

BLM 3.3

RULES OF INFERENCE AND REPLACEMENT

RULES OF INFERENCE

1. *Modus ponens* (MP)

$$\frac{p \supset q \quad p}{\therefore q}$$

2. *Modus tollens* (MT)

$$\frac{p \supset q \quad \sim q}{\therefore \sim p}$$

3. Hypothetical syllogism (HS)

$$\frac{p \supset q \quad q \supset r}{\therefore p \supset r}$$

4. Disjunctive syllogism (DS)

$$\frac{p \vee q \quad \sim p}{\therefore q}$$

5. Constructive dilemma (CD)

$$\frac{(p \supset q) \wedge (r \supset s) \quad p \vee r}{\therefore q \vee s}$$

6. Absorption (Abs.)

$$\frac{p \supset q}{\therefore p \supset (p \wedge q)}$$

7. Simplification (Simp.)

$$\frac{p \wedge q}{\therefore p}$$

8. Conjunction (Conj.)

$$\frac{p \quad q}{\therefore p \wedge q}$$

9. Addition (Add.)

$$\frac{p}{\therefore p \vee q}$$

RULES OF REPLACEMENT

De Morgan's Rules (De M.):
 $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$
 $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

Commutation (Com.):
Note: $p \supset q \neq q \supset p$
 $p \vee q \Leftrightarrow q \vee p$
 $p \wedge q \Leftrightarrow q \wedge p$

Association (Assoc.):
 $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$
 $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

Distribution (Dist.):
 $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
 $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

Double negation (DN):
 $p \Leftrightarrow \sim \sim p$

Transposition (Trans.):
 $p \supset q \Leftrightarrow \sim q \supset \sim p$

Material implication (MI):
 $p \supset q \Leftrightarrow \sim p \vee q$

Exportation (Exp.):
 $(p \wedge q) \supset r \Leftrightarrow p \supset (q \supset r)$

