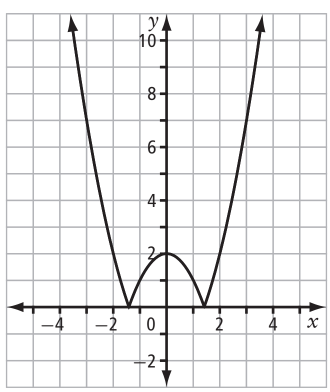


$$y = \begin{cases} \frac{1}{3}x + 2, \text{ if } x \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_} \\ -\left(\frac{1}{3}x + 2\right), \text{ if } x \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_} \end{cases}$$

This question is similar to #9 on pages 377 of *Pre-Calculus 11*.

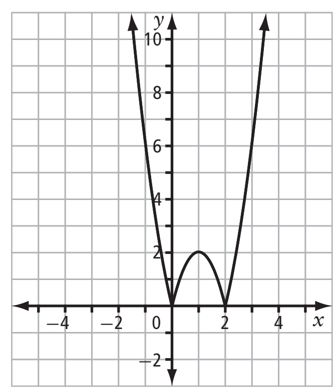
7. Write the piecewise function that represents each graph.

a)



$$y = \begin{cases} -x^2 + 2, \text{ if } \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_} \\ -(x^2 + 2), \text{ if } \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_} \end{cases}$$

b)



$$y = \begin{cases} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}, \text{ if } \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_} \\ \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}, \text{ if } \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_} \end{cases}$$

This question is similar to #10 on page 377 of *Pre-Calculus 11*.

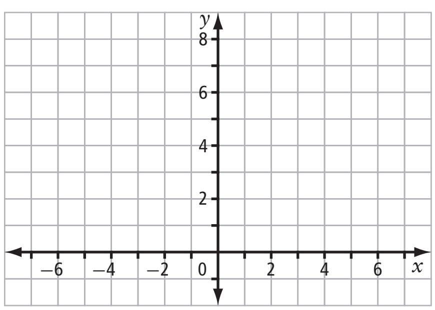
Apply

8. Consider the function $f(x) = \left|\frac{1}{2}x - 6\right|$.

a) Complete the table of values.

x	$y = \left\|\frac{1}{2}x - 6\right\|$
0	
	0
2	
4	
	3
−2	
−6	

b) Sketch the graph of the function.



c) Determine the domain and range.

domain: $\{x \mid$ ____________________ $\}$

range: $\{y \mid$ ____________________ $\}$

d) Express as a piecewise function.

$$y = \begin{cases} \frac{1}{2}x - 6, \text{ if } x \text{ \_\_\_\_\_\_\_\_\_\_} \\ \text{\_\_\_\_\_\_\_\_\_\_\_\_}, \text{ if } x < 12 \end{cases}$$

This question is similar to #12 on page 377 of *Pre-Calculus 11*.

9. Consider the function $f(x) = |x^2 + 4x - 3|$.

a) Express the function in vertex form, $y = |a(x - p)^2 + q|$.

b) What are the coordinates of the vertex?

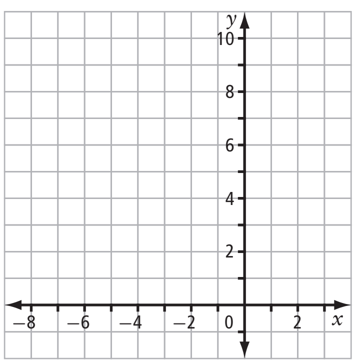
____

Where are the intercepts?

x-intercepts: (________, 0) and (________,0)

y-intercept: (0, ________)

c) Sketch the graph on the grid.



d) Determine the domain and range.

domain: $\{x \mid$ ____________________$\}$

range: $\{y \mid$ ____________________$\}$

e) Express as a piecewise function.

$$y = \begin{cases} x^2 + 4x - 3, \text{ if } x < \underline{\qquad\qquad} \text{ or } x > \underline{\qquad\qquad} \\ -(x^2 + 4x - 3), \text{ if } \underline{\qquad\qquad} \leq x \leq \underline{\qquad\qquad} \end{cases}$$

This question is similar to #13 on page 377 of *Pre-Calculus 11*.

10. Consider these three functions:

$f(x) = x^2 - 2x + 6$ $\qquad$ $g(x) = |x^2 - 2x + 6|$ $\qquad$ $h(x) = |-(x^2 - 2x + 6)|$

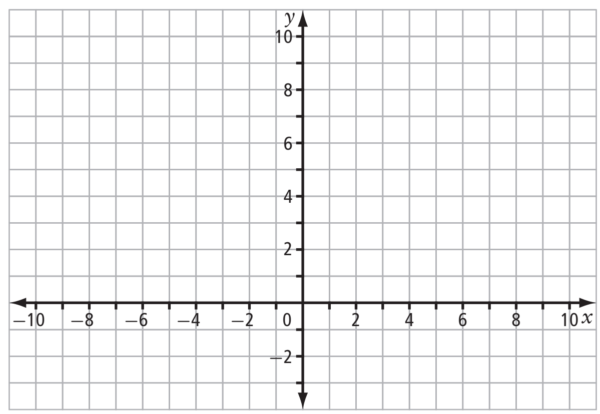
a) For what values of x is $f(x) < 0$? ______________________________

For what values of x is $g(x) < 0$? ______________________________

For what values of x is $h(x) < 0$? ______________________________

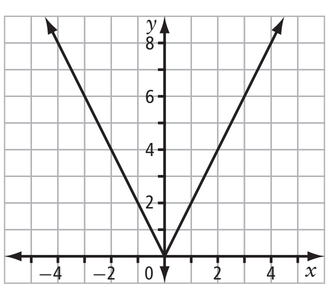
b) Without graphing, predict how the graphs of the three functions compare.

c) Check your prediction using technology. Sketch the display shown on your graphing calculator below.



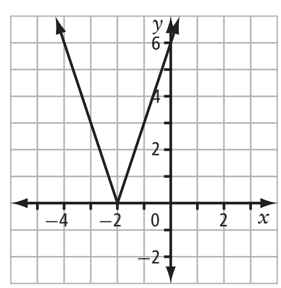
11. State an equation for each graph below.

a)



$y =$ ______________________________

b)



$y =$ ______________________________

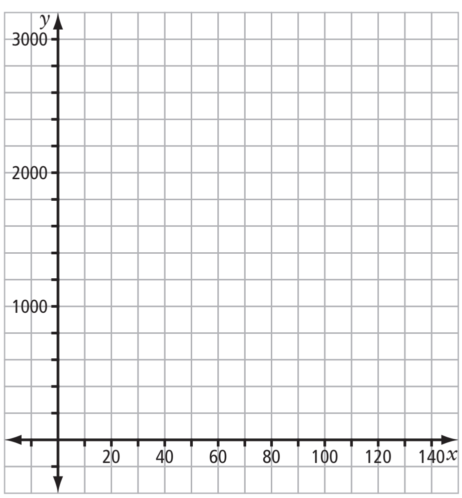
Connect

12. a) Does $|2 - x| = |x - 2|$ for all real values of x? Explain.

b) Does $|x^2| = x^2$ for all real values of x? Explain.

13. The amount of profit that a cell phone company makes is given by the equation $y = -x^2 + 150x - 2600$, where y is the profit in dollars and x is the number of phones sold.

a) Graph the function.



b) Use absolute value to change the function to show only a positive profit. Write the new function with absolute value symbols.

c) Between $x = 20$ and $x = 130$, what is the maximum profit that can be reached?

d) Is there a maximum profit for $x \in R$?

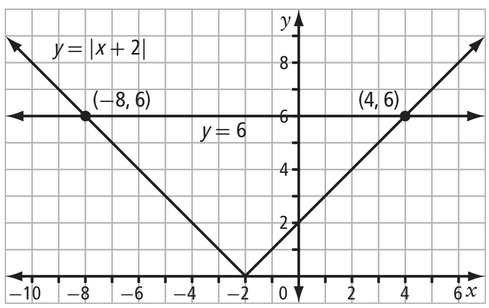
Explain. ______________________________

7.3 Absolute Value Equations

KEY IDEAS

- An absolute value equation includes the absolute value of an expression involving a variable. For example, $|x + 2| = 6$.
- To solve an absolute value equation by graphing:
  - Graph the left side and the right side of the equation on the same set of axes.
  - The point(s) of intersection are the solution(s).

  For example, to solve $|x + 2| = 6$ by graphing, graph $y = |x + 2|$ and $y = 6$ and identify the points of intersection.



  The solutions are $x = -8$ and $x = 4$.

- To solve an absolute value equation algebraically,
  - consider two separate cases:
    Case 1: The expression inside the absolute value symbol is greater than or equal to 0.
    Case 2: The expression inside the absolute value symbol is less than 0.
  - the roots in each case are the solutions
  - there may be extraneous roots that need to be identified and rejected

  For example, to solve $|x + 2| = 6$, consider the cases $x + 2 \geq 0$ and $x + 2 < 0$:

  Case 1
  When $x + 2 \geq 0$:
  $x + 2 = 6$
  $x = 4$

  Case 2
  When $x + 2 < 0$:
  $-(x + 2) = 6$
  $-x - 2 = 6$
  $x = -8$

- Verify solutions by substituting into the original equation.
- Since, by definition, absolute value is greater than or equal to zero, there can be no solution if $|f(x)| = a$, where $a < 0$.

Working Example 1: Solve an Absolute Value Equation

Solve $|x + 3| = 8$.

Solution

Method 1: Use Algebra

Use the definition of absolute value to determine the cases.

$$|x + 3| = \begin{cases} x + 3, \text{ if } x \geq -3 \\ -(x + 3), \text{ if } x < -3 \end{cases}$$

Case 1: $x + 3 \geq 0$

$x + 3 = 8$

$x =$ _____

The value _____ satisfies the condition $x \geq -3$.

Case 2: $x + 3 < 0$

$-(x + 3) = 8$

$-x - 3 = 8$

$-x =$ _____

$x =$ _____

The value _____ satisfies the condition $x < -3$.

Verify the solution algebraically by substitution.

For $x = 5$:

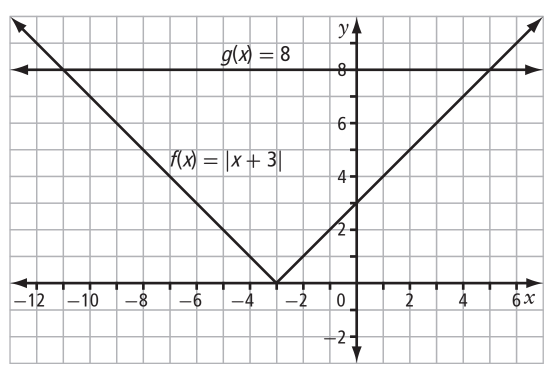
Left Side	Right Side
$\lvert x + 3\rvert$	8
$= \lvert$ _____ $+ 3\rvert$	
$=$ _____	
= Right Side	

For $x = -11$:

Left Side	Right Side
$\lvert x + 3\rvert$	8
$= \lvert$ _____ $+ 3\rvert$	
$= \lvert$ _____ $\rvert$	
$=$ _____	
= Right Side	

The solutions are $x =$ _____ and $x =$ _____.

Method 2: Use a Graph



Write the left side and right side as separate functions, $f(x) = |x + 3|$ and $g(x) = 8$.

Graph $f(x)$ and $g(x)$ on the same grid.

Determine where the functions intersect.

The graphs intersect at (_____, 8)

and (_____, _____).

Thus, $x =$ _____ and $x =$ _____ are the solutions to the equation $|x + 3| = 8$.

Verify by using technology. Enter $f(x) = |x + 3|$ and $g(x) = 8$ into a graphing calculator. Use the intersect feature to determine the points of intersection.

This question is similar to Example 1 on pages 382–383 of *Pre-Calculus 11*.

Working Example 2: Solve an Absolute Value Equation With an Extraneous Solution

Solve $|3x + 2| = 4x + 5$.

Solution

Determine the cases. Then, determine the restrictions on the domain.

$$|3x + 2| = \begin{cases} 3x + 2, \text{ if } x \geq \underline{\hspace{2cm}} \\ -(3x + 2), \text{ if } x < \underline{\hspace{2cm}} \end{cases}$$

Case 1: $3x + 2 \geq 0$

$3x + 2 = 4x + 5$

$x =$ ________

The value $x =$ ________ ______________ (*does* or *does not*) satisfy the condition $x \geq$ ________.

Case 2: $3x + 2 < 0$

$-(3x + 2) = 4x + 5$

$x =$ ________

The value $x =$ ________ ______________ (*does* or *does not*) satisfy the condition $x <$ ________.

Verify the solution algebraically by substitution.

For $x =$ ________:

Left Side	Right Side

For $x =$ ________:

Left Side	Right Side

Therefore, the solution is $x =$ ________.

Working Example 3: Solve an Absolute Value Equation With No Solution

Solve $|5x + 1| + 7 = 3$.

Solution

Isolate the absolute value expression.

$|5x + 1| + 7 = 3$

$|5x + 1| =$ ________

This statement is ______________________ (*always* or *never*) true because of the definition of absolute value.

Since the absolute value of a number must be greater than or equal to ________, by inspection this equation has ________ solution(s).

Working Example 4: Solve an Absolute Value Equation Involving a Quadratic Expression

Solve $|x^2 - 6x| = 8$.

Solution

Use the definition of absolute value to determine the cases.

$$|x^2 - 6x| = \begin{cases} x^2 - 6x, \text{ if } x \le 0 \text{ or } x \ge 6 \\ -(x^2 - 6x), \text{ if } \_\_\_\_ < x < \_\_\_\_ \end{cases}$$

Case 1: $x^2 - 6x \ge 0$

$x^2 - 6x = 8$

$x^2 - 6x$ ________ $= 0$ Move all terms to one side, equal to zero.

Use the quadratic formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a =$ ________, $b =$ ________, $c =$ ________

$$x = \frac{-(\_\_\_\_) \pm \sqrt{(\_\_\_\_)^2 - 4(\_\_\_\_)(\_\_\_\_)}}{2(\_\_\_\_)}$$

$x =$ ______________

Verify.

For $x =$ ______________:

Left Side	Right Side

For $x =$ ______________:

Left Side	Right Side

Case 2: $x^2 - 6x < 0$

$-(x^2 - 6x) = 8$

$-x^2$ ________ $= 8$

$-x^2 + 6x - 8 = 0$ Move all terms to one side, equal to zero.

(______________)(______________) $= 0$ Factor.

(______________) $= 0$ or (______________) $= 0$

$x =$ ______________ or $x =$ ______________

Verify.

For $x =$ ________:

Left Side	Right Side

For $x =$ ________:

Left Side	Right Side

The solutions are $x =$ ________, $x =$ ________, $x =$ ________, and $x =$ ________.

Working Example 5: Solve an Absolute Value Equation Involving a Linear and a Quadratic Equation

Solve $|x^2 - 1| = x$.

Solution

Use the definition of absolute value to determine the cases.

$$|x^2 - 1| = \begin{cases} x^2 - 1, \text{ if } x \le \_\_\_\_\_\_\_\_ \text{ or } x \ge \_\_\_\_\_\_\_\_ \\ -(x^2 - 1), \text{ if } \_\_\_\_\_\_\_\_ < x < \_\_\_\_\_\_\_\_ \end{cases}$$

Case 1: $x^2 - 1 \ge 0$

$$x^2 - 1 = x$$

x^2 ________________________ $= 0$ Move all terms to one side, equal to zero.

Use the quadratic formula to solve for x.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a =$ ________, $b =$ ________, $c =$ ________

$$x = \frac{-(\_\_\_\_\_\_\_\_) \pm \sqrt{(\_\_\_\_\_\_\_\_)^2 - 4(\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_)}}{2(\_\_\_\_\_\_\_\_)}$$

$x =$ ______________

Verify.

For $x =$ ______________:

Left Side	Right Side

For $x =$ ______________:

Left Side	Right Side

Case 2: $x^2 - 1 < 0$

$-(x^2 - 1) = x$

Set the equation equal to zero.

$-x^2$ ________ $= 0$ Move all terms to one side, equal to zero.

Use the quadratic formula to solve for x.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a =$ ________, $b =$ ________, $c =$ ________

$$x = \frac{-(\_\_\_\_\_\_\_\_) \pm \sqrt{(\_\_\_\_\_\_\_\_)^2 - 4(\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_)}}{2(\_\_\_\_\_\_\_\_)}$$

$x =$ ________________

Verify.

For $x =$ ________________:

Left Side	Right Side

For $x =$ ________________:

Left Side	Right Side

The solutions are $x =$ ________ and $x =$ ________.

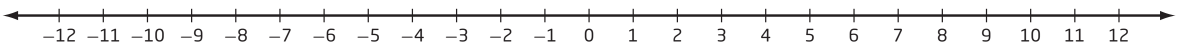
The solutions can also be verified graphically by using technology.
Enter $f(x) = |x^2 - 1|$ and $g(x) = x$ into your graphing calculator.
Use the intersect function to read the solutions. (Note: The values may be given as decimal approximations).

This question is similar to Example 6 on pages 387–388 of *Pre-Calculus 11*.

Check Your Understanding

Practise

1. Use the number line to solve each equation geometrically.



a) $|x| = 11$

b) $|x| - 6 = 0$

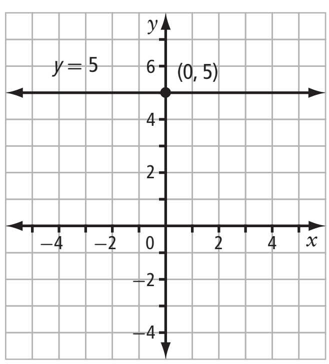
c) $|x| + 8 = 10$

d) $|x| - 2.5 = 7$

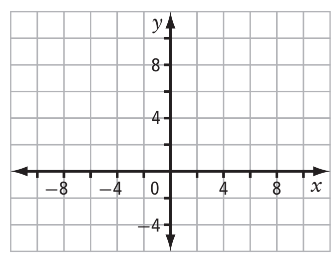
This question is similar to #1 on page 389 of *Pre-Calculus 11*.

2. Solve each absolute value equation by graphing.

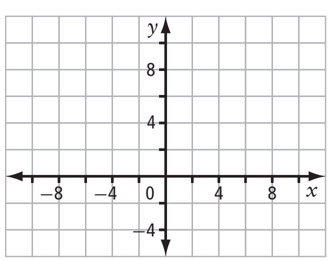
a) $|-2 + x| = 5$



b) $9 = |x + 3|$



c) $|5 - x| = 8$



3. Solve each absolute value equation. Verify your solutions.

a) $|3 + 7x| = 73$

Case 1: $3 + 7x \geq 0$

$3 + 7x = 73$

Case 2: $3 + 7x < 0$

$-(3 + 7x) = 73$

Verify.

For $x =$ ________:

Left Side	Right Side

For $x =$ ________:

Left Side	Right Side

The solutions are $x =$ ________ and $x =$ ________.

b) $|x + 3| = -5$

There is ________ solution because ________.

c) $|-5x| + 4 = -11$

4. Solve each absolute value equation. Verify your solutions.

a) $\dfrac{|x + 4|}{10} = 1$

Case 1: $x + 4 \geq 0$

Case 2: $x + 4 < 0$

b) $|x^2 + x - 2| = x + 3$

Apply

5. A machine fills containers with 32 ounces of oatmeal. After the containers are filled, another machine weighs them. If the container's weight differs from the desired 32-ounce weight by more than 0.5 ounces, the container is rejected.

a) Write an absolute value equation that can be used to find the heaviest and lightest acceptable weights for the container.

b) Solve the equation.

6. Your supervisor allows you to clock-in for your 8 a.m. shift up to 15 min before and up to 15 min after.

a) Write an absolute value equation that can be used to find the acceptable limits for your clock-in times.

b) Solve the equation.

7. To fit correctly, the width of a machine part can vary no more than 0.01 mm from the ideal width of 2.5 mm.

a) Write an absolute value equation to find the acceptable limits to the width in millimetres for this part.

b) Solve the equation.

8. A technician measures an electric current that is 0.036 amperes with a possible error of $\pm$ 0.002 amperes.

a) Write this current, *i*, as an absolute value equation that finds the limits to the current.

b) Solve the equation.

9. A manufacturer has a tolerance of 0.35 lb for a bag of potting soil advertised as weighing 9.6 lb. Write and solve an absolute value equation that finds the limits to the weight of each bag.

10. A company is building a movie theatre. Based on similar movie theatres, an analyst tells the company that its earnings per day are based on the solution to the equation $y = |-x^2 + 48x - 512|$, where y is thousands of dollars and x is the number of movie screens.

a) If the company can build between 16 and 32 movie screens, what is the number of screens within this range that will give the maximum profit?

b) What is the amount of profit?

11. Is the following absolute value equation solved correctly? Explain. If it is not correct, solve it correctly.

$4 - 9|-6 - x| = -15$

$-5|-6 - x| = -15$

$|-6 - x| = 3$

Case 1: $-6 - x \geq 0$

$-6 - x = 3$

$x = 9$

Case 2: $-6 - x < 0$

$6 + x = 3$

$x = -3$

Connect

12. A lab worker observes a mouse walking through a straight tunnel. When the mouse moves forward, a positive number is recorded. When it moves backward, a negative number is recorded. The worker records that the mouse has walked a, b, and c feet.

a) Does the equation $d = |a + b + c|$ represent the total distance, d, the mouse walked in feet? If no, explain why not. If yes, replace a, b, and c with values to verify.

b) Use your answer to part a) to determine if $|a + b + c| = |a| + |b| + |c|$.

13. Twin babies have masses of m_1 and m_2 grams. Their mother wants to know how many grams separate the masses of the two babies.

a) Write an equation for this difference, d, in such a way that the result will always be positive no matter which baby is heavier.

b) Write an application problem that would use absolute values as part of the solution.

c) Write an equation and solve your problem.

14. a) Explain without solving why the equation $|4x + 7| + 8 = 2$ has no solution.

b) Write an absolute value equation that has no solution. Explain why it has no solution.

7.4 Reciprocal Functions

KEY IDEAS

- For any function $f(x)$, the reciprocal function is $\frac{1}{f(x)}$.

  The reciprocal function is not defined when the denominator is 0, so $f(x) \neq 0$.

  For example, if $f(x) = x$, then $\frac{1}{f(x)} = \frac{1}{x}$, $x \neq 0$.

- Use these guidelines when graphing $y = \frac{1}{f(x)}$ given the graph of $y = f(x)$.
  – An asymptote is a straight line that the graph approaches, but never touches.

  – The general equation of a vertical asymptote is $x = a$, where a is a non-permissible value of $\frac{1}{f(x)}$.

  – As $|f(x)|$ gets very large, the absolute value of the reciprocal function, $\left|\frac{1}{f(x)}\right|$, approaches zero.

  This means the graph will approach the x-axis, but not meet the x-axis.

  The x-axis, defined by the equation $y = 0$, is a horizontal asymptote.

  – An invariant point does not change when a transformation is applied.

  – For a reciprocal function, invariant points occur when $f(x) = 1$ and when $f(x) = -1$.

  Why is this true?
  What is the reciprocal of 1?
  What is the reciprocal of –1?

  – To find the x-values of the invariant points, solve the equations $f(x) = \pm 1$.

  – If a point (x, y) satisfies the function $y = f(x)$, then the point $\left(x, \frac{1}{y}\right)$ satisfies the reciprocal function.

  – As the value of x approaches a non-permissible value, the absolute value of the reciprocal function, $\left|\frac{1}{f(x)}\right|$, gets very large.

Working Example 1: Compare the Graphs of a Function and Its Reciprocal

a) Sketch the graphs of $y = f(x)$ and its reciprocal function, $y = \frac{1}{f(x)}$, where $f(x) = 2x$.

b) State how the functions are related.

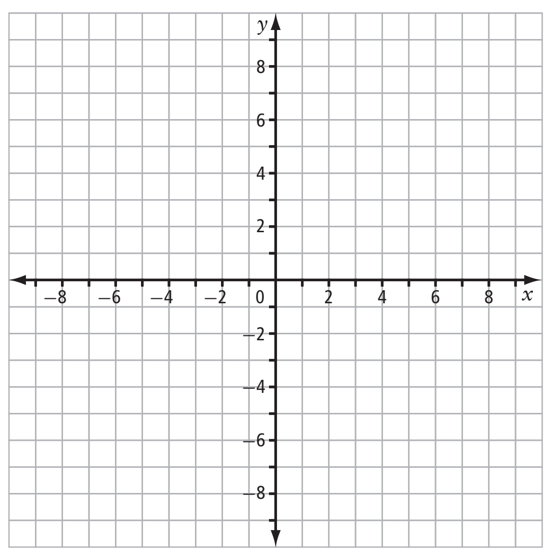
Solution

a) Complete the table of values for $y = 2x$ and $y = \frac{1}{2x}$.

x	$y = 2x$	$y = \frac{1}{2x}$
−4	−8	$-\frac{1}{8}$
−3		
−2		
−1		
$-\frac{1}{2}$		
$-\frac{1}{4}$		
$-\frac{1}{6}$		
0	0	Undefined
$\frac{1}{6}$		
$\frac{1}{4}$		
$\frac{1}{2}$		
1		
2		
3		
4		

- What is the product of each pair of numbers in columns 2 and 3?
- What happens to the values of $\frac{1}{f(x)}$ as the values of x increase?
- What happens to the values of $\frac{1}{f(x)}$ as the values of x get closer to zero?
- Why is the value of $\frac{1}{f(x)}$ undefined for $x = 0$?

Graph each function.



b) The function $y = 2x$ is a function of degree ________.

The vertical asymptote is defined by the non-permissible value of x.

The equation of the vertical asymptote is $x =$ ______________.

The equation of the horizontal asymptote is $y =$ ________.

Complete the following table.

Characteristic	$y = 2x$	$y = \frac{1}{2x}$
Domain		
Range		
Asymptotes		
Can $x = 0$?		
Can $y = 0$?		
What happens to the value of y as x increases in value?		
What happens to the value of y as x decreases in value?		
Invariant point(s)		

Working Example 2: Graph the Reciprocal of a Linear Function

Consider the function $f(x) = 2x + 1$.

a) What is the reciprocal function of $f(x)$?

b) Determine the equation of the vertical asymptote for $y = \frac{1}{f(x)}$.

c) Graph the function $f(x)$ and its reciprocal function, $y = \frac{1}{f(x)}$.

Solution

a) If $f(x) = 2x + 1$, the reciprocal of $y = f(x)$ is $y =$ ______________.

b) The vertical asymptote(s) occurs at any non-permissible values of $y = \frac{1}{2x + 1}$.

Determine the non-permissible value(s).

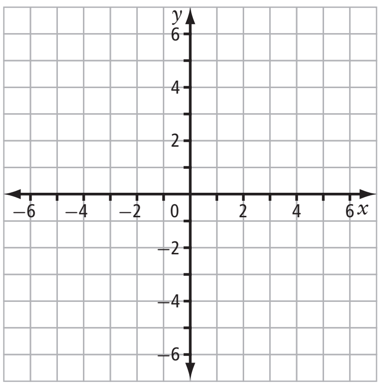
The non-permissible value is $x =$ ________.

The equation of the vertical asymptote is $x =$ ________.

c) Method 1: Use Pencil and Paper

To graph $y = 2x + 1$, use the slope and y-intercept.

The slope is ________ and the y-intercept is ________.



Complete the table to help sketch the graph of $y = \frac{1}{2x + 1}$.

Characteristic	$y = 2x + 1$	$y = \frac{1}{2x + 1}$
x-intercept	(________, 0)	None
Asymptotes	None	$x =$ ________ $y =$ ________
Invariant points	(________, 1) and (________, −1)	
As x increases in value, y ________ in value.		

Add the graph of $y = \frac{1}{f(x)}$ to the grid.

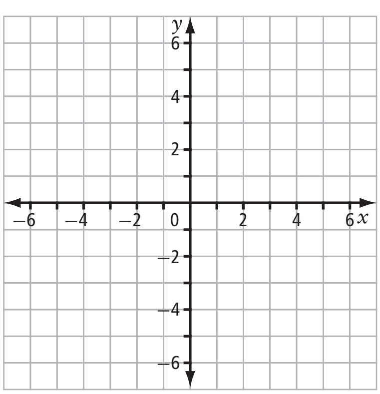
Method 2: Use Technology

Graph the function using a graphing calculator. Enter $y = 2x + 1$ and $y = \frac{1}{2x + 1}$.

What window settings allow you to see both branches of the graph?

____

Sketch the display shown on your calculator below.



Use the table feature of the calculator and the graph to complete the table below.

Characteristic	$y = 2x + 1$	$y = \frac{1}{2x + 1}$
x-intercept	(________, 0)	None
Asymptotes	None	$x =$ ________ $y =$ ________
Invariant points	(________, 1) and (________, −1)	
As x increases in value, y ________ in value.		

Working Example 3: Graph the Reciprocal of a Quadratic Equation

Consider the function $f(x) = x^2 - 1$.

a) What is the reciprocal function of $y = f(x)$?

b) Determine the non-permissible values of x for $y = \frac{1}{f(x)}$.

c) State the equation(s) of the vertical asymptote(s) of $y = \frac{1}{f(x)}$.

d) Determine the x-intercept(s) and y-intercept of $y = \frac{1}{f(x)}$.

e) Graph $y = f(x)$ and $y = \frac{1}{f(x)}$.

Solution

a) If $f(x) = x^2 - 1$, the reciprocal of $y = f(x)$ is $y =$ ________________.

b) The reciprocal function, $y = \frac{1}{f(x)}$, is not defined when $f(x) =$ ________.

Determine the non-permissible values.

The non-permissible values are $x =$ ________ and $x =$ ________.

c) The vertical asymptotes are located at the non-permissible values of x.

The equations of the vertical asymptotes are $x =$ ________ and $x =$ ________.

d) At the x-intercept(s), the y-coordinate is ________.

There are no x-values for which $y =$ ________, so there are ________ x-intercepts.

At the y-intercept(s), the x-coordinate is ________.

$y = \frac{1}{x^2 + 1}$

$y = \frac{\text{\_\_\_\_\_\_\_\_\_\_}}{(\text{\_\_\_\_\_\_\_\_})^2 - 1}$

$y =$ ________

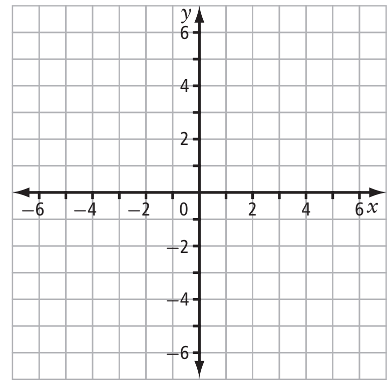
The y-intercept is located at (0, ________).

e) Method 1: Use Pencil and Paper

For $f(x) = x^2 - 1$, the coordinates of the vertex are (0, ________).

The x-intercepts are at (________, 0) and (________, 0).

Use this information to graph $y = x^2 - 1$.



To graph $y = \frac{1}{f(x)}$:

- Draw the asymptotes, $x =$ ________, $x =$ ________ $y =$ ________.
- Determine and plot the invariant points.
  To find the invariant points, solve $f(x) = \pm 1$.

  $x^2 - 1 = 1$ or $x^2 - 1 = -1$

  $x^2 =$ ________ $x^2 =$ ________

  $x =$ ________ $x =$ ________

  The invariant points are (________, 1), (________, 1), and (________, −1).

- Complete the following table and plot the points.

x	$y = x^2 - 1$	$y = \frac{1}{x^2 - 1}$
3	8	$\frac{1}{8}$
2		
1		Undefined
0		
−1		Undefined
−2	−3	
$\frac{1}{2}$		
$\frac{1}{3}$		
$\frac{1}{4}$		
$-\frac{1}{2}$		
$-\frac{1}{3}$		
$-\frac{1}{4}$		

Add the graph of $y = \frac{1}{f(x)}$ to the grid above.

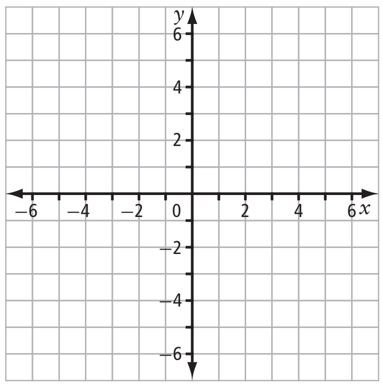
Method 2: Use Technology

Enter the functions $y = x^2 - 1$ and $y = \frac{1}{x^2 - 1}$ into your graphing calculator.

What window settings allow you to see the vertex and the intercepts?

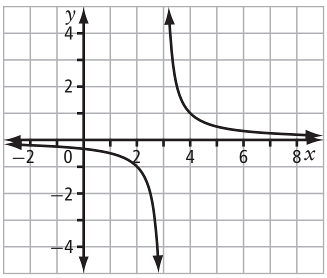
____

Sketch the display on your calculator.



Working Example 4: Graph $y = f(x)$ Given the Graph of $y = \frac{1}{f(x)}$

The graph of a reciprocal function, $y = \frac{1}{ax + b}$, where $a \neq 0$ and $b \neq 0$, is shown.



a) Sketch the graph of $y = f(x)$.

b) State the equation of $y = f(x)$ in the form $y = ax + b$.

Solution

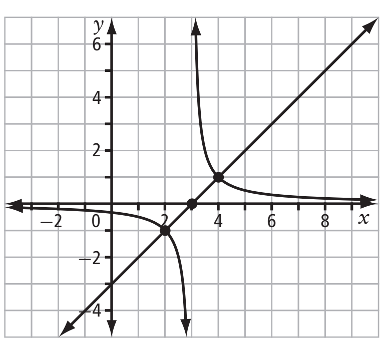
a) $y = \frac{1}{f(x)}$ is of the form $y = \frac{1}{ax + b}$, so $f(x)$ is of the form $y = ax + b$.

This is a ____________________ function.
(*linear* or *quadratic*)

The graph of $y = \frac{1}{f(x)}$ has a vertical asymptote at $x =$ ________, so $f(x)$ has an x-intercept at (________, 0).

The graph of $y = \frac{1}{f(x)}$ has invariant points at (________, 1) and (________, −1).

Plot the x-intercept and the invariant points. Draw a line through the points.



b) Method 1: Use the Slope and y-Intercept

Use the coordinates of the x-intercept and one of the invariant points from part a) to determine the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\square - \square}{\square - \square}$$

$$m = \frac{\square}{\square}$$

$m =$ ________

From the graph, the y-intercept, b, is −3.

The equation for $y = f(x)$ in the form $y = mx + b$ is $y =$ ____________________.

Method 2: Use the x-Intercept

With an x-intercept of 3, the function $y = f(x)$ is based on a factor of $x - 3$. That is, $y = a(x - 3)$, where a is another factor of y.

To determine a, substitute the coordinates of an invariant point from part a) for x and y.

$y = a(x - 3)$

________ $= a($________ $- 3)$

________ $=$ ________a

________ $= a$

The equation for $y = f(x)$ is $y =$ ____________________.

This question is similar to Example 4 on pages 401–402 of *Pre-Calculus 11*.

Check Your Understanding

Practise

1. Complete the table with either the missing original function, $f(x)$, or its corresponding reciprocal function, $\frac{1}{f(x)}$.

$y = f(x)$	$y = \frac{1}{f(x)}$
$y = -x$	
	$y = \frac{1}{3x - 1}$
$y = x^2 - 4x + 4$	
	$y = \frac{1}{x^2}$
$y = \frac{1}{2}x$	

This question is similar to #1 on page 403 of *Pre-Calculus 11*.

2. Write the equation(s) of the vertical asymptote(s) for each reciprocal function.

a) $y = \frac{1}{x}$; $x =$ ______

b) $y = \frac{1}{x + 5}$; $x =$ ______

c) $y = \frac{1}{7x - 2}$; $x =$ ______

d) $y = \frac{1}{x^2 - 16}$; $x =$ ______ and $x =$ ______

e) $y = \frac{1}{x^2 + x - 6}$; $x =$ ______ and $x =$ ______

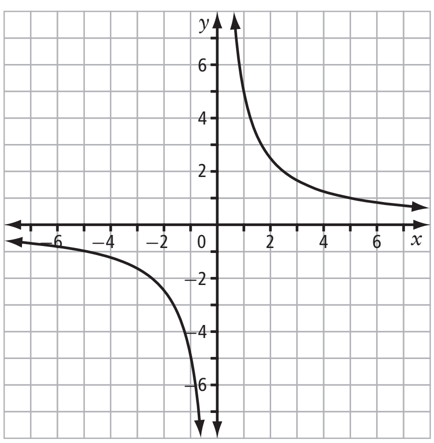
This question is similar to #3 on page 404 of *Pre-Calculus 11*.

3. Complete the table with the reciprocal function and the invariant point(s)

$y = f(x)$	$y = \frac{1}{f(x)}$	Invariant Point(s)
$y = x - 8$		(_______, 1), (_______, −1)
$y = 3x - 4$		(_______, 1), (_______, −1)
$y = x^2 - 25$		(_______, 1), (_______, 1), (_______, −1), (_______, −1)
$y = x^2 - x - 29$		(_______, 1), (_______, 1), (_______, −1), (_______, −1)

4. For the following graphs, identify
- the domain
- the range
- the equations of the vertical asymptotes

a)

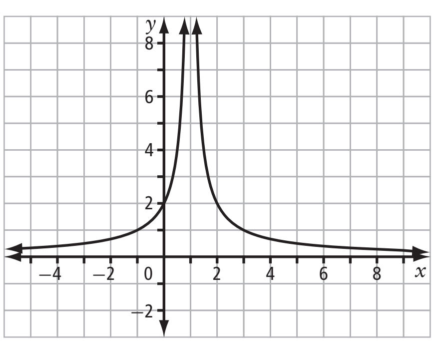


Domain: ________________________

Range: ________________________

Vertical asymptote: ____________________

b)



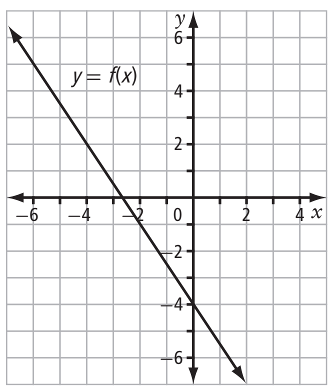
Domain: ________________________

Range: ________________________

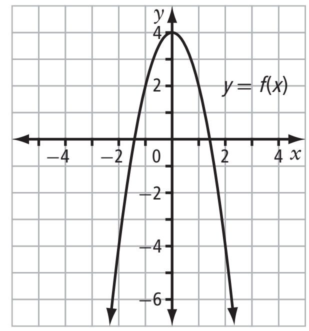
Vertical asymptote: ____________________

5. For each graph of $y = f(x)$, sketch the graph of the reciprocal function, $y = \frac{1}{f(x)}$, on the same set of axes.

a)



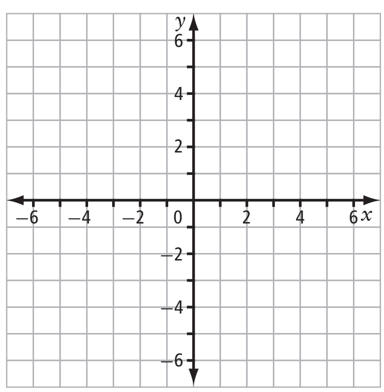
b)



This question is similar to #6 on page 404 of *Pre-Calculus 11*.

6. Sketch the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the same set of axes below.
State the asymptotes, the invariant points, and the intercepts.

a) $f(x) = x + 3$



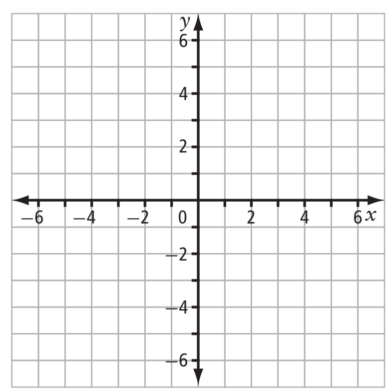
Asymptotes: ______________________

Invariant points: ______________________

Intercepts: $f(x)$: ______________________

$\frac{1}{f(x)}$: ______________________

b) $f(x) = x^2 + x - 6$



Asymptotes: ______________________

Invariant points: ______________________

Intercepts: $f(x)$: ______________________

$\frac{1}{f(x)}$: ______________________

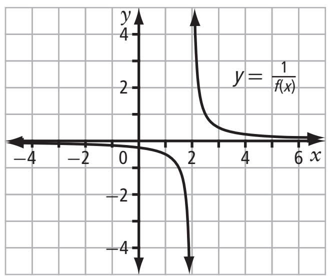
This question is similar to #7 and 8 on page 404 of *Pre-Calculus 11*.

Apply

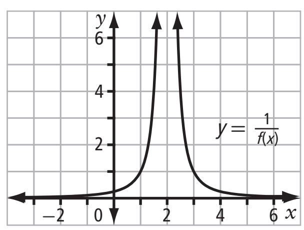
7. Consider this graph of a reciprocal function, $y = \dfrac{1}{f(x)}$.

- On the same set of axes, sketch the graph of the original function, $y = f(x)$.
- Explain the steps you used to obtain the graph.
- State the original function, $f(x)$, showing how you obtained your answer.

a)



b)



This question is similar to #10 on page 406 of *Pre-Calculus 11*.

8. One factor that affects the speed, s, of a tsunami is the depth of the water the tsunami is travelling through. The speed of a particular tsunami, in kilometres per hour, multiplied by the reciprocal of the square root of the water depth, d, in kilometres, is equal to 365.

a) Write an equation to find the speed.

____

b) Rewrite the formula to isolate d.

____

c) Use the formula from part b) to find the depth of the water at each speed.

i) 800 km/h **ii)** 580 km/h **iii)** 190 km/h

9. Find and correct the errors in the following table:

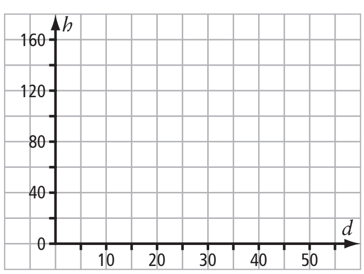
Characteristic	$y = 2x + 5$	$y = \frac{1}{2x + 5}$
x-intercept	$-\frac{5}{2}$	$-\frac{2}{5}$
y-intercept	5	$\frac{1}{5}$
Invariant points	$\left(\frac{-5}{2}, 0\right)$, (0, 5), and $\left(0, \frac{1}{5}\right)$	

10. Your height, h, above ground in metres and the distance, d, you can see to the horizon, in kilometres, are related to the constant 12.759 as follows: $\frac{d^2}{h} = 12.759$

a) Complete the table.

Height, h (m)	Distance, d (km)
2	
	12
40	
100	
	40

b) Sketch the graph.



c) If you are at a height of 20 m, what distance can you see to the horizon?

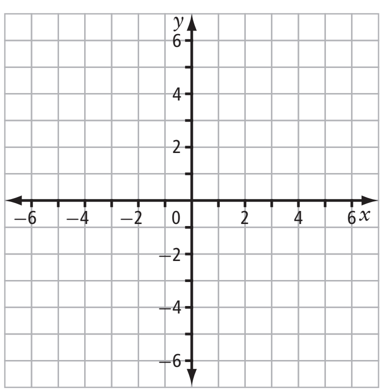
d) If you see a boat on the ocean at a distance of 8 km, what is your viewing height?

Connect

11. Compare the function $f(x) = x^2 - 3$ to $|f(x)|$ and to $\frac{1}{f(x)}$ by completing the table below.

	$f(x) = x^2 - 3$	$g(x) = \|x^2 - 3\|$	$h(x) = \frac{1}{x^2 - 3}$
x-intercept(s)			
y-intercept(s)			
Domain			
Range			
Piecewise function			
Invariant points			
Asymptote(s)			

Check the accuracy of your answers by using technology. Enter the functions in your graphing calculator and use the trace function to verify your answers. Sketch the display shown on your calculator of all three functions on the grid below.



Chapter 7 Review

7.1 Absolute Value, pages 278–284

1. Evaluate.

a) $|-17| =$ ________

b) $-\left|-1\frac{1}{2}\right| =$ ________

c) $|1.02| =$ ________

2. Arrange the numbers in order from greatest to least.

$|-20.1|, -|20|, \left|\frac{41}{2}\right|, -20.2, \left|-19\frac{3}{4}\right|, -19.65$

3. Evaluate.

a) $|0 - 18| =$ ____

b) $-2|10.5| + |(-3)^3| =$ ____

c) $|-20 + 3(-2)^2| =$ ____

4. Insert absolute value symbols to make each statement true.

a) $9 - 12 + (-2)(4) = -5$

b) $(1.3 - 3.3)^3 = 8$

c) $8 - 11(12 - 15) = -25$

5. The deepest point in Lake Superior is 732 feet below sea level. The surface elevation is 600 feet above sea level.

a) Write an absolute value statement to determine the maximum depth.

b) What is the depth?

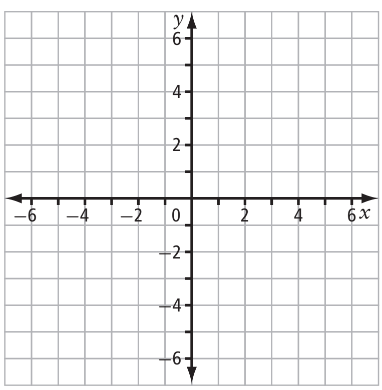
7.2 Absolute Value Functions, pages 285–296

6. Consider the functions $f(x) = 2x - 1$ and $g(x) = |2x - 1|$.

a) Complete the table.

x	$f(x)$	$g(x)$
−1		
0		
1		
2		
3		
4		

b) Sketch the graphs of $f(x)$ and $g(x)$.



c) Complete the table.

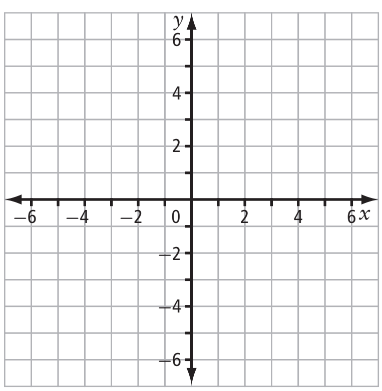
Characteristic	$f(x)$	$g(x)$
Domain		
Range		
x-intercepts		
y-intercepts		
Piecewise function	N/A	

7. Consider the functions $f(x) = -x^2 + 4$ and $g(x) = |-x^2 + 4|$.

a) Complete the table.

x	$f(x)$	$g(x)$
−3		
−2		
−1		
0		
1		
2		
3		

b) Sketch the graphs of $f(x)$ and $g(x)$.

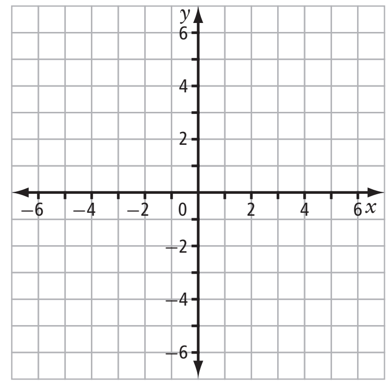


c) Complete the table.

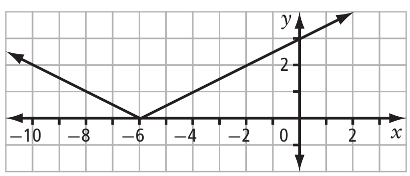
Characteristic	$f(x)$	$g(x)$
Domain		
Range		
x-intercepts		
y-intercepts		
Similarities		
Differences		
Piecewise function	N/A	

8. a) Without graphing, predict how the graphs of $f(x) = x^2 + 3x + 6$ and $g(x) = |x^2 + 3x + 6|$ are related.

b) Check your prediction by using technology. Sketch the graph(s) below.



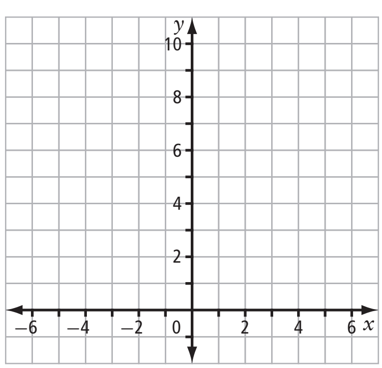
9. Write an absolute value function of the form $y = |ax + b|$ that has the following graph.



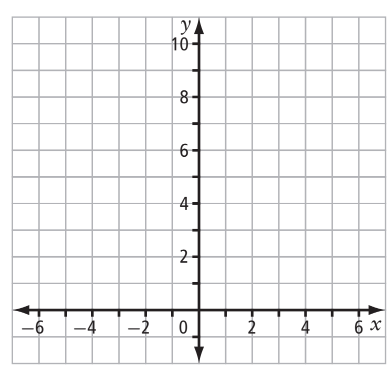
7.3 Absolute Value Equations, pages 297–308

10. Solve each absolute value equation by graphing.

a) $|-5x| + 4 = 9$



b) $|x^2 - 1| = 8$



11. Solve each absolute value equation algebraically.

a) $|2x + 10| = 14$

b) $|3x - 6| = x + 4$

c) $|x^2 - 8x + 12| = 20$

12. The adult dose of a medicine is 50 mg, but it is acceptable for the dose to vary by 0.5 mg.

a) Write an absolute value equation to determine the limits of an acceptable dosage.

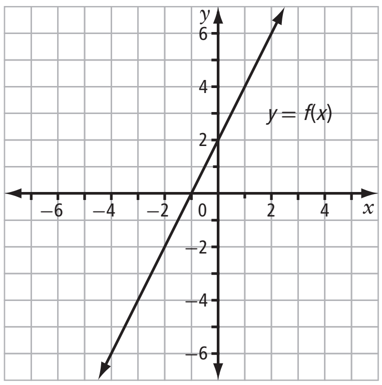
b) What are the limits?

7.4 Reciprocal Functions, pages 309–323

13. For each graph of $y = f(x)$ below,

- Sketch the graph of the reciprocal function, $y = \frac{1}{f(x)}$, on the same grid.
- Find the asymptotes, invariant points, and intercepts.

a)

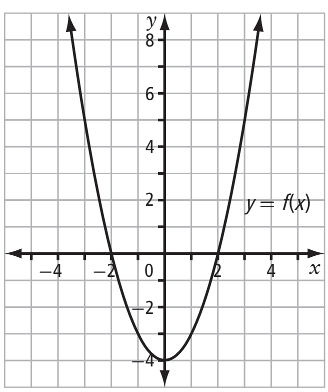


Asymptotes: ______________________________

Invariant points: ______________________________

x-intercept: ______________; y-intercept: ______________

b)



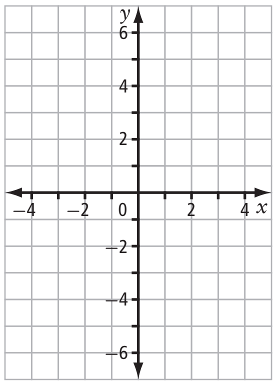
Asymptotes: ______________________________

Invariant points: ______________________________

x-intercept: ______________; y-intercept: ______________

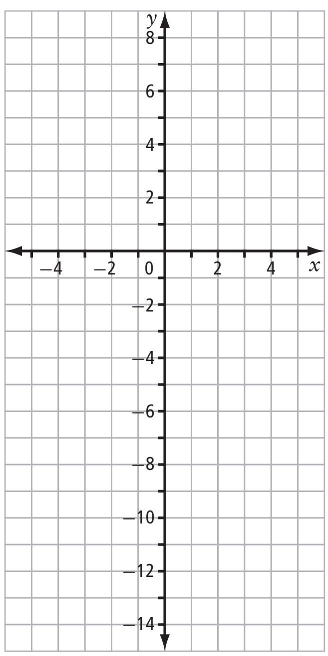
14. Sketch the graphs of $y = f(x)$ and $y = \dfrac{1}{f(x)}$ on the same set of axes. Then, complete the table.

a) $f(x) = 3x + 4$



Characteristic	$f(x)$	$\dfrac{1}{f(x)}$
Asymptotes	N/A	
x-intercept		
y-intercept		
Invariant points		
Domain		
Range		

b) $f(x) = x^2 - x - 12$



Characteristic	$f(x)$	$\dfrac{1}{f(x)}$
Asymptotes	N/A	
x-intercept		
y-intercept		
Invariant points		
Domain		
Range		

15. The resistance, R, in ohms in an electric circuit is equal to the power, P, in watts, multiplied by the reciprocal of the square of the current, I, in amperes.

a) Write an equation to represent the relationship.

b) What is the resistance, in ohms, for a circuit that uses 500 watts of power with a current of 2 amperes?

c) The resistance in a circuit is 4 ohms. The same circuit uses 100 watts of power. Find the current in the circuit, in amperes.

d) Find the power, in watts, when the current is 0.5 amperes and the resistance is 600 ohms.

Chapter 7 Skills Organizer

Complete the table for any function $y = f(x)$

Characteristic	$y = f(x)$	$y = \|f(x)\|$	$y = \frac{1}{f(x)}$
x-intercept(s)			
y-intercept(s)			
Domain			
Range			
Piecewise function			
Invariant points			
Asymptote(s)			